TRENTOOL 3.1 – Using the Ragwitz Criterion in TRENTOOL

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What is TRENTOOL? TRENTOOL (<u>TRansfer ENtropy TOOL</u>box) is an open-source MATLAB toolbox that allows the user to easily handle the considerable complexity of transfer entropy (TE) estimation from time series. For the use with neural data TRENTOOL seamlessly integrates with the popular FieldTrip toolbox (Oostenveld, Fries, Maris, & Schoffelen, 2011).

State Space Reconstruction TE estimation requires the reconstruction of the state space from a scalar time series. A state is defined as the collection of random variables that uniquely describe the state of a process. The set of all possible realizations of such state variables is called the state space of the process. State spaces can be (appropriatively) reconstructed using a time-delayed embedding (Takens, 1981), such that reconstructed states take the form

$$\mathbf{x}_{t}^{d_{x}} = (x_{t}, x_{t-1}, \dots, x_{t-(d-1)\tau}),$$

where d is called the embedding dimension and τ the embedding delay. To find optimal values for d and τ , TRENTOOL tests how well user provided pairs of candidate values for d and τ optimize a local predictor (Ragwitz & Kantz, 2002). This is done in the function TEprepare.m

The Ragwitz Criterion We call the dimension d and embedding delay τ the embedding parameters. Embedding parameters are found by optimizing a local predictor proposed in (Ragwitz & Kantz, 2002). The Ragwitz criterion uses candidate values for d and τ and tests how well these candidate values embed the time series (Fig. 1), such that for each time point the next point in time can be predicted with minimum error (Fig. 2).

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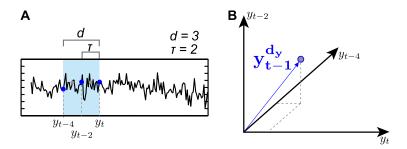


Figure 1: Time-delayed embedding. Example for the dime-delayed embedding of a point at time t using embedding parameters d=3 and $\tau=2$. Scalar time points at times t, t-2, and t-4 (A) are combined into an time-delayed embedded vector (B).

The Ragwitz criterion proceedes as follows (Fig. 2):

For all pairs of candidate values d and τ do the following:

- 1. Embed the time series using d and τ
- 2. Do the following for each embedded point $\mathbf{y_t^{d_y}}$:
 - (a) conduct a nearest neighbour search
 - (b) predict the next point $\mathbf{\hat{y}_{t+1}^{d_y}}$ as the average of the nearest neighbours
 - (c) calculate the local error $e_t = \hat{\mathbf{y}}_{t+1}^{\mathbf{d_y}} \mathbf{y}_{t+1}^{\mathbf{d_y}}$
- 3. Sum the local errors over all embedded points

As output, the Ragwitz criterion will return the pair of values d and τ , that minimize the error over all embedded time points.

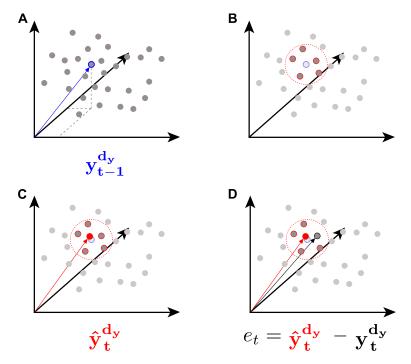


Figure 2: Ragwitz Criterion. (A) Each point in the time series is embedded using the current values for d and τ . (B) A nearest neighbour search is performed for each point. (C) For each point, the next point in time is predicted as the average of the reference point's nearest neighbours. The prediction error is computed as the distance of the predicted and the actual next point.

Using the Ragwitz Criterion in TRENTOOL See table ?? for parameters that define the behaviour of the Ragwitz Criterion in TRENTOOL. TRENTOOL requires the user to provide candidate values for d and τ to use the Ragwitz Criterion for parameter optimization (parameters cfgTEP.ragdim and cfgTEP.ragtaurange respectively). Note, that τ is given as a factor to be multiplied with the autocorrelation time (ACT) of the time series. For example, cfgTEP.ragtaurange = [0.2 0.5] would scan τ values from 20 % to 50 % of the ACT of the time series. The number of steps within the ragtaurange is specified in ragtausteps.

TRENTOOL further allows to specify the type of neighbour search to be conducted for local prediction. The user may chose from a range search cfgTEP.flagNei = 'Range' or a k-Nearest-Neighbour search cfgTEP.flagNei = 'Mass'. Accordingly, the parameter cfgTEP.sizeNei has to either define the range to be searched or the number of nearest neighbours.

As a last parameter, the user can set the number of points to be used for local prediction in cfgTEP.repPred. TRENTOOL will use the first repPred values to predict the next point in time from nearest neighbours and to calculate the prediction error.

Table 1
Prameters for the configuration structure cfgTEP. specific to the use of the Ragwitz Criterion for embedding parameter optimization (TRENTOOL Version 3.0).

field name	data type	units	description
optimizemethod	string		define method for parameter optimization: 'ragwitz'
ragdim	integer	samples	candidate embedding dimensions to be scanned
ragtaurange	double	units of act	1x2-vector of min and max embedding delays
ragtausteps	integer		number of equidistant steps in ragtaurange with a minimum of 5
flagNei	string		'Range' or 'Mass' (kNN) type of neighbor search
sizeNei	integer		Radius or mass for the neighbor search depending on flagNei
repPred	integer	samples	number of sample points for which the prediction is performed (has to be smaller than $length(timeSeries) - (embeddingdimension-1)* tau*ACT-u$)

References

- Oostenveld, R., Fries, P., Maris, E., & Schoffelen, J.-M. (2011). Fieldtrip: Open source software for advanced analysis of MEG, EEG, and invasive electrophysiological data. *Computational Intelligence and Neuroscience*, 2011, 1–9.
- Ragwitz, M., & Kantz, H. (2002, Apr). Markov models from data by simple nonlinear time series predictors in delay embedding spaces. *Phys. Rev. E*, 65, 056201. Available from http://link.aps.org/doi/10.1103/PhysRevE.65.056201
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