

Image filtering

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Outline

Point operations

Image filtering

Frequential decomposition

Mathematical operations applied to images

Mathematical operations (addition, subtraction, multiplication, division, log, ...) applied to **each pixel** of the image:

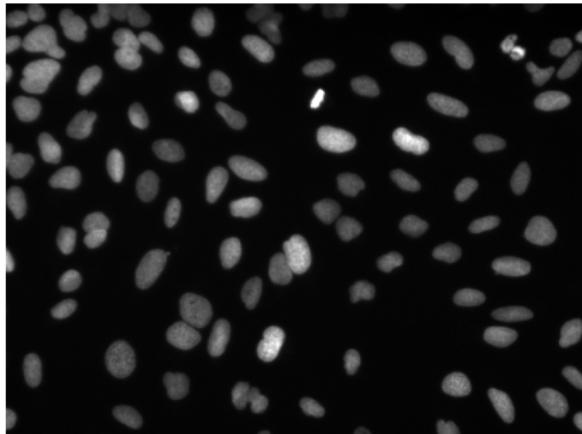
$$\mathbf{I}_1 = f(\mathbf{I}_0), \text{ where } f(I_0(x, y)) = I_0(x, y) + a \text{ for example} \quad (1)$$

Mathematical operations applied to images

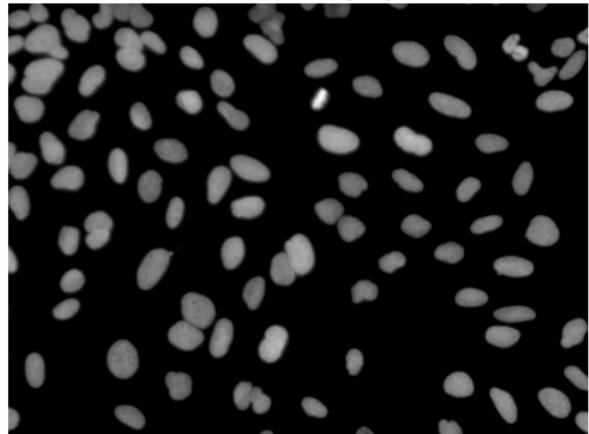
Mathematical operations (addition, subtraction, multiplication, division, log, ...) applied to **each pixel** of the image:

$$\mathbf{I}_1 = f(\mathbf{I}_0), \text{ where } f(I_0(x, y)) = I_0(x, y) + a \text{ for example} \quad (1)$$

\mathbf{I}_0



$\mathbf{I}_1 = \log(\mathbf{I}_0)$



Practice

<https://youtu.be/0hV0FHz70HQ>

Mathematical operations applied between images

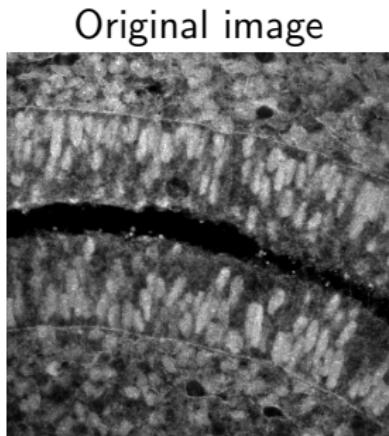
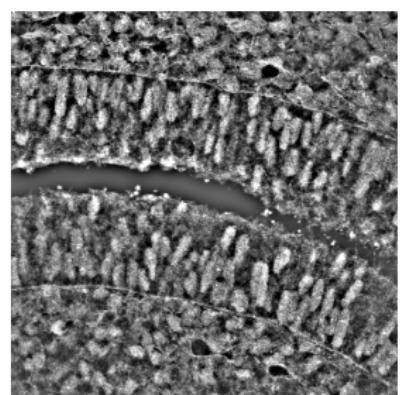
Mathematical operations (addition, subtraction, multiplication, division, ...) applied between the **pixels of two images** with the same size

$$\mathbf{I}_3 = g(\mathbf{I}_1, \mathbf{I}_2), \text{ where } g(\mathbf{I}_1, \mathbf{I}_2) = I_1(x, y) + I_2(x, y) \text{ for example} \quad (2)$$

Mathematical operations applied between images

Mathematical operations (addition, subtraction, multiplication, division, ...) applied between the **pixels of two images** with the same size

$$\mathbf{I}_3 = g(\mathbf{I}_1, \mathbf{I}_2), \text{ where } g(\mathbf{I}_1, \mathbf{I}_2) = I_1(x, y) + I_2(x, y) \text{ for example} \quad (2)$$

 \mathbf{I}_1  $f(\mathbf{I}_1) \text{ where } f \text{ is a Gaussian blur}$  $\mathbf{I}_2 = g(\mathbf{I}_1, f(\mathbf{I}_1)) \text{ where } g \text{ is a subtraction}$

Practice

<https://youtu.be/EFZQTzM2c6s>

Outline

Point operations

Image filtering

Frequential decomposition

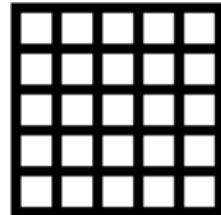
Linear filtering

Image

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	5	5	5	0	0
0	0	5	10	5	0	0
0	0	5	5	5	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Filter

1	1	1
1	1	1
1	1	1

Filtered image

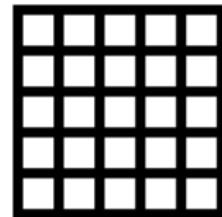
Linear filtering

Image

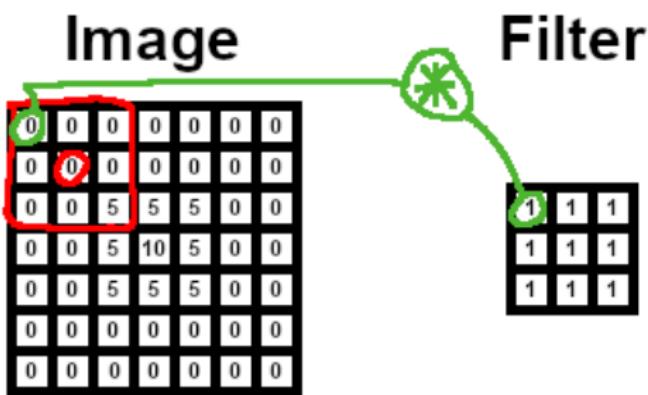
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	5	5	5	0	0	0
0	0	5	10	5	0	0	0
0	0	5	5	5	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Filter

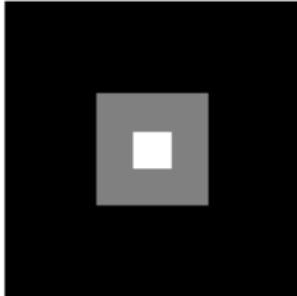
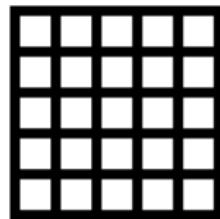
1	1	1
1	1	1
1	1	1

Filtered image

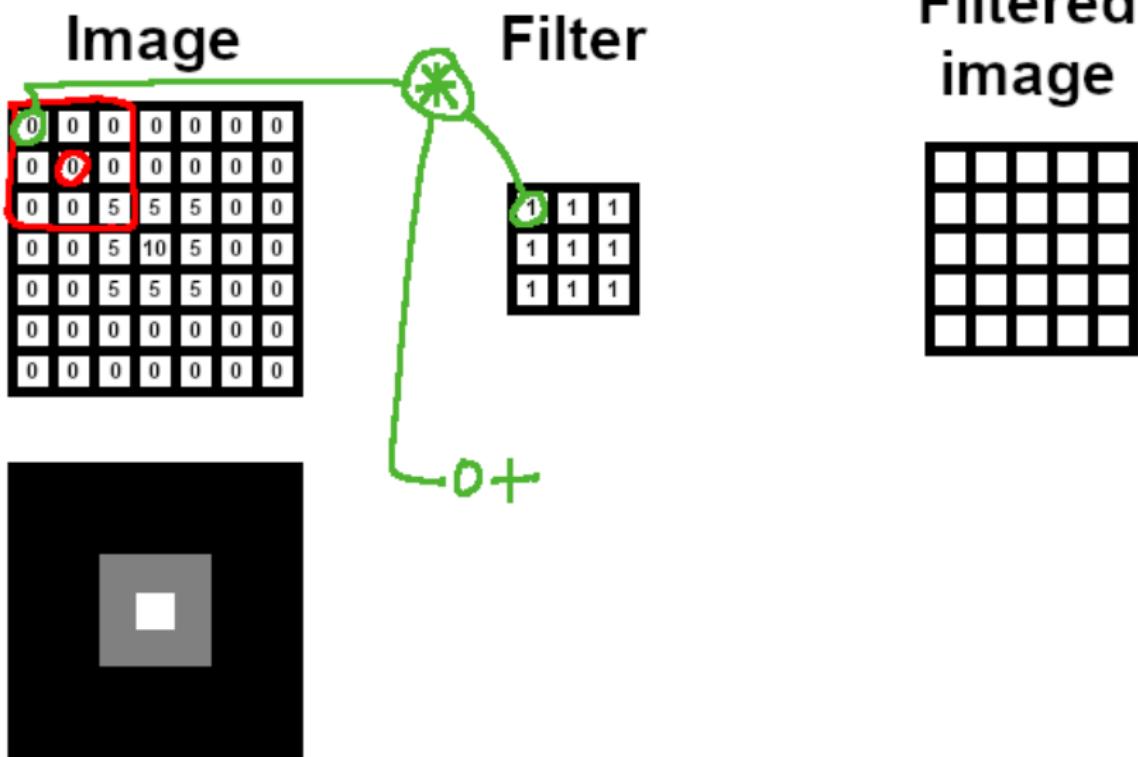
Linear filtering



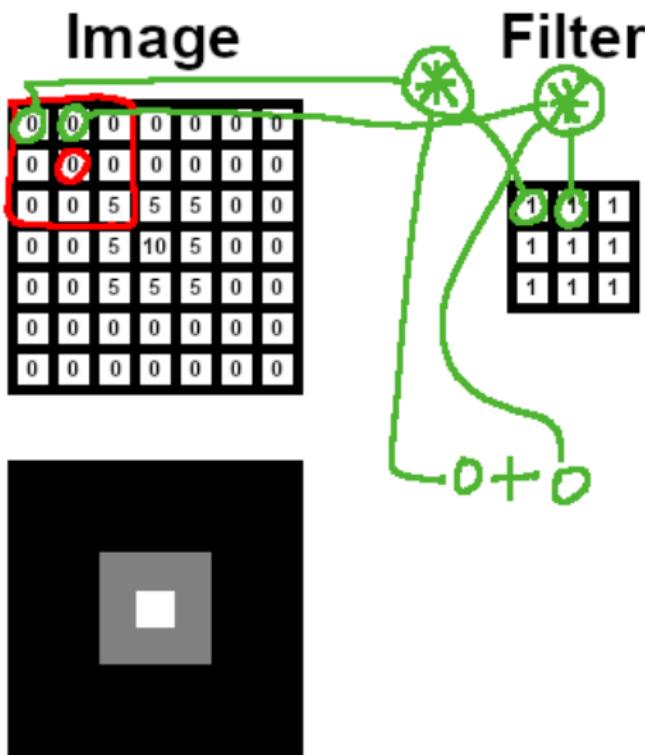
Filtered image



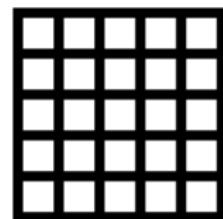
Linear filtering



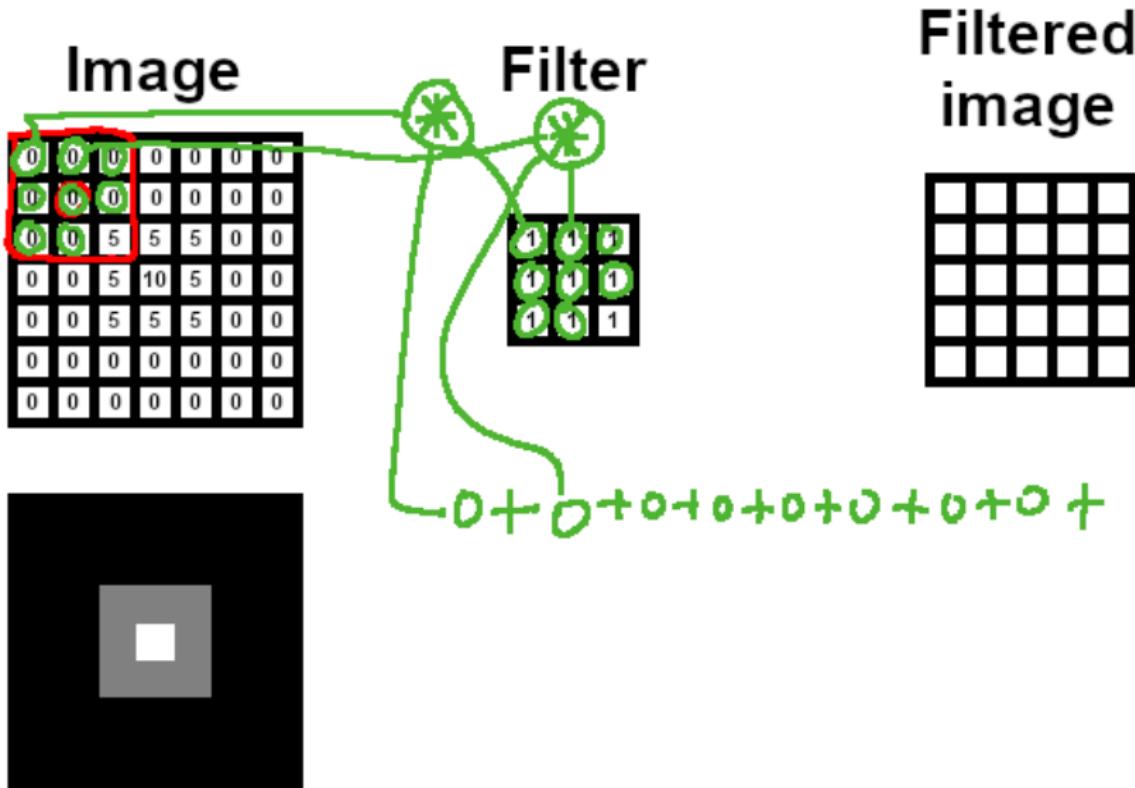
Linear filtering



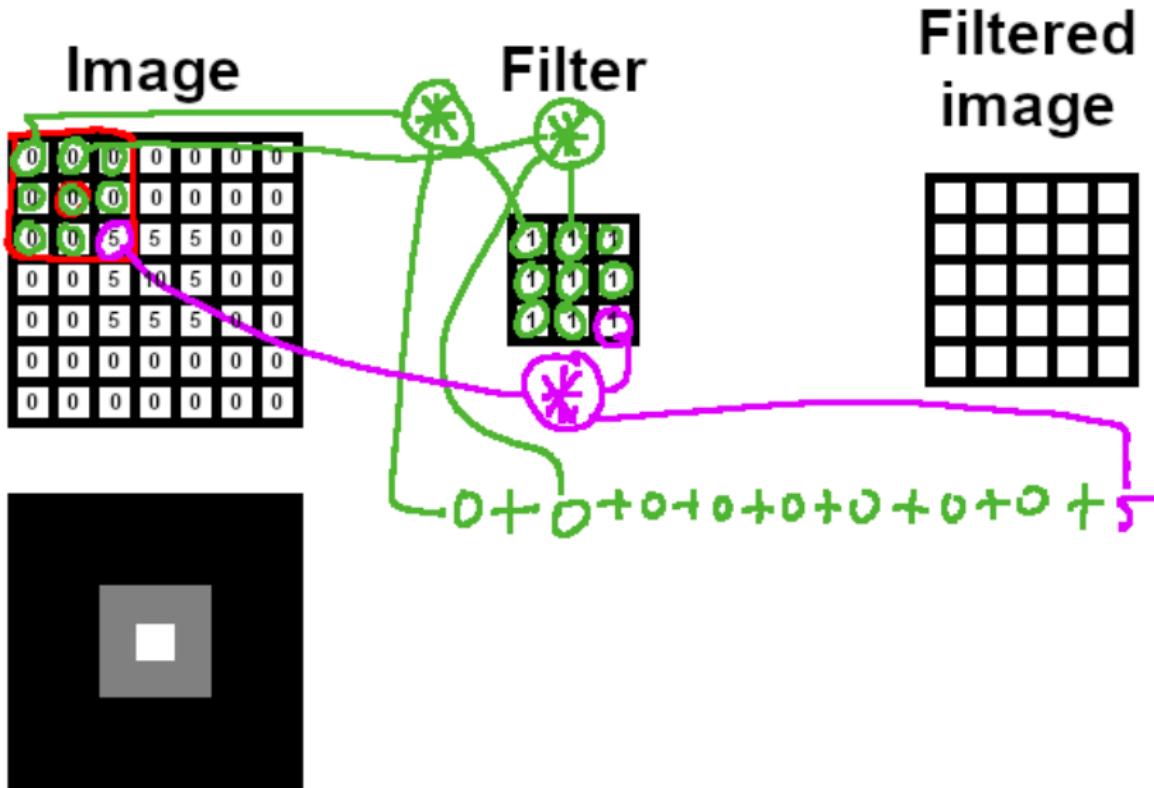
Filtered
image



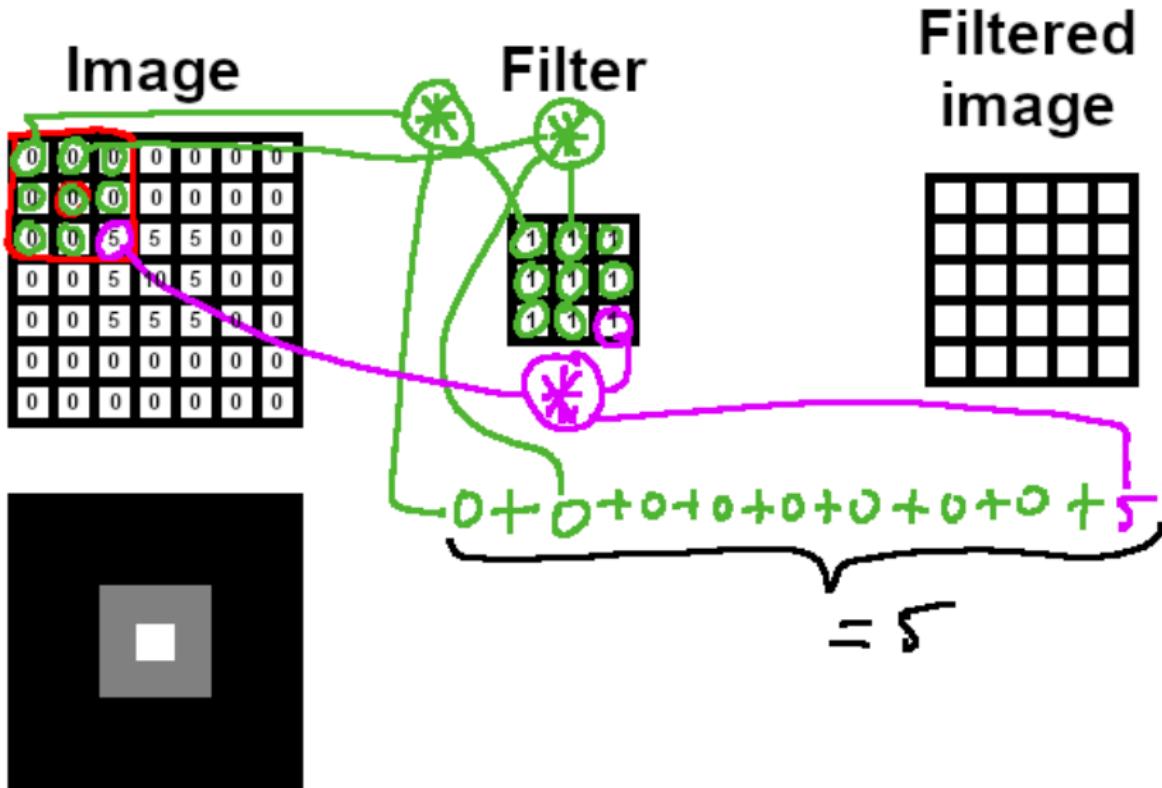
Linear filtering



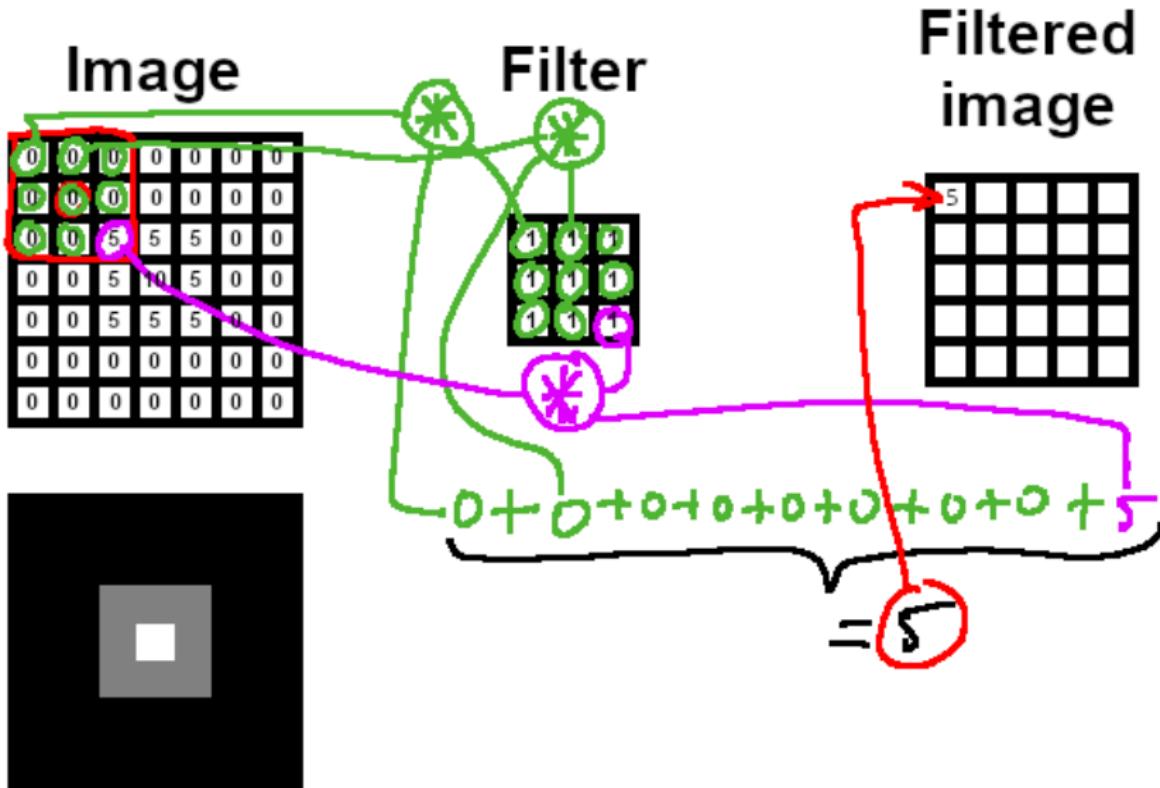
Linear filtering



Linear filtering



Linear filtering



Linear filtering

Image

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	5	5	5	0	0
0	0	5	10	5	0	0
0	0	5	5	5	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Filter

1	1	1
1	1	1
1	1	1

Filtered image

5		



Linear filtering

Image

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	5	5	5	0	0
0	0	5	10	5	0	0
0	0	5	5	5	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Filter

1	1	1
1	1	1
1	1	1

Filtered image

5		



Linear filtering

Image

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	5	5	5	0	0
0	0	5	10	5	0	0
0	0	5	5	5	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Filter

1	1	1
1	1	1
1	1	1

Filtered image

5		

$$0+0+0+0+0+0+\frac{5}{9}$$



Linear filtering

Image

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	5	5	5	0	0
0	0	5	10	5	0	0
0	0	5	5	5	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0



Filter

1	1	1
1	1	1
1	1	1

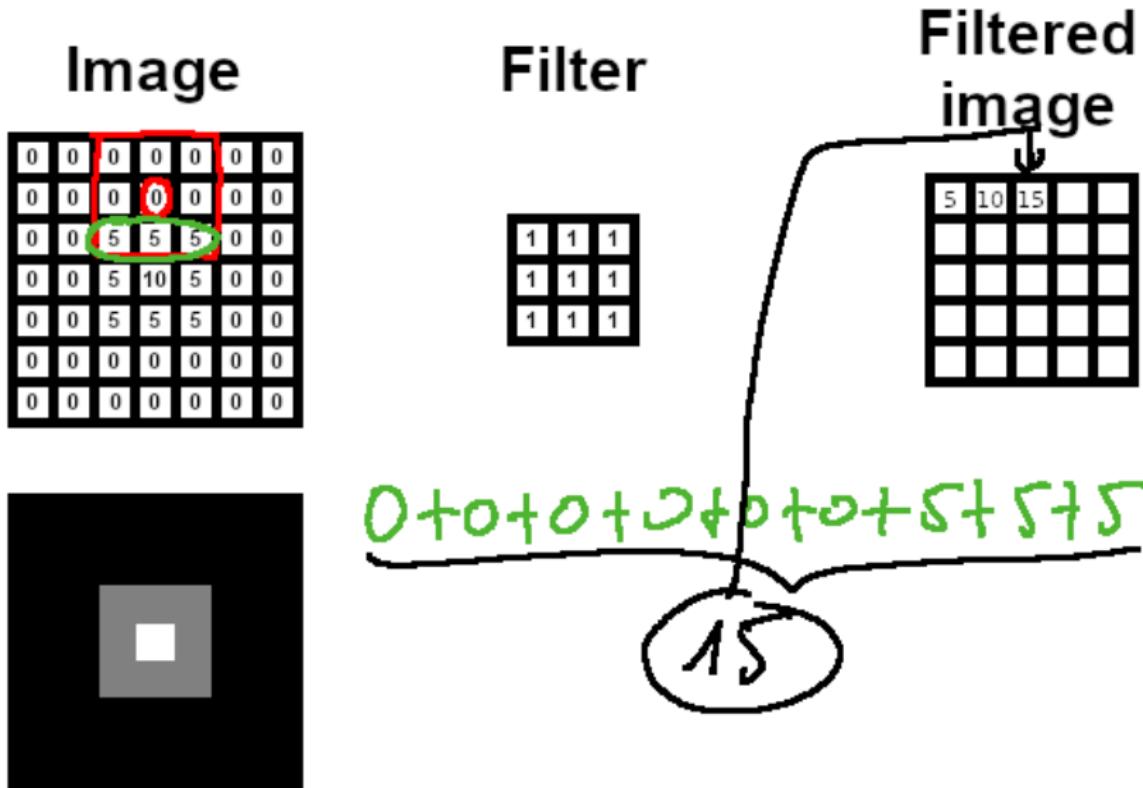
Filtered image

5	10		

$$0 + 0 + 0 + 0 + 0 + 0 + 0 + 10$$

10

Linear filtering



Linear filtering

Image

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	5	5	5	0	0
0	0	5	10	5	0	0
0	0	5	5	5	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Filter

1	1	1
1	1	1
1	1	1

Filtered image

5	10	15	10	5
10	25	35	25	10
15	35			



Linear filtering

Image

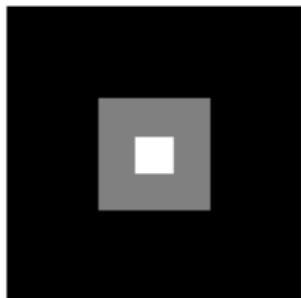
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	5	5	5	0	0
0	0	5	10	5	0	0
0	0	5	5	5	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Filter

1	1	1
1	1	1
1	1	1

Filtered image

5	10	15	10	5
10	25	35	25	10
15	35	50		



$$S + S + S + F + 10 + S + S + S + F$$

50

Linear filtering

Image

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	5	5	5	0	0
0	0	5	10	5	0	0
0	0	5	5	5	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Filter

1	1	1
1	1	1
1	1	1

Filtered image

5	10	15	10	5
10	25	35	25	10
15	35	50	35	15
10	25	35	25	10
5	10	15	10	5



Linear filtering

Image

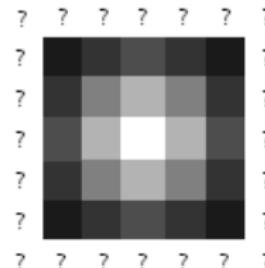
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	5	5	5	0	0
0	0	5	10	5	0	0
0	0	5	5	5	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Filter

1	1	1
1	1	1
1	1	1

Filtered image

5	10	15	10	5
10	25	35	25	10
15	35	50	35	15
10	25	35	25	10
5	10	15	10	5



Linear filtering

Image

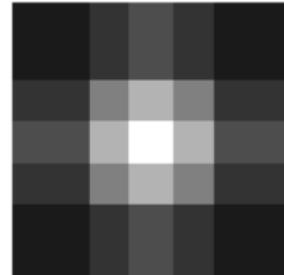
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	5	5	5	0	0
0	0	5	10	5	0	0
0	0	5	5	5	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Filter

1	1	1
1	1	1
1	1	1

Filtered image

5	10	15	10	5
10	25	35	25	10
15	35	50	35	15
10	25	35	25	10
5	10	15	10	5



Linear filtering

Original image



1	1	1
1	1	1
1	1	1

1	2	1
2	4	2
1	2	1

-1	0	1
-2	0	2
-1	0	1

-1	-2	-1
0	0	0
1	2	1

Linear filtering

Original image



Mean filter



1	1	1
1	1	1
1	1	1

1	2	1
2	4	2
1	2	1

-1	0	1
-2	0	2
-1	0	1

-1	-2	-1
0	0	0
1	2	1

Linear filtering

Original image



Mean filter



1	1	1
1	1	1
1	1	1

Gaussian filter



1	2	1
2	4	2
1	2	1

-1	0	1
-2	0	2
-1	0	1

-1	-2	-1
0	0	0
1	2	1

Linear filtering

Original image



Mean filter



1	1	1
1	1	1
1	1	1

Gaussian filter



1	2	1
2	4	2
1	2	1

x contour



-1	0	1
-2	0	2
-1	0	1

y contour



-1	-2	-1
0	0	0
1	2	1

Practice

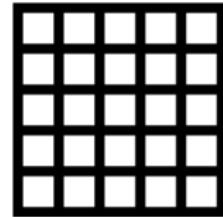
<https://youtu.be/F4xRAaTk1j0>

Non linear filtering

Image

0	0	0	0	0	0	0
0	10	0	0	0	0	0
0	0	5	5	0	0	0
0	0	5	10	5	0	0
0	0	5	5	5	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Median

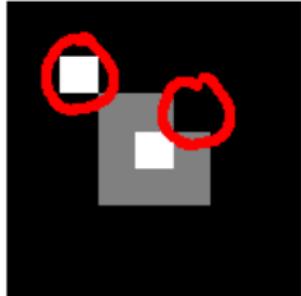
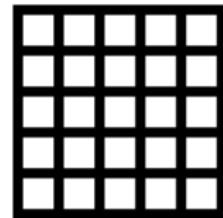
Filtered image

Non linear filtering

Image

0	0	0	0	0	0	0
0	10	0	0	0	0	0
0	0	5	5	0	0	0
0	0	5	10	5	0	0
0	0	5	5	5	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Median

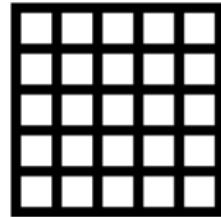
Filtered image

Non linear filtering

Image

0	0	0	0	0	0	0	0
0	10	0	0	0	0	0	0
0	0	5	5	0	0	0	0
0	0	5	10	5	0	0	0
0	0	5	5	5	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Median

Filtered image

Non linear filtering

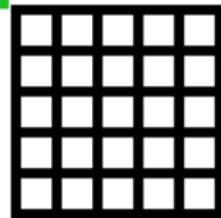
Image

0	0	0	0	0	0	0	0
0	10	0	0	0	0	0	0
0	0	5	5	5	0	0	0
0	0	5	10	5	0	0	0
0	0	5	5	5	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Median

0,0,0,0,10,5,0,0,5

Filtered image



Non linear filtering

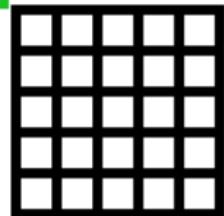
Image

0	0	0	0	0	0	0	0
0	10	0	0	0	0	0	0
0	0	5	5	5	0	0	0
0	0	5	10	5	0	0	0
0	0	5	5	5	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

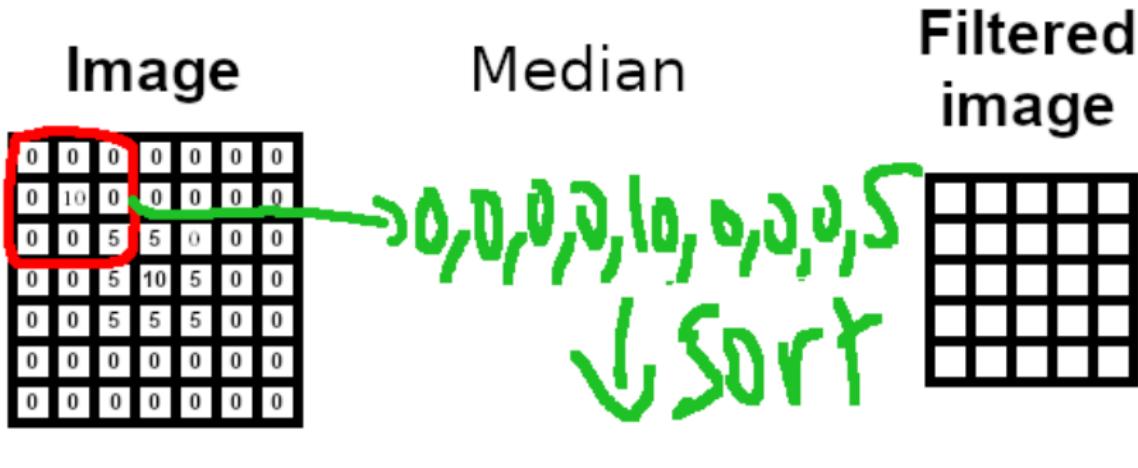
Median

0,0,0,0,10,5,0,0,5
↓ Sort

Filtered image

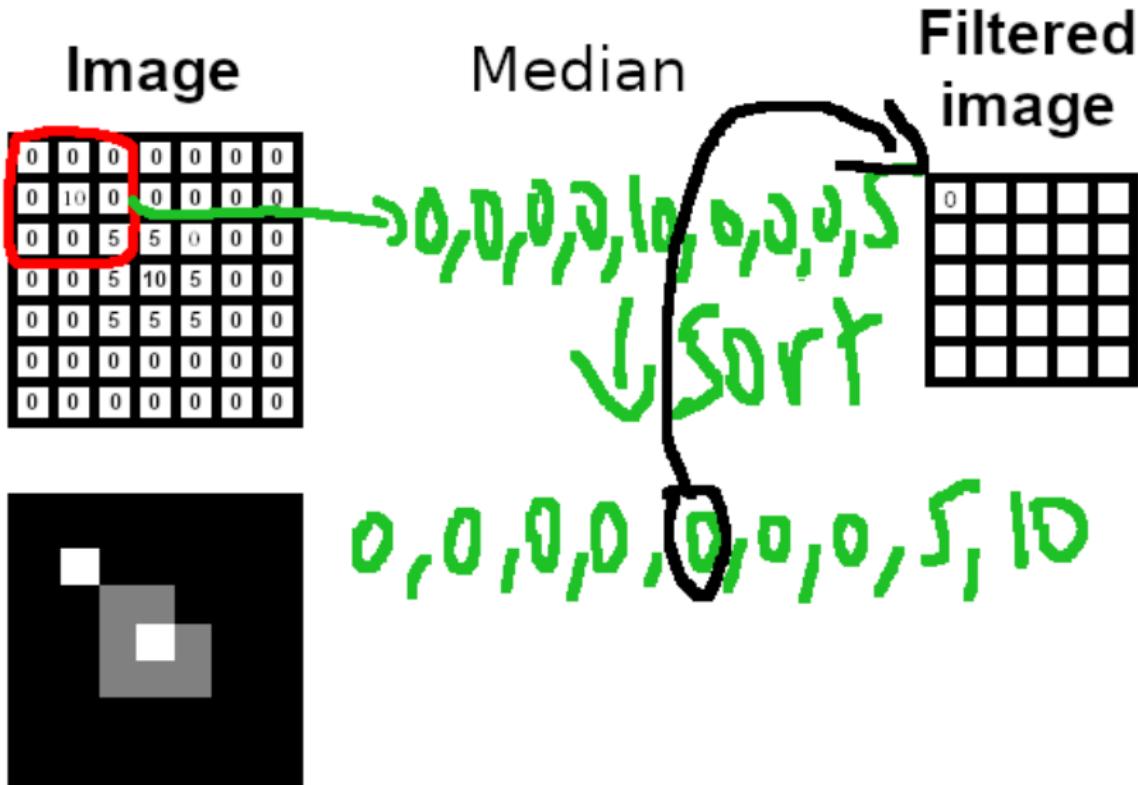


Non linear filtering



0, 0, 0, 0, 0, 0, 5, 10

Non linear filtering



Non linear filtering

Image

0	0	0	0	0	0	0
0	10	0	0	0	0	0
0	0	5	5	0	0	0
0	0	5	10	5	0	0
0	0	5	5	5	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Median

Filtered image

0			



Non linear filtering

Image

0	0	0	0	0	0	0
0	10	0	0	0	0	0
0	0	5	5	0	0	0
0	0	5	10	5	0	0
0	0	5	5	5	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Median

Filtered image

0	0	0	0	0
0				



Non linear filtering

Image

0	0	0	0	0	0	0
0	10	0	0	0	0	0
0	0	5	5	0	0	0
0	0	5	10	5	0	0
0	0	5	5	5	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Median

0, 0, 0, 0, 5, 5, 0, 5, 10

↓ sort

Filtered image

0	0	0	0	0
0				



0, 0, 0, 0, 5, 5, 5, 10, 10

Non linear filtering

Image

0	0	0	0	0	0	0
0	10	0	0	0	0	0
0	0	5	5	0	0	0
0	0	5	10	5	0	0
0	0	5	5	5	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Median

0, 0, 0, 0, 5, 5, 0, 5, 10

↓ sort

Filtered image

0	0	0	0	0
0	5			

0, 0, 0, 0, 5, 5, 5, 10, 10



Non linear filtering

Image

0	0	0	0	0	0	0	0
0	10	0	0	0	0	0	0
0	0	5	5	0	0	0	0
0	0	5	10	5	0	0	0
0	0	5	5	5	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Median

Filtered image

0	0	0	0	0	0
0	5	5	0	0	0
0	5	5	5	0	0
0	0	5	0	0	0
0	0	0	0	0	0



Non linear filtering

Image

0	0	0	0	0	0	0	0
0	10	0	0	0	0	0	0
0	0	5	5	0	0	0	0
0	0	5	10	5	0	0	0
0	0	5	5	5	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Median

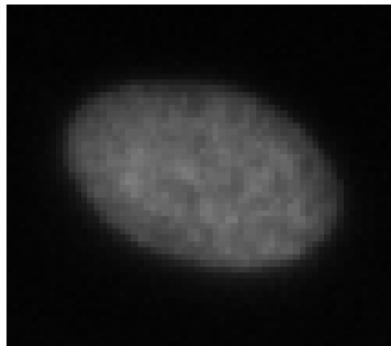
Filtered image

0	0	0	0	0	0
0	5	5	0	0	0
0	5	5	5	0	0
0	0	5	0	0	0
0	0	0	0	0	0

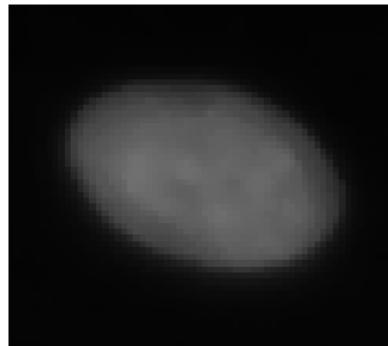


Non linear filtering

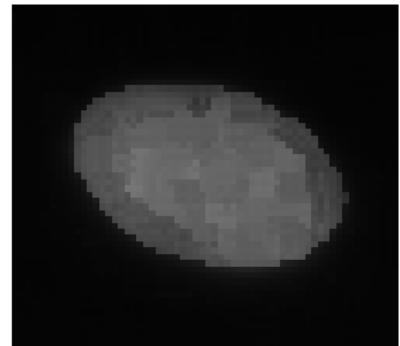
Original image



Median filter



Kuwahara filter



Practice

<https://youtu.be/NOiNLsJ93Jk>

Outline

Point operations

Image filtering

Frequential decomposition

Fourier transform

In signal processing, **Fourier analysis** consists in approximating a function by **sums** of simple **trigonometric** functions

Fourier transform

In signal processing, **Fourier analysis** consists in approximating a function by **sums** of simple **trigonometric** functions

Fourier transform of $f(t)$:

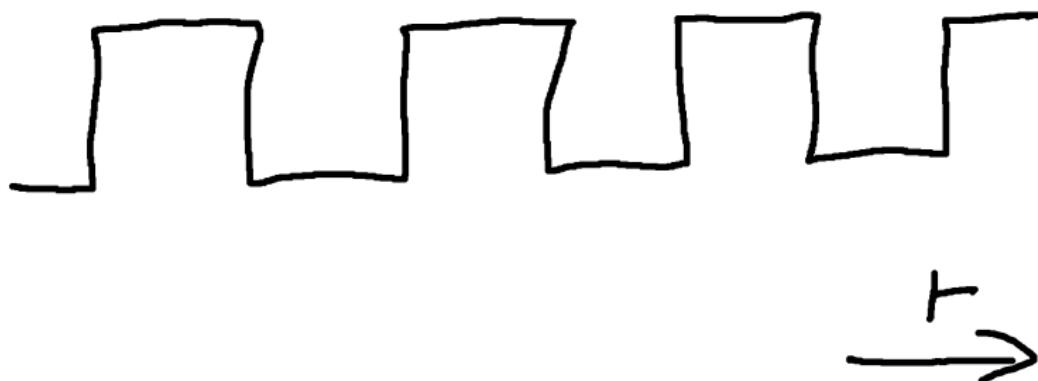
$$\hat{f}(\nu) = \sum_{-\infty}^{+\infty} f(t) e^{-i2\pi\nu t} dt \quad (3)$$

Inverse Fourier transform of $\hat{f}(\nu)$:

$$f(t) = \sum_{-\infty}^{+\infty} \hat{f}(\nu) e^{i2\pi\nu t} dt \quad (4)$$

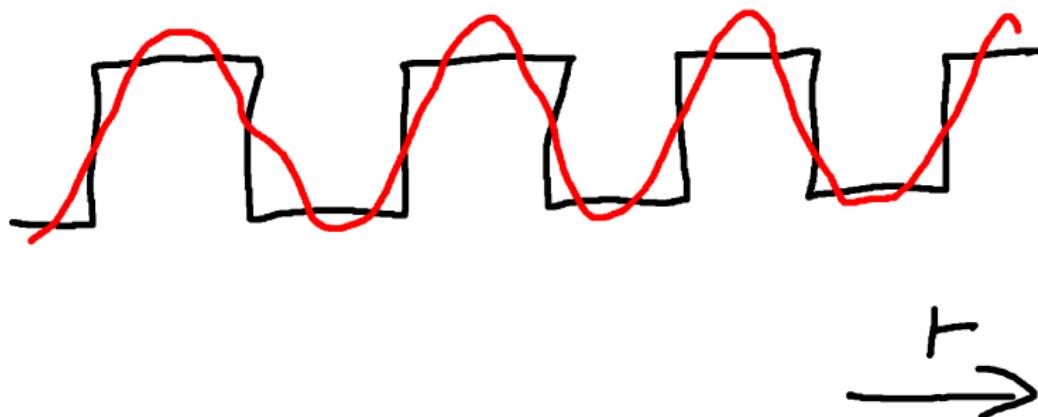
Fourier transform

In signal processing, **Fourier analysis** consists in approximating a function by **sums** of simple **trigonometric** functions



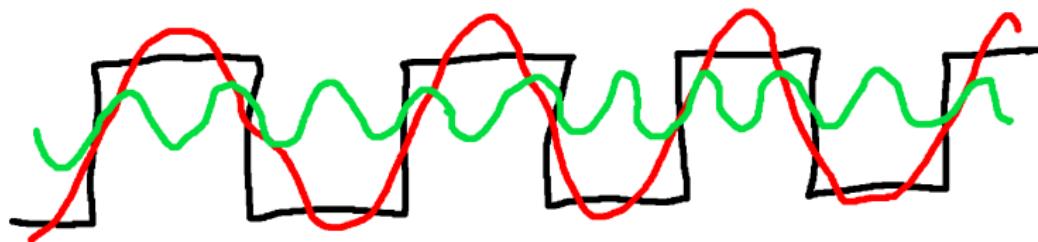
Fourier transform

In signal processing, **Fourier analysis** consists in approximating a function by **sums** of simple **trigonometric** functions



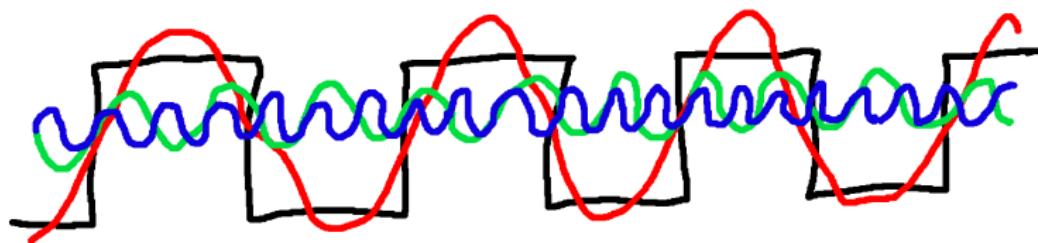
Fourier transform

In signal processing, **Fourier analysis** consists in approximating a function by **sums** of simple **trigonometric** functions



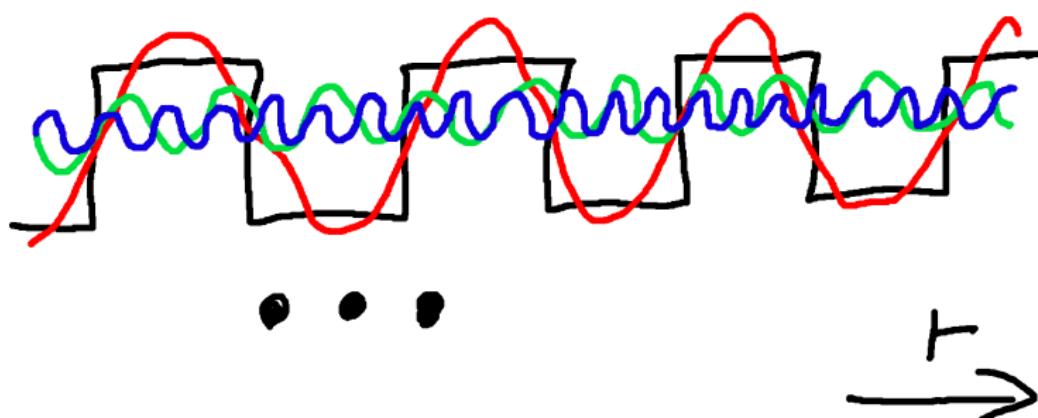
Fourier transform

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Fourier transform

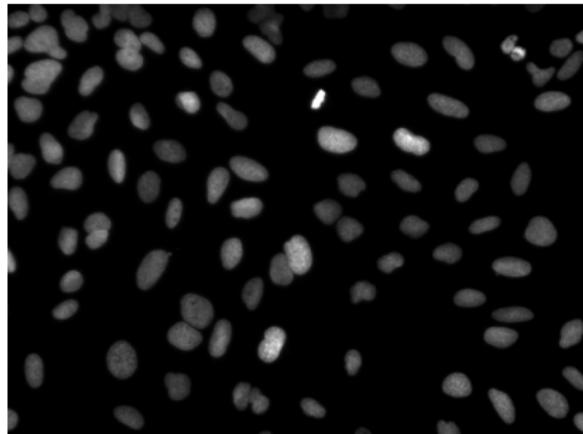
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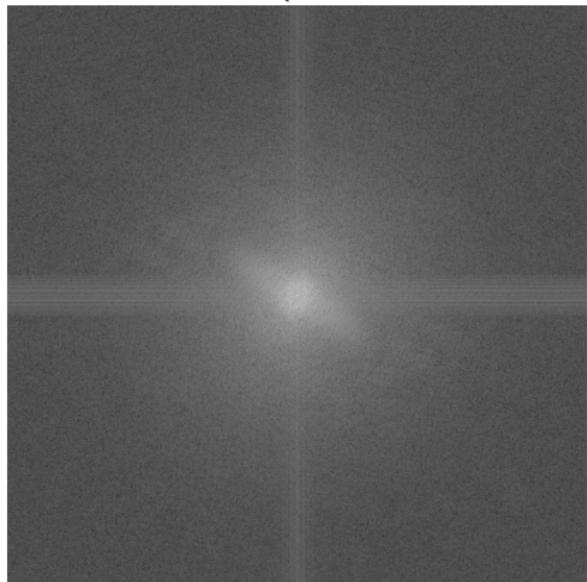
Fourier transform

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Original image (spatial domain)



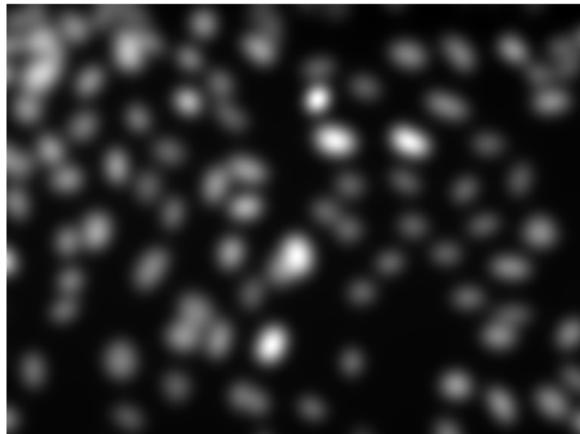
Fourier transform (frequential domain)



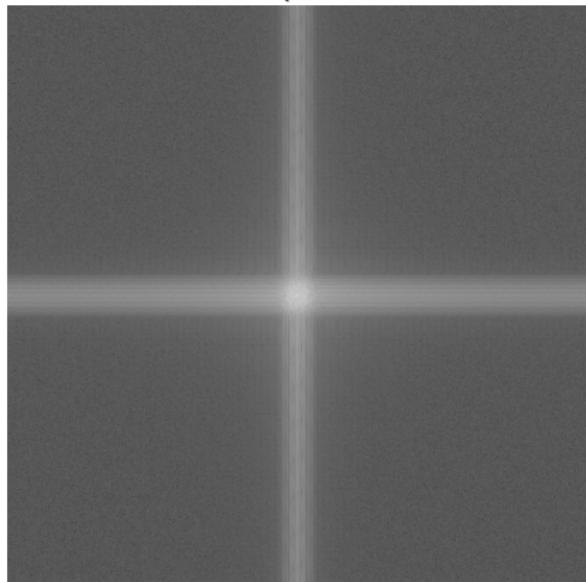
Fourier transform

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Blurred image (spatial domain)



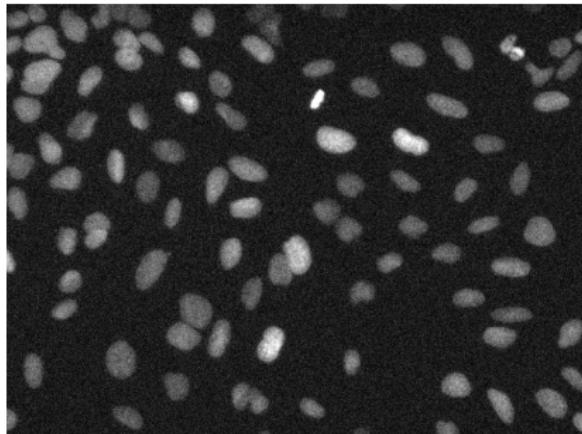
Fourier transform (frequential domain)



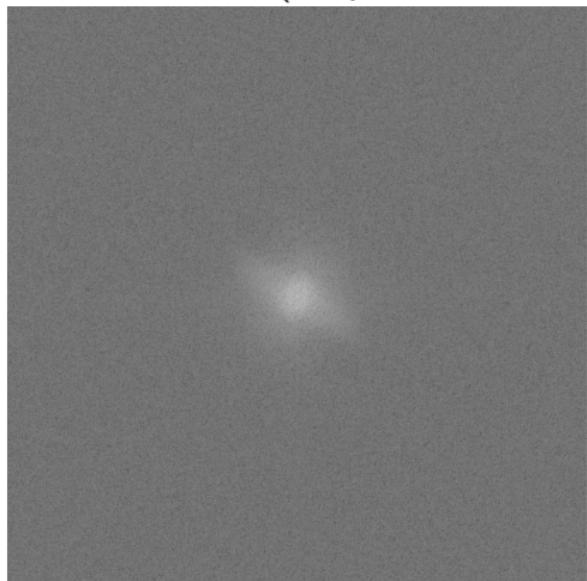
Fourier transform

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Noisy image (spatial domain)



Fourier transform (frequential domain)



Practice

https://youtu.be/E_Vt5djPCfE