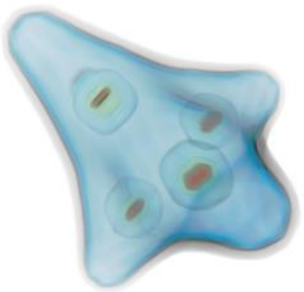
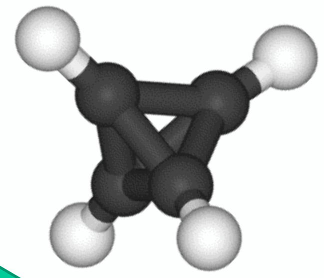
Introduction to topology-based analysis

Attila Gyulassy
SCI Institute, University of Utah

Why do we do topological analysis?

Data



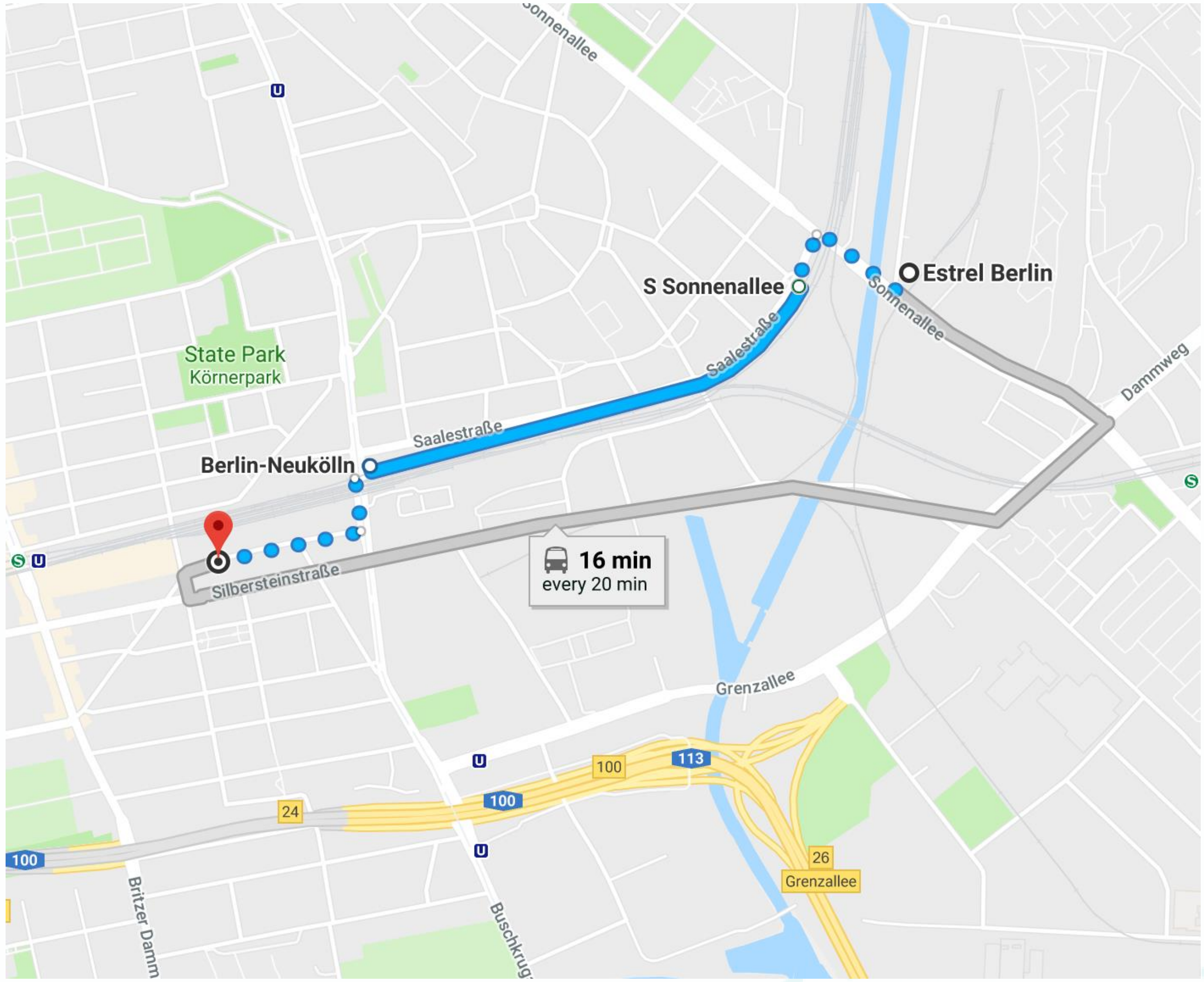


C_4H_4

Tetrahedrane

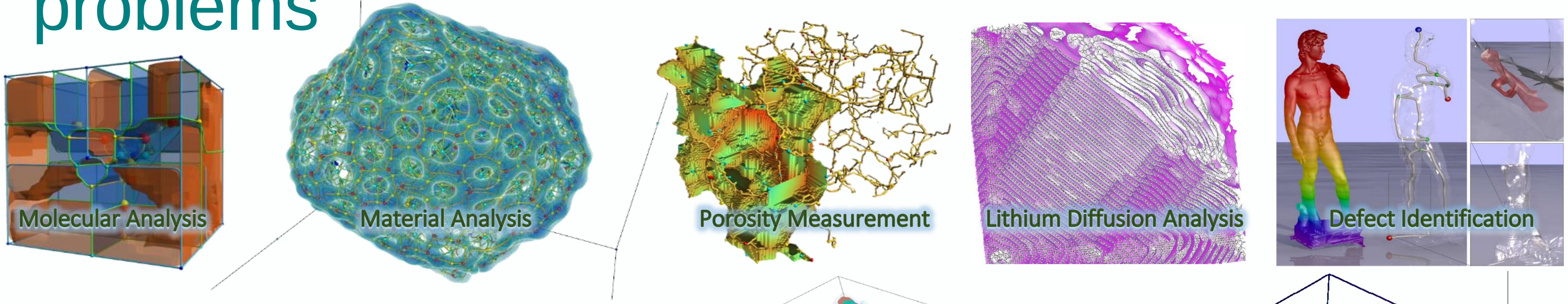
Language

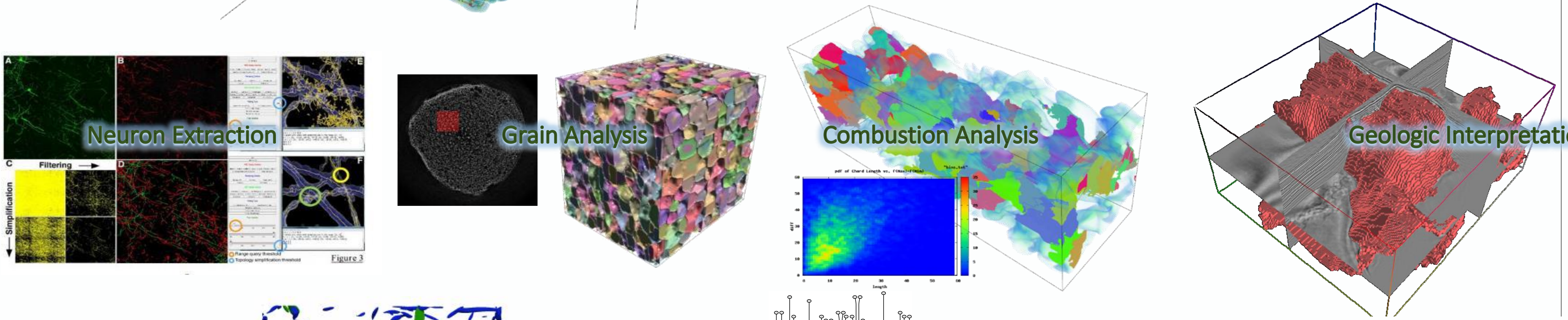
We need the right abstraction for the task

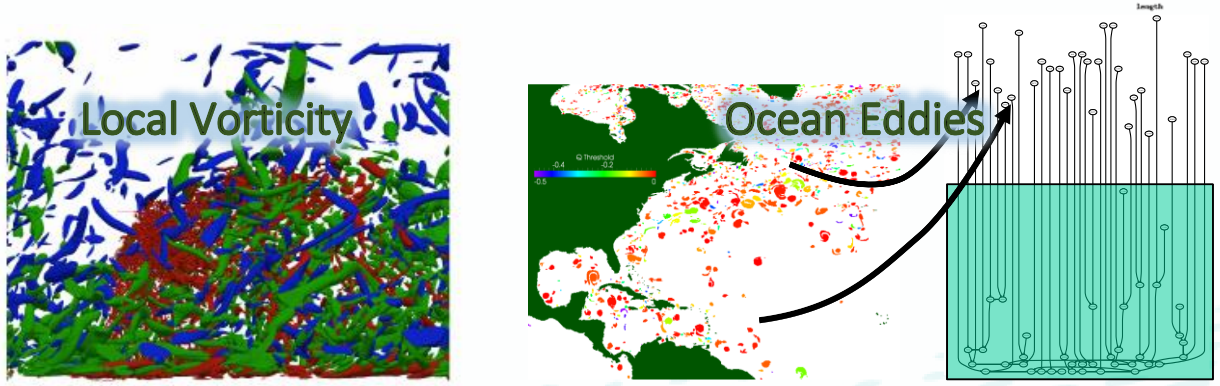


VIS 2018

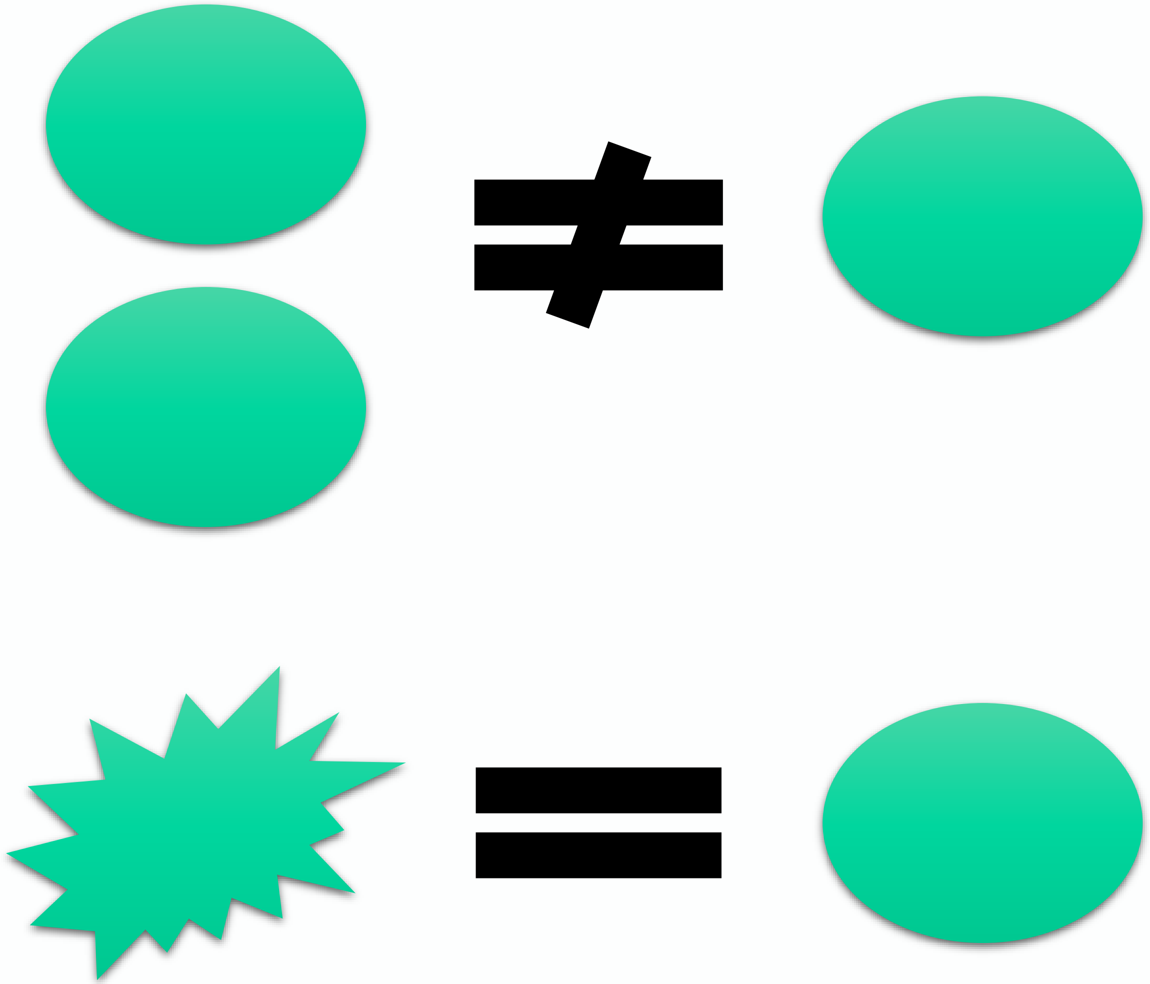
Topological abstractions can solve varied problems





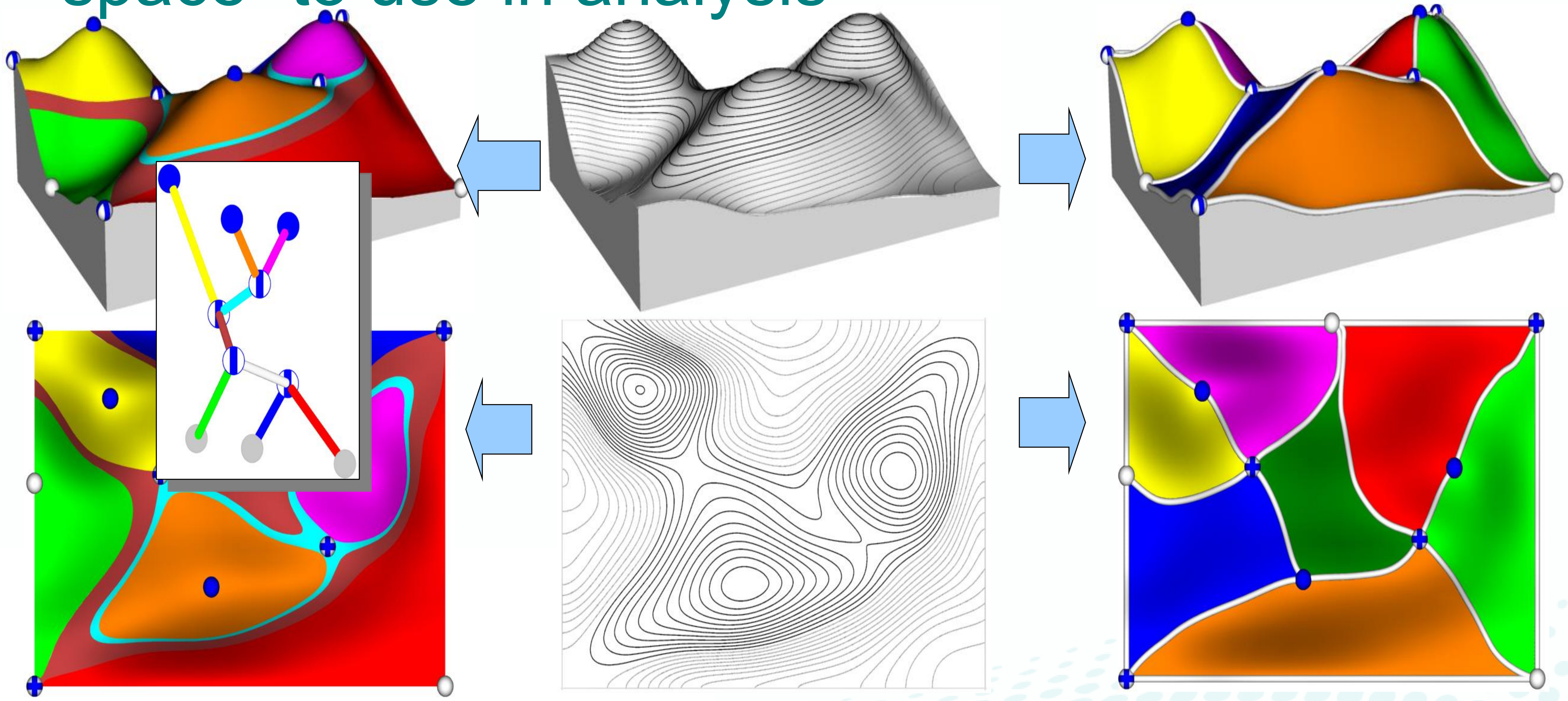


“Topological” analysis is a broad term relating to connectedness, arrangement, and cycles



Art by Henry Segerman and Keenan Crane

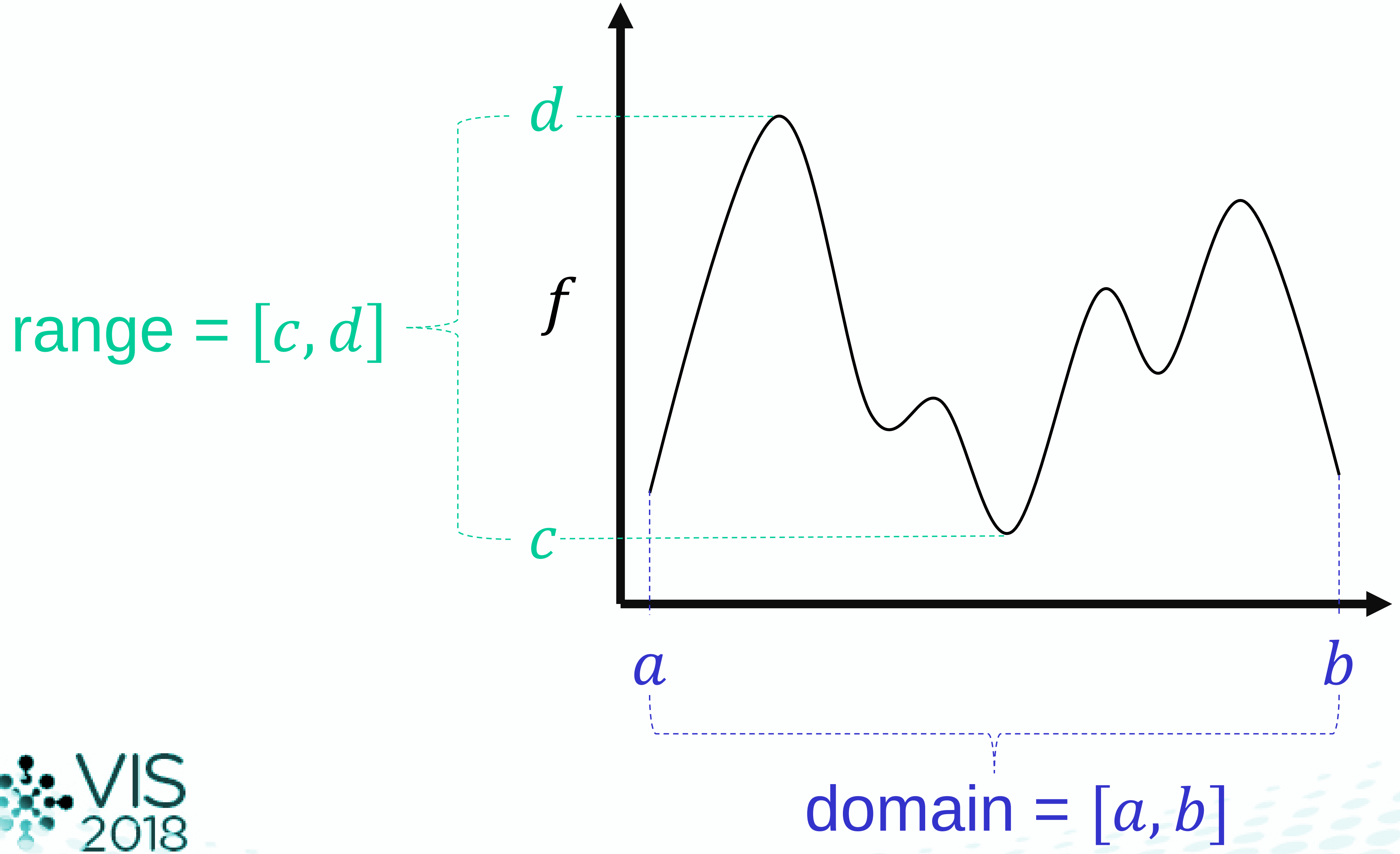
“Topological” approaches provide a “feature space” to use in analysis



Contour Trees/Reeb Graphs/Merge Trees

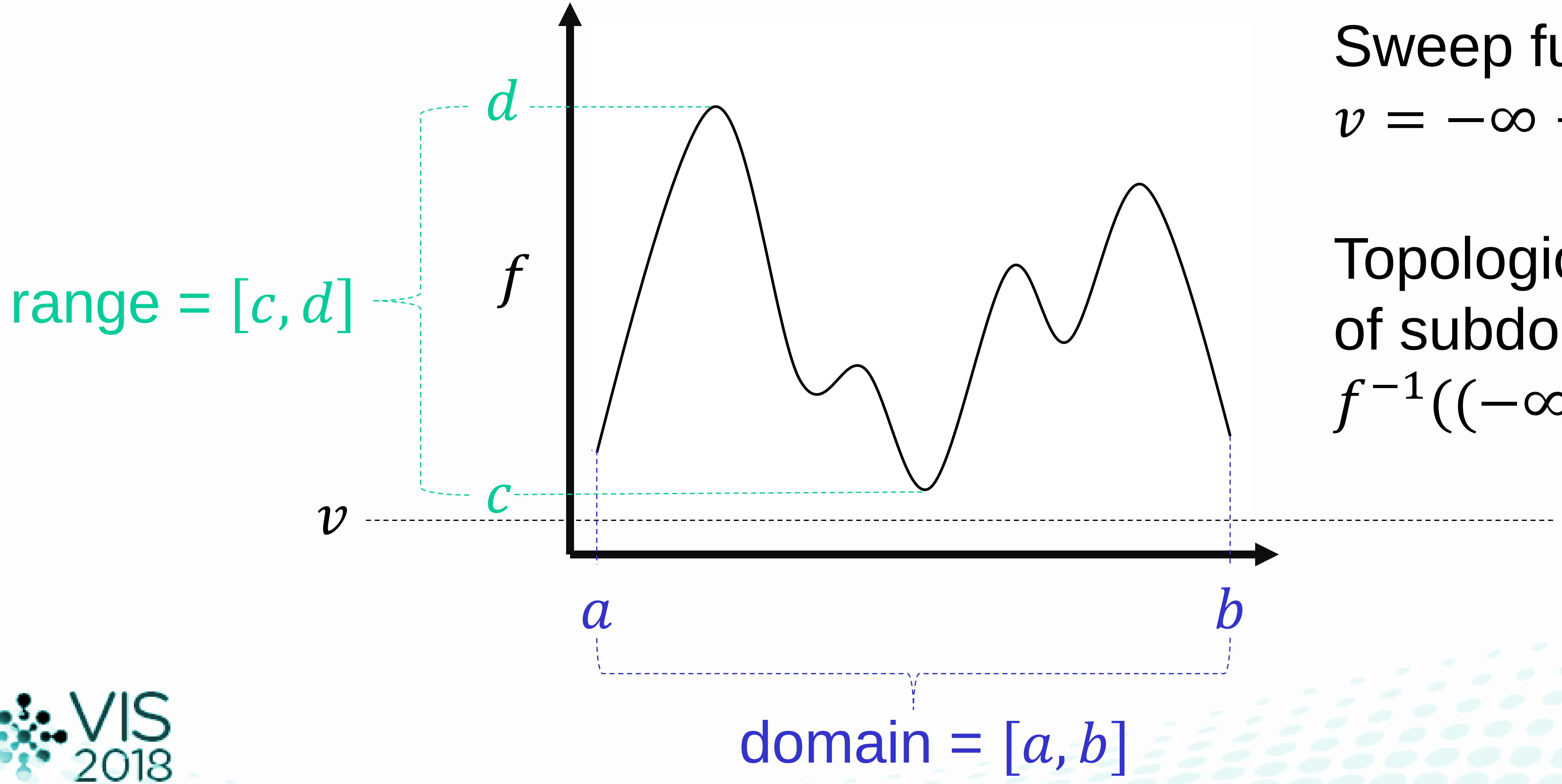
Morse/Morse-Smale Complexes

The “topology” of a scalar function relates to topological changes during a *filtration*



VIS 2018

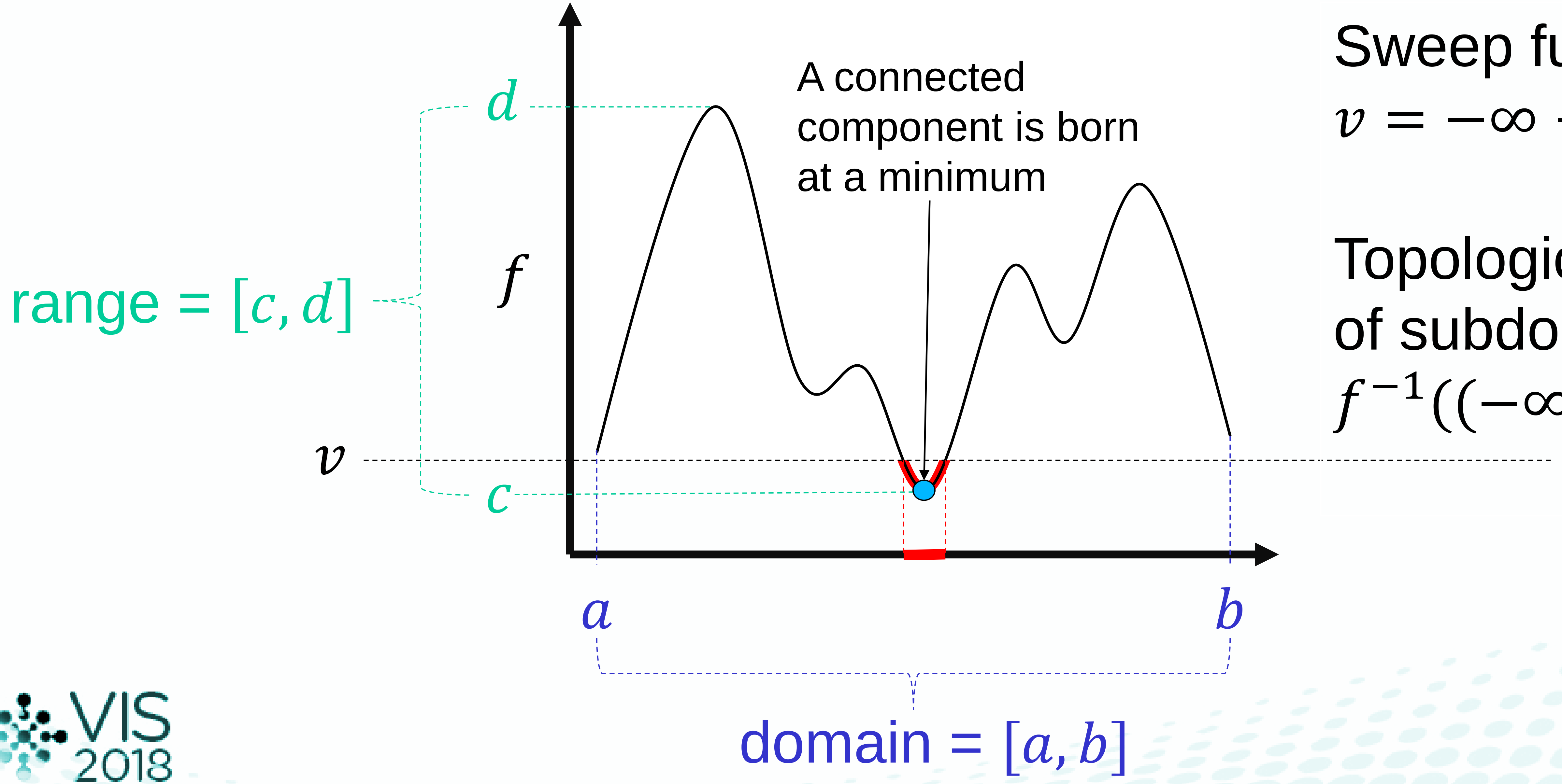
The “topology” of a scalar function relates to topological changes during a *filtration*



Sweep function from $v = -\infty \rightarrow \infty$

Topological changes of subdomain $f^{-1}((-\infty, v])$

The “topology” of a scalar function relates to topological changes during a *filtration*

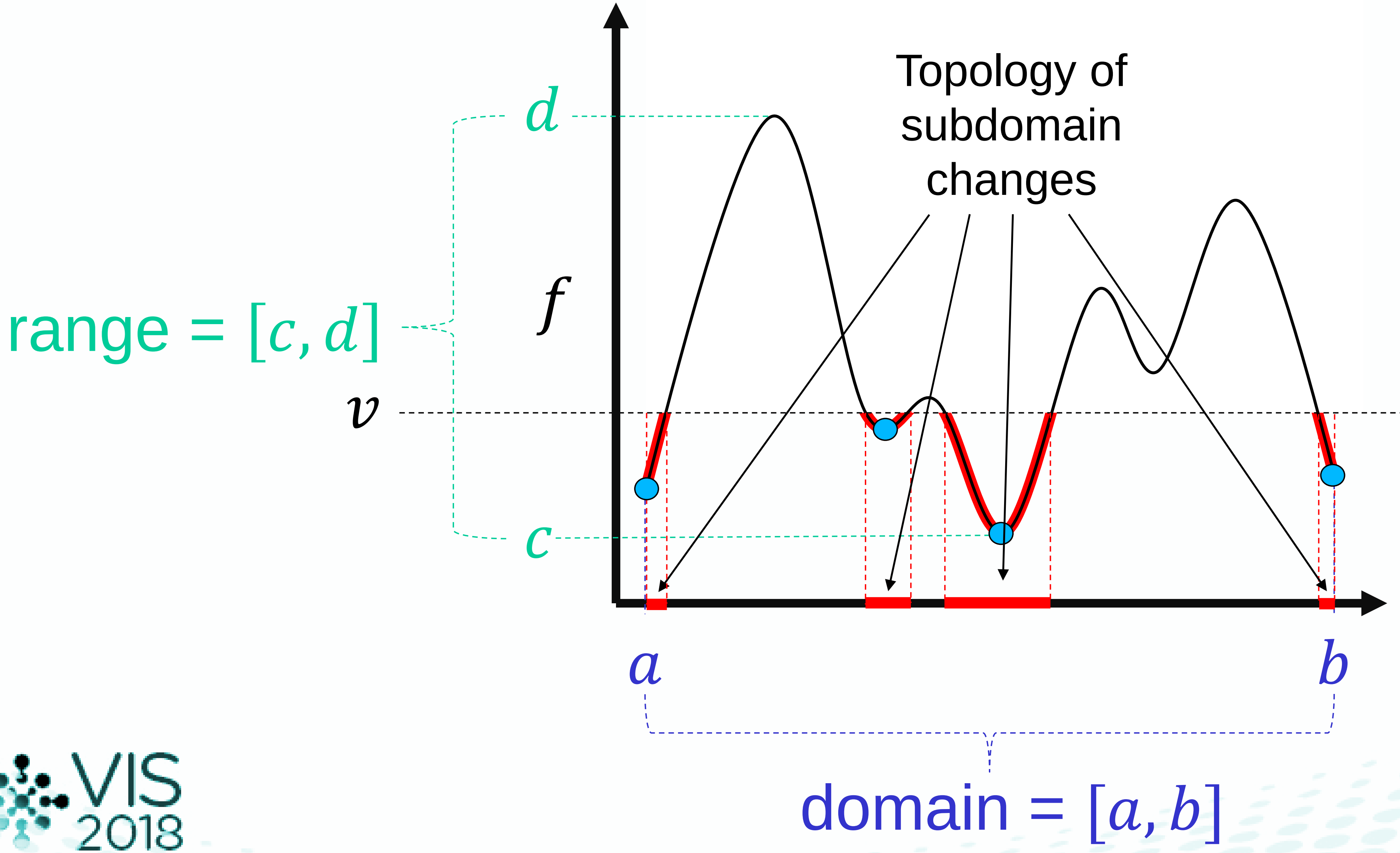


Sweep function from $v = -\infty \to \infty$

Topological changes of subdomain $f^{-1}((-\infty, v])$

VIS 2018

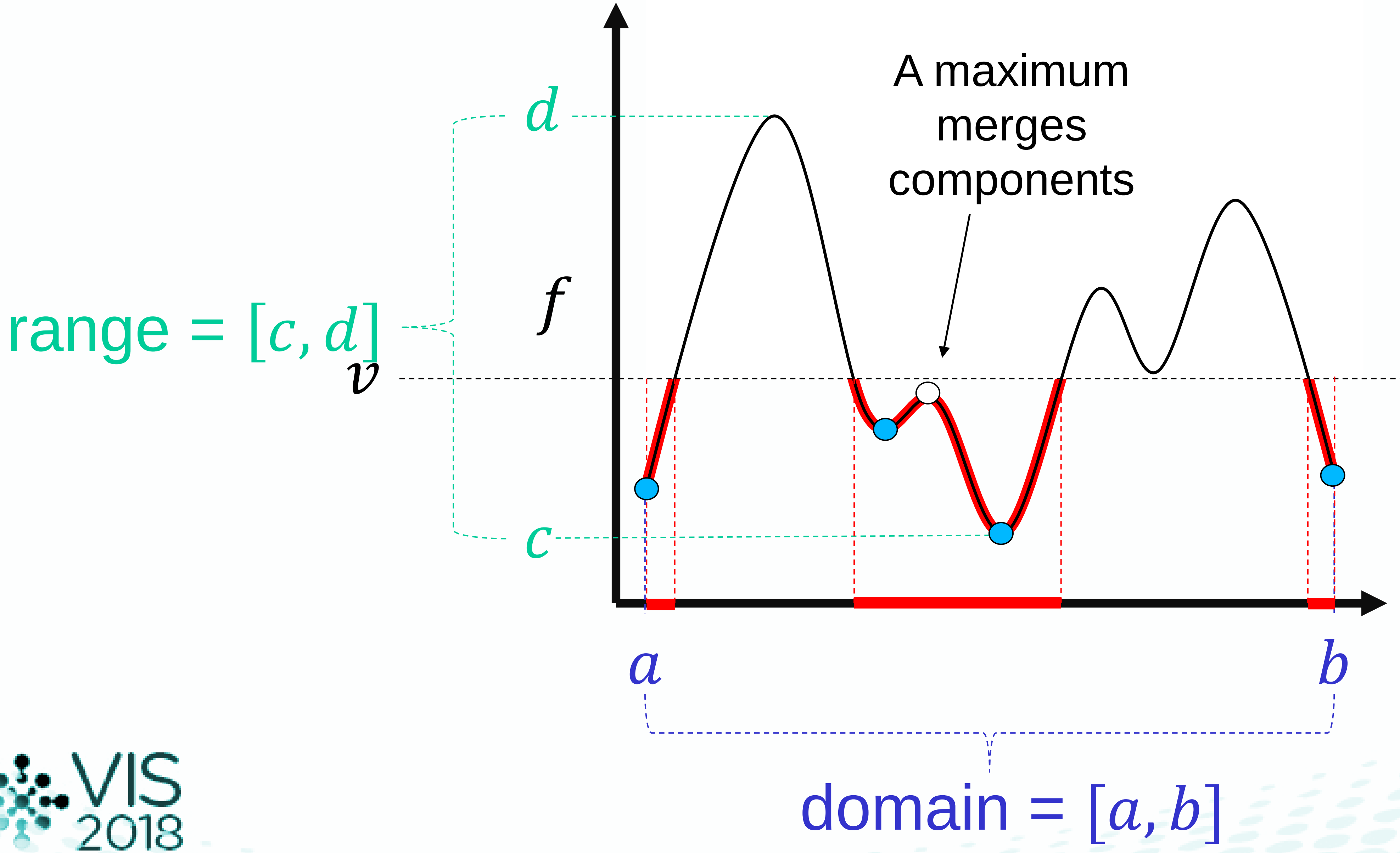
The “topology” of a scalar function relates to topological changes during a *filtration*



Sweep function from $v = -\infty \to \infty$

Topological changes of subdomain $f^{-1}((-\infty, v])$

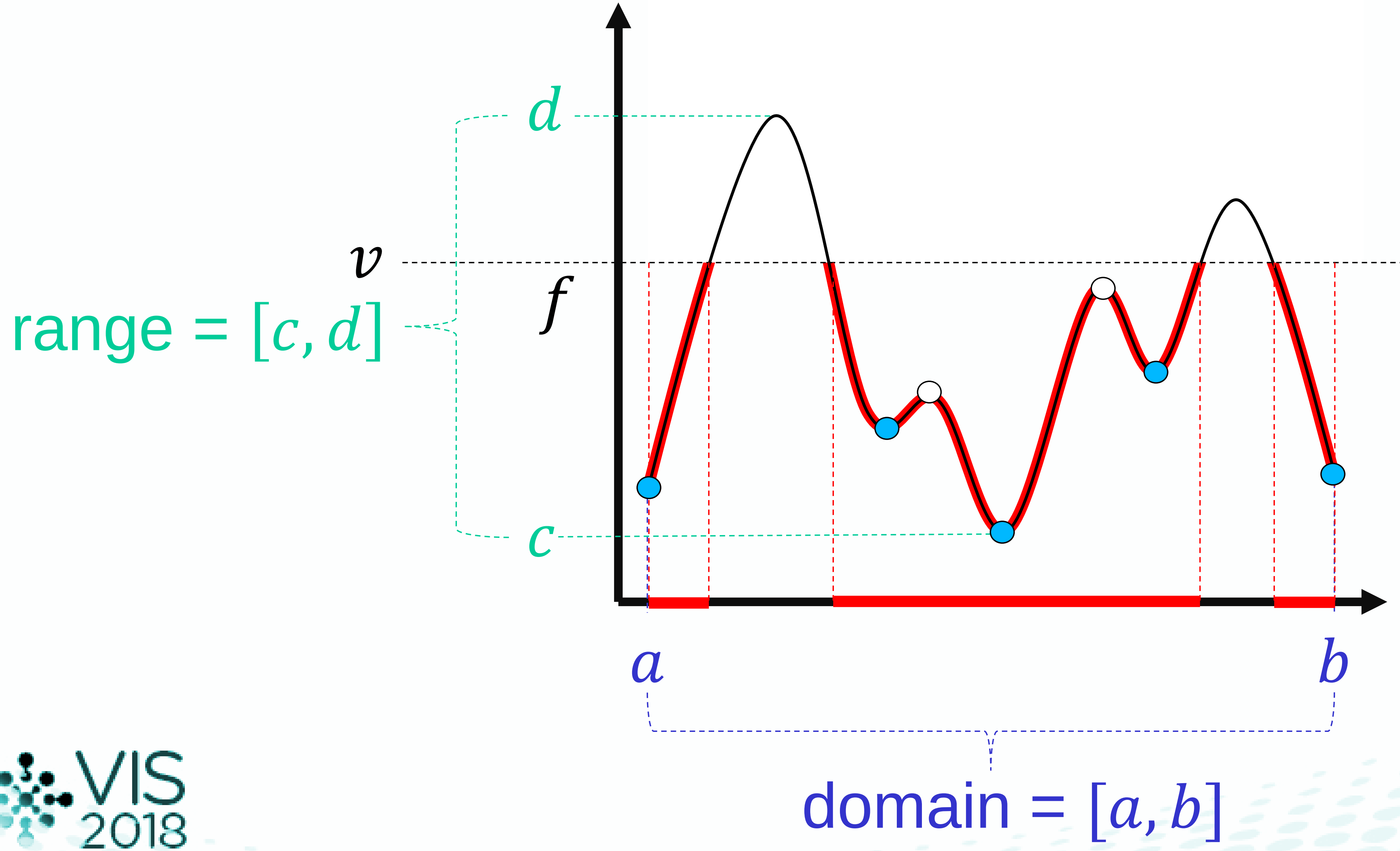
The "topology" of a scalar function relates to topological changes during a *filtration*



Sweep function from $v = -\infty \to \infty$

Topological changes of subdomain $f^{-1}((-\infty, v])$

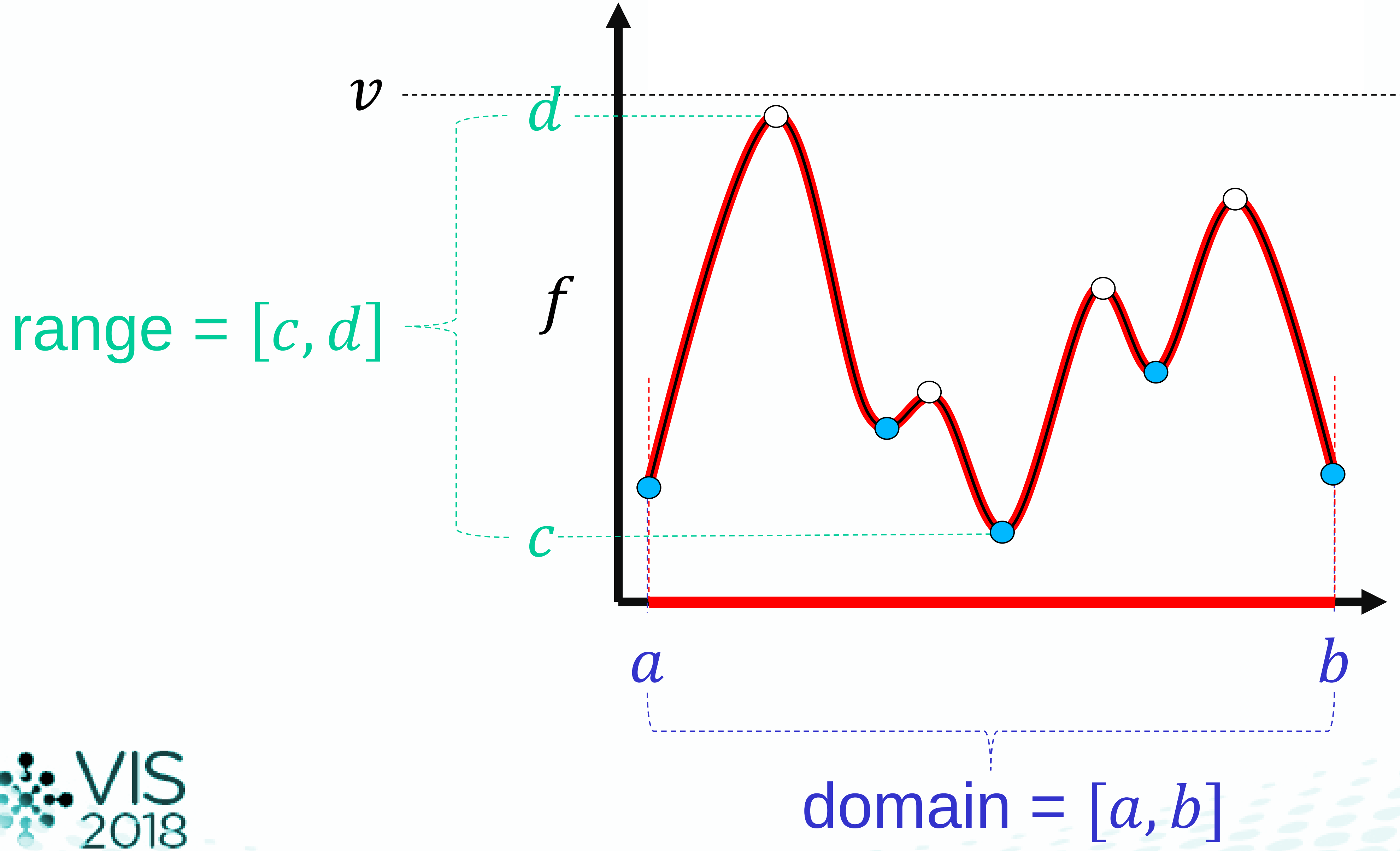
The "topology" of a scalar function relates to topological changes during a *filtration*



Sweep function from $v = -\infty \to \infty$

Topological changes of subdomain $f^{-1}((-\infty, v])$

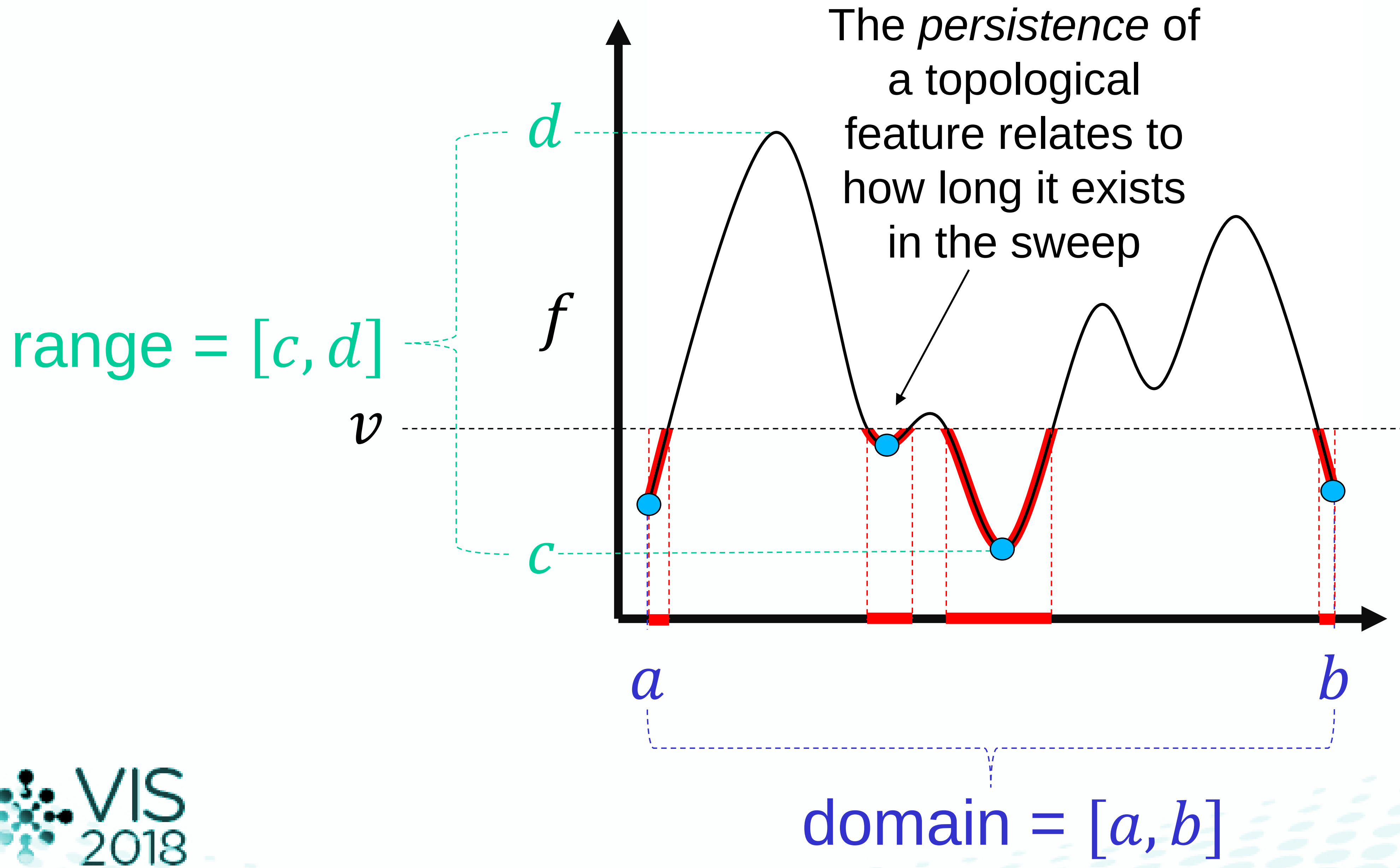
The “topology” of a scalar function relates to topological changes during a *filtration*



Sweep function from $v = -\infty \rightarrow \infty$

Topological changes of subdomain $f^{-1}((-\infty, v])$

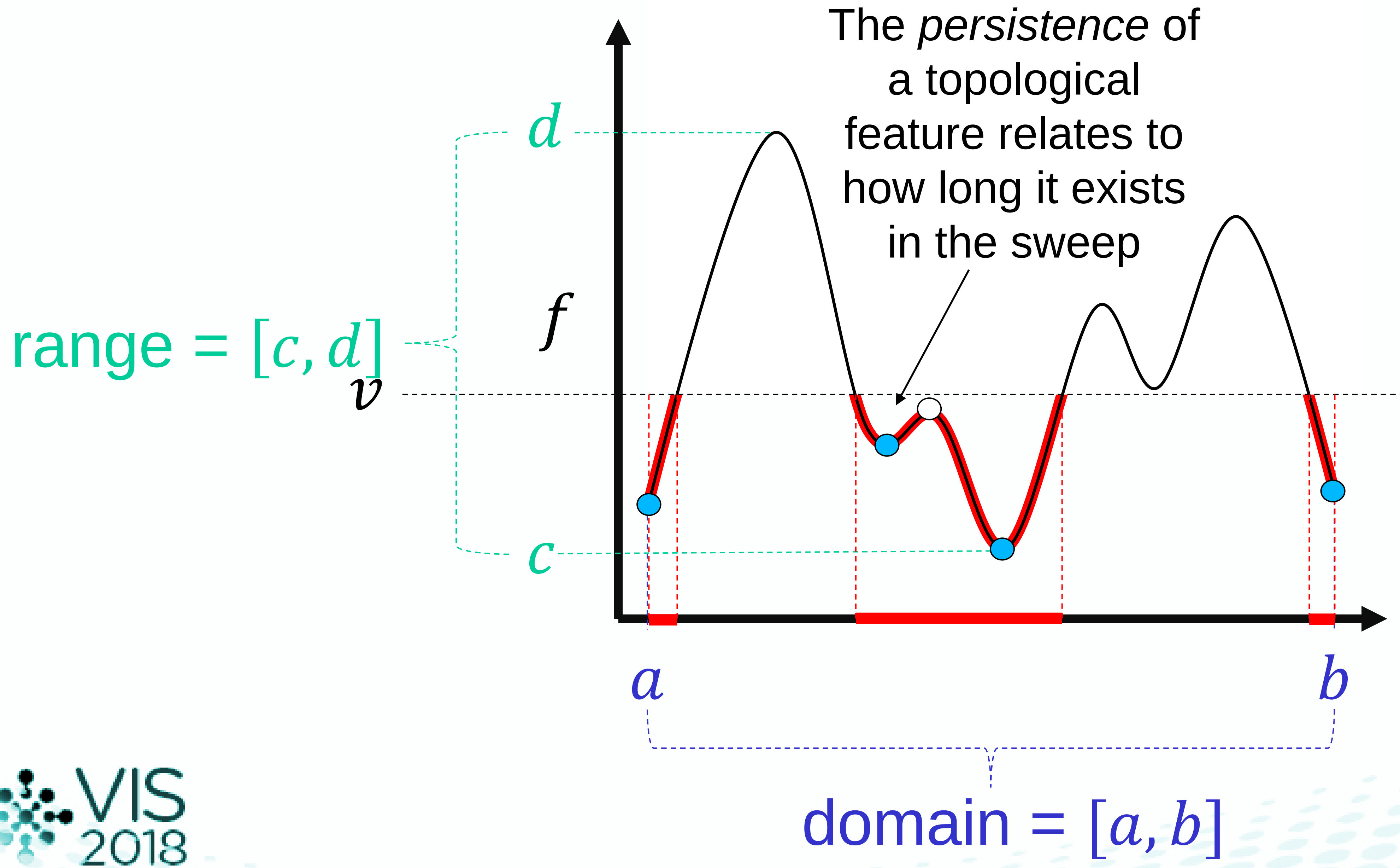
The “topology” of a scalar function relates to topological changes during a *filtration*



Sweep function from $v = -\infty \to \infty$

Topological changes of subdomain $f^{-1}((-\infty, v])$

The “topology” of a scalar function relates to topological changes during a *filtration*

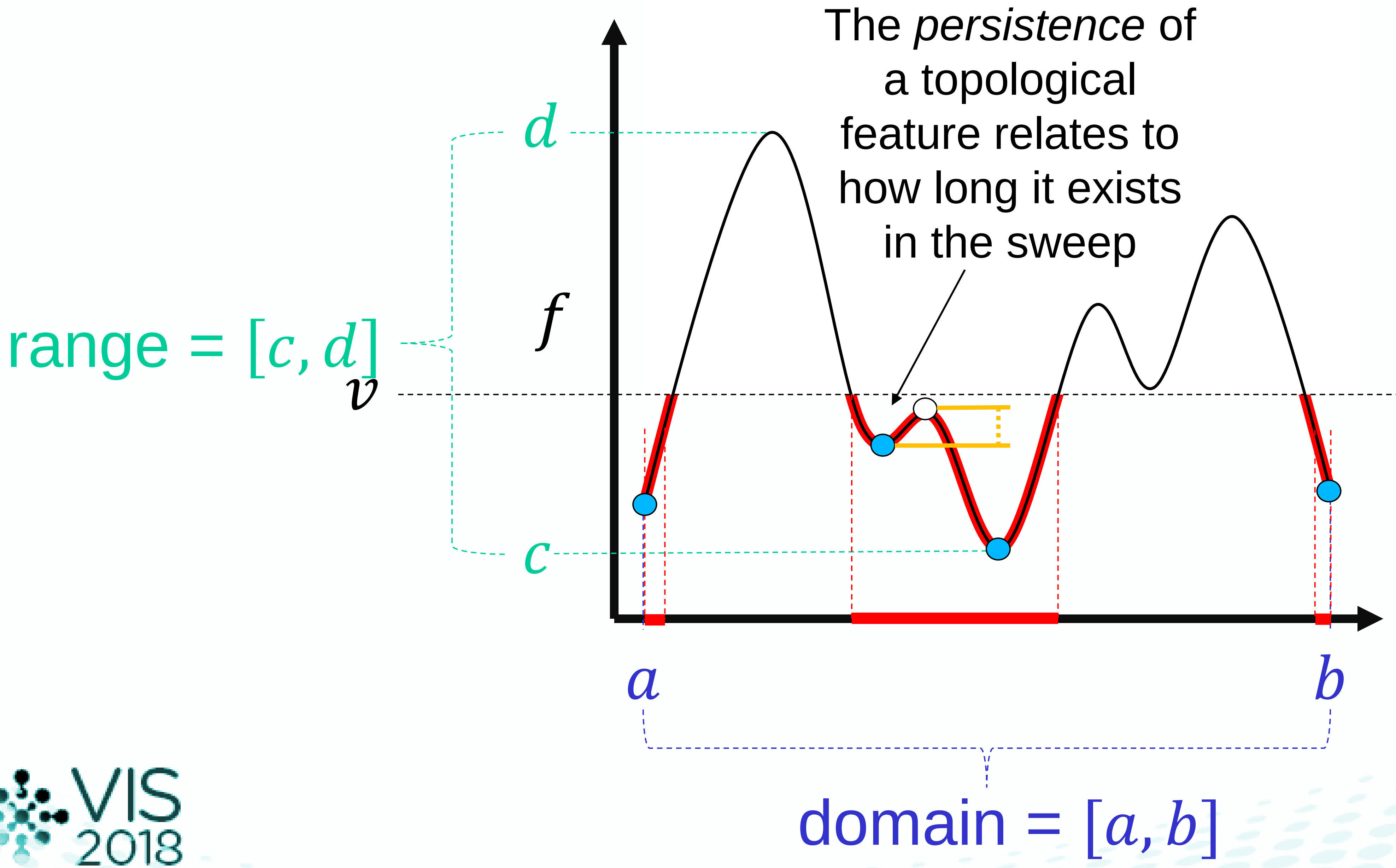


Sweep function from $v = -\infty \to \infty$

Topological changes of subdomain $f^{-1}((-\infty, v])$

VIS 2018

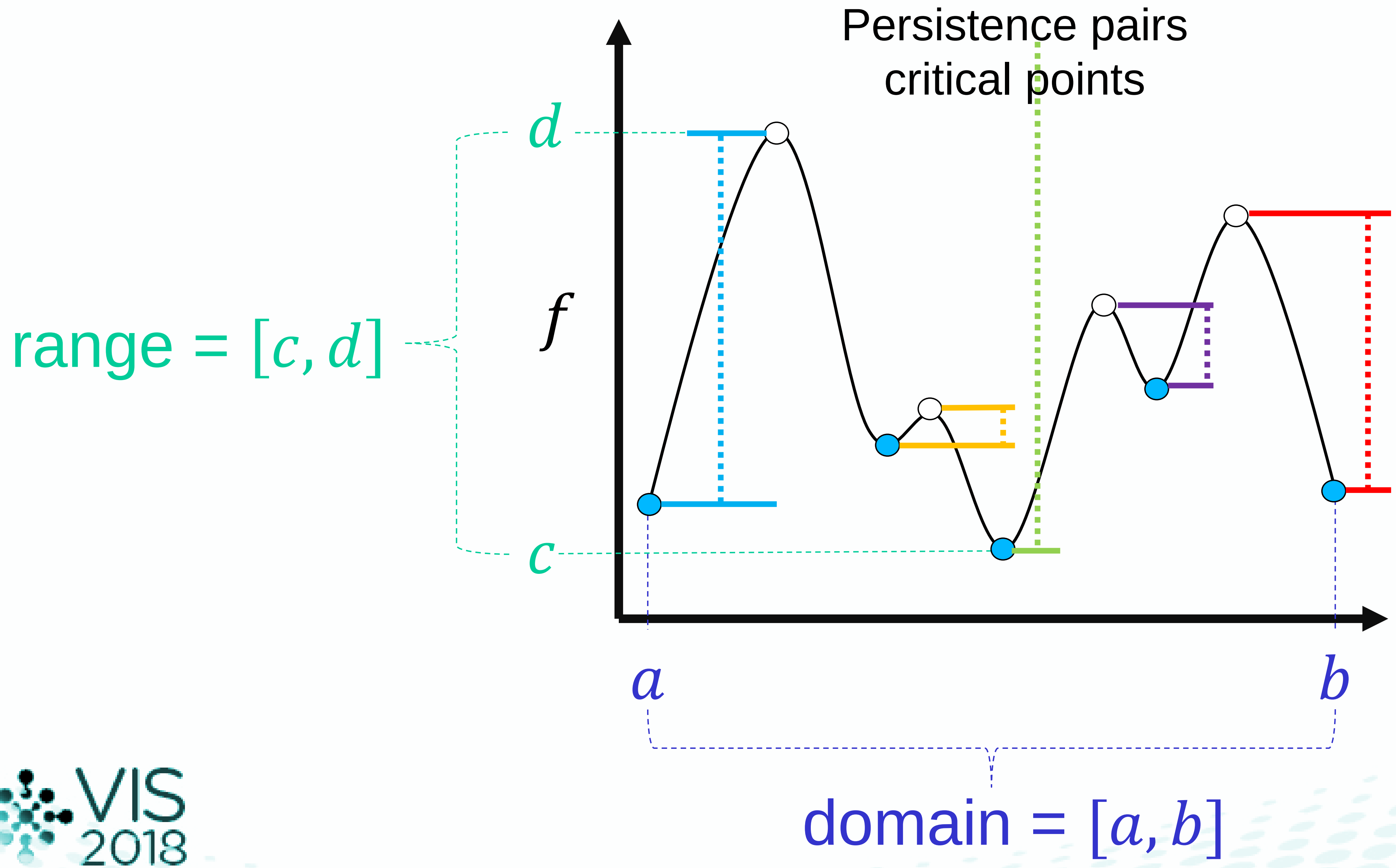
The "topology" of a scalar function relates to topological changes during a *filtration*



Sweep function from $v = -\infty \to \infty$

Topological changes of subdomain $f^{-1}((-\infty, v])$

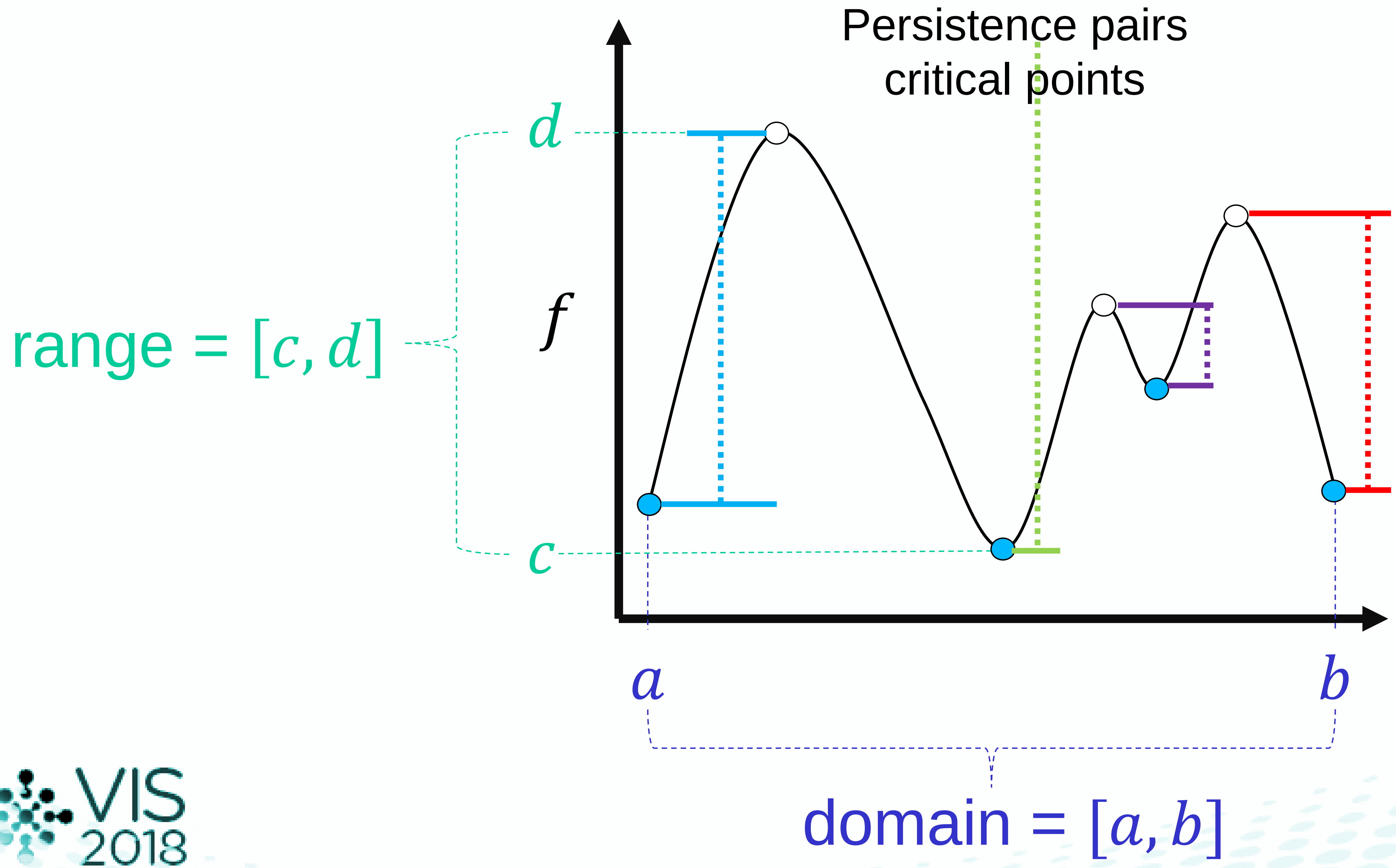
The “topology” of a scalar function relates to topological changes during a *filtration*



Sweep function from
$v = -\infty \rightarrow \infty$

Topological changes of subdomain $f^{-1}((-\infty, v])$

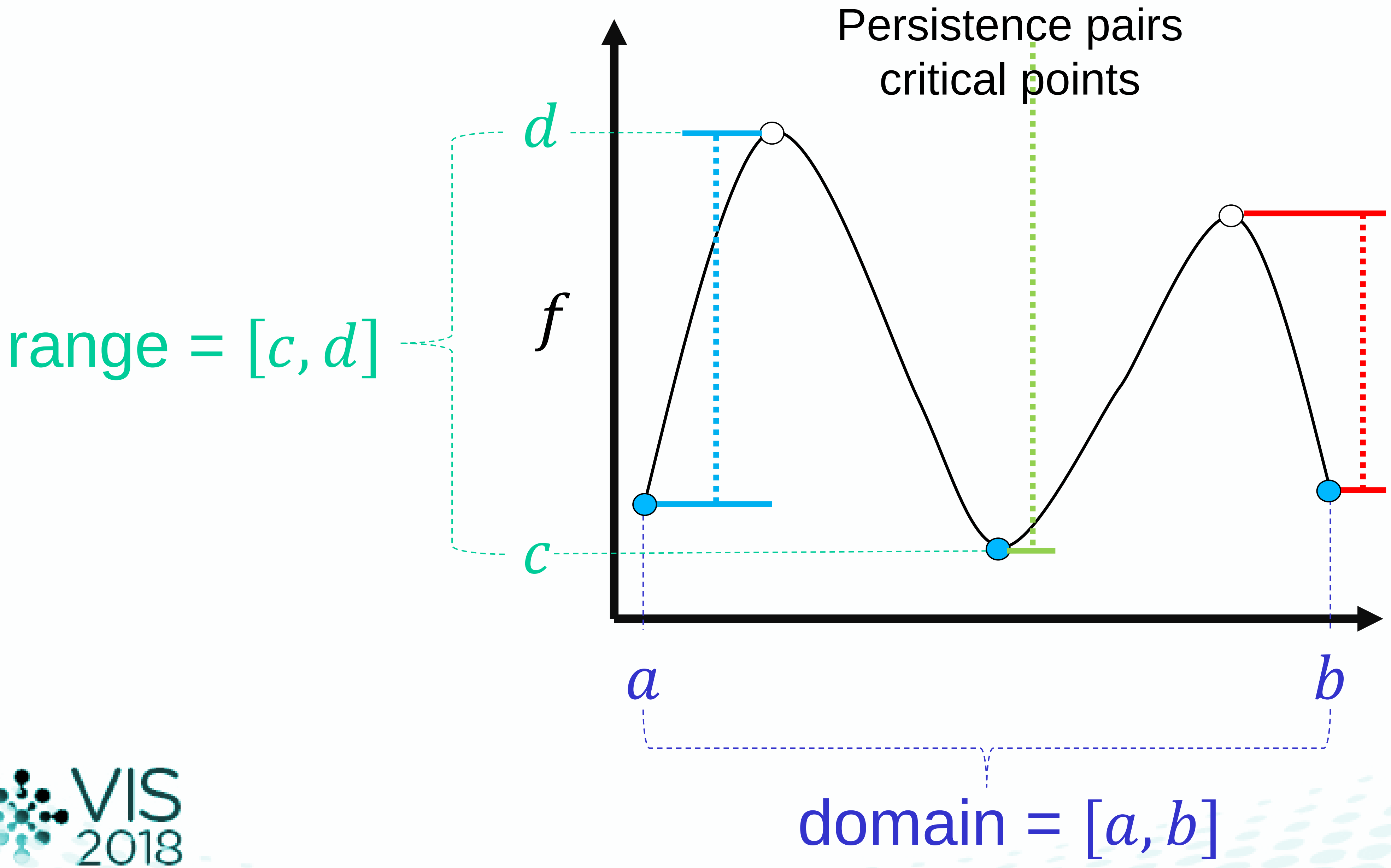
The "topology" of a scalar function relates to topological changes during a *filtration*



Sweep function from
$v = -\infty \to \infty$

Topological changes of subdomain $f^{-1}((-\infty, v])$

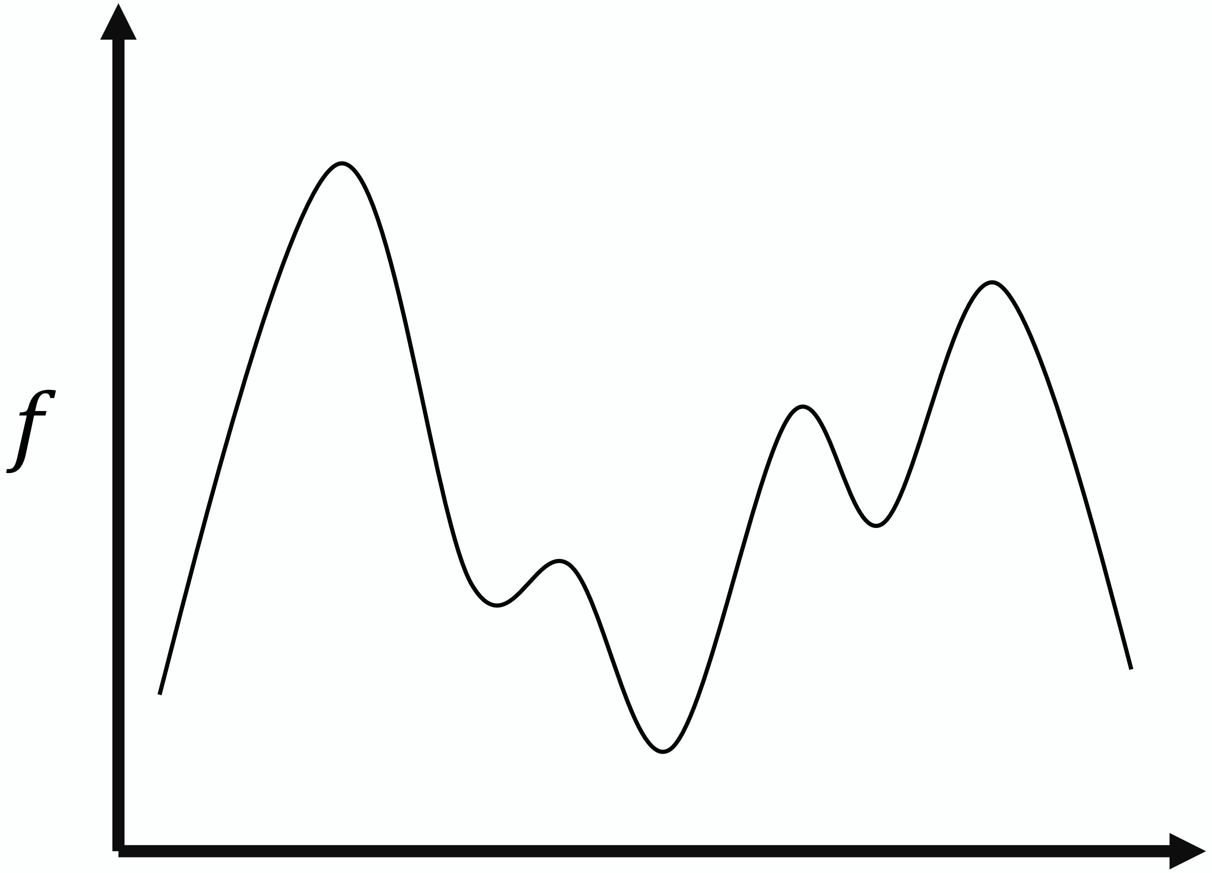
The "topology" of a scalar function relates to topological changes during a *filtration*



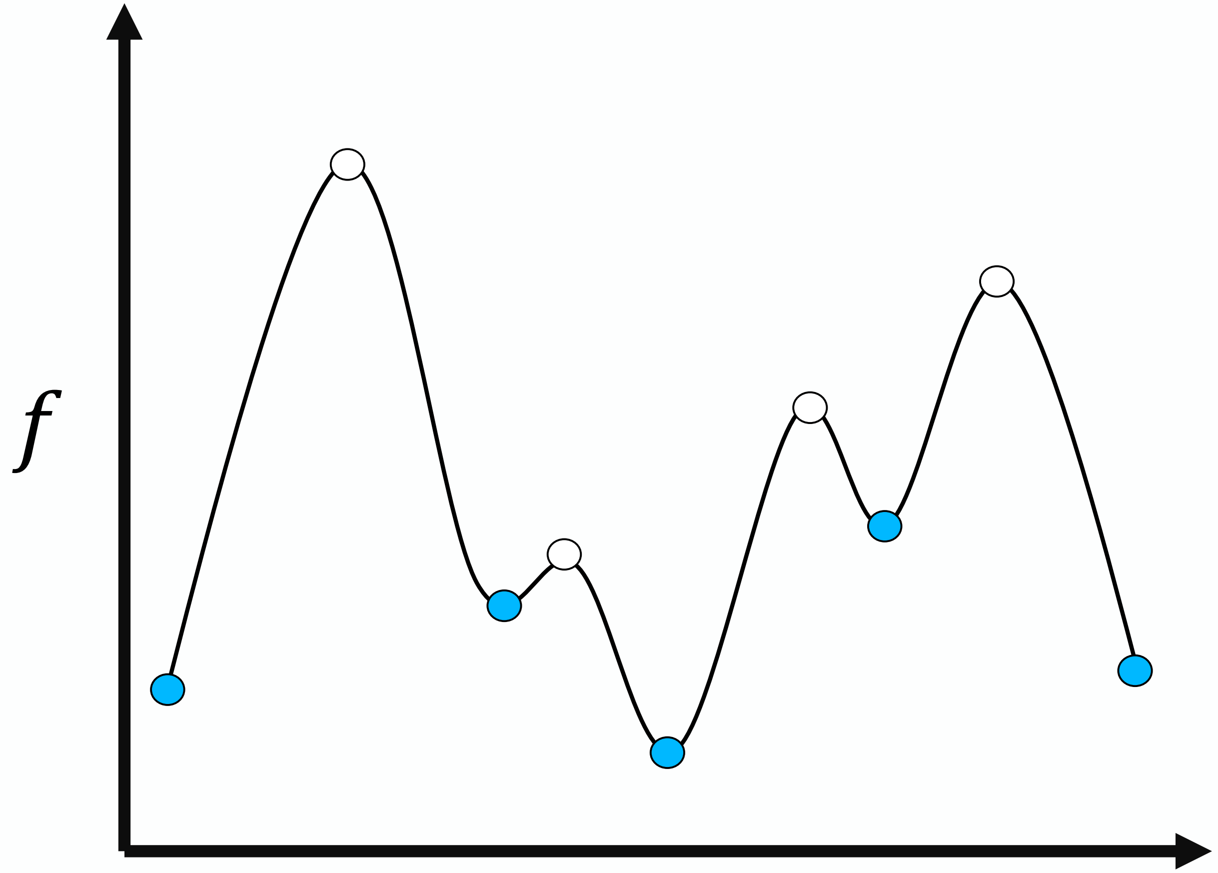
Sweep function from $v = -\infty \rightarrow \infty$

Topological changes of subdomain $f^{-1}((-\infty, v])$

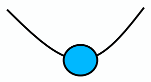
Features of a 1-dimensional function



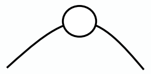
Features of a 1-dimensional function



- Critical points where $\nabla f = 0$



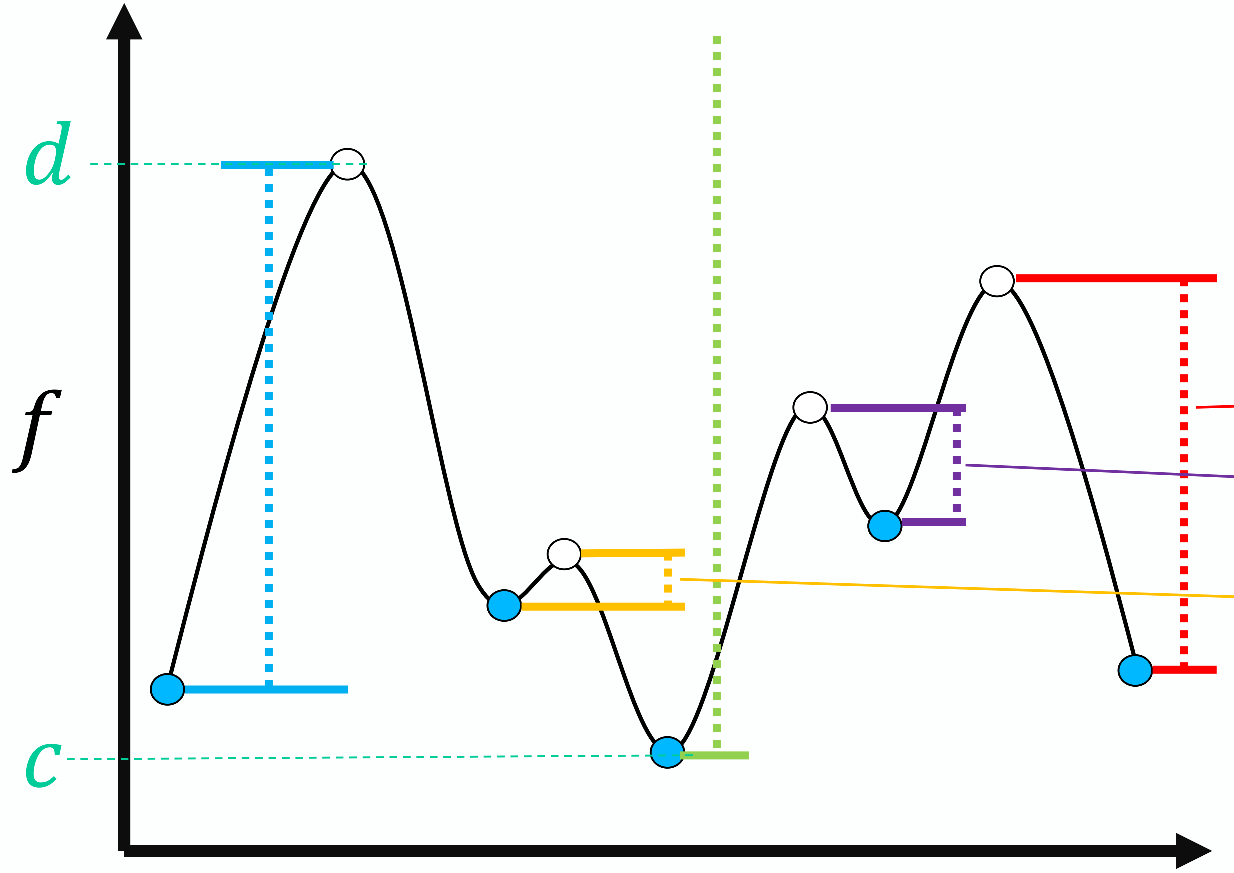
Index-0
minimum

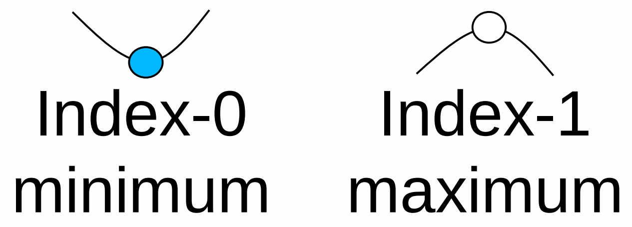


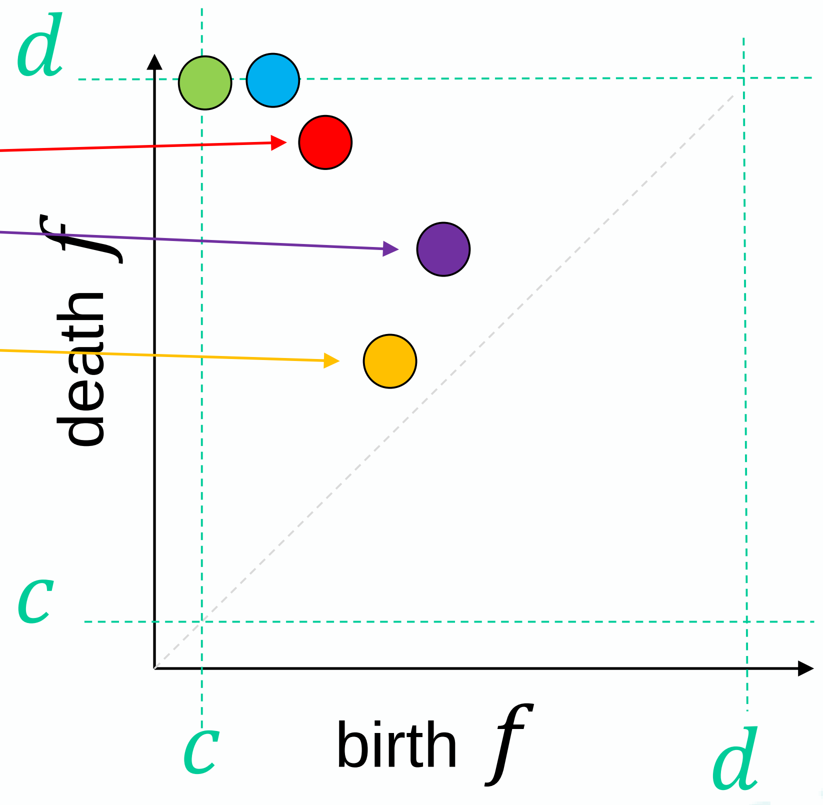
Index-1
maximum

Features of a 1-dimensional function

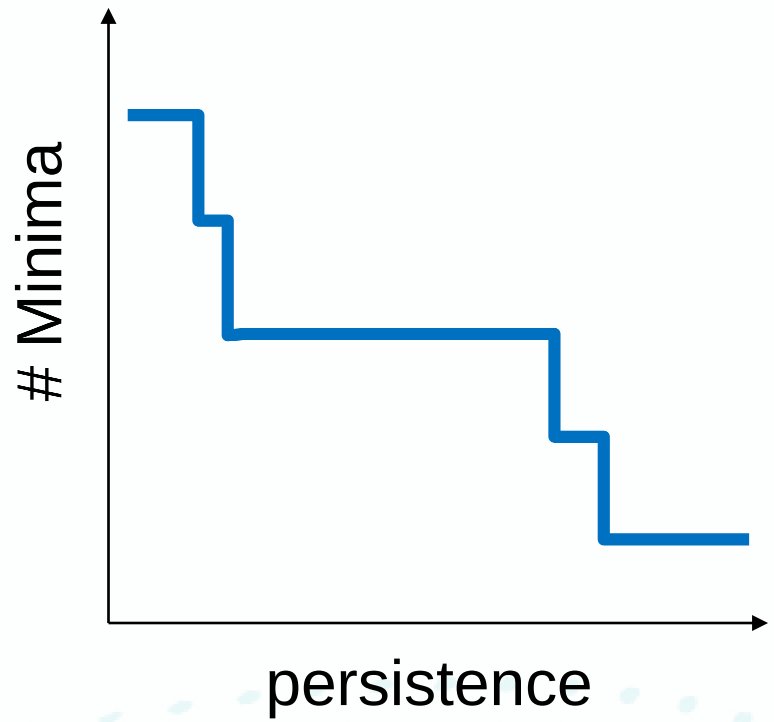
- Critical points where $\nabla f = 0$



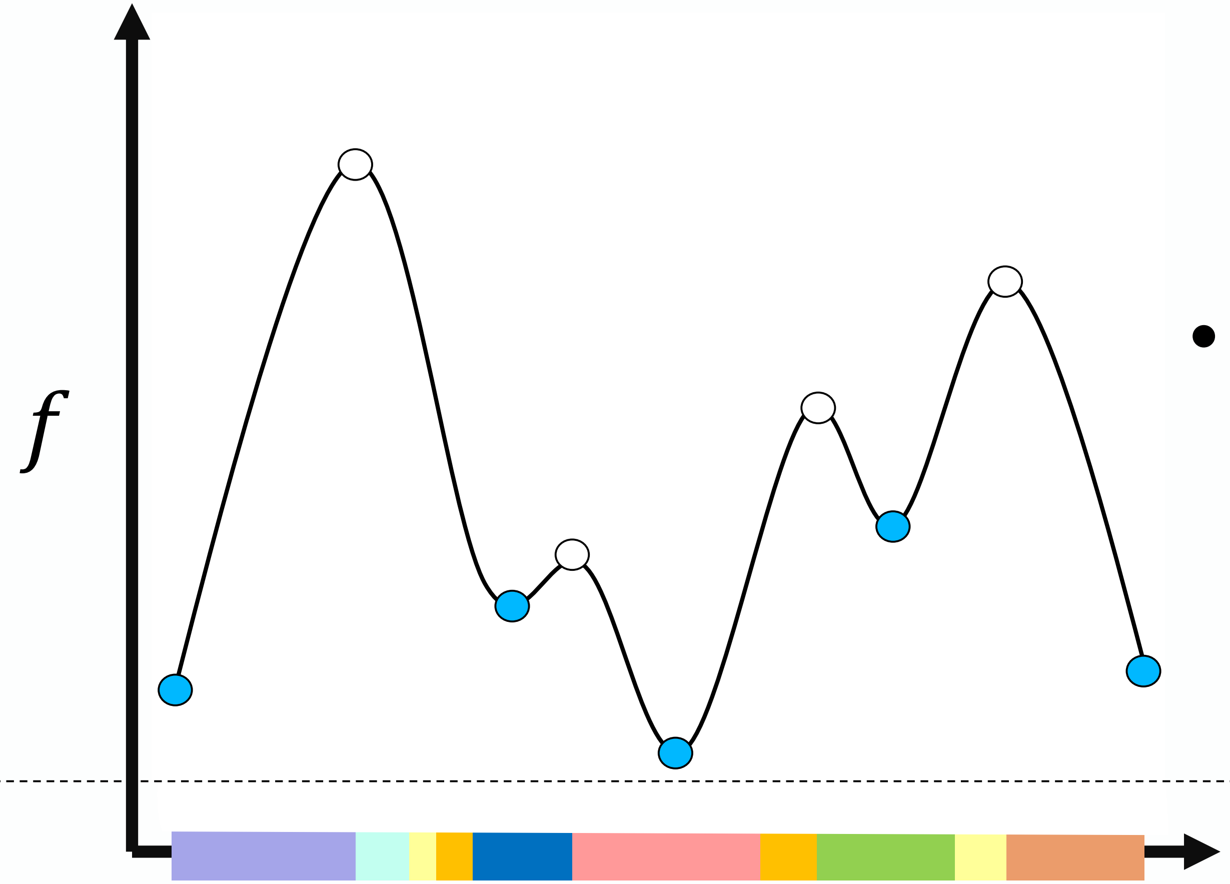




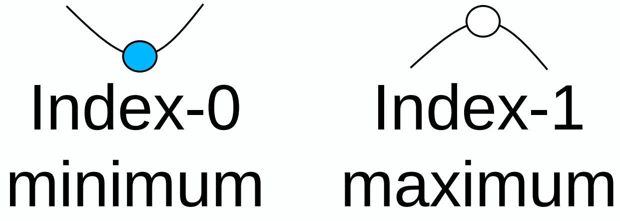
Persistence diagram

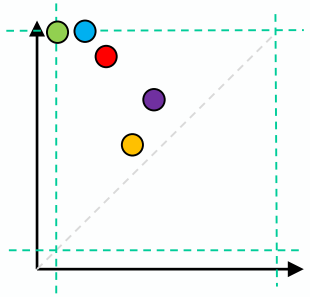


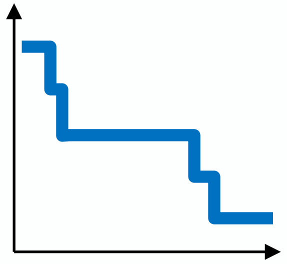
Features of a 1-dimensional function



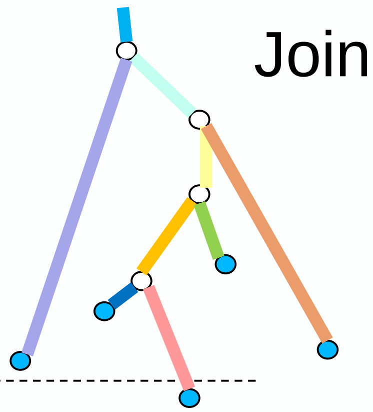
- Critical points where $\nabla f = 0$



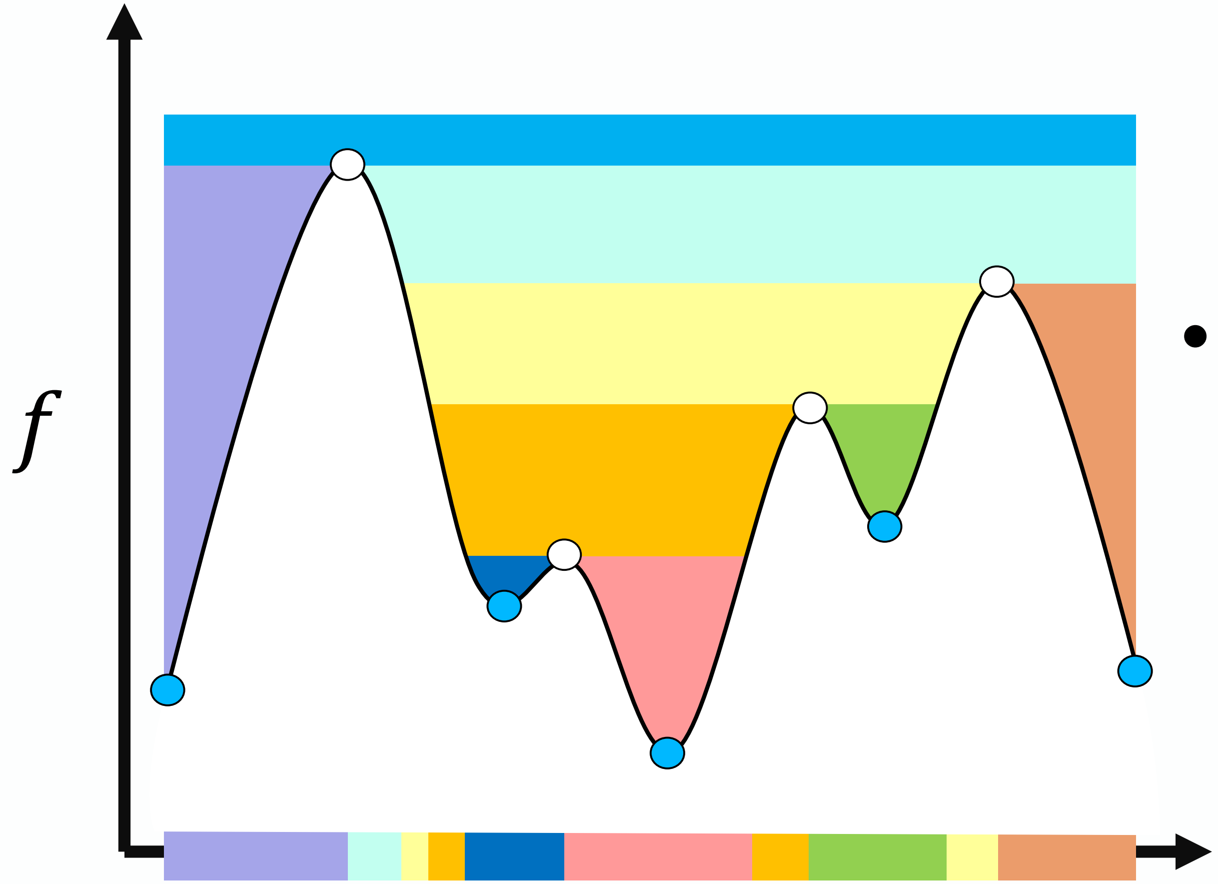




- Components existing during the filtration



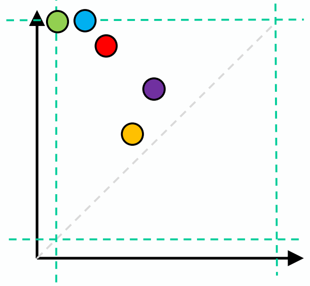
Features of a 1-dimensional function

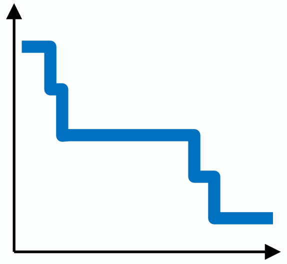


- Critical points where $\nabla f = 0$

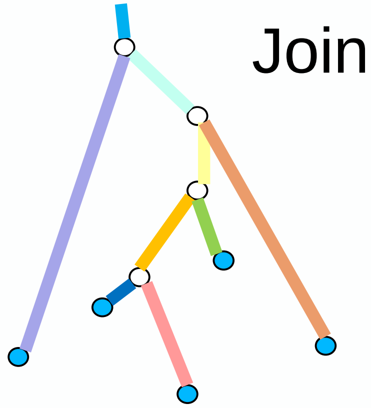
Index-0 minimum

Index-1 maximum





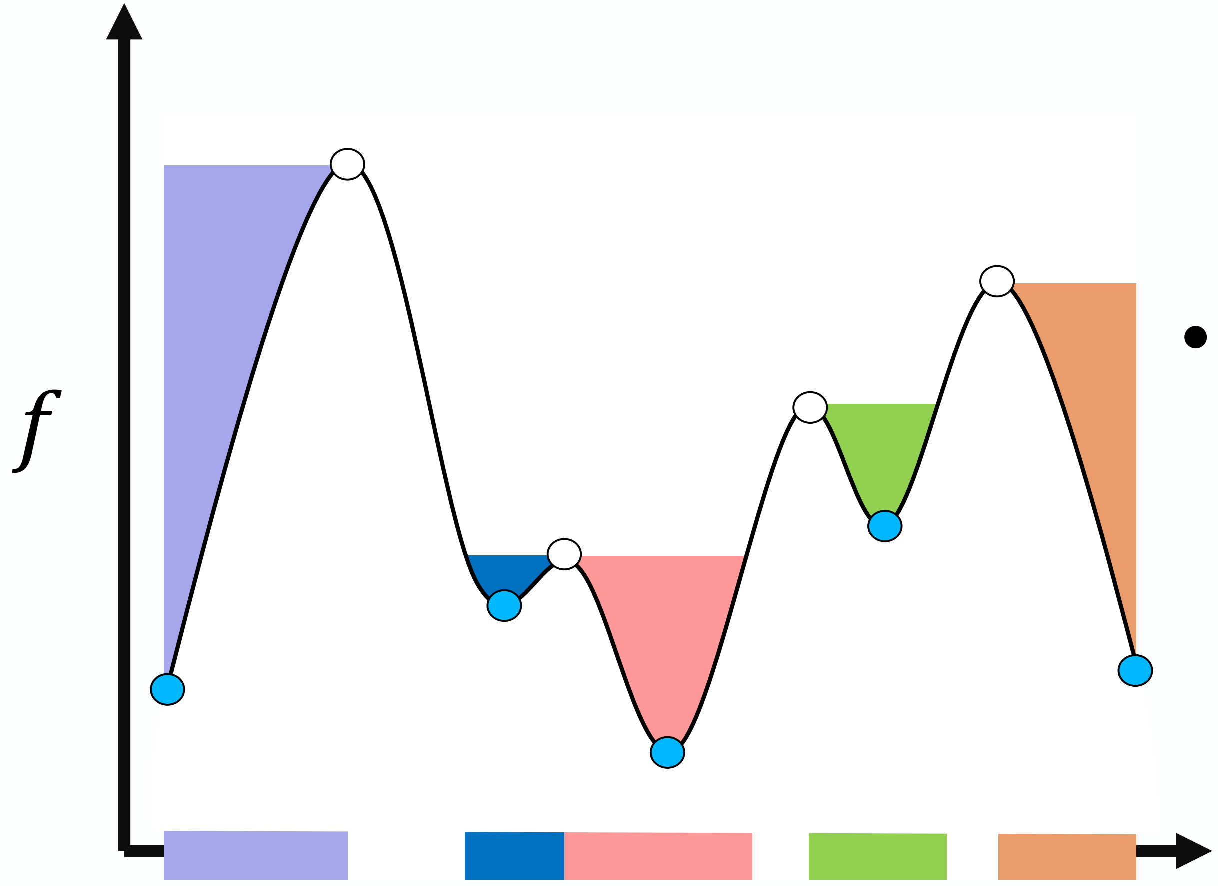
- Components existing during the filtration



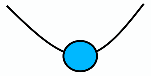
Join tree

segmentation

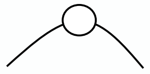
Features of a 1-dimensional function



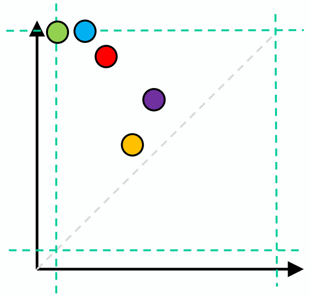
- Critical points where $\nabla f = 0$

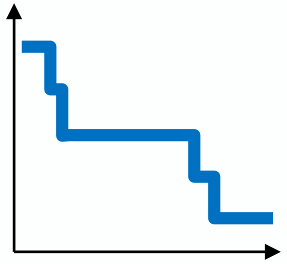


Index-0 minimum

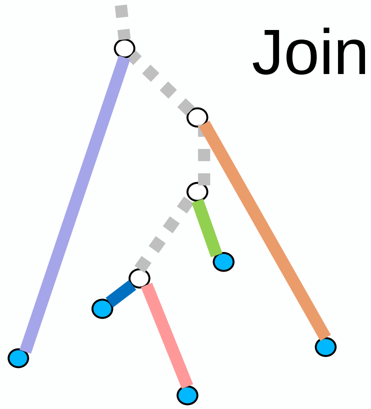


Index-1 maximum





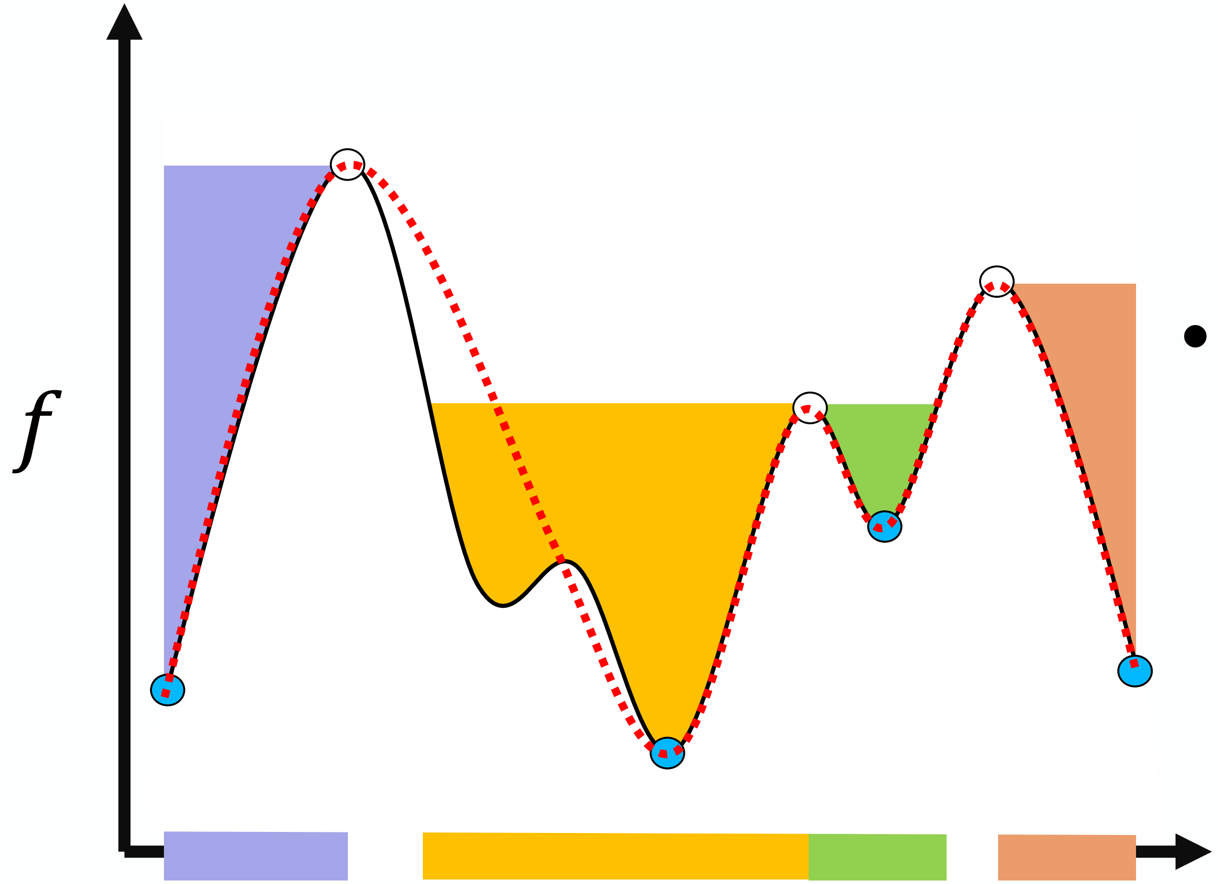
- Components existing during the filtration



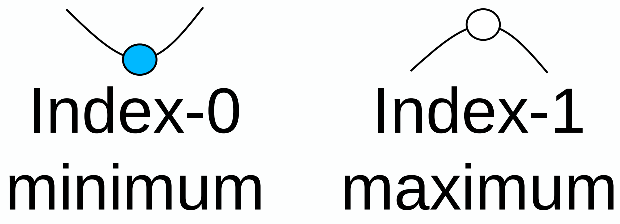
Join tree

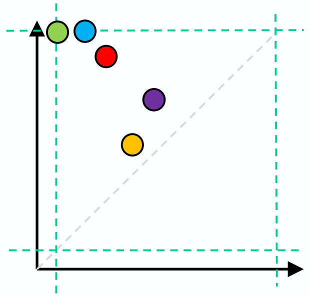
“local valley”, “Extremal regions”

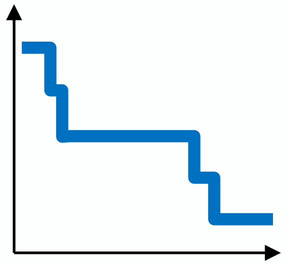
Features of a 1-dimensional function



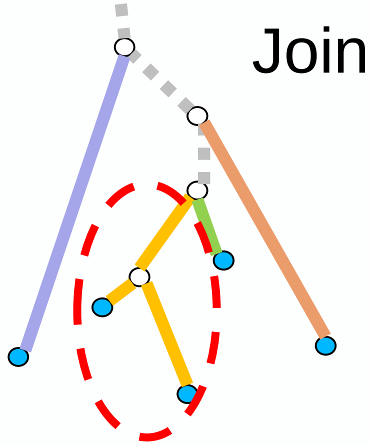
- Critical points where $\nabla f = 0$







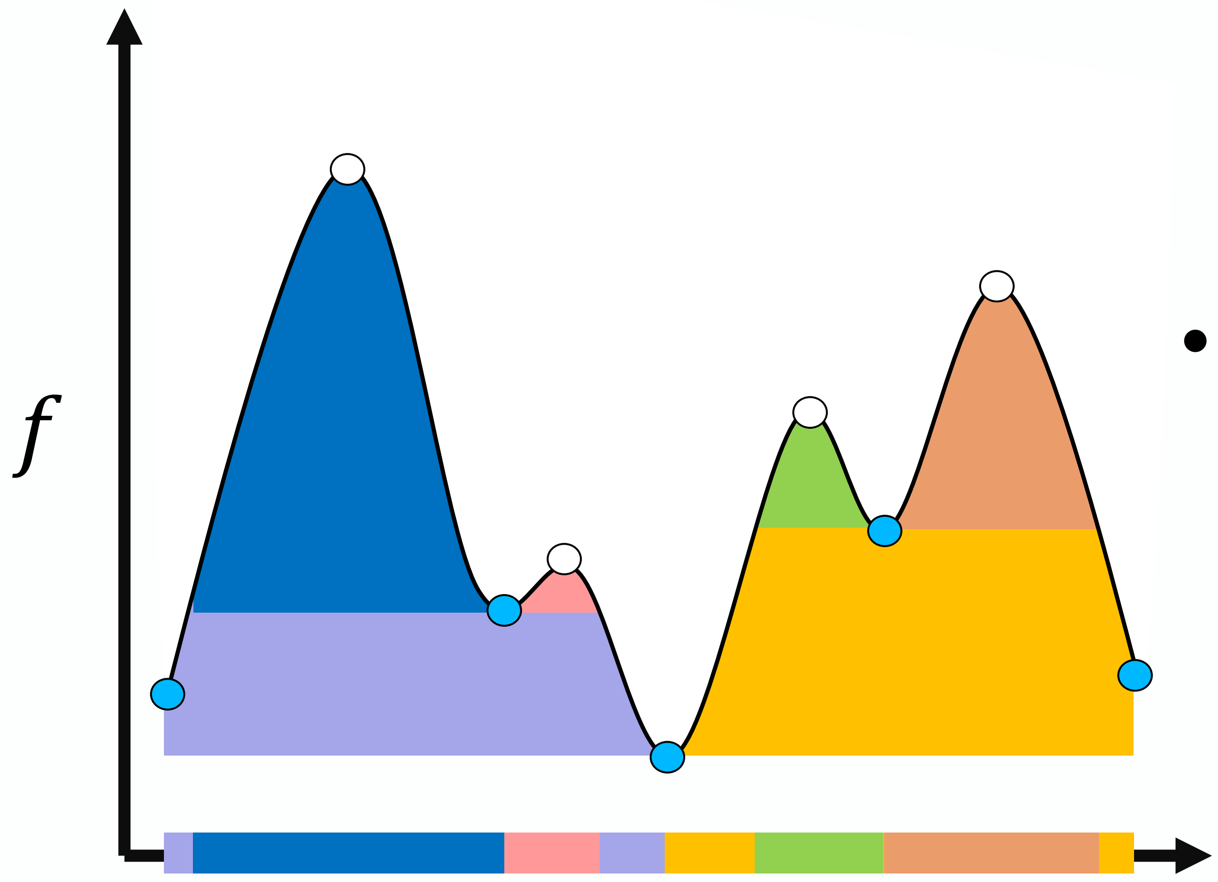
- Components existing during the filtration



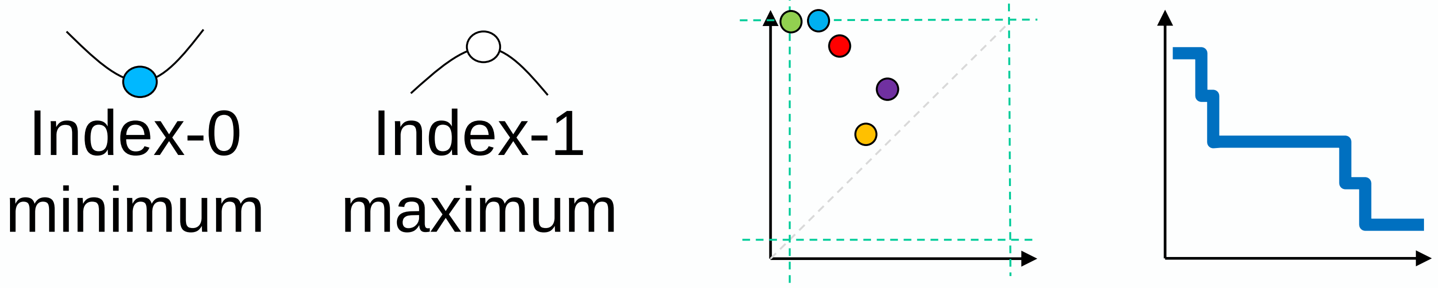
Join tree “local valley”, “Extremal regions”

“at persistence *x*”

Features of a 1-dimensional function

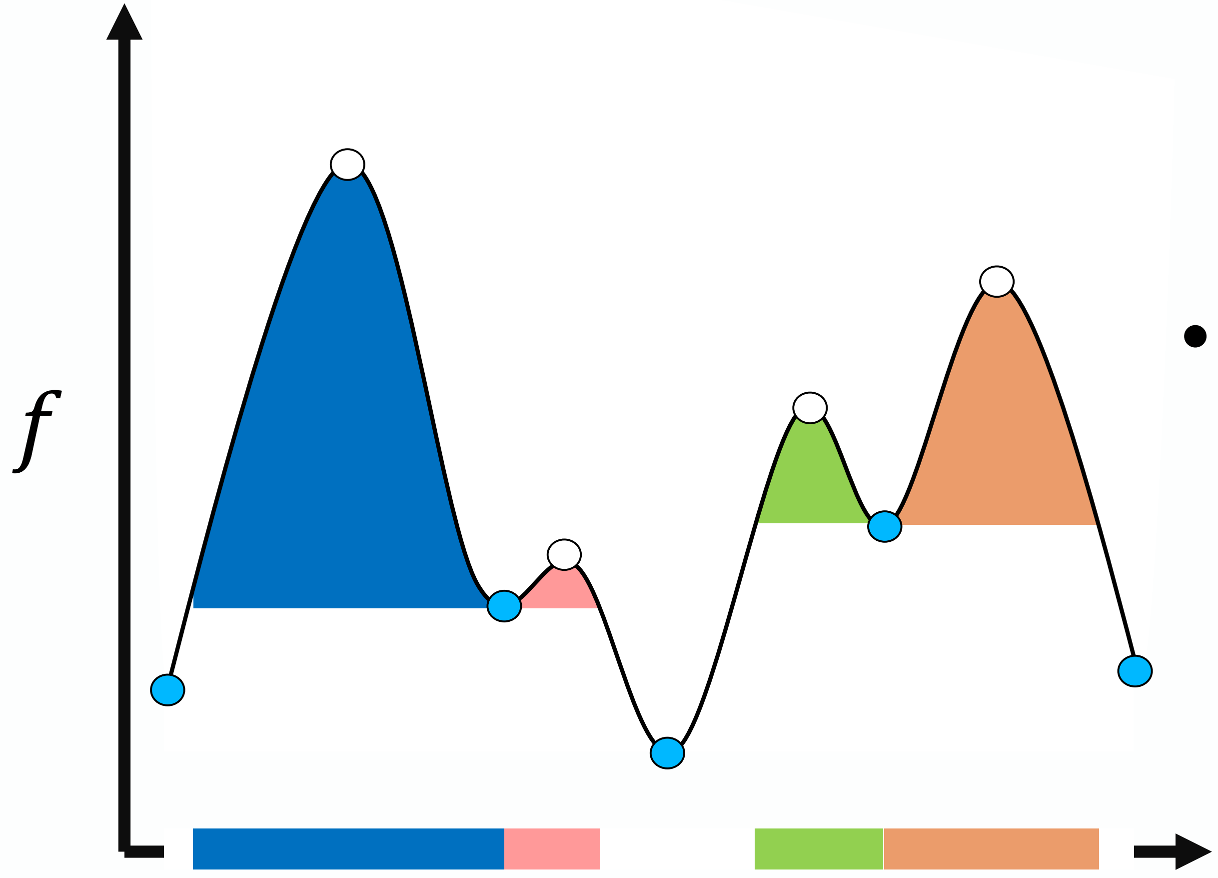


- Critical points where $\nabla f = 0$

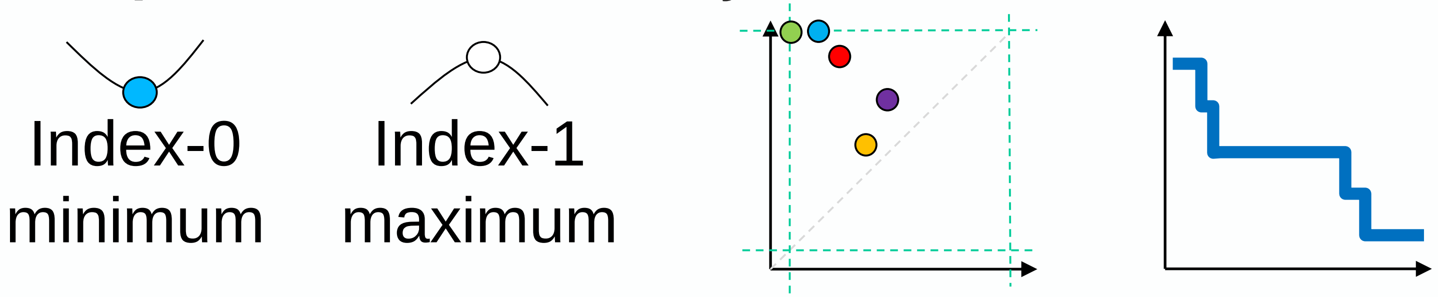


- Components existing during the filtration

Features of a 1-dimensional function

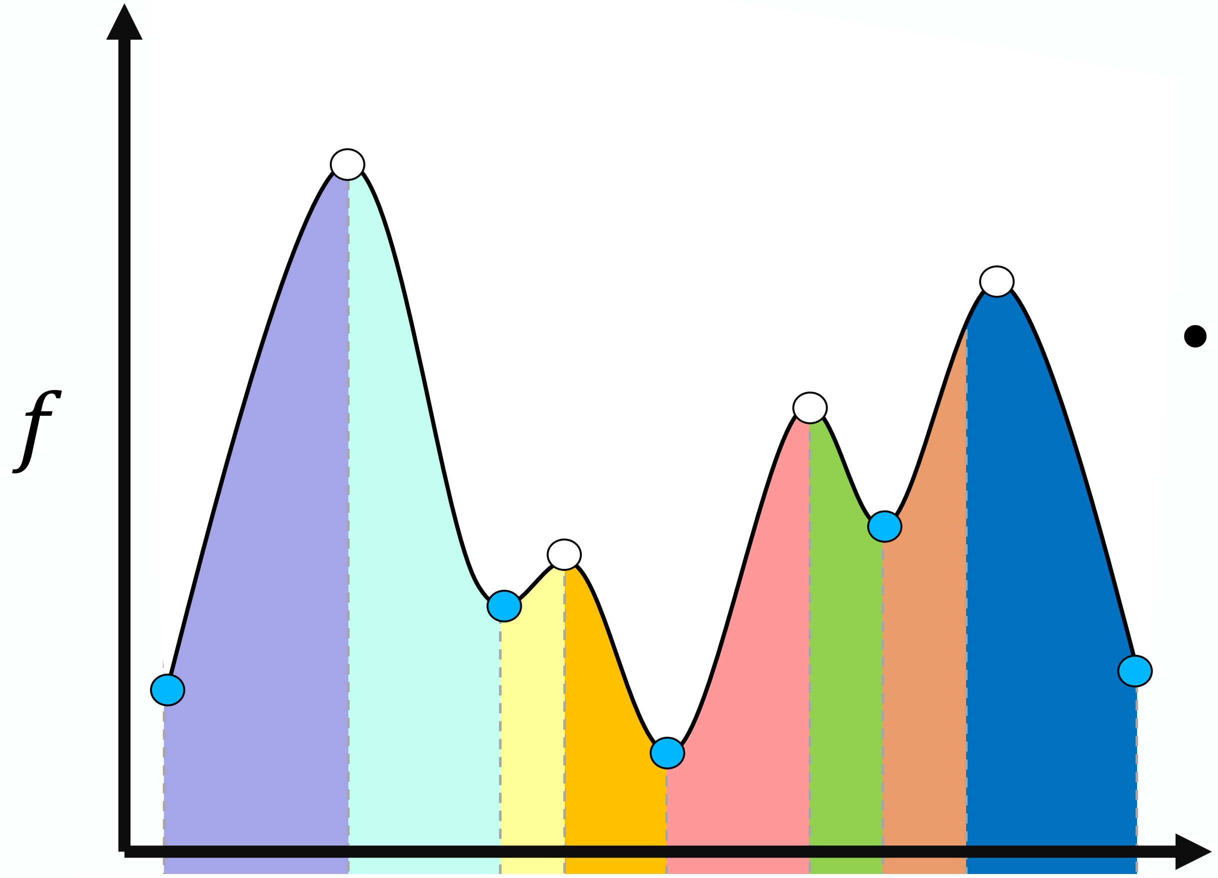


- Critical points where $\nabla f = 0$

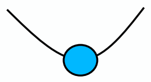


- Components existing during the filtration

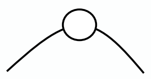
Features of a 1-dimensional function



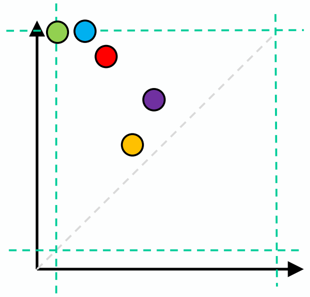
- Critical points where $\nabla f = 0$

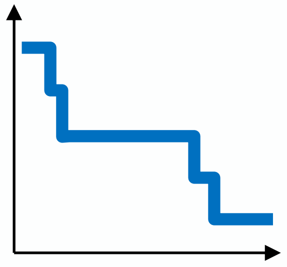


Index-0 minimum

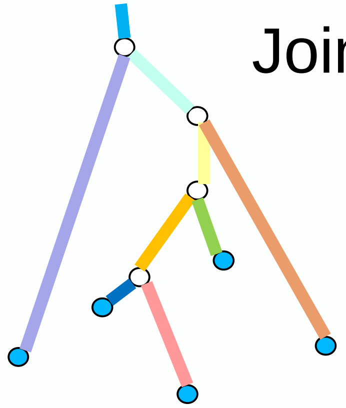


Index-1 maximum

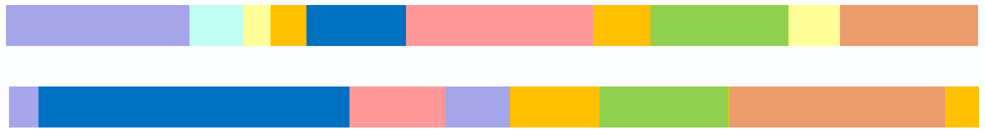




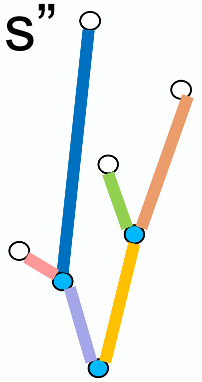
- Components existing during the filtration



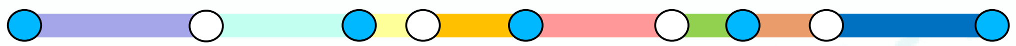
Join tree "local valley", "Extremal regions"



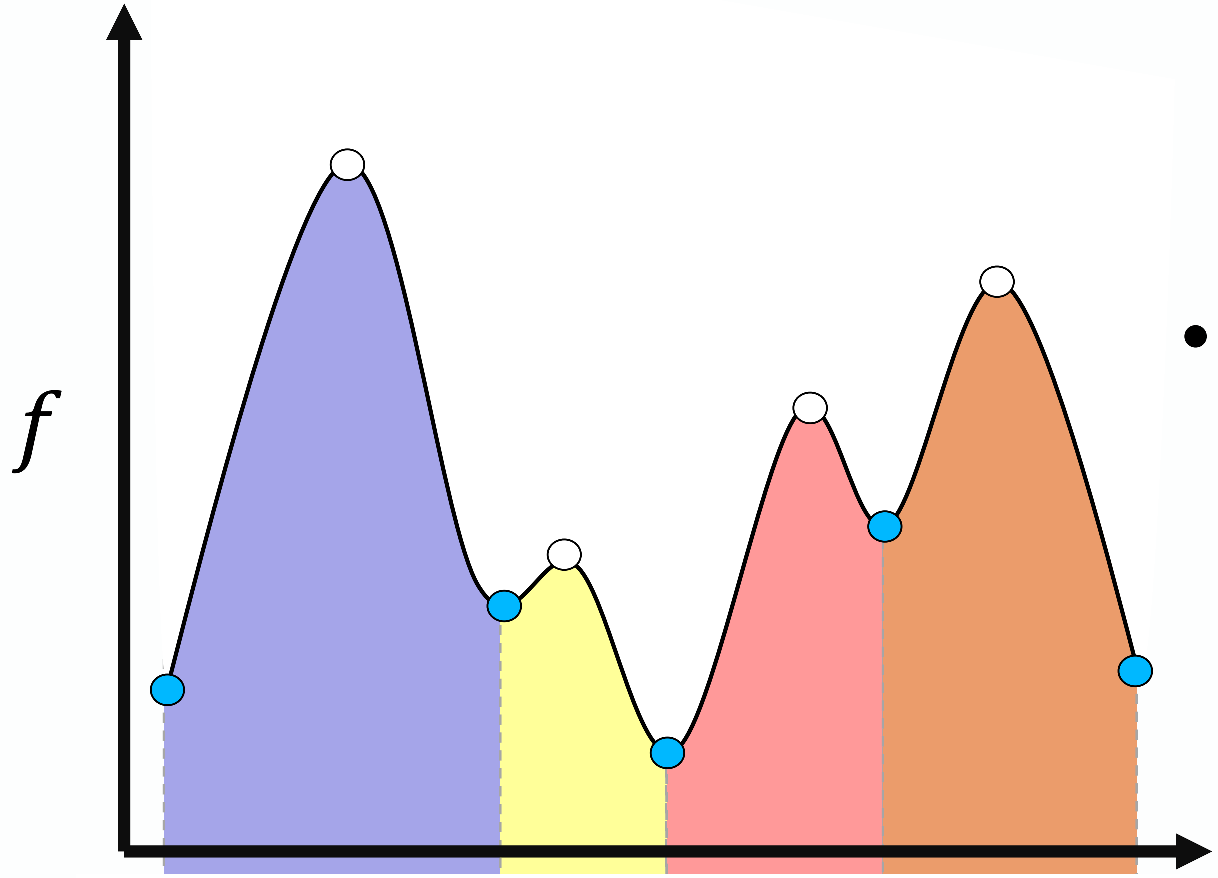
"local peak" Split tree



- Locally monotonic regions



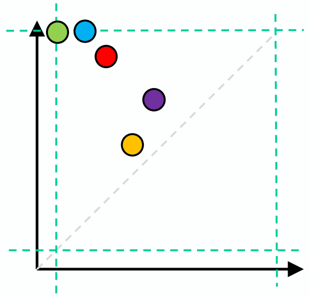
Features of a 1-dimensional function

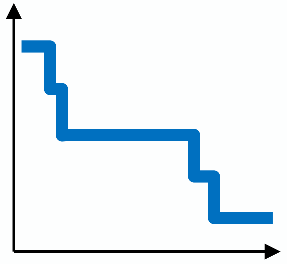


- Critical points where $\nabla f = 0$

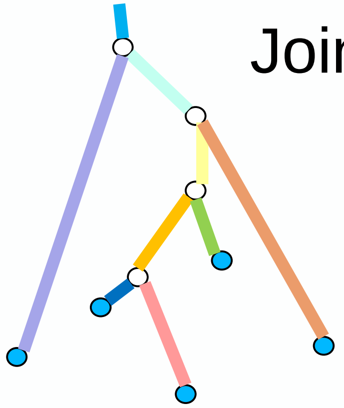
Index-0 minimum

Index-1 maximum



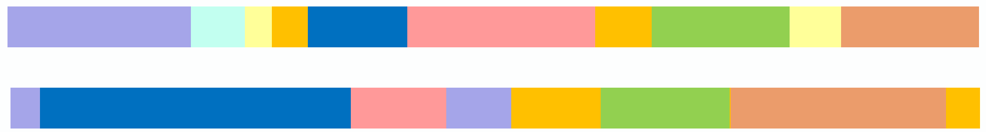


- Components existing during the filtration



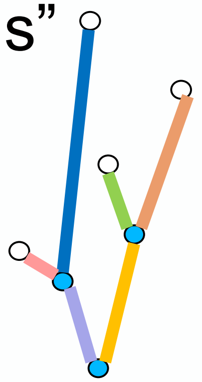
Join tree

"local valley", "Extremal regions"



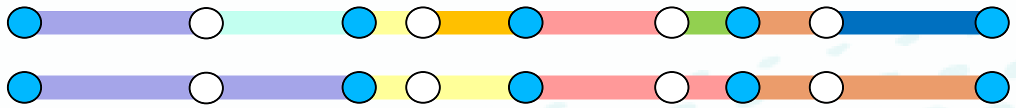
"local peak"

Split tree

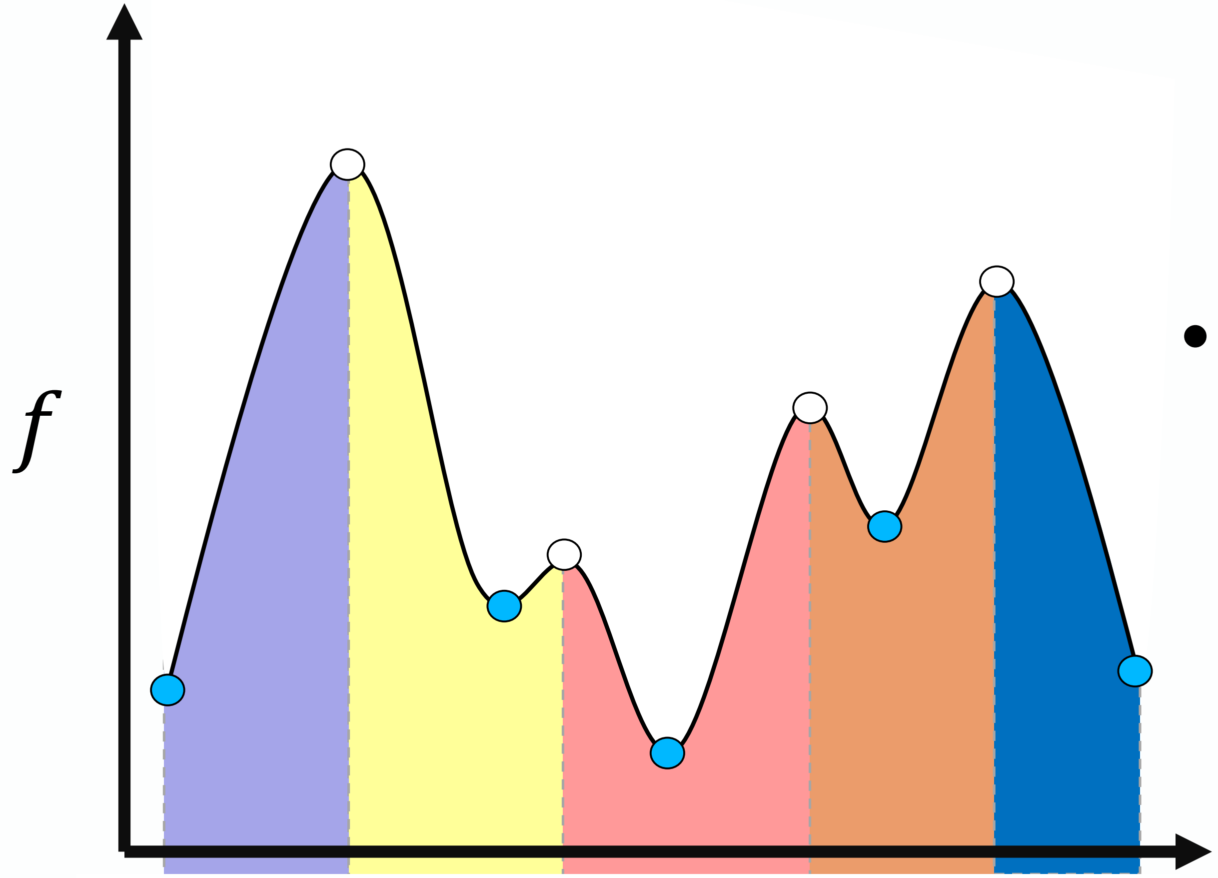


- Locally monotonic regions

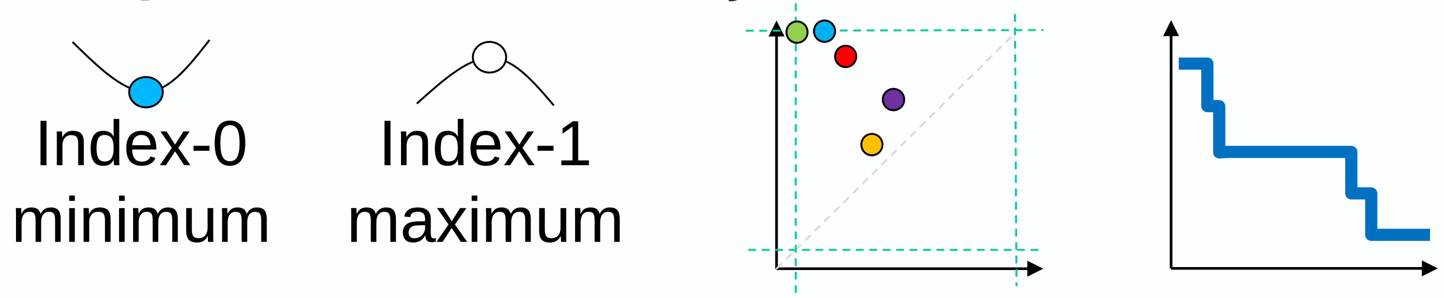
"Mountains"



Features of a 1-dimensional function

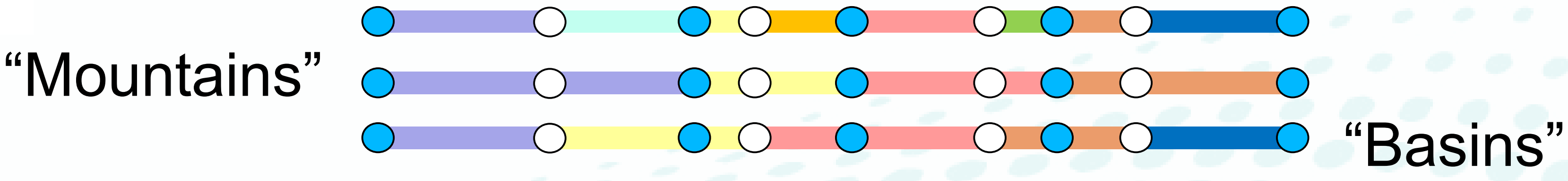


- Critical points where $\nabla f = 0$

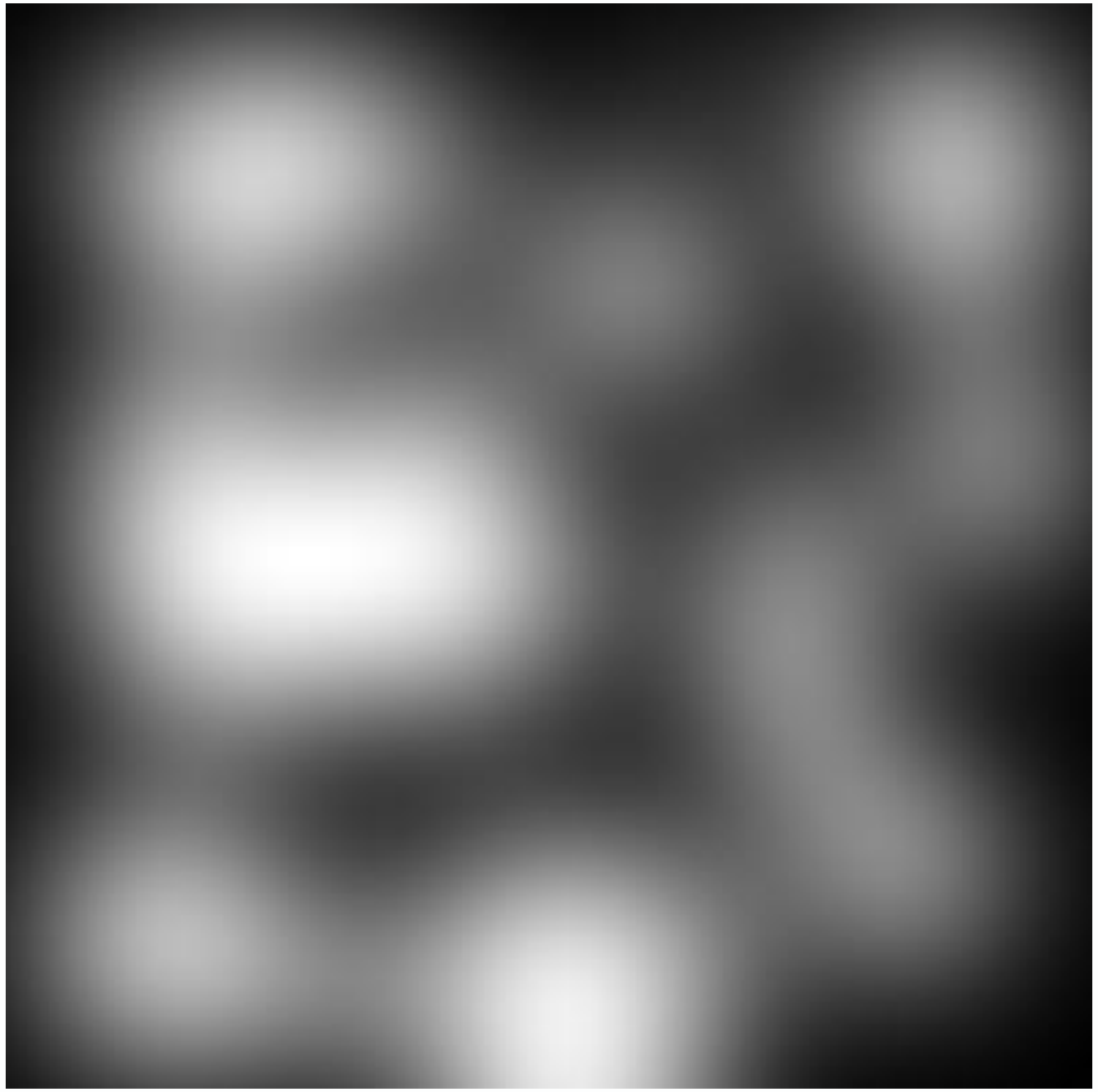


- Components existing during the filtration

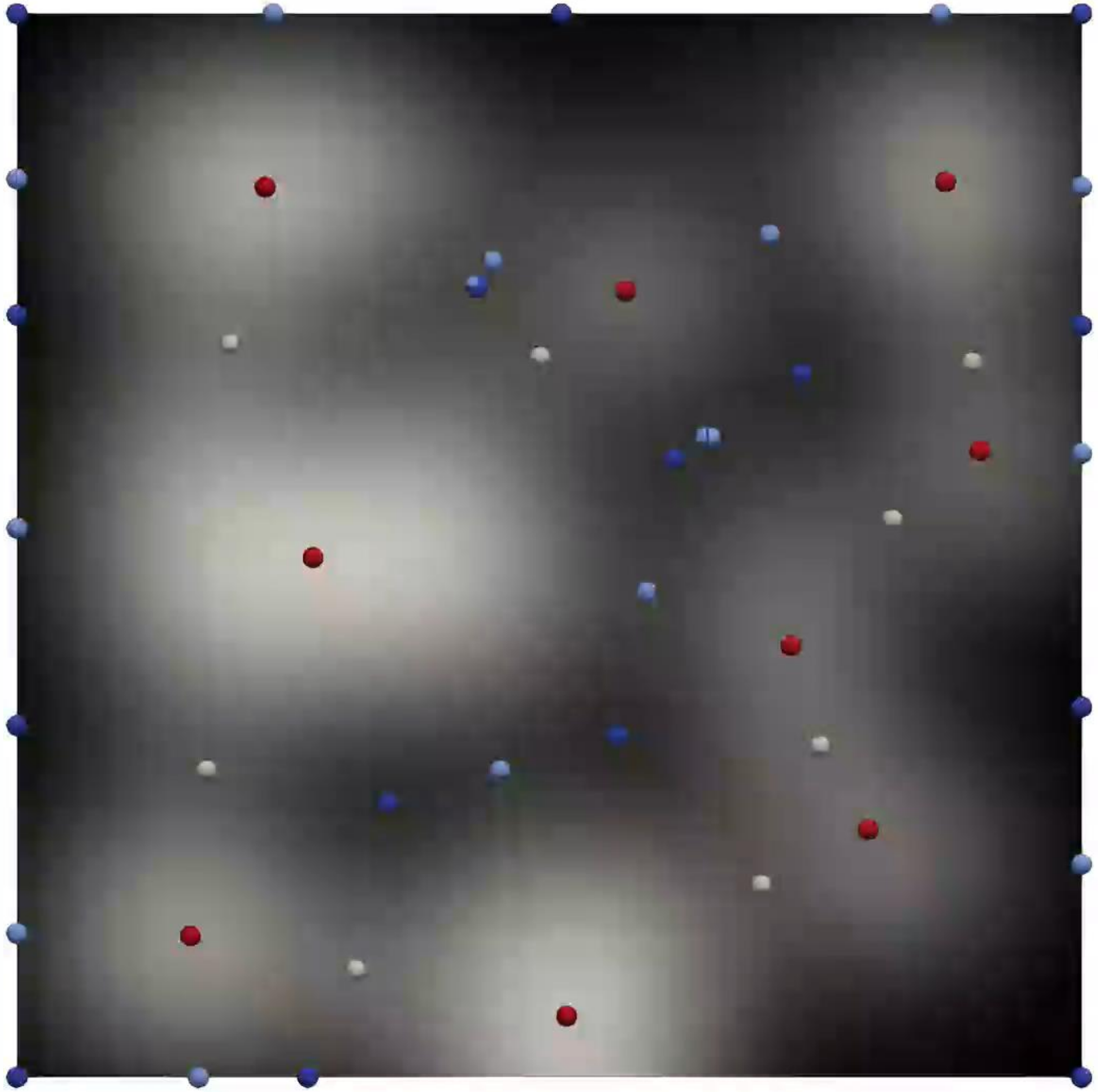
- Locally monotonic regions



Features of a 2-dimensional function



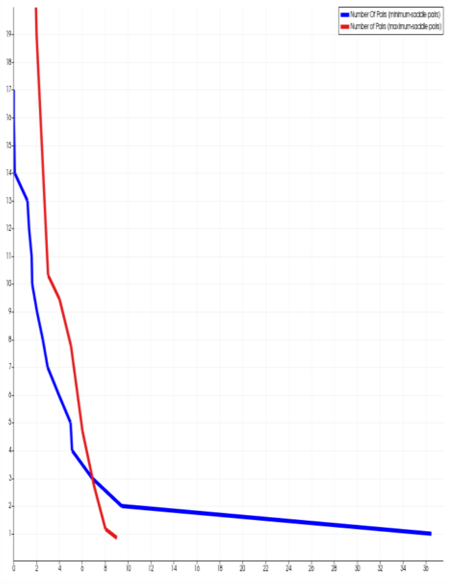
Grayscale rendering
Of 2d scalar function



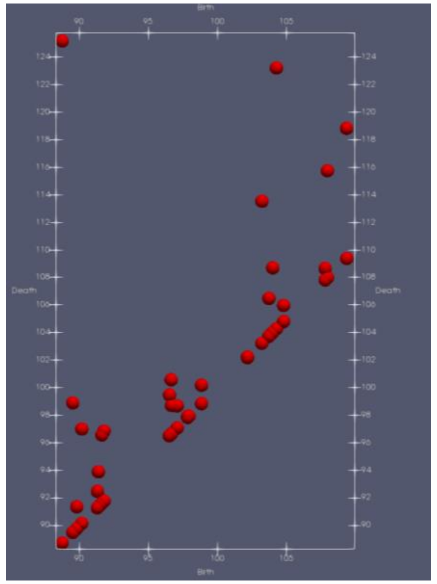
$f^{-1}((-\infty, v])$

VIS 2018

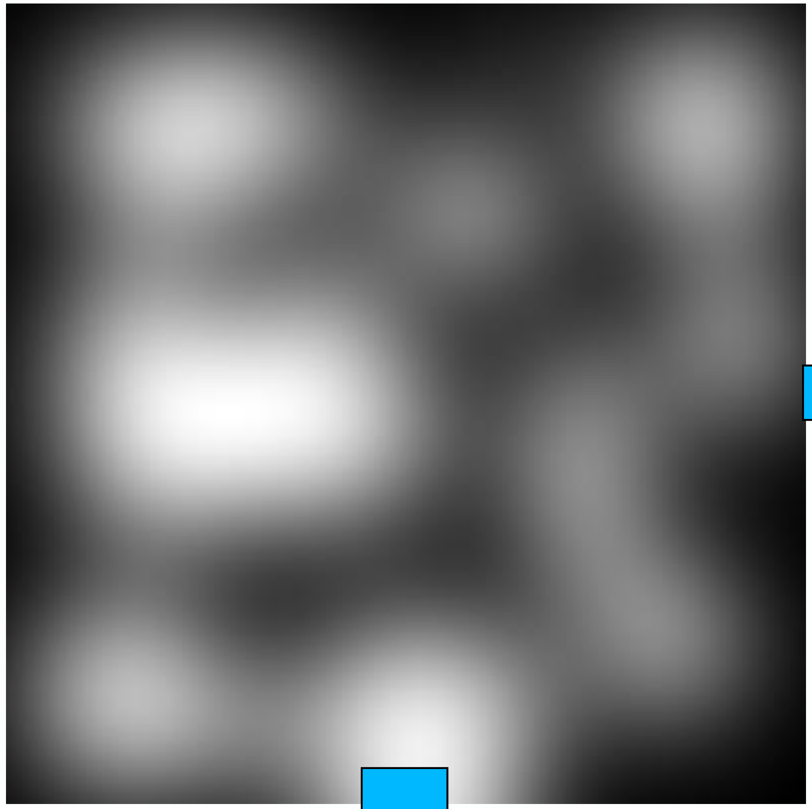
Features of a 2-dimensional function

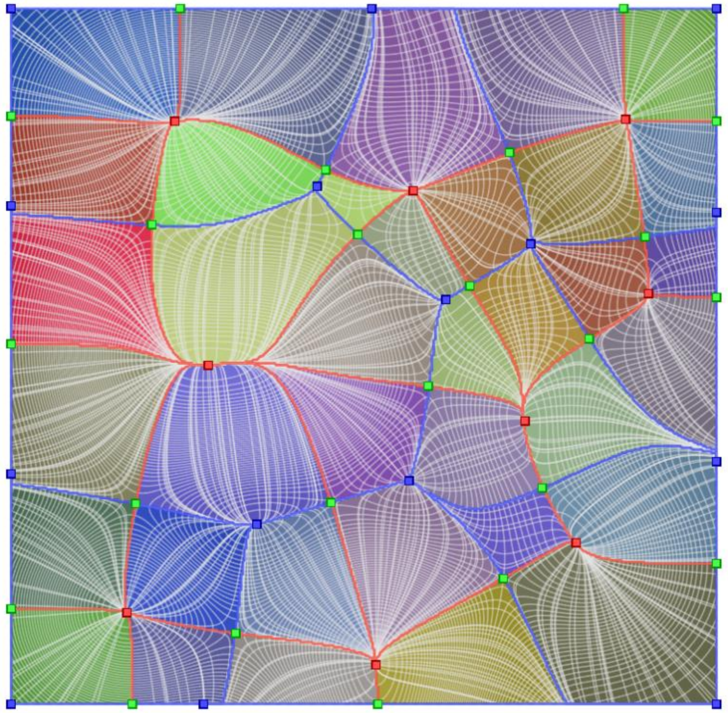


Persistence curve

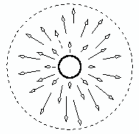


Persistence diagram

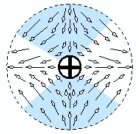




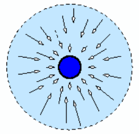
Morse-Smale Complex



Minimum

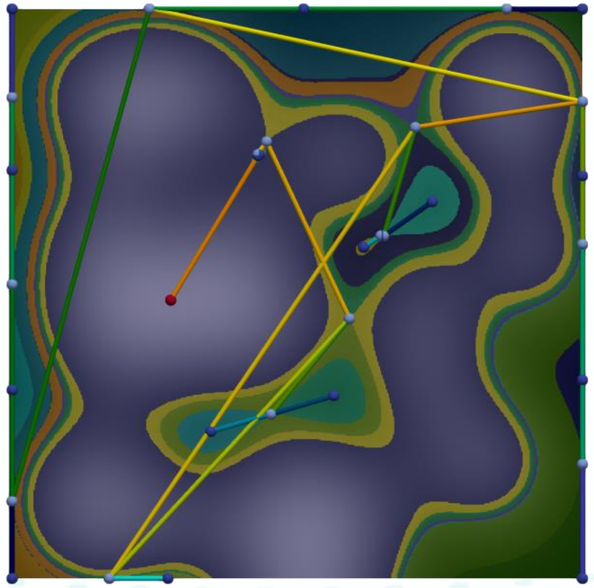


Saddle

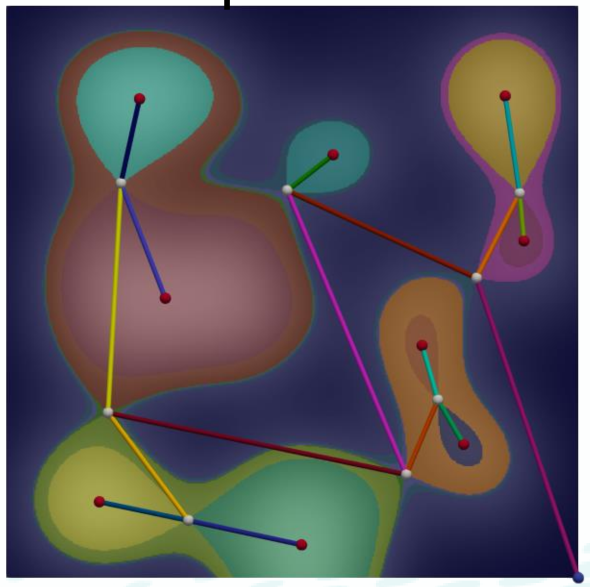


Maximum

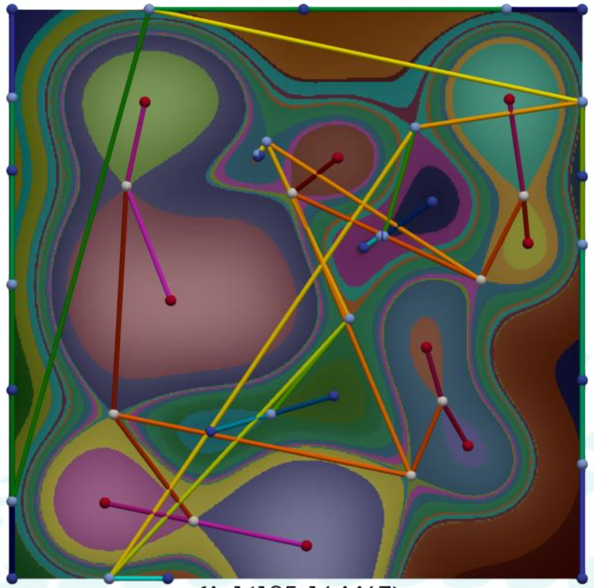
Join tree



Split tree

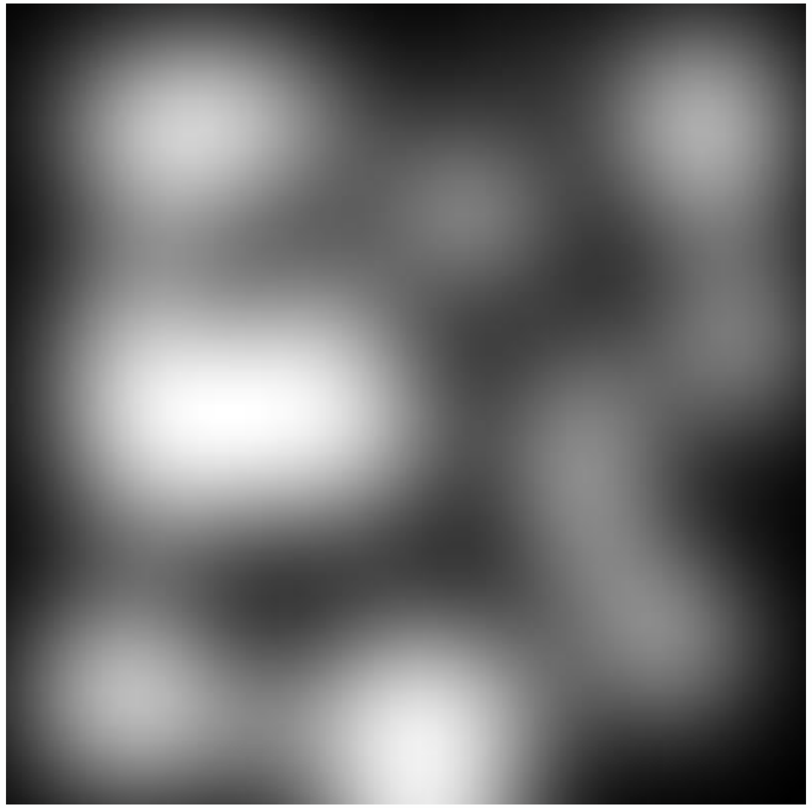


Contour tree

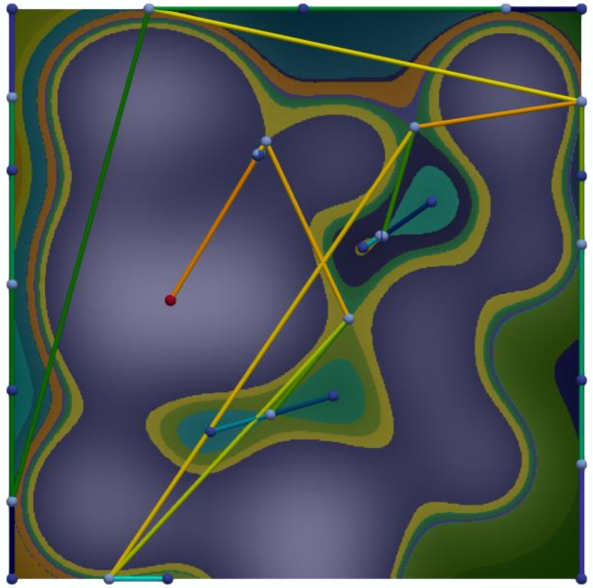


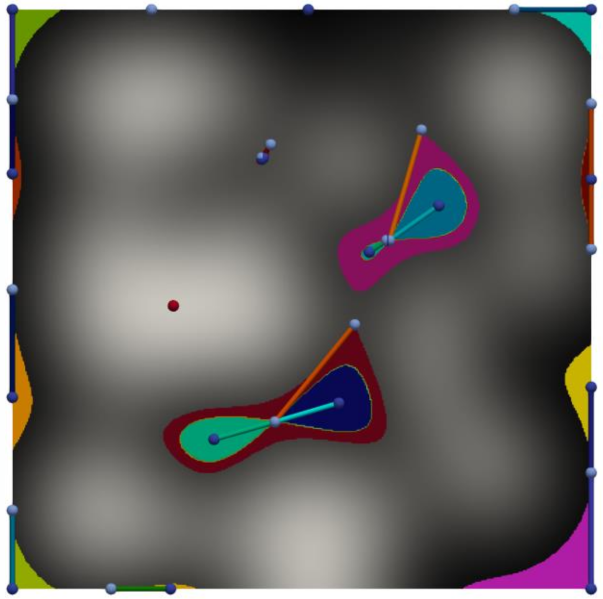
Features of a 2-dimensional function

Contour-based features



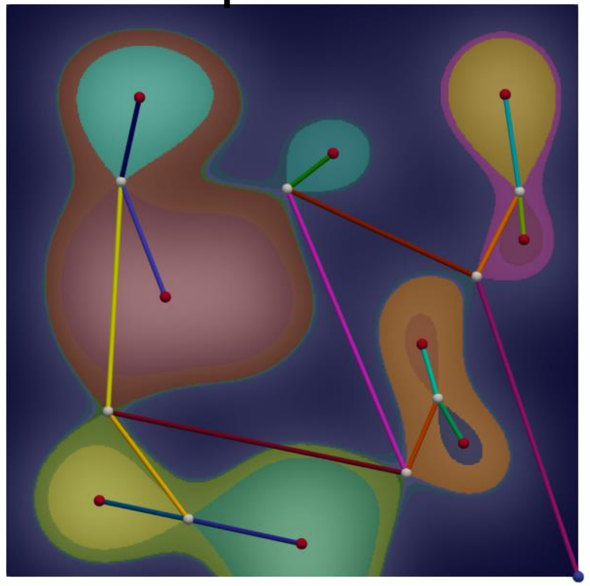
Join tree

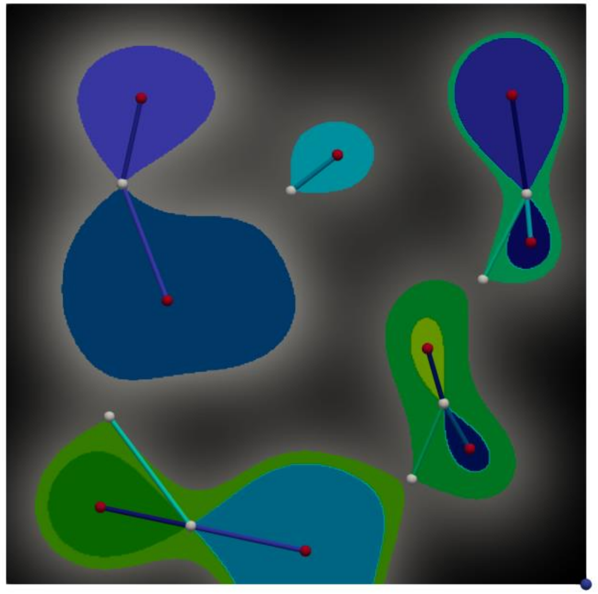




"local valleys"

Split tree

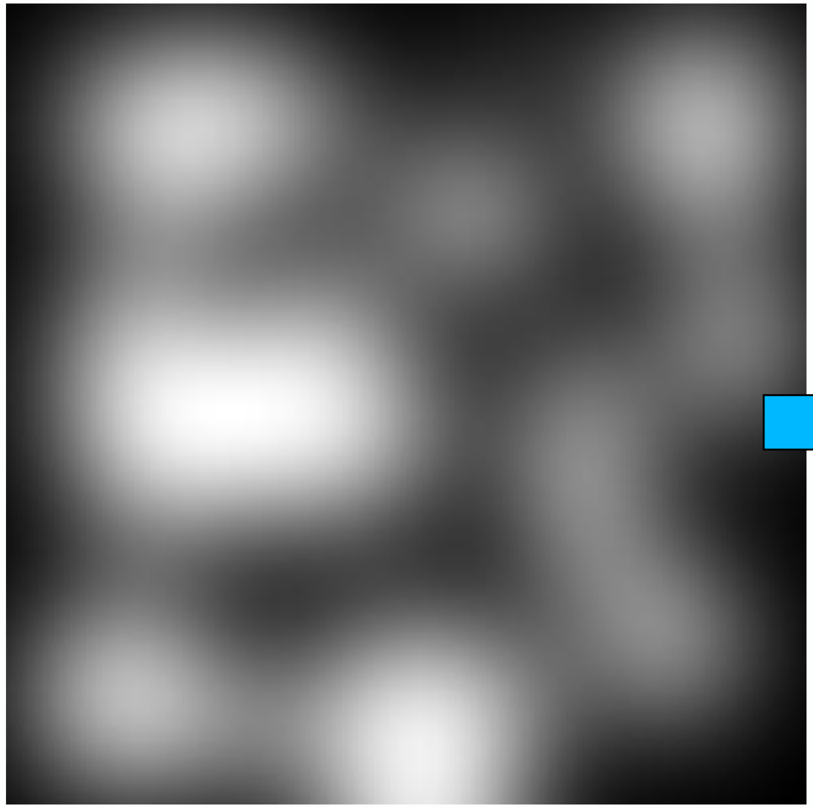


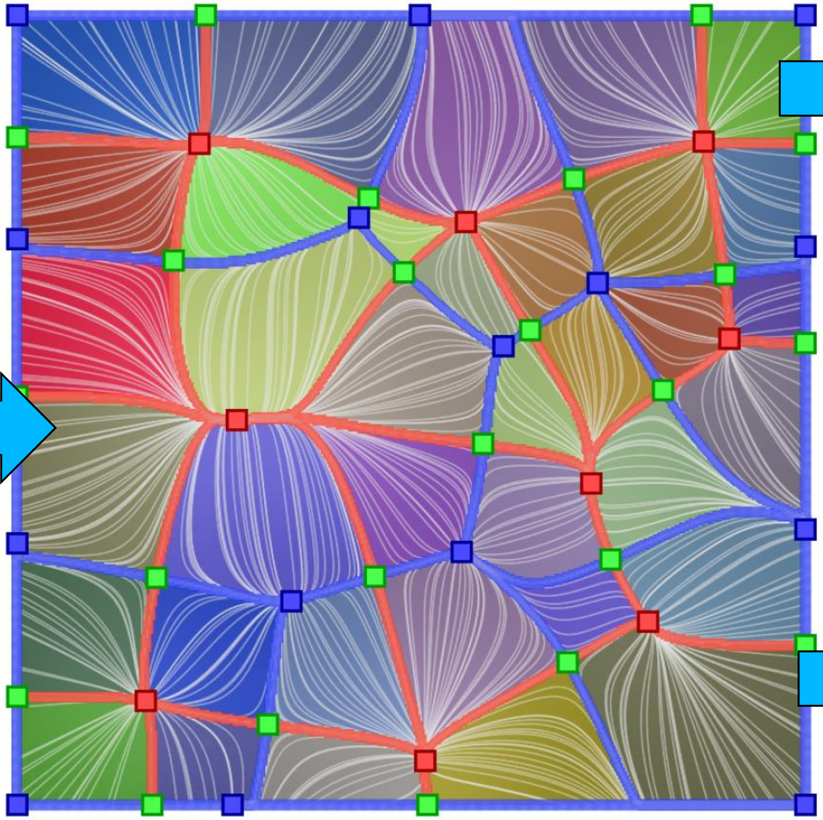


"local peaks"

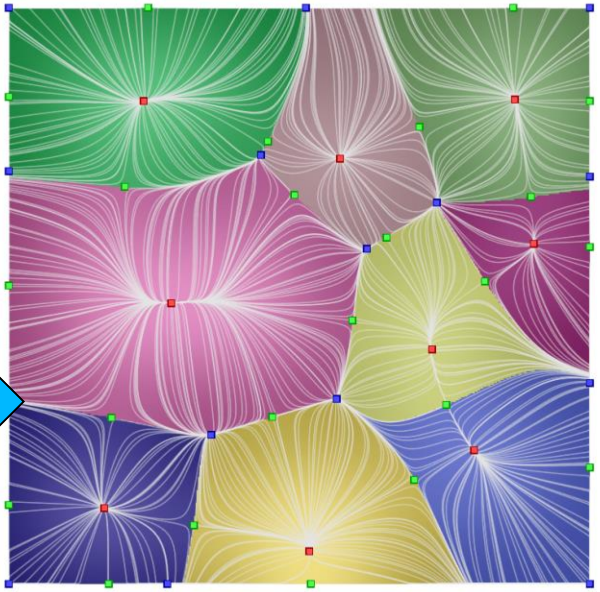
Features of a 2-dimensional function

Gradient based features

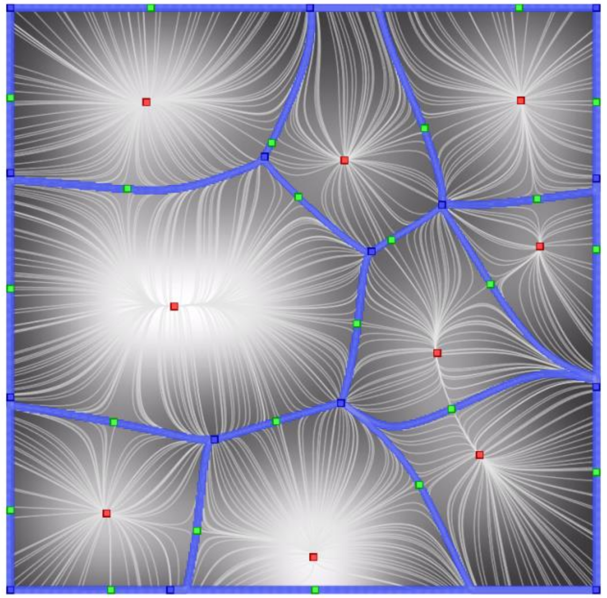




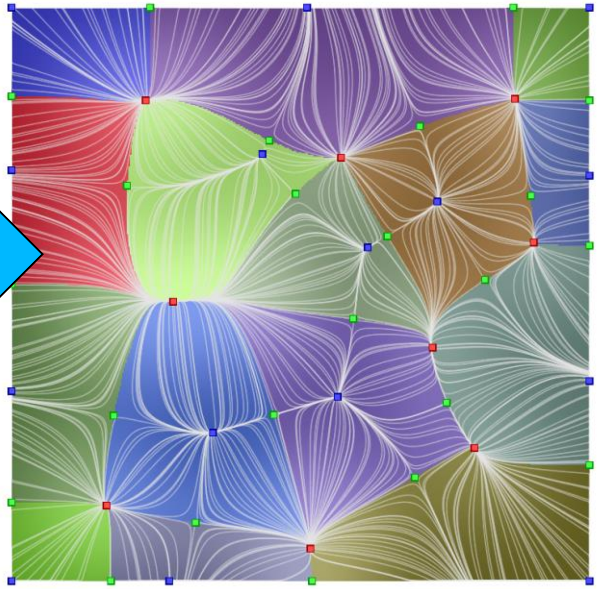
Morse-Smale Complex



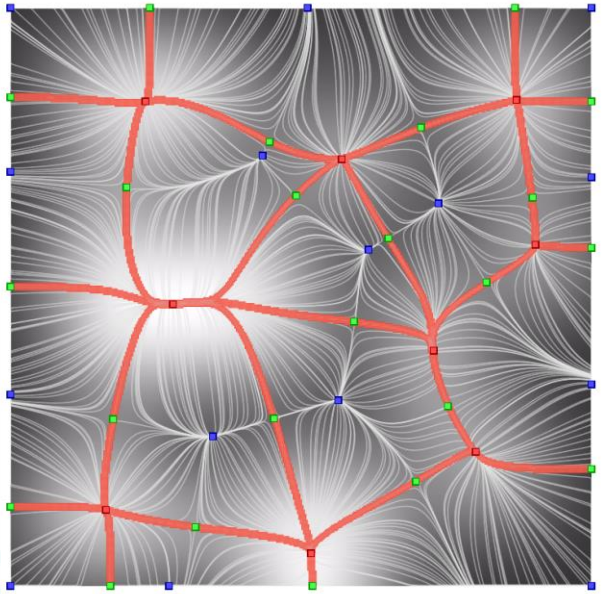
“Mountains”



“Valley lines”

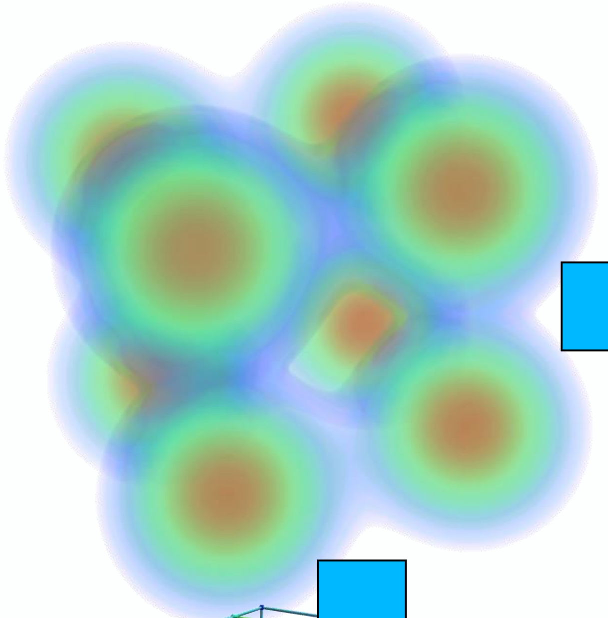


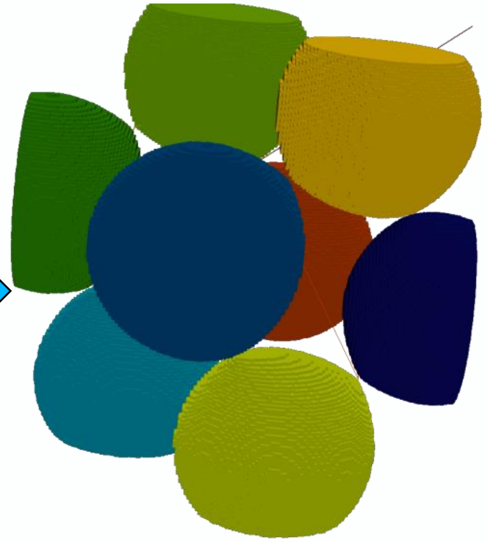
“Basins”



“Ridge lines”

Features of a 3-dimensional function





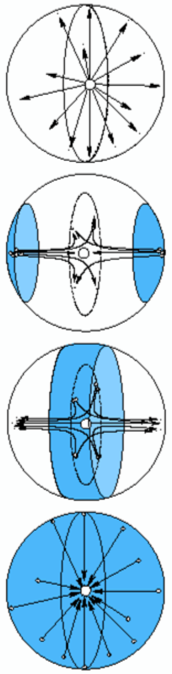
Split tree, Join tree, Contour-trees works the same as 1- and 2d

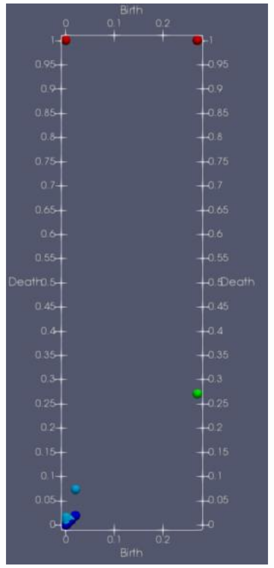
Minimum

1-saddle

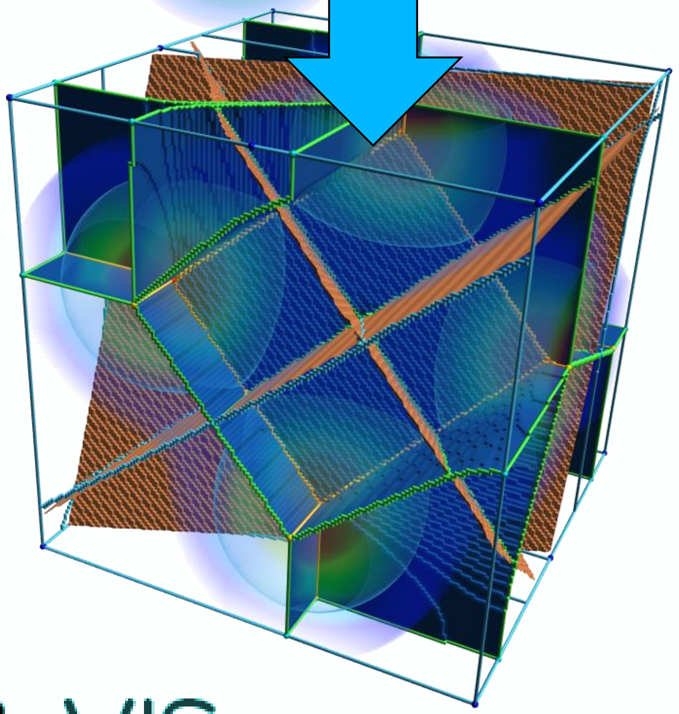
2-saddle

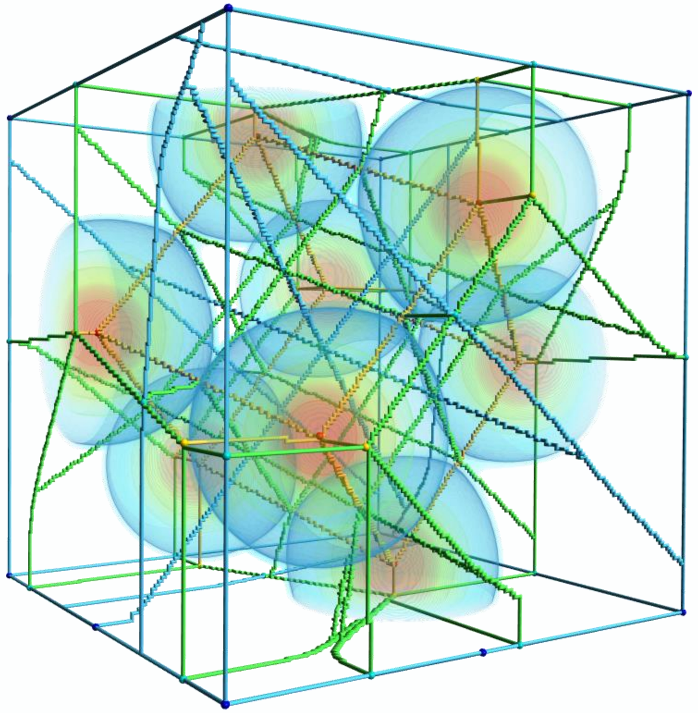
Maximum



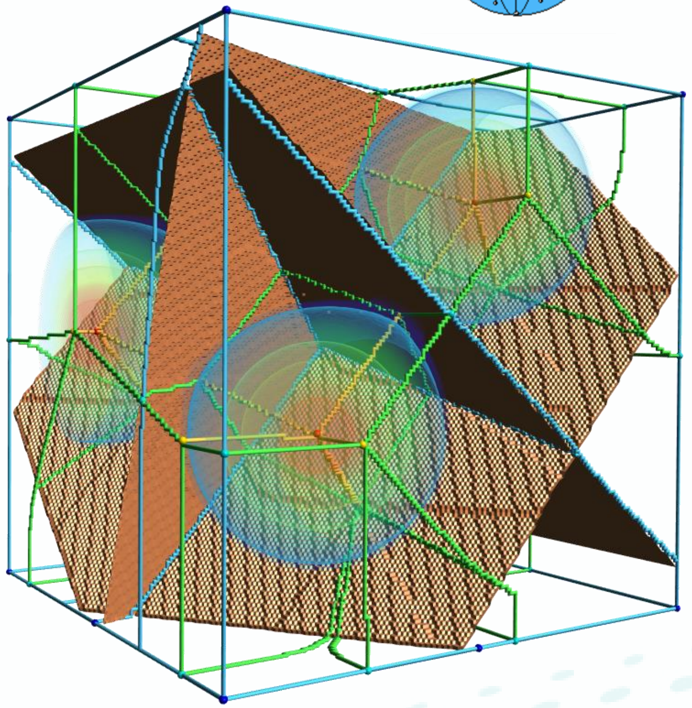


Persistence diagrams also record saddle-saddle pairs

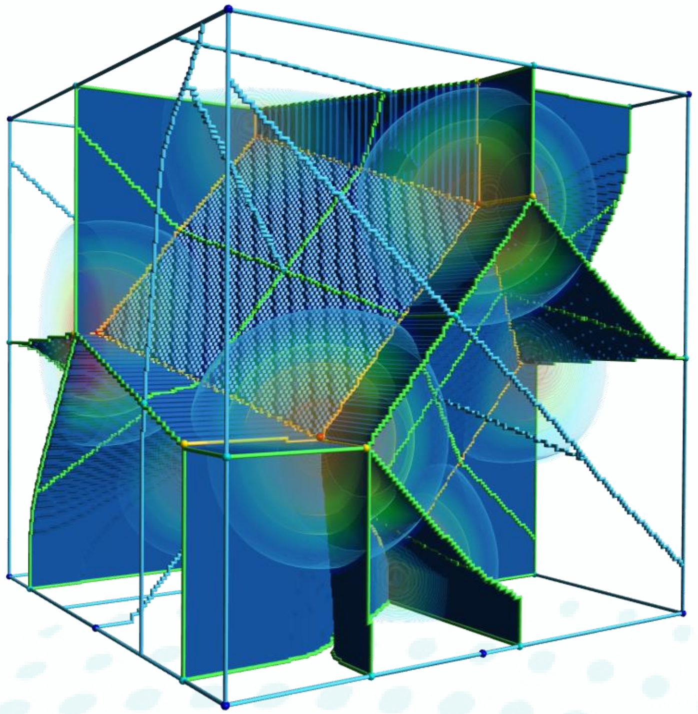




“Ridge lines” “Valley lines”
“Saddle connectors”



“Ridge surfaces”



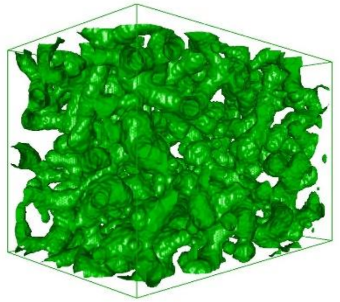
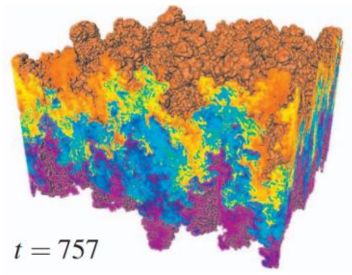
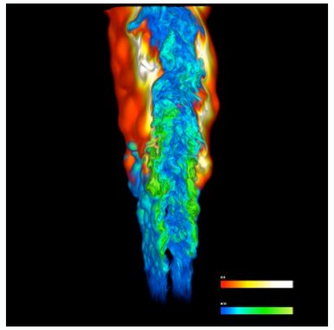
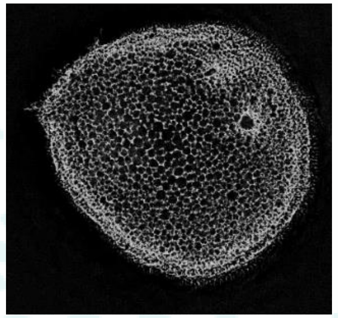
“Valley surfaces”

And in reverse:

"I need to extract/identify/count/measure…."

- Extreme point → Persistence diagram/ persistence plot
- Local peaks/valleys → Leaves/branches of join/split/contour trees
- Ridge/Valley lines → Saddle-extremum arcs of MSC
- Mountains/Basins → Ascending/descending segmentation ids of MSC
- Separating surfaces → Ascending/descending (d-1)-manifolds of MSC

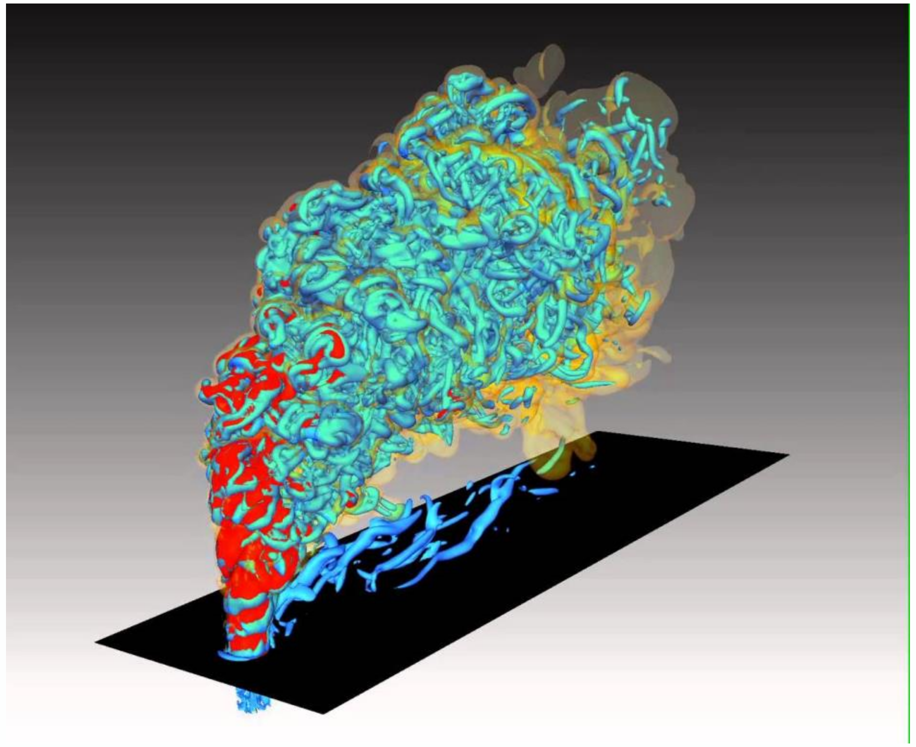
Create a hypothesis for topological features

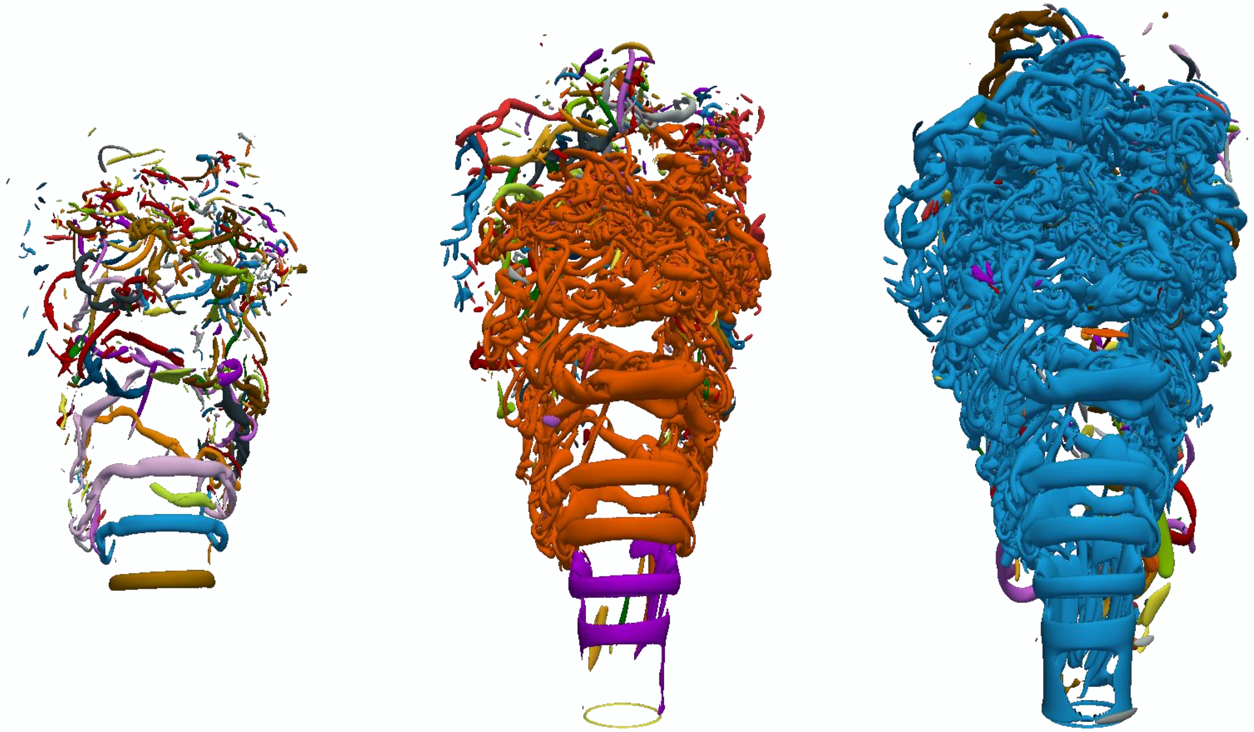
Data		Scientific question		Redefine with abstraction
	→	"Quantify porosity"	→	Measure total length and number of cycles in ridge-like "filaments"
	→	"Measure bubble formation rate"	→	Evolution of stable descending 2-manifolds ("mountains") on mixture fraction isosurface
	→	"Identify ignition kernels"	→	Local peaks and regions in temperature field below flame base
	→	"Measure deformation of cavity walls"	→	Measure deviation from plane for ascending 2-manifolds separating "basins"

Continuously re-evaluate hypothesis

Identifying vorticity structures in turbulent flows

Indicator-based Vortex Detection Remains most Common Approach with Well-Known Problems





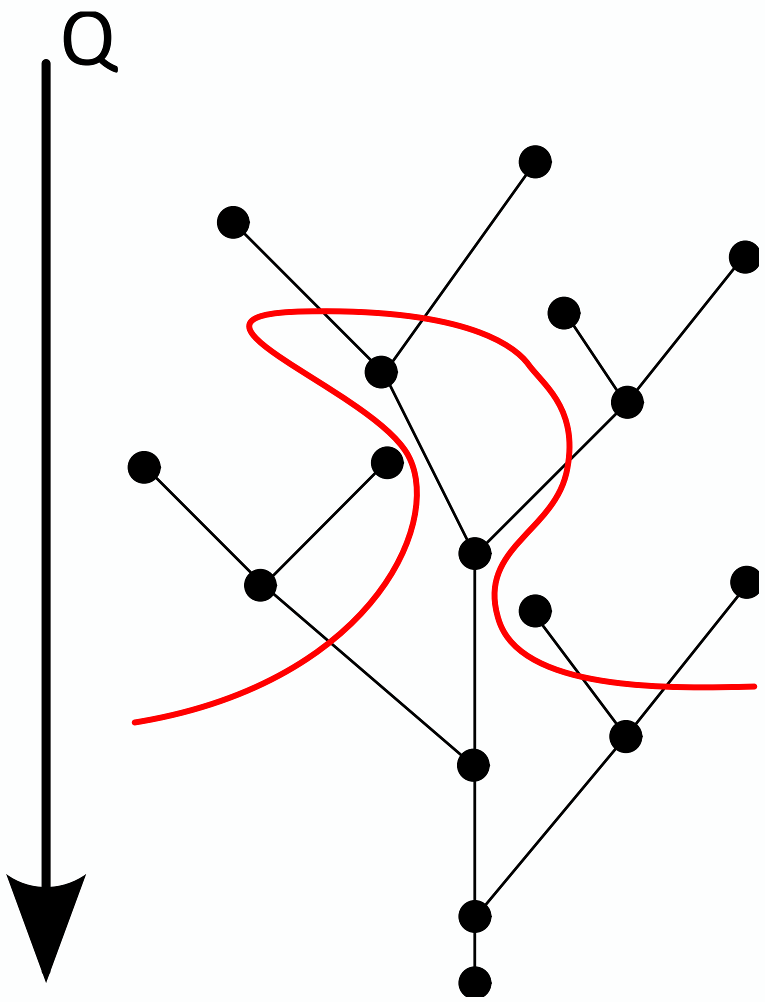
Q = 1000 Q = 100 Q = 5

P.-T. Bremer, A. Gruber, J. Bennett, A. Gyulassy, H. Kolla, J. Chen, R.W. Grout. **"Identifying turbulent structures through topological segmentation,"** In *Com. in App. Math. and Comp. Sci.*, Vol. 11, No. 1, pp. 37-53. 2016.

Identifying vorticity structures in turbulent flows

Cutting the split tree based on *relevance* allows for “local peaks” definition of vortex

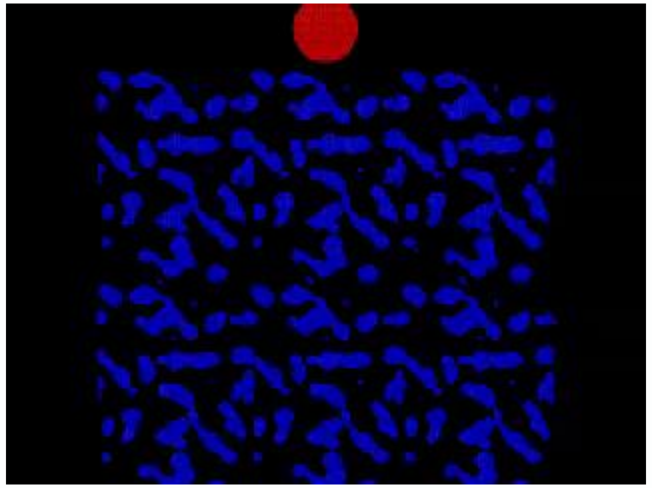
$$relevance(p) = 1 - \frac{localMax(p) - f(p)}{localMax(p) - globalMin}$$

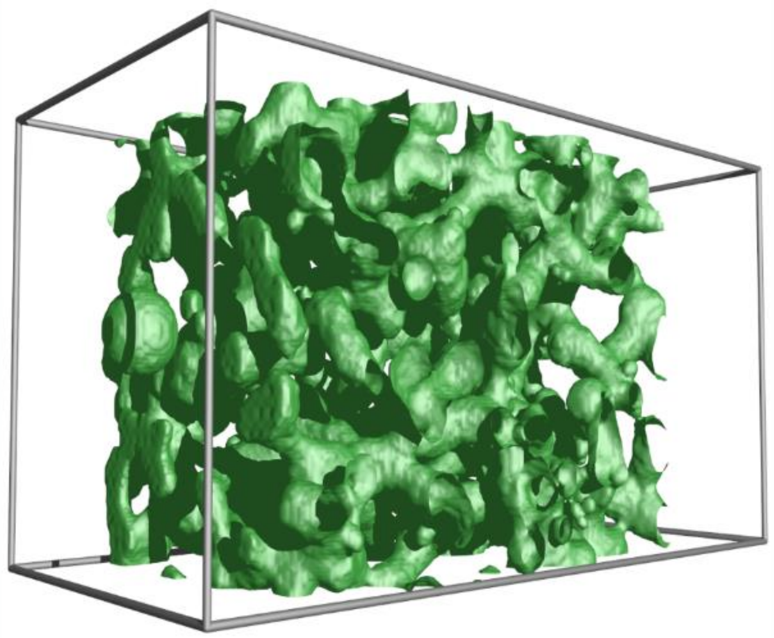


P.-T. Bremer, A. Gruber, J. Bennett, A. Gyulassy, H. Kolla, J. Chen, R.W. Grout. **“Identifying turbulent structures through topological segmentation,”** In *Com. in App. Math. and Comp. Sci.*, Vol. 11, No. 1, pp. 37-53. 2016.

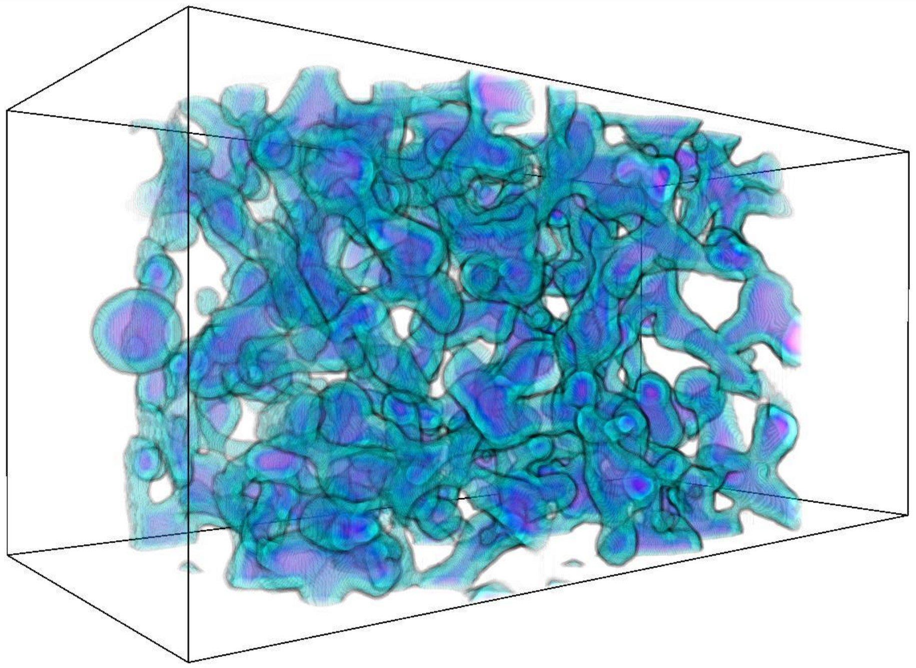
Measure change in porosity of metal foam

Measure filament length and number of cycles, using ridge-like lines of the MS complex





Time varying field of binary data (1 = metal, 0 = vacuum)



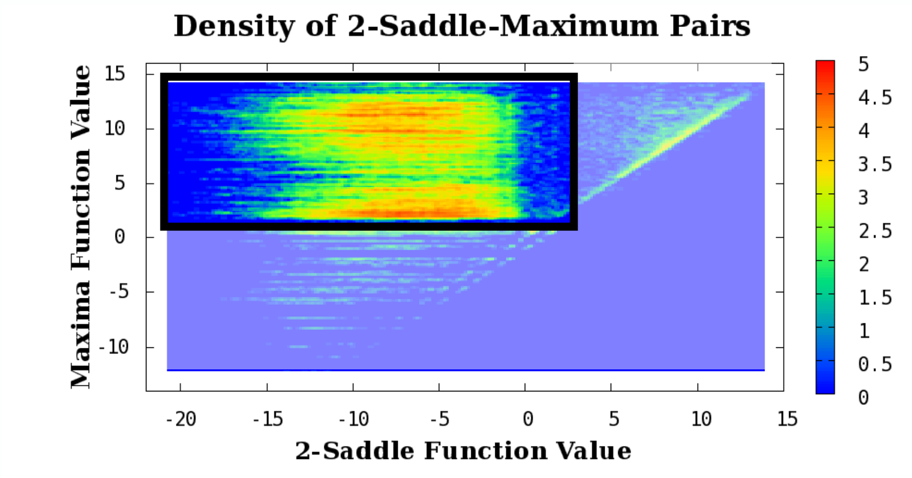
Signed distance field from 0.5 isosurface

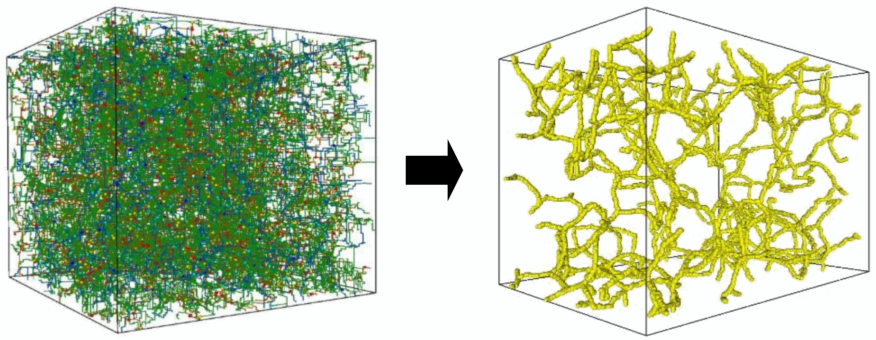
VIS 2018

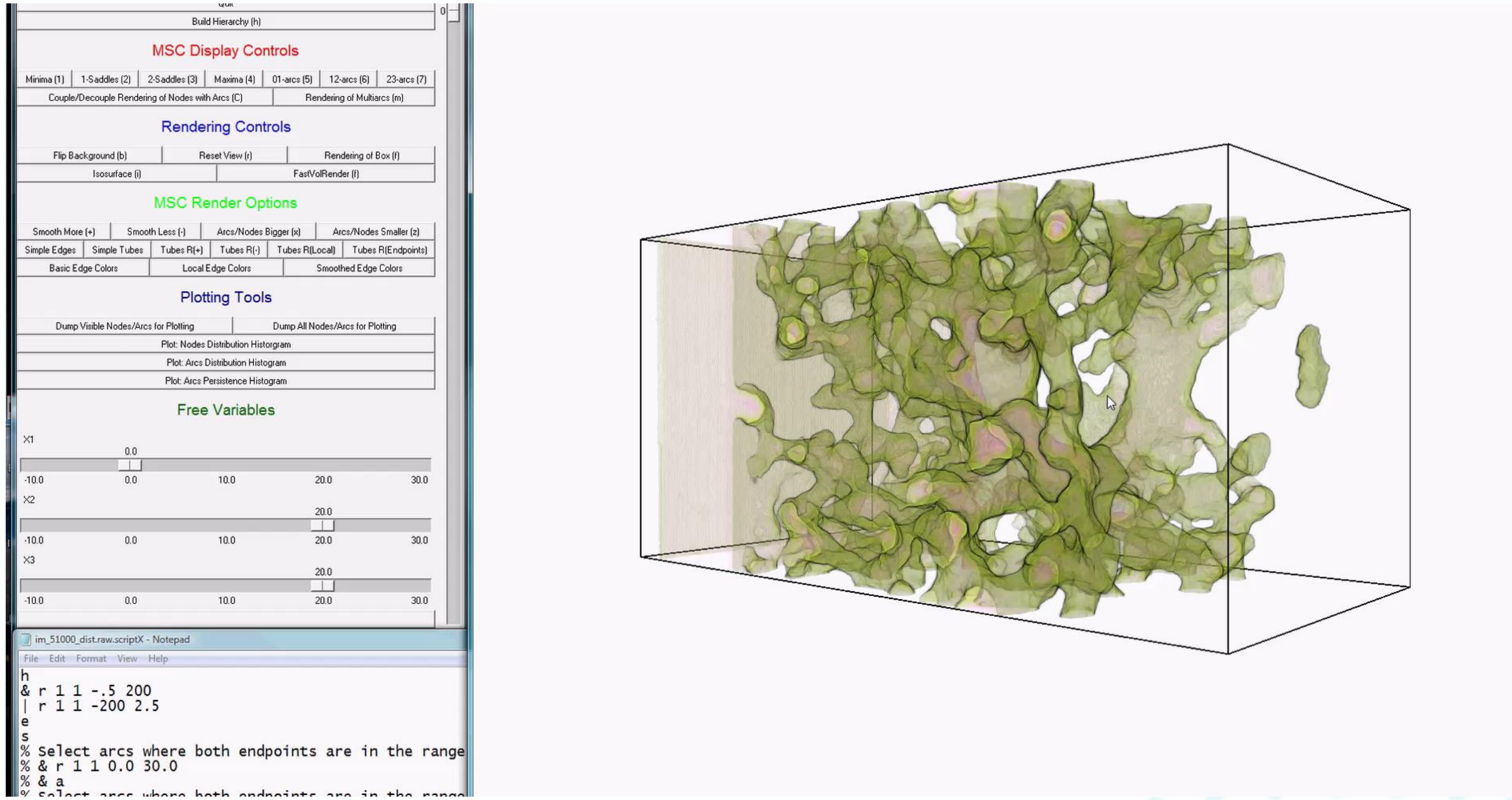
A. Gyulassy et al., **"Topologically Clean Distance Fields,"** in IEEE Transactions on Visualization and Computer Graphics, vol. 13, no. 6, pp. 1432-1439, Nov.-Dec. 2007.

Measure change in porosity of metal foam

Measure filament length and number of cycles, using ridge-like lines of the MS complex

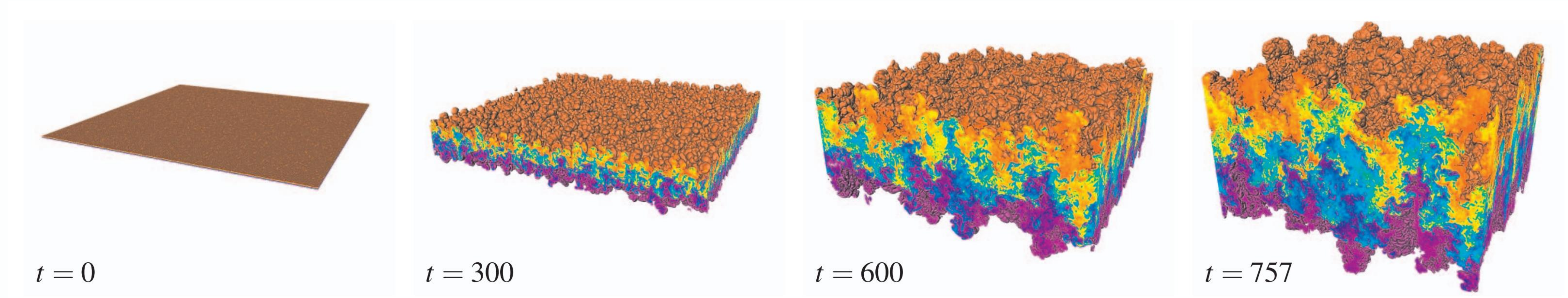




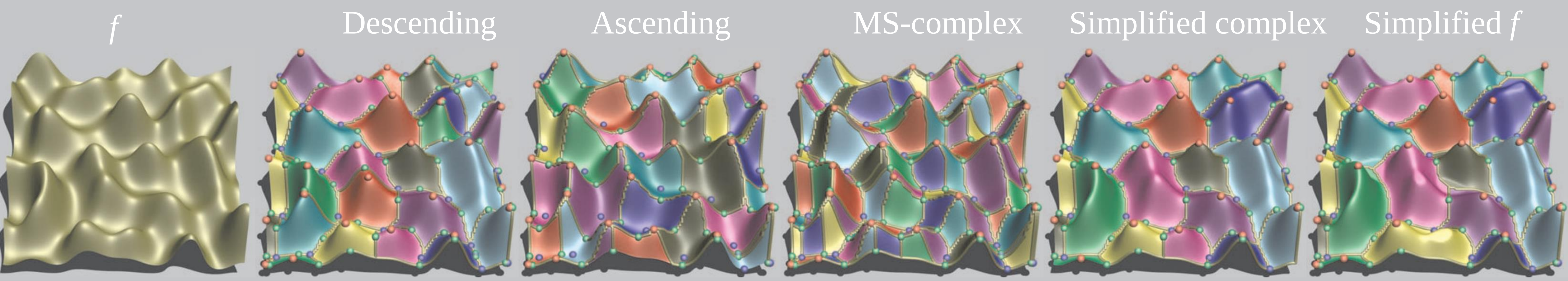


A. Gyulassy et al., **"Topologically Clean Distance Fields,"** in IEEE Transactions on Visualization and Computer Graphics, vol. 13, no. 6, pp. 1432-1439, Nov.-Dec. 2007.

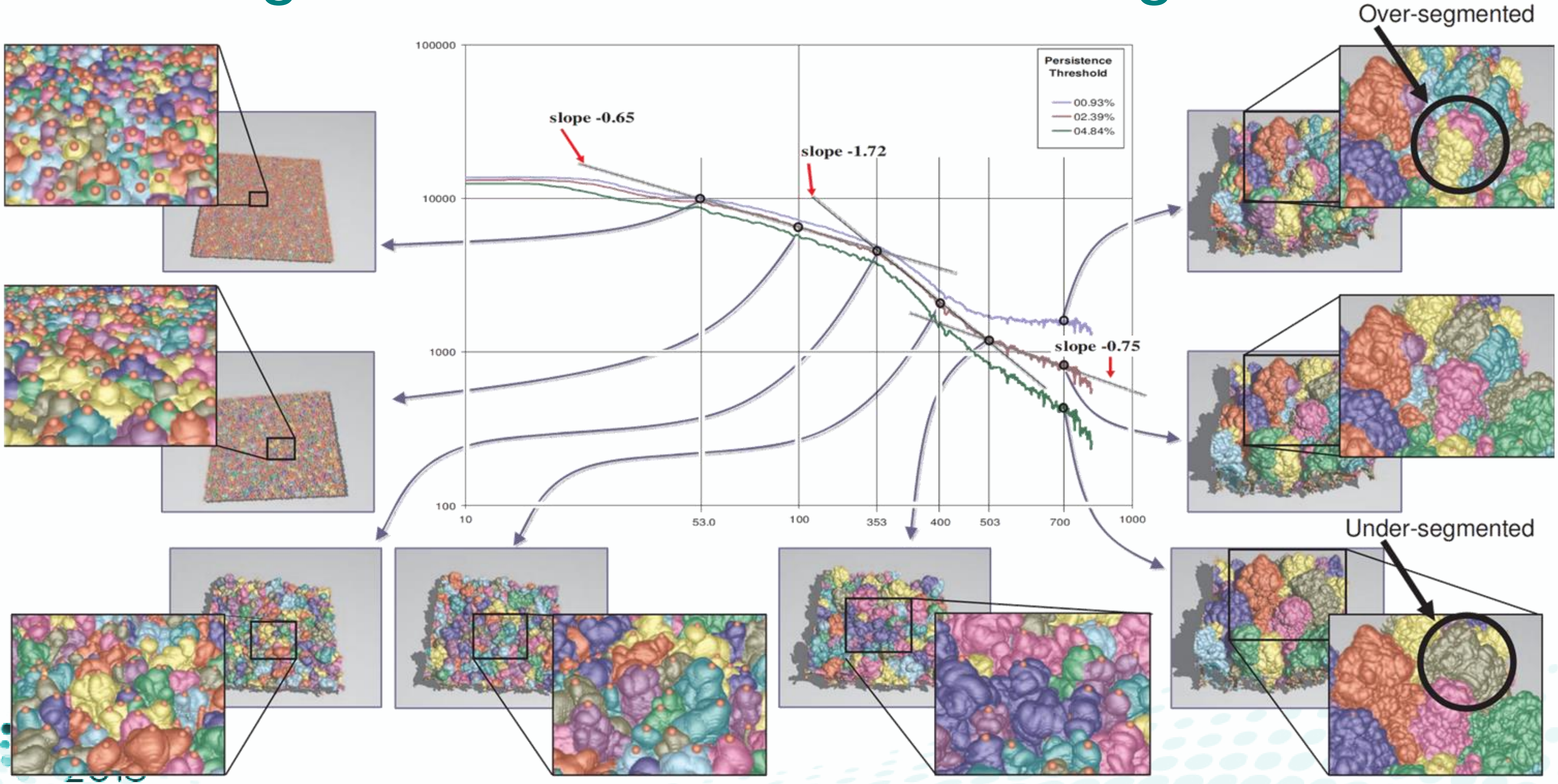
Counting bubbles in turbulent mixing



Pick a value, extract isosurface, analyze height function of it

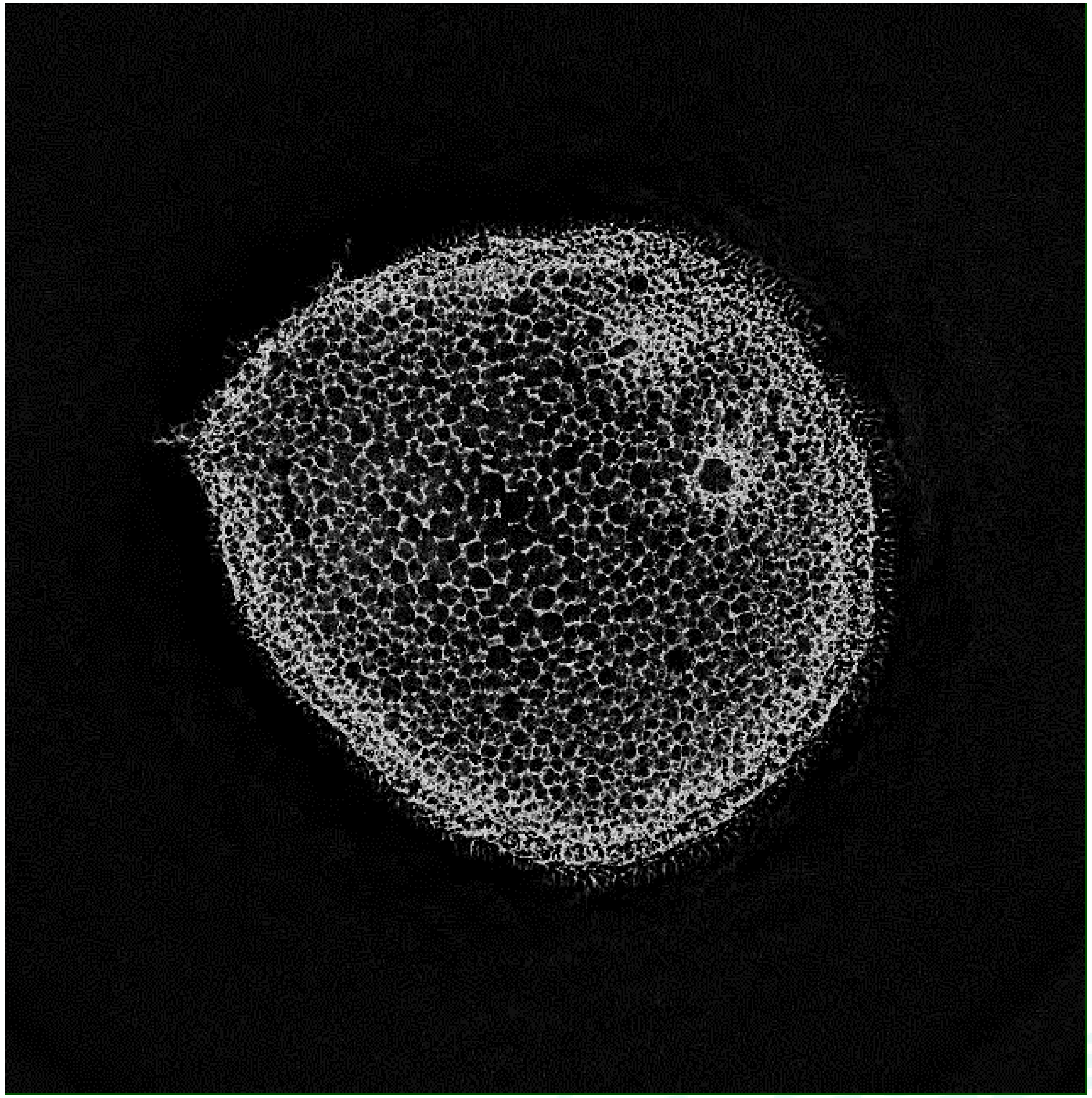


Counting bubbles in turbulent mixing



Deformation of voids in a foam

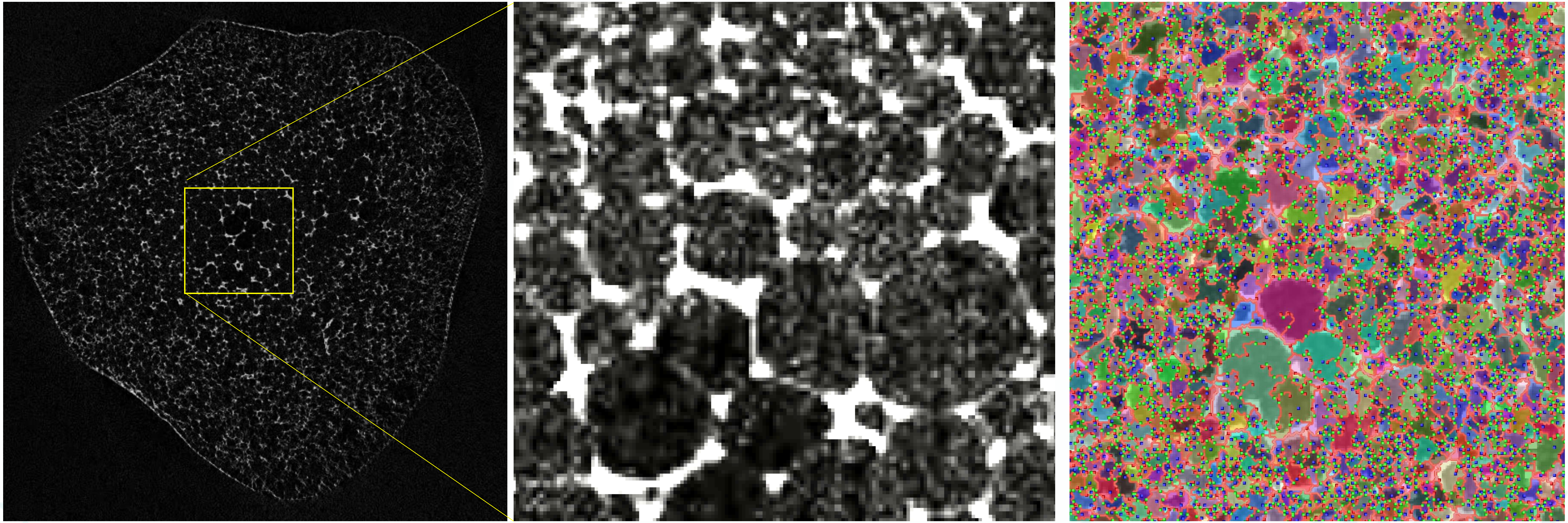
Extract basins from MS complex, measure sizes, boundaries, etc.?



VIS 2018

Deformation of voids in a foam

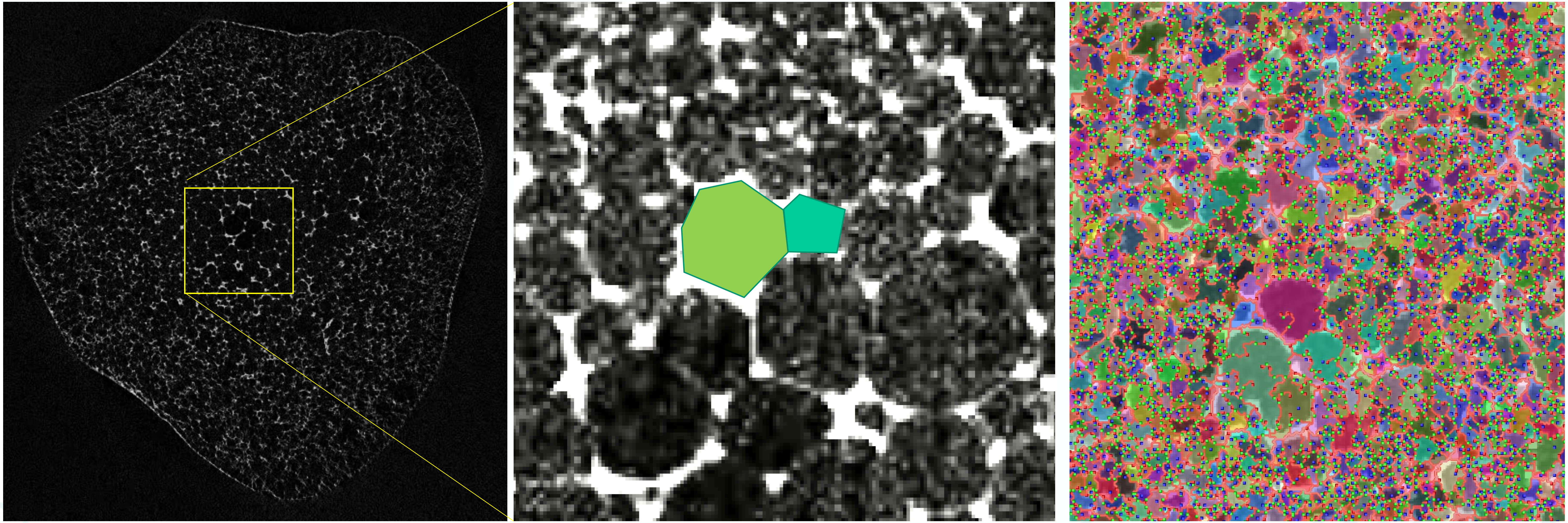
Extract basins from MS complex, measure sizes, boundaries, etc.?



CT scans typically have high-frequency high-amplitude noise

Deformation of voids in a foam

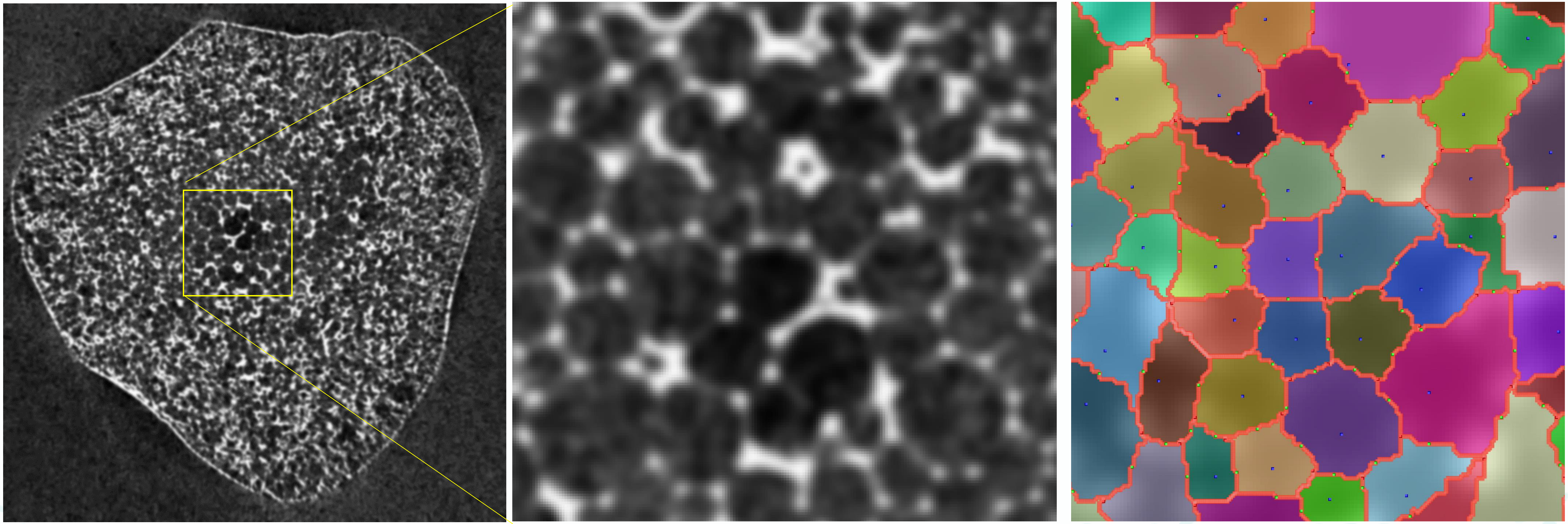
Extract basins from MS complex, measure sizes, boundaries, etc.?



CT scans typically have high-frequency high-amplitude noise

Deformation of voids in a foam

Extract basins from MS complex, measure sizes, boundaries, etc.?



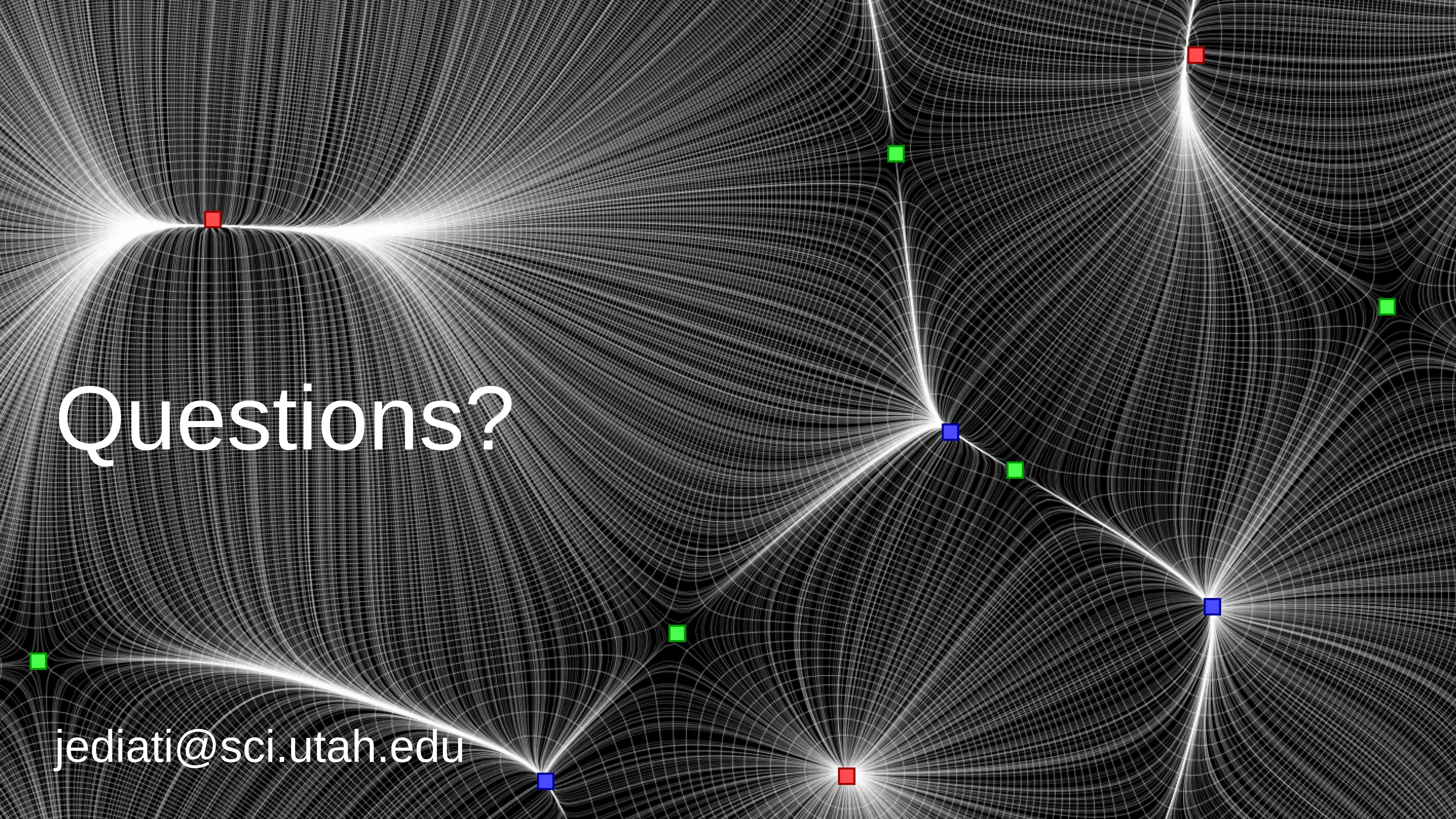
Having a model for noise, and an idea of scales can help

Words of wisdom? More like, *musings*…

I have never gotten the translation to abstraction exactly right on the first attempt (sample size 20+ application domains in 10 years)

Visualize!!! Visual feedback and debugging is invaluable in converging to a robust feature definition

Topology tells you what *IS*, not necessarily what you think there should be



Questions?

jediati@sci.utah.edu