Lab #1 - solutions

Vittorio Zampinetti*

Mauro Gasparini[†]

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Lab 1 (with solutions)

Iris Dataset

Download the dataset at this link (https://archive.ics.uci.edu/ml/datasets/iris) (look for the iris.data file) and place the file together with your R script (for simplicity). At that web page, you can also get some information regarding the origin and nature of the data.

The dataset is available as Comma-Separated Values (CSV) file, which is nothing but a plain text file where each row is a row of a table and every column is separated by a comma. To make it look more like a CSV file, rename it from iris.data to iris.csv.

Import data

Now we're ready to import the data in R and view it as a table.

Hint: use read_csv() function from the readr library. Check the function arguments. Also, you might need to manually add the column names. To check which column names should be added and in which order, check under "attribute information" on the dataset reference page linked above.

^{## #} A tibble: 150 x 5

^{*}Politecnico di Torino, vittorio.zampinetti@polito.it

[†]Politecnico di Torino, mauro.gasparini@polito.it

```
##
      sepal_length sepal_width petal_length petal_width class
##
              <dbl>
                           <dbl>
                                         <dbl>
                                                      <dbl> <chr>
##
    1
                5.1
                             3.5
                                           1.4
                                                        0.2 Iris-setosa
    2
                4.9
                             3
                                           1.4
##
                                                        0.2 Iris-setosa
##
    3
                4.7
                             3.2
                                           1.3
                                                        0.2 Iris-setosa
##
    4
                4.6
                             3.1
                                           1.5
                                                        0.2 Iris-setosa
##
    5
                5
                             3.6
                                           1.4
                                                        0.2 Iris-setosa
                                                        0.4 Iris-setosa
##
    6
                5.4
                             3.9
                                           1.7
##
    7
                4.6
                             3.4
                                           1.4
                                                        0.3 Iris-setosa
##
    8
                5
                             3.4
                                           1.5
                                                        0.2 Iris-setosa
##
    9
                4.4
                             2.9
                                           1.4
                                                        0.2 Iris-setosa
                4.9
                                           1.5
## 10
                             3.1
                                                        0.1 Iris-setosa
## # ... with 140 more rows
```

Answer these questions:

- How many observations does the dataset consist of?
- Which are the different classes?

```
iris_df %>%
    nrow()

## [1] 150

iris_df %>%
    pull(class) %>%
    unique()

## [1] "Iris-setosa" "Iris-versicolor" "Iris-virginica"
```

Exploratory Data Analysis (EDA)

First compute the mean and standard deviation of each measure for each class separately.

```
iris_df %>%
  group_by(class) %>%
  summarize(
   msl = mean(sepal_length), msw = mean(sepal_width),
   mpl = mean(petal_length), mpw = mean(petal_width),
   ssl = sd(sepal_length), ssw = sd(sepal_width),
   spl = sd(petal_length), spw = sd(petal_width)
)
```

```
## # A tibble: 3 x 9
##
     class
                                    mpl
                                           mpw
                                                 ssl
##
     <chr>>
                      <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <
## 1 Iris-setosa
                       5.01
                             3.42
                                   1.46 0.244 0.352 0.381 0.174 0.107
## 2 Iris-versicolor
                      5.94
                             2.77
                                   4.26 1.33 0.516 0.314 0.470 0.198
## 3 Iris-virginica
                       6.59
                            2.97 5.55 2.03 0.636 0.322 0.552 0.275
```

• What can you infer? Is there any measure which is more indicative of a certain class?

You can also plot the empirical distribution of the four measures separately, in order to better visualize how far (or close) they are from each other.

• Plot the distributions of the four measures in a 2x2 grid, differentiating the types with color encoding (optional).

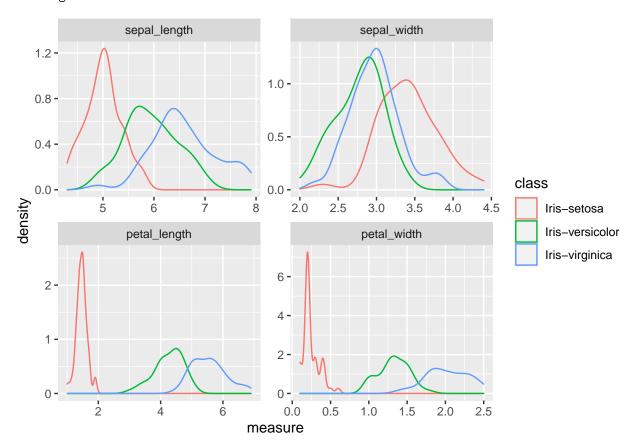
Hint: You can use melt() from the reshape2 library to transform the dataset, and then plot the various densities with color encoding on the four measures.

library(reshape2)

```
##
## Attaching package: 'reshape2'
## The following object is masked from 'package:tidyr':
##
## smiths

iris_df %>%
   melt(value.name = "measure") %>% # collapse all measures in one col
   # then add a column with the measure name
   ggplot() +
   geom_density(aes(measure, color = class)) +
   facet_wrap(vars(variable), nrow = 2, scales = "free")
```

Using class as id variables



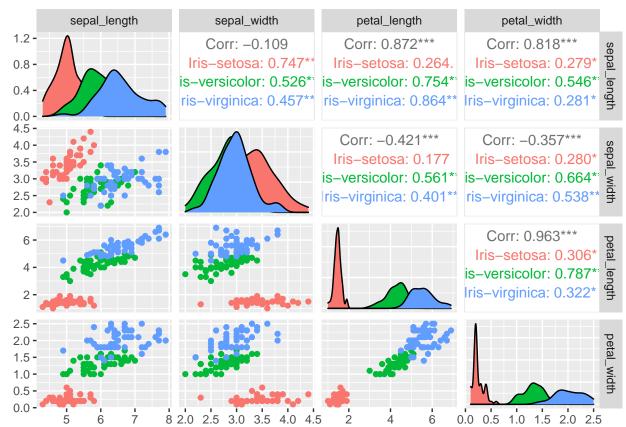
We can also visualize the four measures on a set of plots that shows the correlation between variables, two-by-two. This is easily done with a pair plot.

• Draw a pair plot (optional).

Hint: install the package with install.packages("GGally") and load it with library(GGally), then read the help document of the ggpairs() function. You can also use R base pairs() function.

library(GGally)

```
## Registered S3 method overwritten by 'GGally':
## method from
## +.gg ggplot2
iris_df %>%
    ggpairs(mapping = aes(color = class), columns = 1:4)
```



Confidence intervals

Let's make our decision more statistically relevant. Select only one class, the Setosa type, and one measure, petal length.

Now build a 95% CI around the mean value of the Setosa petal length. Assume σ unknown.

Formally, we want to find x_l, x_u s.t.

$$P(x_l \le \bar{X} \le x_u) = 95\%.$$

• Find such CI, using only R base operators (mean, sd).

To do this, remember that

- $\frac{(\bar{X}-\mu_0)\sqrt{n}}{s} \sim T(n-1)$ where s is the sample standard deviation
- qt() is the R function for the t-distribution quantile
- Validate your result by using t.test() to get the CI.

```
setosa_petal_length <- iris_df %>%
filter(class == "Iris-setosa") %>%
```

```
pull(petal_length)
nspl <- length(setosa_petal_length)</pre>
crit_t \leftarrow qt(.975, nspl - 1)
mu0 <- mean(setosa_petal_length)</pre>
se <- sd(setosa_petal_length) / sqrt(nspl)</pre>
delta <- crit_t * se</pre>
ci <- c(mu0 - delta, mu0 + delta)
## [1] 1.414689 1.513311
With t.test
t.test(setosa_petal_length, mu = mu0, conf.level = 0.95)
##
   One Sample t-test
##
## data: setosa_petal_length
## t = 0, df = 49, p-value = 1
## alternative hypothesis: true mean is not equal to 1.464
## 95 percent confidence interval:
## 1.414689 1.513311
## sample estimates:
## mean of x
       1.464
##
```

Compare this confidence interval to the mean and std-dev values you got for each class for the petal length measure.

• Do you think the petal length is a good measure to differentiate between Setosa and the other two types? Why?

P-value

Imagine you get measurements in terms of all 4 indicators of 5 iris flowers belonging to the same class, but you don't know which. These are the observations.

```
x_sample <- tibble(
    sepal_length = c(4.738759, 5.545983, 5.389729, 4.549803, 5.896723),
    sepal_width = c(3.132478, 3.537232, 3.217107, 3.190097, 3.636949),
    petal_length = c(1.220472, 1.321923, 1.573662, 1.289875, 1.705737),
    petal_width = c(0.23, 0.09, 0.33, 0.22, 0.18)
)</pre>
```

• Just looking at the values of the petal length, can you take advantage of the CI you just computed in order to make a guess about the class of these flowers? (Setosa or not Setosa)

To make this guess more statistically relevant, we have to quantify our confidence.

• What is the *p-value* of this sample, only looking at the petal length, against the null hypothesis that the samples are coming from the Setosa type?

Remember that the p-value is defined as

$$P\left(T \ge \frac{(\bar{\mathbf{x}} - \mu_0)\sqrt{n}}{s} | \mu = \mu_0\right)$$

• Based on the value you found, what can you say about the class of this sample?

```
# manually
nx <- nrow(x sample)</pre>
t_stat <- (mean(x_sample$petal_length) - mu0) * sqrt(nx) /</pre>
  sd(x_sample$petal_length)
pt(t_stat, df = nx - 1, lower.tail = FALSE)
## [1] 0.6619335
# with t-test
t.test(x_sample$petal_length, mu = mu0)
##
##
    One Sample t-test
##
## data: x_sample$petal_length
## t = -0.44983, df = 4, p-value = 0.6761
## alternative hypothesis: true mean is not equal to 1.464
## 95 percent confidence interval:
## 1.165161 1.679506
## sample estimates:
## mean of x
## 1.422334
Now take this other sample:
y_sample <- tibble(</pre>
  sepal length = c(6.303990, 6.705969, 7.795044, 7.015665, 7.056670),
  sepal_width = c(3.152411, 2.499612, 3.293934, 3.275724, 2.843923),
  petal_length = c(6.356729, 5.975576, 4.998114, 5.423811, 5.382871),
  petal\_width = c(1.589342, 2.305014, 2.260900, 2.185519, 1.589370)
)
  • What is the p-value?
  • Is it higher or lower than 0.025?
  • Could we guess the answer to this last question without computing it?
# manually
ny <- nrow(y_sample)</pre>
t_stat <- (mean(y_sample$petal_length) - mu0) * sqrt(ny) / sd(y_sample$petal_length)
pt(t_stat, df = ny - 1, lower.tail = FALSE)
## [1] 3.231502e-05
# with t-test
t.test(y_sample$petal_length, mu = mu0)
##
   One Sample t-test
##
##
## data: y_sample$petal_length
## t = 17.36, df = 4, p-value = 6.463e-05
## alternative hypothesis: true mean is not equal to 1.464
## 95 percent confidence interval:
```

```
## 4.961539 6.293302
## sample estimates:
## mean of x
```

5.62742