

# A REVEALED PREFERENCE RANKING OF U.S. COLLEGES AND UNIVERSITIES

CHRISTOPHER AVERY  
MARK GLICKMAN  
CAROLINE HOXBY,  
AND ANDREW METRICK\*

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## Abstract

We show how to construct a ranking of U.S. undergraduate programs based on students' revealed preferences. We construct examples of national and regional rankings, using hand-collected data on 3,240 high-achieving students. Our statistical model extends models used for ranking players in tournaments, such as chess or tennis. When a student makes his matriculation decision among colleges that have admitted him, he chooses which college "wins" in head-to-head competition. The model exploits the information contained in thousands of these wins and losses. Our method produces a ranking that, unlike rankings based on the matriculation or admission rate, would be difficult for a college to manipulate. If our ranking were used in place of these measures, the pressure on colleges to practice strategic admissions would be relieved. We show how to deal with tuition discounts, alumni preferences, early decision programs, specialty schools, and similar issues.

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The authors' affiliations are, respectively, John F. Kennedy School of Government at Harvard University; Department of Health Services at the Boston University School of Public Health, Department of Economics at Harvard University, and Department of Finance of The Wharton School at the University of Pennsylvania. We thank Bruce Sacerdote, Joel Waldfogel and seminar participants at Columbia, Wharton, Yale, the University of Texas at Austin, University of California Santa Cruz, Harvard, and the National Bureau of Economic Research for helpful comments. We thank Andrew Fairbanks and Jim Barker, who helped to design and implement the College Admissions Project survey. We also thank Michael Behnke, Larry Momo, Jay Matthews, and the 510 high school counselors who made the survey happen. We are grateful for the aid of many hard-working and perspicacious research assistants: Joshua Barro, James Carmichael, Rohit Chandwani, Michael Cuthbert, Suzanne Ko, Ilyana Kuziemko, Michael McNabb, Kathryn Markham, Emily Oster, Chris Park, Jenna Robins, Aaron Roth, Maria Shim, Catherine So, Rania Succar, Michael Thakur, Kenneth Wang, and Jill Zitnik. Scott Resnick deserves very special thanks. The first version of this paper appeared in October 2002.

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## I. Why a Revealed Preference Ranking?

In this study, we show how to construct a ranking of U.S. undergraduate programs based on students' revealed preferences –that is, the colleges students prefer when they can choose among them. The result is a ranking of colleges based on their desirability. We develop a statistical model that logically extends models used for ranking players in tournaments, such as chess and tennis. When a student makes his matriculation decision among colleges that have admitted him, he chooses which college "wins" in head-to-head competition. The model exploits the information contained in thousands of these wins and losses. Along with this central contribution, we show how to account for the potentially confounding effects of tuition discounts, financial aid, and other factors that might make a college "win" when it would lose on the basis of its intrinsic desirability. We explain how to think about specialty colleges, such as California Institute of Technology, whose applicants are self-selected to an unusual degree; and we show rankings that explicitly take self-selection into account. We construct rankings that are unaffected by early decision admissions programs and demonstrate that they are extremely similar to our preferred ranking.

The rankings we construct are based on a survey of 3,240 highly meritorious students that was specifically conducted for this study. Because we do not have a fully representative sample of college applicants, we rank only about a hundred undergraduate programs and our ranking is an *example*, not definitive. Nevertheless, we can show that our ranking has advantages. In particular, it is less manipulable than crude measures of revealed preference, such as the admissions rate and matriculation rate. A ranking constructed according to our method would be a good substitute for the preference indicators that receive substantial weight in formulas of high publicized college rating systems, like that of *U.S. News and World Report*. Many colleges currently feel compelled to engage in strategic admissions behavior in order to maximize their published college ratings. Use of our ranking method would relieve this pressure.

Rankings based on students' revealed preference measure a college's desirability in students' eyes. Such desirability may reflect a college's quality, but it is unlikely to be identical

to quality. Indeed, the notion of what constitutes quality in a college is likely to differ from person to person. Faculty, parents, policy makers, and students may all assign different weights to colleges' characteristics. Why then construct a revealed preference ranking at all, which merely shows the value that *students* (in combination with their parents) put on colleges?

The primary reason that we are motivated to construct a revealed preference ranking is a practical one. Parents and students demand revealed preference information and college guides feel obliged to offer them some. The two measures of preference used by college guides are the crude matriculation rate and crude admissions rate. One objection to these measures is that they are coarse and use information inefficiently. Our revealed preference ranking *efficiently* aggregates the information contained in individual students' decisions. Another serious objection to these measures is that colleges can manipulate them, though at a cost. Colleges do not necessarily want to manipulate their matriculation rate and admissions rate; they feel compelled to do so. A college that does not manipulate these rates, when its competitors do, loses ground in highly publicized college ratings. Such lost ground will eventually have real effects on the college's ability to recruit students, attract donations, and so on.<sup>1</sup> In short, U.S. colleges are in a bad equilibrium: colleges manipulate the rates even though they would all be better off if no college manipulated the rates. If a revealed preference ranking like ours were used, colleges would find it extremely hard to "defect" and the bad equilibrium would not arise. All parties (including the college guides) should be pleased to have a measure of revealed preference that limits or even eliminates manipulation.

We have attempted to justify constructing a good indicator of revealed preference by pointing out that one is demanded. But, why do students and their parents demand such measures? There are a few possible answers.

First, students believe and act as though their peers matter. This may be because peer quality affects the level of teaching that is offered. Alternatively, students may learn directly

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<sup>1</sup> For evidence on the real effects of the ratings, see Ehrenberg and Monks (1999).

from their peers. If such channels for peer effects are important, then it is reasonable for students to care about whether they are surrounded by peers with high college aptitude. Students will want to see a revealed preference ranking because it will show them which colleges can offer the highest concentration of desirable peers. A more preferred college wins more often in matriculation tournaments. Thus, it can afford to be more selective and can offer peers with higher aptitude.

Second, students—especially the high achieving students on whom we focus—are not ignorant about college quality. They gather information about colleges' quality from publications, older siblings, friends who are attending college, college counselors, and their own visits to colleges. A student may place the greatest weight on his own observations of quality, but he will also put some weight on the observations of other students, simply because his own sample of observations is too small to be representative. A revealed preference ranking efficiently aggregates observations about quality from thousands of students. There are parallels to other industries. For instance, people judge restaurant and hotel quality based partly on their own experiences, but they also want to know about other people's experiences. This is why there is a demand for guides like *Zagat's*, which aggregate people's observations about hotels and restaurants.

Third, it has long been hypothesized that specific colleges' degrees serve as signals of a student's aptitude, which is hard for future employers to observe directly [Spence, 1974]. In equilibrium, a college's degree signals the aptitude of the students who actually attend it. For instance, there will be an equilibrium only if a Cornell degree signals aptitude that is consistent with the actual distribution of aptitude among Cornell students. This is another reason for students to care about the ability of their peers and, thus, their college's tendency to attract students.

In Section II of the paper, we discuss the problems inherent in using the matriculation rate and the admissions rate as measures of revealed preference. In Section III, we present our statistical model of college choice as a multiple comparison problem. Sections IV and V describe, respectively, our model fitting and data. Our main results are described in Section

VI. Section VII describes extensions to the basic model that help us deal with early decision and self-selection. Section VIII concludes the paper.

## II. The Manipulability of Various Measures of Revealed Preference

One of the two common measures of revealed preference is the matriculation rate—the share of accepted students who matriculate at a college:

$$(1) \quad \frac{\text{number of students who matriculate}}{\text{number of students who are admitted}}.$$

There are several methods by which a college can manipulate its matriculation rate. The reason that most methods work is that the matriculation rate is just an aggregate statistic and has no way of taking account of the composition of the pool of admittees (higher or lower merit?) or of *which* students within the pool of admittees are matriculating (those with the best alternative offers or those with worst alternative offers?).

An early decision program is the most dramatic means by which a college can manipulate its matriculation rate. Every early decision admittee has a 100 percent probability of matriculating, so –mechanically– the more students whom a college admits under its early decision program, the higher is its matriculation rate. (It is important to distinguish between early decision, in which a student *commits* to matriculate if admitted, and early action, in which a student is admitted early but can apply to numerous other colleges and turn down the early admission offer.) An early decision program is not without costs for the college. As Avery, Fairbanks, and Zeckhauser (2003) show, colleges lower their admissions standards for early decision applicants in order to induce them to pre-commit to matriculating and pre-commit to having no alternative offers when it comes to negotiating over financial aid. As a college admits more and more of its class under early decision, its actual admissions standards fall and students will therefore experience less meritorious peers. Yet, by the standard of the matriculation rate, the college's desirability will have risen.

Another method by which a college can manipulate its matriculation rate is deliberately not admitting students who are likely to be admitted by close competitors or colleges that are

often more highly preferred. A college administrator may say to himself, "My college will ultimately fail to attract good applicants unless I raise its matriculation rate. I can achieve this with a strategic policy that denies admission to students who seem likely to be accepted by colleges more desirable than mine. By systemically denying them admission, my college will of course lose of its some most desirable students (because some percentage of the highly desirable students would have matriculated). However, it is worthwhile to sacrifice the *actual* desirability of my college class in order to *appear* more desirable on a flawed indicator." Students who care about the long-run reputation of the college would almost certainly prefer that it *not* pursue such a policy because it would reduce peer quality. Yet, by the standard of the matriculation rate, the college's measured appeal would rise just as its actual appeal fell.

Golden, in "Glass Floor: How Colleges Reject The Top Applicants—And Boost Their Status—Accepting Only the Students Likely to Enroll Makes A School Look Selective—"They Were Simply Too Good" (2001), provides numerous examples of colleges' practicing strategic admissions. We need not rely on anecdotes, however, we can look for evidence in our data.

If a college is not practicing strategic admissions, then the probability that a student is admitted ought to rise monotonically in his or her merit. In contrast, a college that is strategic will have non-monotonic admissions probabilities. A student's probability of admission will first rise in his or her merit and then *fall* as his or her merit moves into the range in which the strategic college faces stiff competition. In other words, the college will avoid admitting students in the range in which it is likely to lose in a matriculation tournament. Finally, if the student's merit is high enough, a strategic college will probably admit the student even if the competition will be stiff. This is because the prospective gains from enrolling a "star" will more than make up for the prospective losses from a higher admissions rate and lower matriculation rate. (Recall that the crude admissions rate and matriculation rate do not record *who* is admitted or matriculates.) Although we realize that it is not a definitive measure of a student's merit, for the sake of these purely illustrative figures, we use a student's combined SAT I score, measured in national percentiles. This measure is at least readily understood and reasonably continuous. It is also wholly unrelated to our ranking method.

Consider Figure 1, which shows regular admissions at Harvard, Yale, Princeton, and MIT.<sup>2</sup> We focus on these schools simply because, owing to our having many observations for each of them, they display clear patterns. The probability of a student's being admitted to MIT rises steeply and monotonically in his or her combined SAT score. Now examine Harvard admissions. The probability of admission rises from close to zero at the 88<sup>th</sup> percentile to about 10 percent at the 93<sup>rd</sup> percentile. It then increases very gradually to the 98<sup>th</sup> percentile, and finally rises steeply to 20 percent. In other words, if every student admitted to Harvard matriculated, then Harvard would have a class in which 35 percent of students came from the 99<sup>th</sup> percentile and above, about 55 percent of students came from the 94<sup>th</sup> through the 98<sup>th</sup> percentile, and the remaining 10 percent of students came from the 93<sup>rd</sup> percentile and below. Now examine Yale, which displays a very slight non-monotonicity: the probability of admission rises up through the 93<sup>rd</sup> percentile, then falls just a bit, and finally rises steeply above the 97<sup>th</sup> percentile. Finally, examine Princeton, which displays more non-monotonicity. Its probability of admission rises to 20 percent at the 93 percentile, falls to 10 percent at the 98 percentile, and then rises steeply in the top 2 percentiles.

In short, it appears that MIT and Harvard do not engage in strategic admissions although Harvard probably cares more about factors uncorrelated with the SAT score than MIT does.<sup>3</sup> Yale appears to practice a bit of strategic admission—avoiding the area where Harvard competition is tough and students are not worth fighting (and perhaps often losing) for. Princeton appears to practice more strategic admission—avoiding the same area that Yale avoids, but more so. If we were to show more schools—such as Stanford or Brown—that we later rank similarly to Princeton, we would see that their figures display similar patterns.<sup>4</sup> While

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<sup>2</sup> Everything we have just said about strategic admission applies to regular admission, not Early Decision where a school faces no competition. Thus, Figure 1 shows regular admissions.

<sup>3</sup> The monotonic but nevertheless quite flat portion of Harvard's line suggests that the college is, within this range, looking for student characteristics that are only weakly correlated with the SAT score.

<sup>4</sup> However, similarly ranked schools' figures display a range of behavior, with some looking more like Yale and others looking more like Princeton. The range may be due to sampling variation in

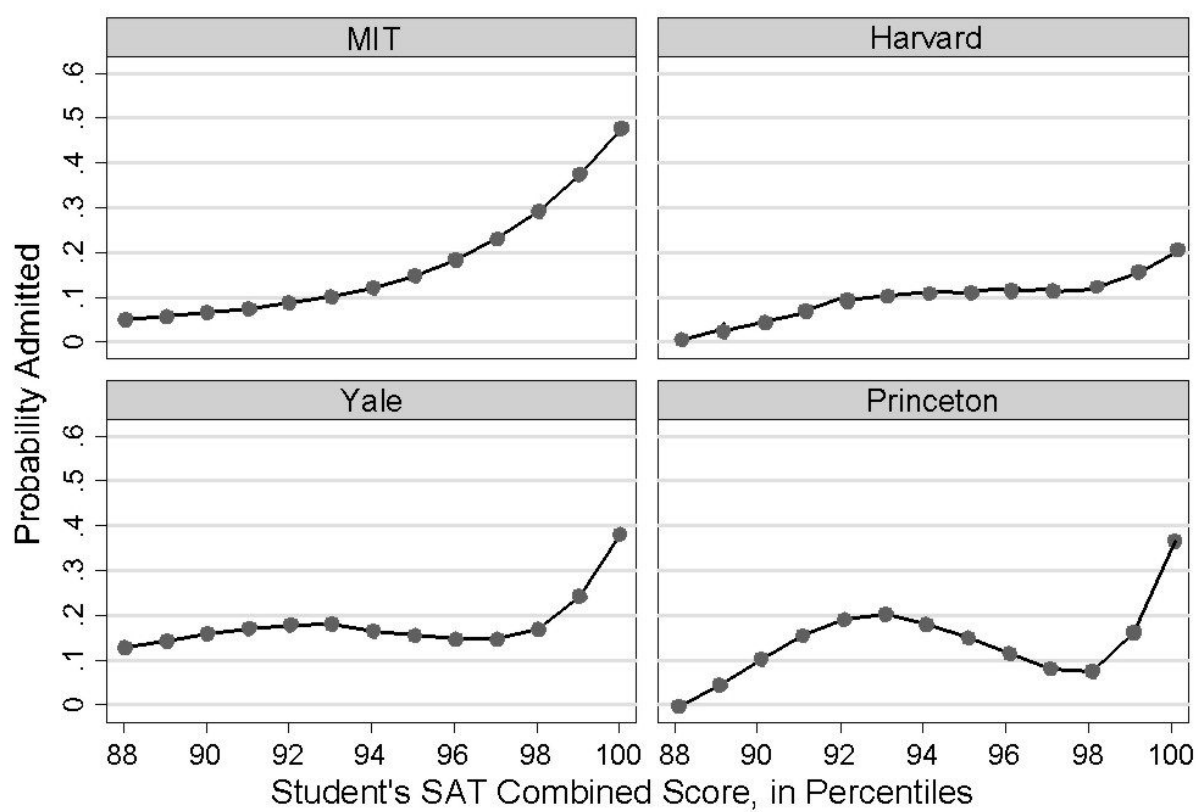




Figure 1 is hardly definitive, it provides suggestive evidence that even a highly prestigious school may practice strategic admissions. Such behavior is potentially costly to the actual quality of an admissions class, with no clear benefit beyond a higher reported matriculation rate.

The second of the two common proxies for revealed preference is the admission rate—that is, the share of applicants who are admitted by a college:

$$(2) \quad \frac{\text{number of students who are admitted}}{\text{number of students who apply}} .$$

There are several methods by which a college can manipulate its admissions rate. The reason that most methods work is that the admissions rate is just an aggregate statistic. It does not account for the composition of the pool of applicants (are they high or low merit?). It does not account for *which* applicants a college admits.

In forming a class of a given size, a college can admit fewer students if its matriculation rate is higher. Therefore, the methods discussed above for manipulating the matriculation rate are also methods for manipulating the admissions rate. For instance, if a college makes heavy use of an early decision program, it only needs to admit only slightly more students than the number that it actually wishes to enroll. This is because the early decision admittees are pre-committed to enrolling. The technique of *not* admitting applicants who are likely to be admitted by close competitors also allows a college to publish a lower (better) admissions rate.

In addition, colleges can manipulate their admissions rate by encouraging applications from students who have little chance of actually gaining admission. A college can advertise less stringent criteria than it actually applies. By doing so, it encourages marginal students to apply, increases its number of applications, decreases its admissions rate, and raises its apparent desirability, even though its real desirability has not changed. For instance, this is

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our data or real variation in colleges' practice of strategic admissions.

how Toor (2000) described her job as an admissions officer at Duke University: "The job of admissions officers is to recruit, to boost application numbers. The more applications, the lower the admit rate, the higher the institutional ranking. Increasing application numbers is usually the No. 1 mandate of the recruiting season. Partly, that means trying to get the very best students to apply. But it also means trying to persuade those regular, old Bright Well-Rounded Kids (B.W.R.K.'s, in admissionese) to apply -- so that the college can reject them and bolster its selectivity rating."

In short, the two conventional measures are manipulable by colleges, though at a cost. If the goal of college admissions is to admit the optimal class, then colleges must systemically deviate from this goal in order to manipulate their matriculation and admissions rates. Colleges must sacrifice actual desirability for apparent desirability. Even if all colleges prefer not to manipulate the crude rates, each college will lose if it refrains from manipulation when other colleges do not refrain.

How might colleges escape this bad equilibrium? If the measure of revealed preference is not manipulable (or manipulable only by very complex, costly means), then all parties could be better off. In the next section, we formally describe the statistical method we use to create a revealed preference ranking of colleges. Here, we can give some intuition into why a ranking based on our method is not prey to simple forms of manipulation. For this exercise, it may be helpful for readers to think of some game familiar to them.

Our method is based on "wins" and "losses" in thousands of "tournaments" in which students are choosing the college at which to matriculate. Under this method, a college's ranking vis-a-vis a competitor is based on its record of wins and losses. Colleges that rarely compete directly in tournaments (because they are of very different selectivity) are ranked using the win/loss records of intermediate colleges that link them through series of tournaments: A routinely competes with B, B routinely competes with C, C routinely competes with D, etc. Given our methods, there is no easy way for a college to artificially boost its ranking with no true change in its appeal to students. For instance, recall the example in which Princeton alters its acceptance decisions in order to avoid match-ups with Harvard, Yale,

Stanford and so on. We would be unable to rank Princeton vis-a-vis its close competitors because its match-ups would always be against less selective colleges. That is, our estimates would reflect the fact that Princeton was not admitting the highly meritorious students for whom it should have been competing. We would see that, while it was consistently "winning," it was winning only among students who failed to get admitted to close competitors.

Readers might also find it helpful if we stated what a college would need to do if it were to manipulate our ranking successfully. None of the crude methods of manipulation described above would work. A college would need to do something more subtle. Return to the Princeton example, for concreteness. Princeton would need to find students in its applicant pool who were likely to attend Princeton even if admitted to Harvard, Yale, MIT, Stanford, and so on. Such students would have to exist exogenously; they could not be "created" by Princeton's giving them extra aid to induce them to matriculate. (Giving them extra aid would not work because we can observe and account for aid.) Moreover, Princeton would have to identify these students using characteristics not observable to other colleges. If the trait that Princeton used to pick out likely matriculators was observable (such as being a Princeton alumnus' child), then this trait could be used as a control in any revealed preference ranking, as we will do below with some characteristics collected in our study. Without an early decision program to bind students or "secret" traits that distinguished its likely matriculators, a college could not identify students whose matriculation tournaments it would win.

### **III. A Model of College Choice**

#### **A. The Desirability of Colleges**

The exercise of ranking colleges is necessarily predicated on the notion that there are latent indices of desirability on which colleges *can* be ranked. In the language of econometrics, the exercise is based on the assumption there are latent variables that indicate the desirability of each college (perhaps on multiple dimensions). Our measure of desirability encompasses all characteristics of a school, including (perceived) educational quality, campus location, and

tuition. We do not claim to know how latent desirability is constructed. We simply assert that, to the extent that students act in accordance with it, we can construct rankings.

We suspect that latent desirability is well defined on a national basis for the most academically elite colleges in the United States. We also suspect that latent desirability is defined on a national basis for the most elite specialized colleges in the United States: engineering schools, music schools, and so on. We would not be surprised to find, however, that once we move below the most academically elite colleges, latent desirability is only well-defined within regions of the country and perhaps within other dimensions. If we had a very large, random sample of all college applicants, we could construct rankings within regions and specialties and show where they joined up to become a national ranking. Given that the data we use for our exercise is focused on high achieving students who do not apply much outside the group of the most academically elite colleges, we will start by constructing a national ranking of such colleges. We will rank only those that the data suggest have a national draw. Subsequently, we construct regional rankings and discuss specialized rankings. Until then, however, we encourage the reader to think of a college's latent desirability as being unidimensional.

For our exercise, is it necessary that all students identically perceive a college's desirability? No. We will allow students' perception of a college's desirability to be distributed around a mean level. Indeed, if there were no such distributions, all students would make identical matriculation decisions when offered the same choices. We know that this does not occur. What we need for our exercise is a pattern of wins and losses that would arise if colleges had latent desirabilities that were perceived with idiosyncratic noise added in.

Finally, note that our exercise does not *impose* the existence of latent desirability; our method simply will not work if widespread agreement on desirability does not exist. To see this, suppose that there were no uniformity in how students perceived colleges' desirability. Each student would act as though he had been randomly assigned a ranking of colleges, where his ranking was independent of all other students' rankings. We would find no pattern in the "wins" and "losses" because it would be random whether a college won or lost in head-to-head

competition for a student. Overall, we can afford to be agnostic about how students develop preferences over colleges. Our data will only reveal such preferences to the extent that they are systematic.

The problem of ranking colleges can be framed as a collection of multiple comparisons. Comparison data come from competitions in which alternatives are compared and an outcome indicates that one alternative has been preferred over the others. Many games fall into this framework because players are compared via competition, and the winner of a competition is deemed the "preferred alternative." Also, marketing applications, including experiments in which consumers choose among products, are well-suited to multiple comparison models. An important problem addressed by multiple comparison models is how to rank objects when direct comparisons do not take place. For example, in the context of a "Swiss system" chess tournament, every competitor competes against only a few other individuals rather than against every other competitor. That is, player A competes against B, and B competes against C, but A does not compete against C. Yet, an inference is still desired for the comparison of A versus C. In the context of college choice, every college does not compete directly with every other college, though the goal is to draw conclusions about all colleges' desirability.

### B. Matriculation Tournaments as a Multiple Comparison Problem

To understand how college choice can be viewed as a multiple comparison problem, suppose that a collection of students has been admitted to a set of schools. Each school's desirability is modeled as a latent distribution of values. Each student effectively holds a tournament among the collection of schools that have admitted him; in our model this tournament is played by taking one draw from each school's distribution. The school with the highest draw has "won" the multi-player tournament, and the student matriculates at that school. Assuming that there are no confounding variables, a reasonable inference is that the school that wins the multi-player tournament is preferred to the other schools in that competition. By aggregating the information from all students' tournaments, inferences about the desirability of schools can be constructed.

David (1988) surveys the rich body of work on multiple comparison modeling, which mainly focuses on paired comparison models, where each tournament contains only two players. While no one has previously attempted to rank colleges using comparison models, there are abundant applications for divining chess ability from tournament data-- see, for example, Zermelo (1929), Good (1955), Elo (1978) and Glickman (1993, 1999, 2001).<sup>5</sup>

We build on the Bradley-Terry (1952) and the Luce (1959) models in which the distribution of desirability is an extreme value distribution. The assumption of an extreme value distribution for potentially observed desirability leads to a logit model. The main alternative to the assumption of an extreme value distribution for potentially observed desirability is a normal distribution. This leads to a class of models studied by Thurstone (1927) and Mosteller (1951) in the context of paired comparisons. When analyzing paired comparison data in practice, it makes almost no difference whether one assumes that the distribution of potentially observed desirability is extreme value or normal (see Stern, 1992). Models based on extreme value distributions tend to be more tractable and computationally efficient, which guides our choice.

It is worth noting that the extreme-value or normal distribution of potential desirabilities is a probabilistic assumption about the merit of an individual school, not an assumption about the distribution of mean desirabilities across schools. Because college comparison data can provide strong information about the relative desirabilities of colleges, any assumption made about the distribution of mean desirabilities should be weak. Our modeling approach allows for the possibility, for example, that a small number of schools are estimated to have mean desirabilities substantially greater than the remaining schools considered.

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<sup>5</sup> While this study is the first to use statistical comparison models to rank colleges, statistical models have been used to study which student characteristics colleges like and which college characteristics students like. See, for example, Manski and Wise (1983), Long (2003), and Avery and Hoxby (2004).

### C. The Matriculation Model

Assuming that each college's potentially observed desirability follows an extreme value distribution with the same scale and shape, the relevant parameter is the location parameter of the distribution. The latent variable is:

$$\theta_i = \text{the desirability parameter of college } i,$$

where we index colleges with  $i=1,2,\dots,I$ .

Students prefer colleges with higher desirability, among those in their choice set. Suppose that student  $j$  is admitted to a set of colleges  $S_j$  consisting of  $m_j$  schools. Let the indicator variable  $Y_{ij}$  tell us which college the student chooses:

$$(3) \quad Y_{ij} = \begin{cases} 1 & \text{if student } j \text{ matriculates at college } i \\ 0 & \text{otherwise} \end{cases}.$$

The result of the multi-player competition among the  $m_j$  colleges that admitted student  $j$  is assumed to follow a multinomial distribution:

$$(4) \quad (Y_{1j}, \dots, Y_{m_j j}) \sim \text{Multinomial}(1, (p_{1j}, \dots, p_{m_j j})) ,$$

where  $p_{ij}$  is the probability that student  $j$  chooses to matriculate at college  $i$  among his  $m_j$  college choices.<sup>6</sup> We assume Luce's choice model, of the form:

$$(5) \quad p_{i^*j} = \frac{\exp(\theta_{i^*})}{\sum_{i \in S_j} \exp(\theta_i)} , \quad i^* \in S_j .$$

This model can be rewritten as a conditional logit model, sometimes called McFadden's choice model.

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<sup>6</sup> For expositional convenience, we have reindexed the colleges in student  $j$  set  $S_j$ , so that they can be written  $1, \dots, m_j$ .

The  $\theta_i$ s embody all characteristics that do not vary within each college: whether it is a liberal arts college, the faculty, a rural as opposed to urban location, and so on. Such characteristics include *average* perceptions about the quality of the education and the *average* cost of attendance. Put another way, a college's desirability is the amalgam of the characteristics that its average admittee experiences. It is logically impossible to identify a college's  $\theta_i$  separately from the effects of characteristics that are constant within the college.

#### D. College Characteristics that Vary Across Admittees

Some college characteristics vary across admittees: the tuition charged (as opposed to the "list price"), grants or scholarships, loans, the college's distance from the student's home, its being in-state, its being the *alma mater* of one or more of the student's parents, and so on. We add these and other individually-varying characteristics to the model because they should improve the model's explanatory power.

To understand this point, consider a college C whose list price is \$20,000 . It offers large tuition discounts of \$15,000 a year to a few admittees and smaller discounts of \$5,000 a year to a slightly larger number of admittees. Most admittees are offered the list price. Suppose that when college C is in a tournament with colleges A and B for an admittee who faces its \$20,000 list price, it always loses to college A and loses to college B seventy percent of the time. Suppose, however, that when the admittee has the \$15,000 discount, college C loses to college A only eighty percent of the time and loses to college B only twenty percent of the time. Suppose that when the admittee has the \$5,000 discount, college C loses to college A ninety percent of the time and loses to college B fifty percent of the time. Our method ranks college C based on its *average* admittee—in other words, an appropriately weighted combination of the cases listed above. If we did not know how the individual admittee's tuition differed from the average admittee's, we might find the win-loss record among colleges A, B, and C somewhat confusing. Knowing the tuition discounts, we can correctly infer that certain students' increased likelihood of choosing college C is due to their being offered a discount that the average admittee does not experience. Put bluntly, we can tell whether college C has done



something to become more desirable in the eyes of its average admittee or has simply "bribed" a particular student to matriculate.

In more abstract terms, by comparing tournaments with different values of individually-varying characteristics, we can derive better estimates of the  $\theta_i$ s (which reflect colleges' average characteristics) because we have simultaneously estimated the effects of individually-varying characteristics.

Let the vector  $\mathbf{x}_{ij} = (x_{1ij}, x_{2ij}, \dots, x_{Kij})'$  be the  $K$  characteristics that can vary among admittees and that are faced by admittee  $j$  who is considering whether to matriculate at college  $i$ . Note well that each characteristic is de-measured so that we obtain the college's desirability at its average level in the data. It is not possible to separately identify the effect of these average characteristics from the  $\theta_i$  for each school. We treat  $\mathbf{x}_{ij}$  as a vector of covariates which are allowed to enter the model linearly. Specifically, the probabilities for the matriculation model become:

$$(6) \quad p_{i^*j} = \frac{\exp(\theta_{i^*} + \mathbf{x}_{ij}'\boldsymbol{\delta})}{\sum_{i \in S_j} \exp(\theta_i + \mathbf{x}_{ij}'\boldsymbol{\delta})}, \quad i^* \in S_j.$$

In practice, we shall find that estimating the  $\boldsymbol{\delta}$  along with the  $\theta_i$  has only a very small effect on our ranking. Nevertheless, understanding the role of individually-varying characteristics is useful.

#### IV. Model Fitting

We summarize estimated college desirability by computing the posterior means of the  $\theta_i$  where the likelihood is the product of multinomial logit probabilities derived from equation (6). The posterior means of the  $\theta_i$  can be computed by maximum likelihood—specifically, a Newton-Raphson algorithm for multinomial logit models as implemented in Stata—and this is

the method we mainly use because it is the least time-intensive computationally. In particular, we use this method to compute our main results and variants on them, such as regional rankings and rankings for subgroups of students.<sup>7</sup>

A difficulty with maximum likelihood is that it does not provide us with answers to questions like "is there a meaningful distinction in desirability between the college ranked 15th and the college ranked 20th?" This is because such questions cannot be answered by using the standard errors generated by maximum likelihood. The statistical significance of the difference between any two colleges' ranks depends on the degree of overlap between their two groups of admittees. A simple standard error does not embody this information.

Therefore, we also use Markov chain Monte Carlo (MCMC) simulation to compute estimates of the posterior means. The ranking produced by MCMC simulation is identical to that produced by maximum likelihood, which shows that the priors used in the simulation have a vanishingly small effect.<sup>8</sup> Because, however, MCMC produces simulated values from the posterior distribution of model parameters, MCMC can generate quantities of interest that are more complex than the mean. In particular, if we want to compare colleges 15 and 20, we can extract pairs of values from the simulated posterior distribution of  $(\theta_{15}, \theta_{20})$ . The probability that  $\theta_{15}$  is greater than  $\theta_{20}$  can be evaluated by computing the proportion of pairs in which  $\theta_{15}$  is greater than  $\theta_{20}$ . An answer of 95 percent or more is analogous to a 95 percent significance test.

To conduct the MCMC simulation, we assume a locally uniform but proper prior distribution that factors into independent densities and that consists of the following components:

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<sup>7</sup> Specifically, we compute our main results both by maximum likelihood and Markov chain Monte Carlo simulation (see below). We, however, compute variants such as the regional and subgroup rankings only by maximum likelihood. We do this purely to save on computational time when we do not need estimates akin to those shown in Table 4.

<sup>8</sup> More precisely, they produce identical rankings for the schools we attempt to rank. We do not attempt to rank schools for which we have only a few matriculation tournaments (see below for details).

$$\theta_i \sim N(0, \sigma^2)$$

18

$$(8) \quad \frac{1}{\sigma^2} \sim \text{Gamma}(0.1, 0.1)$$

$$\delta_k \sim N(0, 100) \quad \text{for } k = 1, 2, \dots, K$$

where  $\delta_k$  indexes the  $k^{\text{th}}$  element of the vector  $\delta$ .

The MCMC algorithm proceeds as follows. Initial values of all parameters are set to the prior mean values. Then values are simulated from the conditional posterior distributions of each model parameter. This process is repeated until the distributions of values for individual parameters stabilize. The values simulated beyond this point can be viewed as coming from the posterior distribution.

We implemented the MCMC algorithm using the program BUGS (Spiegelhalter et al., 1996). For each model, a burn-in period of 10,000 iterations was run, and parameter summaries were based on every 5th iteration of a subsequent 30,000 iterations. Based on trace plots from our data analyses, 10,000 iterations was sufficient to reach the stationary distribution. Every 5th iteration was sampled to reduce the autocorrelation in successive parameter draws. This process produced 6000 values per parameter on which to calculate parameter summaries.

## V. Data

To construct an example of our revealed preference ranking, we use from the College Admissions Project survey, in which we surveyed high school seniors in the college graduating class of 2004.<sup>9</sup> We designed the survey to gather data on students with very high college aptitude who are likely to gain admission to the colleges with a national or broad regional draw that are most appropriate for ranking. While such students are represented in surveys that attempt to be nationally representative, such as the National Educational Longitudinal Survey, they are a very small share of the population of American students. As a result, the number of such students is so small in typical surveys that their behavior cannot be analyzed, even if the survey contains a large number of students. By focusing on students with very

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<sup>9</sup> See Avery and Hoxby [2000] for additional detail.

strong academic credentials, we end up with a sufficient number of tournaments among colleges with a national draw to construct a revealed preference ranking among them.

We reemphasize that we use the College Admissions Project data to construct an *example* of a revealed preference ranking. If we had had much greater resources, we would have surveyed a more fully representative sample of students in the United States. With more data, our national ranking would be more definitive, and we would be able to rank many more colleges (most of them in regional or specialized rankings, not the national ranking). At the end of this section, we describe the cut-offs we used to determine which colleges we could reasonably rank.

#### A. Survey Design

In order to find students who were appropriate candidates for the survey, we worked with counselors from 510 high schools around the United States. The high schools that were selected had a record of sending several students to selective colleges each year, and they were identified using published guides to secondary schools and the experience of admissions experts.<sup>10</sup> Each counselor selected ten students at random from the top of his senior class as measured by grade point average. Counselors at public schools selected students at random from the top 10 percent of the senior class, while counselors at private schools (which tend to be smaller and have higher mean college aptitude) selected students at random from the top 20 percent of the senior class.<sup>11</sup> The counselors distributed the surveys to students, collected the

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<sup>10</sup> The experts are thanked in our acknowledgments. The guides included Peterson's (1999) and Newsweek's annual list (the complete version of which is posted on the internet only).

<sup>11</sup> The counselors were given detailed instructions for random sampling from the top 20, 30, 40, or 50 students in the senior class depending on the size of the school. For example, a counselor from a public school with 157 students was asked to select 10 students at random from the top 20 students in the senior class, with the suggestion that the counselor select students ranked #1, 3, 5, 7, 9, 11, 13, 15, 17, and 19.

completed surveys, and returned them to us for coding.<sup>12</sup> Students were tracked using a randomly assigned number; we never learned the names of the students who participated.

Survey participants completed two questionnaires over the course of the academic year. The first questionnaire was administered in early December 1999. It asked for information on the student's background and college applications; the majority of the questions were taken directly from the Common Application, which is accepted by many colleges in lieu of their proprietary application forms. Each student listed up to ten colleges to which he hoped to apply, in order of preference. In addition, each student listed his credentials (such as test scores), race or ethnicity, and the colleges and graduate schools attended by parents and siblings.

The second questionnaire was administered in May 2000 and asked for information about the student's admission outcomes, financial aid offers, scholarship offers, and matriculation decision. We obtained detailed information on aid, grants, scholarships, and loan. On a third questionnaire distributed to a parent of each survey participant, we collected information on parents' income range (see Table 1 for the income categories.)

We matched the survey to colleges' administrative data on tuition, room and board, location, and other college characteristics. In all cases, the ultimate source for the administrative data was the college itself and the data were for the 2000-01 school year, which corresponds to the survey participants' freshmen year.<sup>13</sup>

The College Admissions Project survey produced a response rate of approximately 65 percent, including full information for 3,240 students from 396 high schools.<sup>14</sup> The final sample

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<sup>12</sup> The exception was the parent survey, which parents mailed directly to us in an addressed, postage-paid envelope so that they would not have to give possibly sensitive financial information to the high school counselor.

<sup>13</sup> See Avery and Hoxby [2004] for a complete description of administrative data sources.

<sup>14</sup> The most common reasons for failure to return the survey were changes of high school administration, an illness contracted by the counselor, and other administrative problems that were unrelated to the college admissions outcomes of students who had been selected to participate.

contains students from 43 states plus the District of Columbia.<sup>15</sup> Although the sample was constructed to include students from every region of the country, it is intentionally representative of applicants to highly selective colleges and therefore non-representative of American high school students as a whole. Of course, all of the students in the sample have very strong academic records.

Because the students are drawn from schools that send several students to selective colleges each year, the students in the sample are probably slightly better informed than the typical high aptitude applicant. However, in other work [Avery and Hoxby, 2004], we have found that students in the sample act substantially like one another when they make college decisions, regardless of whether they come from more or less advantaged backgrounds. This suggests that a revealed preference ranking based on our sample may reflect slightly more information than one based on the typical applicant, but the difference in the information embodied in the ranking is probably small.

## B. Sample Statistics

The summary statistics shown in Tables 1 and 2 demonstrate show the students in the sample are high achieving. The average (combined verbal and math) SAT score among participants was 1357, which put the average student in the sample at the 90th percentile of all SAT takers.<sup>16</sup> About 5 percent of the students won a National Merit Scholarship; 20 percent of them won a portable outside scholarship; and 46 percent of them won a merit-based grant from at least one college. 45 percent of the students attended private school, and their parents'

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<sup>15</sup> The states missing from the sample are Alaska, Delaware, Iowa, Mississippi, North Dakota, South Dakota, and West Virginia.

<sup>16</sup> We converted American College Test (ACT) scores to SAT scores using the cross-walk provided by The College Board. We converted all college admissions scores into national percentile scores using the national distribution of SAT scores for the freshman class of 2000-01.

income averaged \$119,929 in 1999.<sup>17</sup> However, 76 percent of the sample had incomes below the cut-off where a family is considered for aid by selective private colleges, and 59 percent of the students applied for need-based financial aid.<sup>18</sup> Among survey participants, 73 percent were white, 16 percent Asian, 3.5 percent black, and 3.8 percent Hispanic.

Table 1				
Description of the Students in the College Admission Project Data				
Variable	Mean	Std. Dev.	Minimum	Maximum
Male	0.41	0.49	0.00	1.00
White, non-Hispanic	0.73	0.44	0.00	1.00
Black, non-Hispanic	0.04	0.18	0.00	1.00
Asian	0.16	0.36	0.00	1.00
Hispanic	0.04	0.19	0.00	1.00
Native American	0.00	0.03	0.00	1.00
Other race/ethnicity	0.04	0.19	0.00	1.00
Parents are married	0.83	0.38	0.00	1.00
Sibling(s) enrolled in college	0.23	0.42	0.00	1.00
Parents' income	119,929	65,518	9,186	240,000
Expected family contribution	27,653	16,524	0	120,000
Applied for financial aid?	0.59	0.49	0.00	1.00
National Merit Scholarship winner	0.05	0.22	0.00	1.00
Student's combined SAT score	1357	139	780	1600
Student's SAT score, in national percentiles	90.4	12.3	12.0	100.0
Median SAT score at <i>most</i> selective college to which student was admitted	86.4	10.4	33.5	98.0
Median SAT score at <i>least</i> selective college to which student was admitted	73.8	14.6	14.3	97.0
Student's high school was private	0.45	0.50	0.00	1.00

Looking at Table 2, which shows descriptive statistics on the colleges where the

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<sup>17</sup> See Avery and Hoxby [2004] for descriptions of how the aid variables were hand checked and how some parents' income was estimated based on their Expected Family Contribution, a federal financial aid measure.

<sup>18</sup> The cut-off was approximately \$160,000, but the actual cut-off depends on family circumstances.

students applied, were admitted, and matriculated; we can see that the survey participants made logical application decisions. The mean college to which they *applied* had a median SAT score at the 83<sup>rd</sup> percentile; the mean college to which they were *admitted* had median SAT score at the 81<sup>st</sup> percentile. 47.5 percent applied to at least one Ivy League college. These statistics suggests that students aimed a little high in their applications but included "safety schools," a procedure that is optimal.

**Table 2**  
**Description of the Colleges in the College Admission Project Data**

Variable	Colleges at Which Students					
	Applied		Were Admitted		Matriculated	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Matriculated at this college	0.28	0.45	0.18	0.39	1.00	0.00
Admitted to this college	1.00	0.00	0.66	0.47	1.00	0.00
Grants from this college	2720	5870	1778	4933	4029	7051
Loans from this college	641	2282	413	1856	1020	2722
Work study amount from this college	172	593	111	483	296	768
Father is an alumnus of this college	0.04	0.20	0.03	0.17	0.07	0.25
Mother is an alumna of this college	0.03	0.17	0.02	0.14	0.04	0.19
Sibling attended or attends this college	0.05	0.21	0.04	0.19	0.08	0.28
College is public	0.3325	0.4711	0.2631	0.4403	0.2843	0.4512
College's median SAT score, in percentiles	80.5947	12.5188	83.8816	12.0390	83.4215	12.5494
In-state tuition	16435	9594	18181	9199	17432	9513
Out-of-state tuition	19294	6191	20498	5891	19841	6371
Tuition that applies to this student	17671	8492	19277	7965	18340	8599
College is in-state	0.3270	0.4691	0.2666	0.4422	0.3368	0.4727
Distance between student's high school and this college, in miles	597	809	673	873	576	827

Students matriculated at colleges that were more selective, on average, than the colleges to which they were admitted: the median SAT score of matriculation colleges was at the 83.4<sup>th</sup> percentile, as opposed to the 81<sup>st</sup> percentile for colleges to which students were admitted. These facts imply that students included "safety schools" in their choice sets but infrequently matriculated at them. One measure of the high college aptitude of the survey participants is



the list of colleges at which the largest numbers of participants matriculated. Seventeen institutions enrolled at least 50 students from the sample: Harvard, Yale, University of Pennsylvania, Stanford, Brown, Cornell, University of Virginia, Columbia, University of California–Berkeley, Northwestern, Princeton, Duke, University of Illinois, New York University, University of Michigan, Dartmouth, and Georgetown.

## VI. A National Ranking

We show a college in the national ranking if it was not a military academy and if, in our sample, it competed in matriculation tournaments in at least six of the nine regions of the U.S. 110 colleges met these criteria. The mean college shown in the national ranking competed in 73 matriculation tournaments, and the median college competed in 57. Admittedly, the six region cut-off is somewhat arbitrary, but we show regional rankings below that pick up extra colleges. Note that if a small college fails to appear in the rankings, one should not conclude that its ranking is below those of colleges that appear in the ranking or that it does not have a national draw. Our sample might fail to pick up enough applicants to include a small college in the national ranking, even if its draw were national in character.

### A. National Ranking

Table 3 presents the revealed preference ranking of colleges and universities with a national draw. For each college, we present its mean desirability among students. The table also shows the estimate of  $\theta_i$  associated with each college. While the  $\theta_i$  are not based solely or even largely on head-to-head tournaments (they are based on multiplayer tournaments and on inferences drawn from "indirect" tournaments), there is a way to translate the point estimates into a prediction of the probability that any college will be picked over another college in a head-to-head matriculation tournament. Therefore, purely as an aid to the reader in interpreting the point estimates, we show some of these translations, which are computed as follows:

$$(9) \quad Prob(i \rightarrow i') = \frac{\exp(\theta_i)}{\exp(\theta_i) + \exp(\theta_{i'})},$$

where  $\rightarrow$  denotes the relation "is ranked higher than".

Some predictions of this type are shown the two right-hand columns, as examples. For instance, the top row shows that, if a student were choosing between Harvard and California Institute of Technology (Cal Tech), her probability of matriculating at Harvard is predicted to be 59 percent. If the student were choosing between Harvard and Wellesley (the college listed ten places below Harvard), her probability of matriculating at Harvard is predicted to be 93 percent. Readers can compute other examples for themselves.

All of the top twenty, except for the University of Virginia, are private institutions. About four-fifths of the top twenty are universities—the exceptions being Amherst, Wellesley, Williams, and Swarthmore. The next twenty institutions are, however, a mix of public and private, small and large, colleges and universities. They are also more geographically diverse. They include private schools from middle and southern states: University of Chicago, Furman, Carleton, Davidson, Northwestern, Oberlin, Vanderbilt. There are also several public universities: the University of California - Berkeley, the University of California - Los Angeles, Georgia Institute of Technology, the University of Texas at Austin, and the University of North Carolina at Chapel Hill. The colleges ranked from 41 to 106 include a good number of states' "flagship" universities, numerous liberal arts colleges, several private universities, and a few more institutes of technology.<sup>19</sup>

The results for Cal Tech are somewhat problematic. Because students self-select into applying to Cal Tech based on an orientation toward math and science, Cal Tech's pool of admittees overlaps only slightly with that of most other institutions, except for MIT, with which Cal Tech has substantial overlap. MIT's pool, on the other hand, overlaps substantially

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<sup>19</sup> The students in our sample who had a Florida resident as a parent were the first cohort to receive Florida A-Plus Scholarships, which allowed them to attend public universities in Florida for free. The initiation of the scholarships generated considerable excitement and may have raised the ranking of public universities in Florida, such as Florida State, among students in our sample.

with other top schools. These facts have two implications. First, Cal Tech is ranked largely *through* MIT's ranking vis-a-vis other institutions. It depends to an unusual degree on indirect tournaments. Second, much more than any other institution in the top twenty, Cal Tech draws upon a self-selected group of applicants. We address the problem of self-selection into specialty schools in a later section. For now, we merely flag the issue and note that a few institutions, most obviously Cal Tech, may have a ranking positively affected by self-selection.

In the far right-hand column of Table 3, we show the ranking we obtain when we do not estimate parameters on the individually-varying characteristics simultaneously with the  $\theta_i$ . It is immediately clear that the individually-varying covariates make very little difference to our estimates. Some schools move up or down a few places—for instance, Yale and Cal Tech trade places, Stanford and MIT trade places—but the rankings are extremely similar. In fact, the correlation between the rankings with and without the covariates is 99 percent.<sup>20</sup>

It would not be informative to attach standard errors to the estimates of  $\theta_i$  shown in Table 3 because the test of interest is whether any two colleges' rankings are statistically distinct. Instead, we present Table 4, which shows the percentage of posterior draws in MCMC simulation in which one college's  $\theta$  is higher than another. These percentages are the Bayesian analog of paired significance tests. For instance, in 70 percent of the draws from the posterior distribution, Harvard's  $\theta$  was higher than Cal Tech's and, in 98 percent of the draws, Harvard's  $\theta$  was higher than Yale's. For all other colleges, Harvard's ranking was higher in at least 99.5 percent of the draws. It is important not to confuse these significance test analogs with the estimated probability that a college "wins" in a head-to-head competition. For example, Harvard's ranking is higher than Princeton's in more than 99.5 percent of draws, but the probability that a student matriculates at Harvard rather than Princeton in a head-to-head tournament is only 75 percent of the time. In other words, our confidence about the relative

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<sup>20</sup> The covariates may make little difference because students are insensitive to them, but we think that it is more likely that *similarly desirable* colleges match one another's offers in an overall way even if the exact features of the offer differ. Thus, a student who is offered a grant by one college may be offered a generously subsidized loan by a second college that is ordinarily competitive with the first.

position of the  $\theta_i$ s can be close to one hundred percent but the  $\theta_i$ s need not imply that students are anywhere close to one hundred percent likely to choose a college.

As a rule, the lower one goes in the revealed preference ranking, the less distinct is a college's desirability from that of its immediate neighbors in the ranking. Among the top ten colleges, we generally enjoy confidence of about 75 percent that a college is ranked higher than the college listed one or two below it. This confidence falls to about 65 for colleges ranked eleven to twenty and falls further to 55 to 60 percent for colleges ranked twenty-one to forty. This is not surprising: in many ordinal rankings, cardinal desirability is more bunched the lower one goes in the ranking. That is, there may be less consensus among students about colleges' desirability as we move from the best known colleges to those that are less widely known. However, in our case, there is another, independent reason why the distinctness of colleges' desirability falls off. It is the nature of our sample that our data are thickest for the most selective colleges. We did a simple test to determine the degree to which data thickness by itself was responsible for the fall off in confidence: we randomly selected only 20 observations per college. With these data, we found that about two-thirds of the drop-off in confidence disappeared. That is, if our data were equally representative for all colleges, our confidence about the exact rank order would still fall, but it would probably fall only about one third as fast as it does.

Table 3

## A Revealed Preference Ranking of Colleges Based on Matriculation Decisions

Rank Based on Matriculation (covariates)	College Name	$\theta$	Implied Prob. of "Winning" vs. College Listed...		Rank Based on Matriculation (no covariates)
			1 Row Below	10 Rows Below	
1	Harvard University	9.13	0.59	0.93	1
2	California Institute of Tech	8.77	0.56	0.92	3
3	Yale University	8.52	0.59	0.92	2
4	MIT	8.16	0.51	0.89	5
5	Stanford University	8.11	0.52	0.90	4
6	Princeton University	8.02	0.73	0.90	6
7	Brown University	7.01	0.56	0.78	7
8	Columbia University	6.77	0.54	0.73	8
9	Amherst College	6.61	0.51	0.71	9
10	Dartmouth	6.57	0.52	0.72	10
11	Wellesley College	6.51	0.53	0.71	12
12	University of Pennsylvania	6.39	0.56	0.71	11
13	University of Notre Dame	6.13	0.51	0.70	16
14	Swarthmore College	6.07	0.55	0.69	13
15	Cornell University	5.87	0.53	0.67	17
16	Georgetown University	5.77	0.50	0.64	15
17	Rice University	5.75	0.50	0.64	19
18	Williams College	5.75	0.51	0.66	14
19	Duke University	5.72	0.52	0.65	18
20	University of Virginia	5.65	0.51	0.67	21
21	Brigham Young University	5.61	0.53	0.68	20
22	Wesleyan University	5.48	0.55	0.67	24
23	Northwestern University	5.30	0.51	0.64	23
24	Pomona College	5.27	0.52	0.65	22
25	Georgia Institute of Technology	5.17	0.50	0.63	30
26	Middlebury College	5.17	0.50	0.64	27
27	U. of California: Berkeley	5.17	0.51	0.64	25
28	University of Chicago	5.11	0.51	0.63	29
29	Johns Hopkins University	5.08	0.54	0.63	26
30	U. of Southern California	4.92	0.52	0.60	32
31	Furman University	4.86	0.52	0.60	28
32	U. North Carolina Chapel Hill	4.77	0.52	0.58	34
33	Barnard College	4.70	0.51	0.57	35
34	Oberlin College	4.67	0.51	0.57	38
35	Carleton College	4.63	0.51	0.56	31
36	Vanderbilt University	4.61	0.51	0.56	36
37	Davidson College	4.58	0.50	0.55	42
38	UCLA	4.57	0.50	0.58	44
39	University of Texas at Austin	4.56	0.51	0.57	33
40	University of Florida	4.53	0.52	0.57	40
41	New York University	4.46	0.51	0.55	46

Table 3

## A Revealed Preference Ranking of Colleges Based on Matriculation Decisions

Rank Based on Matriculation (covariates)	College Name	$\theta$	Implied Prob. of "Winning" vs. College Listed...		Rank Based on Matriculation (no covariates)
			1 Row Below	10 Rows Below	
42	Tufts University	4.43	0.51	0.57	50
43	Washington and Lee University	4.41	0.51	0.57	45
44	Vassar College	4.39	0.50	0.58	47
45	Grinnell College	4.38	0.50	0.57	43
46	University of Michigan	4.37	0.50	0.58	48
47	U. Illinois Urbana-Champaign	4.36	0.53	0.59	41
48	Carnegie Mellon University	4.26	0.50	0.56	59
49	U. of Maryland: College Park	4.26	0.50	0.57	51
50	College of William and Mary	4.25	0.50	0.57	62
51	Bowdoin College	4.25	0.52	0.57	53
52	Wake Forest University	4.16	0.51	0.56	54
53	Claremont Mckenna College	4.14	0.51	0.56	39
54	Macalester College	4.08	0.50	0.55	65
55	Colgate University	4.07	0.51	0.56	55
56	Smith College	4.05	0.51	0.57	49
57	Boston College	4.04	0.50	0.57	56
58	University of Miami	4.02	0.50	0.58	37
59	Mount Holyoke College	4.01	0.50	0.58	60
60	Haverford College	3.99	0.51	0.60	63
61	Bates College	3.96	0.50	0.60	52
62	Connecticut College	3.95	0.51	0.60	61
63	Kenyon College	3.92	0.51	0.59	57
64	Emory University	3.88	0.51	0.59	66
65	Washington University	3.86	0.51	0.60	64
66	Occidental College	3.83	0.52	0.62	68
67	Bryn Mawr College	3.77	0.52	0.61	67
68	Southern Methodist University	3.70	0.50	0.59	58
69	Lehigh University	3.69	0.53	0.59	71
70	Holy Cross College	3.59	0.51	0.58	69
71	Reed College	3.57	0.50	0.58	78
72	Rensselaer Polytechnic Institute	3.55	0.50	0.57	77
73	Florida State University	3.55	0.51	0.57	73
74	Colby College	3.50	0.51	0.56	72
75	UC: Santa Barbara	3.45	0.53	0.56	83
76	Miami U.: Oxford Campus	3.34	0.50	0.54	75
77	George Washington University	3.34	0.50	0.57	79
78	Fordham University	3.33	0.50	0.57	74
79	Dickinson College	3.33	0.51	0.59	70
80	Sarah Lawrence College	3.28	0.51	0.57	80
81	Catholic University of America	3.26	0.50	0.58	91
82	Bucknell University	3.26	0.50	0.58	85

Table 3

**A Revealed Preference Ranking of Colleges Based on Matriculation Decisions**

Rank Based on Matriculation (covariates)	College Name	$\theta$ Implied Prob. of "Winning" vs. College Listed...			Rank Based on Matriculation (no covariates)
		1 Row Below	10 Rows Below		
83	U. of Colorado at Boulder	3.26	0.51	0.60	81
84	U. of Wisconsin-Madison	3.24	0.51	0.61	84
85	Arizona State University	3.22	0.51	0.61	86
86	Wheaton College	3.17	0.53	0.61	76
87	Trinity College	3.07	0.51	0.59	82
88	Rose-Hulman Inst. of Tech.	3.04	0.51	0.59	99
89	U. of California: Santa Cruz	2.99	0.50	0.58	94
90	Boston University	2.98	0.51	0.65	88
91	U. of California: San Diego	2.96	0.50	0.66	98
92	Tulane University	2.94	0.52	0.66	90
93	University of Richmond	2.86	0.51	0.67	97
94	Case Western Reserve U.	2.80	0.51	0.71	89
95	Colorado College	2.76	0.51	0.70	95
96	Indiana U. Bloomington	2.71	0.50	0.72	93
97	Penn State University Park	2.71	0.51	0.73	92
98	American University	2.68	0.51	0.74	87
99	Hamilton College	2.65	0.57	0.73	96
100	University of Washington	2.36	0.52	0.70	100
101	University of Rochester	2.30	0.51	0.91	101
102	Michigan State University	2.27	0.53		108
103	Lewis & Clark College	2.16	0.56		104
104	Clark University	1.92	0.51		102
105	Skidmore College	1.90	0.53		103
106	Purdue University	1.76	0.52		110
107	Colorado State University	1.70	0.51		107
108	Syracuse University	1.65	0.50		106
109	University of Vermont	1.63	0.53		105
110	Scripps College	1.50	0.82		109

Estimates of other parameters:

Tuition (In Thousands, In-state or Out-of-state, Whichever Applies)	-0.021	(0.019)
Grants (In Thousands)	0.087	(0.007)
Loans (In Thousands)	0.098	(0.017)
Work-study (in Hundreds)	0.050	(0.007)
Indicator: Is Dad's College	0.481	(0.158)
Indicator: Is Mom's College	0.050	(0.202)
Indicator: Is a Sibling's College	0.592	(0.135)
Indicator: College in Home State	0.074	(0.132)
Indicator: College in Home Region	0.005	(0.108)
Distance from Home (Thousands of Miles)	0.035	(0.067)

Notes: Estimates of parameters in equation (6) by maximum likelihood as described in the text. Numbers in parentheses are standard errors. Test statistics for other parameter estimates are in next table.

**Table 4: Share of Draws in Which College in the Row is Ranked Higher than College in the Column**  
**statistical significance analogs: MCMC Estimates of  $\text{Prob}(\theta_i > \theta_j)$ ; *not* probabilities of winning in a head-to-head tournament**

	Cal- tech	Yale	MIT	Stan- ford	Prince- ton	Brown	Colum- bia	Am- herst	Dart- mouth	Wel- lesley	U Penn	Notre Dame	Swarth- more	Cor- nell	Georg e-town	Rice	Wil- liams	Duke	U Vir- ginia
Harvard	0.70	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Caltech	n/a	0.66	0.81	0.82	0.85	0.99	1.00	0.99	0.99	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00
Yale		n/a	0.79	0.84	0.90	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MIT			n/a	0.59	0.66	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Stanford				n/a	0.63	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Princeton					n/a	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Brown						n/a	0.79	0.87	0.90	0.88	0.98	0.98	0.95	1.00	1.00	1.00	0.99	1.00	1.00
Columbia							n/a	0.61	0.66	0.70	0.85	0.90	0.90	1.00	1.00	0.98	0.98	1.00	1.00
Amherst								n/a	0.51	0.56	0.70	0.80	0.79	0.97	0.98	0.93	0.94	0.98	0.98
Dartmouth									n/a	0.56	0.66	0.81	0.79	0.97	0.98	0.95	0.94	0.98	0.99
Wellesley										n/a	0.58	0.72	0.71	0.90	0.93	0.89	0.88	0.92	0.95
U Penn*											n/a	0.71	0.70	0.96	0.98	0.91	0.90	0.98	0.99
Notre Dame												n/a	0.54	0.68	0.75	0.69	0.70	0.75	0.82
Swarthmore													n/a	0.60	0.72	0.68	0.68	0.73	0.76
Cornell														n/a	0.65	0.58	0.59	0.66	0.77
Georgetown															n/a	0.51	0.48	0.50	0.63
Rice																n/a	0.52	0.51	0.60
Williams																	n/a	0.53	0.59
Duke																		n/a	0.60
U Virginia																			n/a

\*Abbreviations: MIT: Massachusetts Institute of Technology; U Penn: University of Pennsylvania.

Notes: The table shows the share of MCMC draws in which the point estimate of  $\theta$  for the college in the row is higher than the  $\theta$  for the college in the column.



**Table 4 continued: Share of Draws in Which College in the Row is Ranked Higher than College in the Column**  
**statistical significance analogs: MCMC Estimates of  $\text{Prob}(\theta_i > \theta_j)$ ; not probabilities of winning in a head-to-head tournament**

	Wes- leyan	NWU	Pomona	Georgia Tech	Middle- bury	Berke- ley	U Chic- ago	JHU	USC	Fur- man	UNC	Bar- nard	Ober- lin	Carle- ton	Van- derbilt	David- son	UCLA	UT Aus- tin	U Flor- ida
BYU*	0.56	0.64	0.63	0.69	0.69	0.68	0.69	0.71	0.76	0.75	0.78	0.82	0.84	0.80	0.85	0.84	0.84	0.86	0.86
Wesleyan	n/a	0.65	0.66	0.74	0.73	0.78	0.78	0.78	0.86	0.78	0.88	0.91	0.93	0.92	0.95	0.93	0.96	0.94	0.96
NWU*		n/a	0.53	0.63	0.63	0.66	0.72	0.70	0.85	0.73	0.84	0.91	0.94	0.89	0.96	0.91	0.98	0.92	0.96
Pomona			n/a	0.58	0.58	0.59	0.63	0.63	0.75	0.69	0.81	0.85	0.89	0.84	0.90	0.88	0.93	0.87	0.92
Georgia Tech				n/a	0.49	0.47	0.52	0.53	0.68	0.63	0.73	0.81	0.83	0.80	0.88	0.82	0.88	0.83	0.90
Middlebury					n/a	0.50	0.54	0.57	0.69	0.65	0.74	0.82	0.86	0.79	0.90	0.86	0.90	0.87	0.89
Berkeley						n/a	0.56	0.55	0.74	0.67	0.79	0.87	0.87	0.83	0.93	0.85	0.99	0.89	0.94
U Chicago							n/a	0.51	0.69	0.62	0.75	0.83	0.87	0.80	0.89	0.82	0.91	0.87	0.92
JHU*								n/a	0.63	0.63	0.72	0.80	0.82	0.76	0.87	0.81	0.88	0.82	0.88
USC*									n/a	0.55	0.60	0.71	0.73	0.69	0.79	0.72	0.83	0.76	0.86
Furman										n/a	0.53	0.58	0.60	0.60	0.61	0.65	0.65	0.64	0.68
UNC*											n/a	0.60	0.59	0.58	0.67	0.64	0.68	0.64	0.72
Barnard												n/a	0.52	0.52	0.56	0.59	0.55	0.56	0.65
Oberlin													n/a	0.49	0.50	0.56	0.56	0.54	0.63
Carleton														n/a	0.54	0.53	0.54	0.53	0.60
Vanderbilt															n/a	0.54	0.51	0.53	0.61
Davidson																n/a	0.50	0.50	0.56
UCLA*																	n/a	0.51	0.58
UT Austin*																		n/a	0.54
U Florida																			n/a

\*Abbreviations: BYU: Brigham Young University; NWU: Northwestern University; JHU: Johns Hopkins University; USC: University of Southern California; UNC: University of North Carolina at Chapel Hill; UCLA: University of California Los Angeles; UT Austin: University of Texas at Austin.

Notes: The table shows the share of MCMC draws in which the point estimate of  $\theta$  for the college in the row is higher than the  $\theta$  for the college in the column.

**Table 4 continued: Share of Draws in Which College in the Row is Ranked Higher than College in the Column**  
**statistical significance analogs: MCMC Estimates of  $\text{Prob}(\theta_i > \theta_j)$ ; not probabilities of winning in a head-to-head tournament**

	Tufts	Wash & Lee	Vassar	Grinnell	U Michigan	U Illinois	CMU	UMD	Wlm & Mary	Bowdoin	Wake Forest	Claremont	Macalester	Colgate	Smith	BC	U Miami	Mt Holyoke	Haverford
NYU	0.50	0.56	0.56	0.59	0.57	0.59	0.68	0.69	0.68	0.62	0.76	0.64	0.75	0.81	0.76	0.82	0.74	0.77	0.83
Tufts	n/a	0.57	0.56	0.58	0.55	0.59	0.67	0.67	0.68	0.64	0.72	0.64	0.73	0.77	0.75	0.79	0.74	0.77	0.81
Wash & Lee*		n/a	0.48	0.52	0.47	0.49	0.55	0.56	0.60	0.52	0.62	0.60	0.65	0.66	0.65	0.70	0.65	0.65	0.70
Vassar			n/a	0.55	0.49	0.51	0.59	0.62	0.62	0.56	0.68	0.62	0.68	0.73	0.71	0.78	0.72	0.73	0.80
Grinnell				n/a	0.45	0.47	0.51	0.53	0.52	0.53	0.57	0.58	0.59	0.60	0.62	0.67	0.62	0.65	0.69
U Michigan					n/a	0.54	0.61	0.65	0.66	0.58	0.73	0.63	0.70	0.73	0.71	0.80	0.73	0.75	0.82
U Illinois						n/a	0.58	0.63	0.63	0.54	0.69	0.60	0.68	0.71	0.70	0.77	0.71	0.71	0.79
CMU*							n/a	0.53	0.56	0.51	0.59	0.55	0.62	0.66	0.67	0.69	0.65	0.64	0.73
UMD*								n/a	0.53	0.46	0.56	0.56	0.59	0.61	0.64	0.66	0.64	0.62	0.73
Wlm & Mary*									n/a	0.46	0.54	0.53	0.56	0.59	0.59	0.62	0.62	0.61	0.66
Bowdoin										n/a	0.58	0.56	0.60	0.63	0.64	0.70	0.65	0.64	0.72
Wake Forest											n/a	0.49	0.54	0.57	0.58	0.62	0.60	0.60	0.67
Claremont*												n/a	0.55	0.50	0.54	0.57	0.56	0.55	0.58
Macalester													n/a	0.49	0.51	0.54	0.53	0.54	0.58
Colgate														n/a	0.54	0.56	0.54	0.54	0.61
Smith															n/a	0.52	0.50	0.50	0.58
BC*																n/a	0.50	0.46	0.54
U Miami																	n/a	0.47	0.54
Mt Holyoke																		n/a	0.55
Haverford																			n/a

\*Abbreviations: Wash & Lee: Washington and Lee University; CMU: Carnegie Mellon University; UMD: University of Maryland; Wlm & Mary: The College of William and Mary; Claremont: Claremont McKenna; BC: Boston College.

Notes: The table shows the share of MCMC draws in which the point estimate of  $\theta$  for the college in the row is higher than the  $\theta$  for the college in the column.

## B. Comparing Measures of Revealed Preference

For the colleges that are in the top twenty based on revealed preference, Table 5 shows what their rankings would be if they were based on, respectively, the admissions and matriculation rates. We use crude admissions and matriculation rates from the College Board's Standard Research Compilation, the same data as form the "Common Data Set" published on colleges' websites and used by college guides like *U.S. News*.

Table 5			
A Comparison of the Revealed Preference Ranking of Colleges and Rankings Based on the Crude Admissions and Matriculation Rates			
	National Rank Based On:		
	Revealed Preference (based on Matriculation Tournaments)	Admissions Rate	Matriculation Rate
Harvard	1	4	139
Cal Tech	2	9	854
Yale	3	12	309
MIT	4	13	422
Stanford	5	7	297
Princeton	6	5	266
Brown	7	14	561
Columbia	8	6	438
Amherst	9	19	916
Dartmouth	10	20	647
Wellesley	11	23	492
U Penn	12	104	794
U Notre Dame	13	58	459
Swarthmore	14	28	1016
Cornell	15	45	649
Georgetown	16	22	703
Rice	17	25	996
Williams	18	29	797
Duke	19	32	859
U Virginia	20	76	630

Notes: Left-hand column shows rank based on Table 3. The admissions and matriculation rates are based on the Common Data Set, used by most college guidebooks.

Looking at Table 5, we observe that most of the top twenty colleges based on revealed preference are not in the top twenty based on the admissions and matriculation rates. Indeed, the admissions rate puts 10 of them outside the top twenty and the matriculation rate puts all of them outside the top 100. Clearly, there are many colleges with low admissions rates or high

matriculation rates that are not perceived to be highly desirable. Some commentators, when examining Table 5, have commented that the crude matriculation and admissions rates are unexpectedly misleading and asked whether they need to be corrected. That is precisely our point: these crude rates are the (misleading) information that forms the basis of common college ranking procedures. We are unable to frame an argument for why the crude rates have any advantage over the procedure that we outline in this paper.

## VI. Extending the Model to Handle Early Decision

At one level, an early decision program is just an extreme version of a individually-varying characteristic. A college offers early decision applicants an easier admissions standard. In return, the college does not merely expect the student to matriculate with a higher likelihood but actually forces the student to *commit* to matriculate if admitted.<sup>21</sup> Early decision applicants also commit not to apply early to more than one college and, if admitted, not to submit any regular admission applications. The problem with early decision, for our purposes, is the commitment not to apply elsewhere. We can estimate the value of a tuition discount, but we cannot estimate the value of a reduced admissions standard for the simple reason that each early decision admittee is precluded from holding a matriculation tournament. We do not know whether the college at which a student was admitted early decision (and at which, therefore, he automatically matriculated) was actually his most preferred college. If he knew the full menu of colleges that would have admitted him, he might have picked another. But, by the nature of his commitment, he never learns what his menu would have been.

Ordinarily, we would not even know the other colleges the student had been considering when he chose to apply early decision. Thus, even if we had exact information on the student's probability of admission at each school, both through early decision and regular admissions, it would be hard to identify the effect of the reduced admissions standard on the

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<sup>21</sup> Also, because an early decision admittee is forbidden from applying to other colleges, the early decision college need not offer financial aid and scholarships that would be competitive with other colleges' offers.

decision to matriculate.

Fortunately, our survey does contain information on the colleges each student considered, in order of preference. This is because the survey asked students to list the colleges to which they planned to apply, in preference order. Students answered the question before learning about their outcomes from early applications. Thus it is to be hoped that the answers supply us with information on the matriculation tournament each student would hold if he could and how that tournament would turn out.

We expect data on preference orderings, as opposed to matriculation tournaments, to produce rankings more favorable to early decision schools for two reasons. First, students with a very strong idiosyncratic preference for a school may be more likely to apply early decision. Second, preference orderings could be contaminated by the strategy involved in early decision. Either because of cognitive dissonance or a wish to appear non-strategic, a student might claim to prefer the school to which he has already decided, *for strategic reasons*, to apply early decision.

We use the preference orderings in two ways. First, we treat them as pseudo matriculation tournaments in the college ranked first is assumed to be the one at which the student would have matriculated. All other colleges are treated as tournament losers. We compute a ranking based on the pseudo matriculation tournaments for all students, even those for whom we observe an actual tournament. Second, we use all of the information in students' preference orderings by estimating a rank-ordered logistic (Plackett-Luce or exploded logit) model. This is an extension of the model already described. The likelihood of observing a certain ordering of, say, ten colleges is modeled as the product of the probability that the college ranked first would have won in a tournament with the nine others, the probability that the college ranked second would have won in a tournament with the eight others, and so on to the probability that the college ranked ninth would have won in a tournament with the college ranked tenth.

That is, the probability that student  $j$  ranks his menu of  $m_j$  colleges in order  $1, 2, \dots, m_j$  is given by:

$$(7) \quad Prob(1 \rightarrow 2 \rightarrow \dots \rightarrow m_j) = \prod_{i=1}^{m_j} \frac{\exp(\theta_i)}{\sum_{r=1}^{m_j} \exp(\theta_r)}, \quad i \in S_j.$$

where  $Prob[1 \rightarrow 2 \rightarrow \dots \rightarrow m_j]$  is the probability of observing the event  $1 \rightarrow 2 \rightarrow \dots \rightarrow m_j$  among the possible permutations of the colleges, and  $r$  indexes the colleges ranked equal to or lower than college  $i$ . Note that, when we estimate the rank-ordered logistic model, we use the expressed preference ordering even if the student's actual matriculation tournament suggests that his preferences were altered by the time he made a decision.

Our results, presented below, show that rankings based on the preference orderings are extremely similar to those based on actual matriculation tournaments. This gives us confidence that, if we were somehow to have known about the matriculation tournaments that early decision admittees would have held if they could, the information would have matched closely to the information we derive from actual matriculation tournaments.

Nevertheless, we believe that preference orderings are no general substitute for actual matriculation tournaments. This is for several reasons. First, matriculation tournaments reveal the choices of students who can actually get into the colleges in question, where preference ordering-based rankings also rely on students for whom the colleges are merely a pipedream. Second, matriculation tournaments occur when a student is most fully informed about his options and colleges' characteristics, whereas preference ordering-based rankings occur before a good deal of information is revealed. Most importantly, matriculation tournaments are based on actual decisions, not wishes that may be expressed for whimsical or strategic reasons. While our results suggest that preference orderings generate rankings very similar to the rankings generated by matriculation tournaments, the students taking our survey had no incentive (beyond cognitive dissonance or a desire to appear non-strategic) to distort their expressed preferences. If rankings based on expressed preferences became prevalent, students' incentives would change and their expressions might become disingenuous.

Table 6 shows our ranking based on matriculation tournaments in the left-hand column.

The two right hand columns show rankings based on students' preference orderings. The rankings based on preference orderings are very similar to the rankings based on matriculation tournaments: the correlation in the rankings is about 0.83. There is some minor place-trading, but the most noticeable differences are two. First, colleges that practice early decision (as opposed to early action or only regular admission) are slightly higher in rankings based on preference orderings. This is what we expected: such rankings are more favorable to them for the reasons we described above. Second, specialty colleges are lower in rankings based on preference orderings.<sup>22</sup> This is probably because they show up rather low in the preference orderings of students who, in fact, are not well suited to them. Such students will either not apply (assuming that they discover the mis-match in time) or not be admitted. We take up specialty colleges in some detail in the next section.

In short, when we compare our basic ranking to rankings that include the preferences of early decision admittees, we conclude that our basic ranking is not unduly affected. We therefore have confidence in our core ranking method.

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<sup>22</sup> Specialty colleges that fall noticeably in at least one of the preference ordering-based rankings include Cal Tech, Wellesley (a women's college), Brigham Young (a Mormon University), and Georgia Institute of Technology (an engineering school). Some colleges known for their regional character also fall somewhat: Furman and Vanderbilt, for instance (both Southern schools).

**Table 6**  
**Preference-Based Rankings for a Variety of Specifications**

College Name	Rank Based on		
	Matriculation Tournaments with Covariates (Table 3)	Highest Listing in Preference Ordering (All planned applications)	Rank Ordered Logit on Preference Ordering (All planned applications)
Harvard University	1	1	1
California Inst. of Technology	2	7	6
Yale University	3	5	2
MIT	4	3	8
Stanford University	5	2	3
Princeton University	6	4	4
Brown University	7	6	5
Columbia University	8	8	7
Amherst College	9	13	10
Dartmouth College	10	11	9
Wellesley College	11	33	29
University of Pennsylvania	12	12	11
University of Notre Dame	13	14	18
Swarthmore College	14	10	17
Cornell University	15	15	14
Georgetown University	16	9	13
Rice University	17	21	20
Williams College	18	22	15
Duke University	19	16	12
University of Virginia	20	25	23
Brigham Young University	21	47	53
Wesleyan University	22	24	22
Northwestern University	23	17	19
Pomona College	24	20	26
Georgia Inst. of Technology	25	43	41
Middlebury College	26	29	16
UC Berkeley	27	42	30
University of Chicago	28	19	25
Johns Hopkins University	29	30	21
U. of Southern California	30	37	58
Furman University	31	49	71
U. North Carolina Chapel Hill	32	39	38
Barnard College	33	26	27
Oberlin College	34	45	51
Carleton College	35	23	34
Vanderbilt University	36	55	59
Davidson College	37	31	37
UCLA	38	44	54



**Table 6**  
**Preference-Based Rankings for a Variety of Specifications**

College Name	Rank Based on		
	Matriculation Tournaments with Covariates (Table 3)	Highest Listing in Preference Ordering (All planned applications)	Rank Ordered Logit on Preference Ordering (All planned applications)
University of Texas Austin	39	53	79
University of Florida	40	84	105
New York University	41	36	31
Tufts University	42	40	32
Washington and Lee	43	61	67
Vassar College	44	28	49
Grinnell College	45	112	45
University of Michigan	46	46	50
U. Illinois Urbana-Champaign	47	41	76
Carnegie Mellon University	48	27	28
U. of Maryland College Park	49	86	102
College of William and Mary	50	69	64
Bowdoin College	51	62	33
Wake Forest University	52	58	47
Claremont Mckenna College	53	85	36
Macalester College	54	70	43
Colgate University	55	83	60
Smith College	56	57	40
Boston College	57	59	55
University of Miami	58	73	82
Mount Holyoke College	59	82	56
Haverford College	60	32	24
Bates College	61	74	35
Connecticut College	62	75	70
Kenyon College	63	65	85
Emory University	64	35	42
Washington University	65	50	46
Occidental College	66	96	90
Bryn Mawr College	67	51	63
Southern Methodist University	68	90	113
Lehigh University	69	79	75
Holy Cross College	70	52	48
Reed College	71	18	44
Rensselaer Polytechnic Institute	72	56	74
Florida State University	73	99	112
Colby College	74	64	39
UC: Santa Barbara	75	106	108
Miami University: Oxford	76	94	93

**Table 6**  
**Preference-Based Rankings for a Variety of Specifications**

College Name	Rank Based on		
	Matriculation Tournaments with Covariates (Table 3)	Highest Listing in Preference Ordering (All planned applications)	Rank Ordered Logit on Preference Ordering (All planned applications)
George Washington University	77	89	77
Fordham University	78	113	85
Dickinson College	79	63	89
Sarah Lawrence College	80	34	66
Catholic University of America	81	111	109
Bucknell University	82	88	87
U. of Colorado at Boulder	83	101	88
U. of Wisconsin at Madison	84	71	69
Arizona State University	85	91	98
Wheaton College	86	48	61
Trinity College	87	72	68
Rose-Hulman Inst. of Tech.	88	54	73
UC: Santa Cruz	89	104	111
Boston University	90	76	83
UC: San Diego	91	81	86
Tulane University	92	93	80
University of Richmond	93	60	65
Case Western Reserve	94	95	81
Colorado College	95	68	57
Indiana U. Bloomington	96	98	101
Penn State University Park	97	87	96
American University	98	100	99
Hamilton College	99	97	72
University of Washington	100	80	95
University of Rochester	101	67	92
Michigan State University	102	107	114
Lewis & Clark College	103	109	91
Clark University	104	110	103
Skidmore College	105	77	78
Purdue University	106	66	106
Colorado State University	107	103	100
Syracuse University	108	105	97
University of Vermont	109	92	104
Scripps College	110	38	52
Corr: column (1) & this column		0.82	0.83
Corr: column (3) & this column	0.82		0.88

## VII. Self-Selection and Rankings for Subgroups of Students

We estimate the  $\theta_i$  from matriculation decisions of admitted applicants. Of course, to be admitted, one must first apply, and students self-select into applying to various colleges. At first glance, such self-selection might appear to pose a pervasive problem, but we think that it poses a narrow problem in practice. To see this, suppose that students apply to all colleges that are in the desirability rank-order "neighborhood" of the college they think they are likely to attend. This is no problem for us: the student's matriculation tournament includes all relevant colleges. Adding additional irrelevant colleges would generate no additional variation.

Now suppose that student evaluates colleges in the "neighborhood" with his personal taste, and applies to all of the colleges therein that suit his taste. Then every matriculation tournament is a competition among colleges based on  $\theta_i + \alpha_j$  where  $\alpha_j$  is student  $j$ 's taste parameter. So long as every school's parameter is shifted equivalently in student  $j$ 's matriculation tournament, there is no problem because a matriculation tournament only identifies  $\theta_i$  up to a constant. That is, since our rankings are unique only up to a constant, it is fine to estimate  $\theta_i$  using matriculation tournaments each of which is run on the basis of  $\theta_i + \alpha_j$ .

Self-selection causes a problem when certain schools are much more taste-intensive than others. Any specialty school could fall into this category, with engineering schools, strongly religious schools, and single-sex schools being the most likely. To see this, consider the concrete example of Cal Tech, which we flagged previously. Cal Tech concentrates on engineering and science and has only minimal course offerings outside these areas. Suppose that a student has a taste for engineering. This taste may induce him to apply to schools with engineering programs: Stanford, Princeton, Cornell, and so on. Each of them play in his tournament with an  $\theta_i + \alpha_j$ . However, such schools are much less engineering-focused than Cal Tech, so the decision to apply to them is less intensive in taste for engineering than is the decision to apply to Cal Tech. Thus, Cal Tech is likely to play with a parameter  $\theta_i + \alpha'_j$  where  $\alpha'_j > \alpha_j$ . This is a problem: Cal Tech is likely to win too often relative to how it would fare in tournaments based on  $\theta_i$ . In other words, if a certain school is so obviously specialized that students who are lukewarm about its specialty don't bother to apply, our estimate of its  $\theta_i$  is

likely to be biased upward.

In an ideal world, we could eliminate these selection issues by estimating the average  $\alpha_j$  among students who apply to each college. Unfortunately, this is not feasible without many further assumptions that would drive our estimates in ways that would be hard to discern. It seems more straightforward to alert readers to this potential bias and provide rankings that acknowledge it explicitly.<sup>23</sup>

We deal explicitly with the taste-intensive draw of specialty schools by creating a few rankings for sub-groups of students who are likely to have a taste for certain specialties. We have picked out a few interesting sub-groups to illustrate the technique: students who plan to major in engineering, math, computer science, or the physical sciences; students who plan to major in the humanities; students from each of the nine Census divisions of the United States. It should be kept in mind, throughout this section, that our subgroup rankings are noisy. This is simply because we are cutting the sample. Thus, it is best to ignore small movements in place except among the top ten institutions where sample size and precision remains substantial.

Table 7 shows our basic ranking and rankings for students from each of the groups of intended majors. Overall, there is a fair degree of similarity among the rankings. This reveals that most schools have reasonably equal strength across the various fields of study. There are, however, some notable differences. Cooper Union for the Advancement of Science and Harvey Mudd, schools that specialize in the hard sciences, are ranked more highly among students who intend to major in the hard sciences. Colgate also moves up considerably. In the ranking based on the choices of student who intend to major in the humanities, Yale occupies the number one slot, displacing Harvard into the third slot. While most changes of a small number of rank places should be ignored because the rank ordering is imprecise, this one

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<sup>23</sup> Self-selection may also affect inference on the  $\delta$  coefficients. For instance, suppose that price sensitivity is heterogeneous and students who are especially price sensitive seek out colleges that offer them substantial discounts. We might overestimate the effects of prices because the variation in the data comes disproportionately from price-sensitive students. For this reason, we do not give strong interpretations to the coefficients on these characteristics.

should not because the analog of statistical significance suggests that place-trading between Harvard and Yale is meaningful. Also, among students who intend to major in the humanities, liberal arts colleges also make a strong showing: there are approximately seven liberal arts colleges among the top twenty-five in the basic ranking, but there are twelve in the rankings based on humanities students. Schools with well-known "great books" programs, such as Columbia and St. John's, are more likely to be picked by humanities students than by other students.

Table 7		
Revealed Preference-based Rankings for Students with Tastes for Various Fields of Study		
Basic Ranking (matriculation tournaments with covariates, students with all preferred majors) (same as Table 3)	Ranking among students who plan to major in engineering, math, computer science, or the physical sciences	Ranking among students who plan to major in the humanities*
Harvard	Harvard	Yale
California Inst. of Tech.	California Inst. of Tech.	Stanford
Yale	Yale University	Harvard
MIT	MIT	Princeton
Stanford	Stanford	Brown
Princeton	Princeton	Columbia
Brown	Wellesley College	Notre Dame
Columbia	Williams College	Amherst College
Amherst College	Dartmouth College	University of Pennsylvania
Dartmouth College	Notre Dame	Dartmouth College
Wellesley College	Amherst College	Swarthmore College
University of Pennsylvania	Brown	Georgetown
Notre Dame	Columbia University	Wellesley College
Swarthmore College	Swarthmore College	Pomona College
Cornell	Cornell	Duke
Georgetown	University of Pennsylvania	St. John's College
Rice	Duke	Kalamazoo College
Williams College	Rice	Middlebury College
Duke	Cooper Union for the Advancement of Science	University of the South
University of Virginia	Colgate	Claremont McKenna
Brigham Young	University of Chicago	Rice
Wesleyan	Harvey Mudd	Cornell
Northwestern	Georgia Inst. of Technology	Carleton College
Pomona College	Northwestern	Wesleyan
Georgia Inst. of Technology	University of Virginia	Northwestern

Table 8 shows the rankings we obtain if we estimate the matriculation model separately for students from each of the nine census divisions of the U.S. The nine divisions are: Division 1: Connecticut, Massachusetts, Maine, New Hampshire, Rhode Island, Vermont; Division 2: New Jersey, New York, Pennsylvania; Division 3: Illinois, Indiana, Michigan, Ohio, Wisconsin; Division 4: Kansas, Minnesota, Missouri, Nebraska; Division 5: D.C., Florida, Georgia, Maryland, North Carolina, South Carolina, Virginia; Division 6: Alabama, Kentucky, Tennessee; Division 7: Arkansas, Louisiana, Oklahoma, Texas; Division 8: Arizona, Colorado, Idaho, Montana, New Mexico, Nevada, Utah, Wyoming; Division 9: California, Hawaii, Oregon, Washington.

We make no great claims for these regional rankings because the sample for each region is small. Indeed, some regional samples are so small that we have left spaces where the estimated  $\theta_i$ s suggest that a school ranked in other regions is missing. For instance, in division 6 (Alabama, Tennessee, Kentucky), neither Cal Tech nor Stanford is ranked. Also, owing to the small samples, we merely group schools outside of the top 30 (see note below the table).

Table 8 shows great consistency, among the regions, in the ranking of the top ten institutions. Each region reproduces the national ranking, with only trivial exceptions. Among institutions ranked 11 to 30, there is considerable consistency among the regional rankings: the differences are likely just noise. In short, the ranking is truly national at the top.

Regionalism is more evident in the colleges ranked 31 to 60, which are shown in the notes below Table 8. While much of the variation in the ranking is noise at this point, owing to the small regional samples, it is notable that Southern colleges do better in the South (U. of the South, Clemson, and Rhodes are the most obvious), Midwestern colleges do better in the Midwest (Bradley is the most obvious), and Western colleges do better in the West (Whitman, Santa Clara, Occidental, and Pitzer are the most obvious). In addition, flagship state universities are likely to show up in their region, even if not in distant regions (U Oregon, U Colorado, and U Arizona are the most obvious). However, even for colleges ranked 31 to 60, the overwhelming impression is that the regional rankings are not very regional. The regional

Table 8: An Example of Regional Preference Rankings of Colleges

Ranking among Students From:								
Region 1: CT, MA, ME, NH, RI, VT	Region 2: NJ, NY, PA	Region 3: IL, IN, MI, OH, WI	Region 4: KS, MN, MO, NE	Region 5: DC, FL, GA, MD, NC, SC, VA	Region 6: AL, KY, TN	Region 7: AR, LA, OK, TX	Region 8: AZ, CO, ID, MT, NM, NV, UT, WY	Region 9: CA, HI, OR, WA
1 Harvard	Harvard	Harvard	Harvard	Harvard	Harvard	Harvard	Harvard	Harvard
2 Cal Tech	Cal Tech	Cal Tech	Cal Tech	Cal Tech		Cal Tech	Cal Tech	Cal Tech
3 Yale	Yale	Yale	Yale	Yale	Yale	Yale	Yale	Yale
4 MIT	MIT	MIT	MIT	MIT	MIT	Stanford	Stanford	Stanford
5 Stanford	Princeton	Stanford	Princeton	Stanford		MIT	Princeton	MIT
6 Princeton	Stanford	Princeton	Stanford	Princeton	Princeton	Princeton	Brigham Young	Princeton
7 Brown	Brown	Brown	Brown	Brown	Brown	Brown	Brown	Brown
8 Columbia	Columbia	Columbia	Amherst	Columbia	Columbia	Columbia	Columbia	Columbia
9 Dartmouth	Dartmouth	Amherst	Dartmouth	Dartmouth	Dartmouth	Dartmouth	Dartmouth	Dartmouth
10 Amherst	Amherst	Dartmouth	Notre Dame	Amherst	Wellesley	Amherst	U Penn	Amherst
11 Wellesley	Wellesley	Wellesley	U Penn	Notre Dame	U Penn	Wellesley	Amherst	U Penn
12 Notre Dame	Notre Dame	U Penn	Swarthmore	Wellesley	Amherst	U Penn	Notre Dame	Wellesley
13 U Penn	U Penn	Notre Dame	Williams	U Penn	Duke	Notre Dame	Williams	Notre Dame
14 Swarthmore	Cooper Union	Swarthmore	Cornell	Swarthmore	Swarthmore	Cornell	Swarthmore	Cornell
15 Rice	Swarthmore	Cornell	Duke	Cornell	Cornell	Rice	Cornell	Swarthmore
16 Cornell	Cornell	Duke	Georgetown	Duke	Georgia Tech	Duke	Duke	Georgetown
17 Georgetown	Georgetown	Rice	U Virginia	Georgetown	Williams	Williams	Rice	Duke
18 Duke	Rice	Williams	Rice	Rice	Georgetown	Georgetown	U Virginia	Rice
19 Williams	Duke	Georgetown	Wesleyan	Williams	Rice	U Virginia	Georgetown	Cooper Union
20 U Virginia	Williams	U Virginia	USC	Harvey Mudd	U Virginia	Wesleyan	Wesleyan	Williams
21 Wesleyan	U Virginia	Wesleyan	Northwestern	U Virginia	Wesleyan	Northwestern	Pomona	U Virginia
22 Harvey Mudd	Harvey Mudd	Harvey Mudd	U Chicago	Wesleyan	Claremont	Berkeley	Middlebury	Harvey Mudd
23 Northwestern	Wesleyan	Northwestern	Pomona	Northwestern	Northwestern	Georgia Tech	Berkeley	Wesleyan
24 Pomona	Northwestern	Pomona	Georgia Tech	Pomona	Fordham	USC	Northwestern	Pomona
25 U Chicago	Pomona	Middlebury	Johns Hopkins	Georgia Tech	Berkeley	U Chicago	USC	Berkeley
26 Middlebury	U Chicago	Johns Hopkins	U Texas	Berkeley	USC	Johns Hopkins	U Chicago	Northwestern
27 Johns Hopkins	Middlebury	Berkeley	UNC	Middlebury	Pomona	Pomona	Georgia Tech	Johns Hopkins
28 USC	Berkeley	USC	Vanderbilt	U Chicago	U Chicago	Middlebury	UNC	USC
29 Berkeley	Johns Hopkins	U Chicago	Carleton	Johns Hopkins	UNC	U Texas	Johns Hopkins	U Chicago
30 Georgia Tech	Georgia Tech	U Texas	Oberlin	USC	Vanderbilt	UNC	Oberlin	Middlebury

### Notes for Table 6

Next 30 colleges, for each region:

Region 1 (CT, MA, ME, NH, RI, VT): UNC, Oberlin, Vanderbilt, U Florida, Barnard, Carleton, Furman, George Mason, Davidson, U Michigan, UCLA, NYU, Tufts, Claremont Mckenna, U Illinois, Vassar, Washington and Lee, Grinnell, Pitzer, Carnegie Mellon, U Maryland, Wake Forest, Kenyon, Bowdoin, William and Mary, Colgate, SMU, Macalester, Boston College.

Region 2 (NJ, NY, PA): USC, U Texas, UNC, Carleton, Barnard, Vanderbilt, Oberlin, Davidson, Washington and Lee, UCLA, NYU, Tufts, U Michigan, U Florida, Furman, Vassar, Grinnell, U Illinois, St. John's, Bowdoin, U Maryland, Kenyon, William and Mary, Carnegie Mellon, Wake Forest, Claremont Mckenna, Smith, Colgate, Boston College, Macalester.

Region 3 (IL, IN, MI, OH, WI): UNC, Claremont Mckenna, Fordham, Carleton, USC, Vanderbilt, Oberlin, Davidson, Barnard, UCLA, U Illinois, SMU, Washington and Lee, Bradley, U Florida, U Michigan, Tufts, Vassar, NYU, Grinnell, U Missouri, Wake Forest, Bowdoin, Carnegie Mellon, Boston College, Illinois Wesleyan, U Oregon, Haverford, Macalester, Smith.

Region 4 (KS, MN, MO, NE): Washington and Lee, Vassar, Davidson, Tufts, Furman, Bowdoin, Colgate, Grinnell, U Michigan, New York, Rhodes, U Illinois, SMU, Haverford, Macalester, Kenyon, Wake Forest, U Missouri, Connecticut College, U Maryland, Carnegie Mellon, Bradley, Sarah Lawrence, Lehigh, Washington U., Bates, Bucknell, College of William and Mary, Boston College, Colby.

Region 5 (DC, FL, GA, MD, NC, SC, VA): UNC, U Texas, U Florida, Fordham, Barnard, Vanderbilt, Carleton, UCLA, Davidson, Oberlin, U Michigan, Tufts, Vassar, U Maryland, Furman, U Illinois, Washington and Lee, NYU, Grinnell, U. of the South, Bowdoin, Kenyon, Carnegie Mellon, William and Mary, Wake Forest, Macalester, Smith, Boston College, U Miami, Colgate.

Region 6 (AL, KY, TN): Furman, Johns Hopkins, Middlebury, UCLA, U Texas, Barnard, Davidson, U the South, Wake Forest, SMU, Carleton, Oberlin, U Michigan, U Illinois, Texas A&M, NYU, Rhodes, Vassar, Occidental, Smith, Clemson, Kenyon, Carnegie Mellon, Bowdoin, William and Mary, Bates, Boston College, U Miami, Washington and Lee, Washington U..

Region 7 (AR, LA, OK, TX): Furman, Oberlin, Carleton, UCLA, Rhodes, Vanderbilt, Barnard, Davidson, Fordham, U Michigan, Washington and Lee, Tufts, NYU, Wake Forest, U Illinois, Bowdoin, Vassar, Carnegie Mellon, Colgate, Smith, U Maryland, SMU, Macalester, Haverford, Washington U., Connecticut College, Emory, Boston College, Mount Holyoke, Bucknell.

Region 8 (AZ, CO, ID, MT, NM, NV, UT, WY): Barnard, Claremont Mckenna, Carleton, Vanderbilt, UCLA, NYU, Wake Forest, Tufts, Macalester, Washington and Lee, U Michigan, Bowdoin, U Oregon, Vassar, Colgate, U Miami, Boston College, Mount Holyoke, Carnegie Mellon, Grinnell, Haverford, William and Mary, Emory, U Missouri, Whitman, U Colorado, Washington U., Santa Clara, U. Arizona, UCSB.

Region 9 (CA, HI, OR, WA): U Texas, SMU, UNC, UCLA, Carleton, Barnard, Oberlin, Davidson, Vanderbilt, NYU, Washington and Lee, Tufts, U Illinois, U Michigan, U Oregon, Pitzer, Vassar, Bowdoin, Carnegie Mellon, Grinnell, Smith, Wake Forest, Macalester, Fordham, St. John's, Claremont Mckenna, William and Mary, Haverford, Emory, Whitman.



favorites never represent more than ten percent of the 30, and most of the colleges that appear show up in every region.

Perhaps the single most interesting college in Table 8 is Brigham Young, which appears in the top 10, between Princeton and Brown, in region 8 (which contains Utah). We have checked and determined that, if we were to compute a Utah-specific ranking, Brigham Young would rank even higher. The dramatic appearance of Brigham Young in the top 10 almost certainly occurs because the college is particularly desirable in the eyes of Mormon students.<sup>24</sup>

In general, what should we think about sub-group rankings? The reason that Brigham Young wins often among Utah students or that the University of the South wins often among Southern students is these colleges are truly more desirable to the students in question. We conclude that specialized tastes are not merely a problem that needs to be "fixed" in the basic ranking. Rather, we are led to the conclusion that, with sufficient data, it would be reasonable to compute a variety of sub-rankings for groups of students with well-defined tastes. We now know that these rankings will tend to join up at the top.

## VI. Conclusions

In this paper, we show how students' college choice behavior can be used to generate revealed preference rankings of American colleges and universities. Using a data set on the college application and matriculation choices of highly meritorious American students, we construct examples of a national revealed preference ranking. We also construct revealed preference rankings by region and students' intended college major. We demonstrate that the our procedure can handle issues like tuition discounts, alumni preferences, and early decision programs. Our procedure also generates a ranking that would be very difficult for a college to manipulate with strategic admissions behavior.

Given the strong demand for measures of revealed preference among parents and students, it is clear that colleges will be forced to provide some such information and college

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<sup>24</sup> This is a conjecture, not a surety, only because we did not ask students about their religion.

guides like *U.S. News* will be forced to give substantial weight to such information. In the absence of a revealed preference ranking method such as ours, colleges and college guides use two flawed, manipulable proxies: the crude admissions rate and crude matriculation rate. These proxies are not only misleading; they induce colleges to engage in distorted conduct that decreases the colleges' *real* selectivity while increasing the colleges' *apparent* desirability, as measured by the proxies. So long as colleges are judged based on the crude admissions and matriculation rates, it is unlikely that all colleges will eliminate strategic admissions or roll back early decision programs, which are key means for manipulating the proxies. Many college administrators correctly perceive that they are in a bad equilibrium. Yet, so long as colleges find it advantageous to use early decision and other costly admissions strategies, the bad equilibrium is likely to persist.

Gathering our data was a moderately costly undertaking for researchers, but the cost would be a trivial share of the revenues associated with college guides. Moreover, much of the data are already compiled by organizations like The College Board and the ACT, so that gathering a nearly universal sample should be feasible. If a revealed preference ranking constructed using our procedure were used in place of manipulable indicators like the crude admissions rate, much of the pressure on colleges to manipulate admissions would be relieved. In addition, students and parents would be informed by significantly more accurate measures of revealed preference. We close by reminding readers that measures of revealed preference are just that: measures of desirability based on students and families making college choices. They do not necessarily correspond to educational quality.

## References

- Avery, Christopher, Andrew Fairbanks, and Richard Zeckhauser. *The Early Admissions Game: Joining the Elite*. Cambridge, MA: Harvard University Press, 2003.
- Avery, Christopher, and Caroline M. Hoxby. *The College Admissions Project: Counselor Report*. Cambridge, MA: The College Admissions Project, 2000.
- Avery, Christopher, and Caroline M. Hoxby. "Do and Should Financial Aid Packages Affect Students' College Choices?" in Caroline M. Hoxby, ed. *College Choice: The Economics of Where to Go, When to Go, and How to Pay for It*. Chicago: University of Chicago Press, 2004.
- David, Herbert. *The Method of Paired Comparisons*. Oxford: Oxford University Press, 1988.
- Ehrenberg, Ronald G., and James W. Monks. "The Impact of US News and World Report College Rankings on Admissions Outcomes and Pricing Decisions at Selective Private Institutions." National Bureau of Economic Research Working Paper Number 7227, 1999.
- Elo, Arpad E. *The Rating of Chessplayers, Past and Present*. London: Batsford, 1978.
- Glickman, Mark E. "Paired Comparison Models with Time Varying Parameters," Doctoral thesis, Harvard University Dept of Statistics, 1993.
- Glickman, Mark E. "Parameter Estimation in Large Dynamic Paired Comparison Experiments." *Applied Statistics*, 48 (1999), pp. 377-394.
- Glickman, Mark E. "Dynamic Paired Comparison Models with Stochastic Variances," *Journal of Applied Statistics*, 28 (2001), pp. 673-689.
- Golden, Daniel. "Glass Floor: How Colleges Reject The Top Applicants—And Boost Their Status—Accepting Only the Students Likely to Enroll Makes A School Look Selective—'They Were Simply Too Good'," *The Wall Street Journal*, 29 May 2001, p. A1.
- Good, Irving J. "On the Marking of Chess Players," *Mathematical Gazette*, 39 (1955), pp. 292-296.
- Long, Bridget T. "Does the Format of a Financial Aid Program Matter? The Effect of State In-Kind Tuition Subsidies," National Bureau of Economic Research Working Paper Number 9720, 2003.

- Luce, R. Duncan. *Individual Choice Behavior*. Wiley: New York, 1959.
- Manski, Charles F., and David A. Wise. *College Choice in America*. Cambridge: Harvard University Press, 1983.
- Mosteller, Frederick. "Remarks on the Method of Paired Comparisons. I. The Least Squares Solution Assuming Equal Standard Deviations and Equal Correlations," *Psychometrika*, 16 (1951), pp. 3-9.
- Peterson's. *Private Secondary Schools 1999-2000*. 20<sup>th</sup> Edition. Princeton, NJ: Peterson's Guides, 1999.
- Spence, Michael. "Education as a Signal," Chapter 2 in *Market Signaling*. Cambridge: Harvard University Press, 1974.
- Spiegelhalter, DJ, A. Thomas, N.G. Best, and W.R. Gilks WR. *BUGS: Bayesian Inference Using Gibbs Sampling*, version 0.6, 1996.
- Stern, Hal. "Are All Linear Paired Comparison Models Empirically Equivalent?" *Mathematical Social Sciences*, 23 (1992), pp. 103-117.
- Thurstone, L.L. "A Law of Comparative Judgment," *Psychological Review*, 34 (1927), pp. 273-286.
- Toor, Rachel. "Pushy Parents and Other Tales of the Admissions Game," *Chronicle of Higher Education*, October 6 2000, p. B18.
- Zermelo, Ernst. "Die Berechnung der Turnier-Ergebnisse als ein Maximumproblem der Wahrscheinlichkeitsrechnung," *Math. Zeit.*, 29 (1929), pp. 436-460.