

# 電腦視覺與應用 Computer Vision and Applications

Lecture08-3D reconstruction

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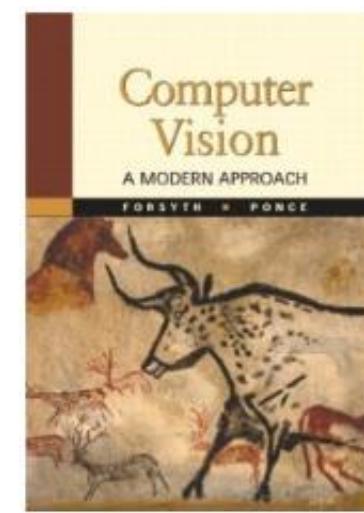
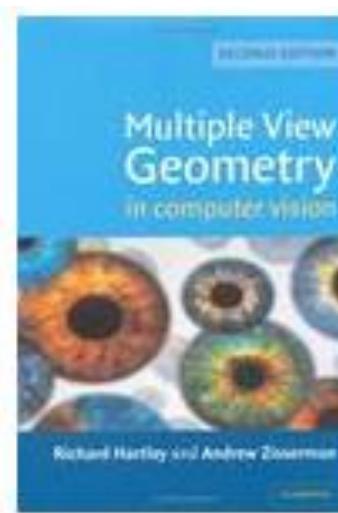
# 3D reconstruction

- Either from calibrated images
- Or from uncalibrated images



# 3D reconstruction

- Lecture Reference at:
  - Multiple View Geometry in Computer Vision, (\*Chapter10 and 12)
  - Computer Vision A Modern Approach, Chapter 13





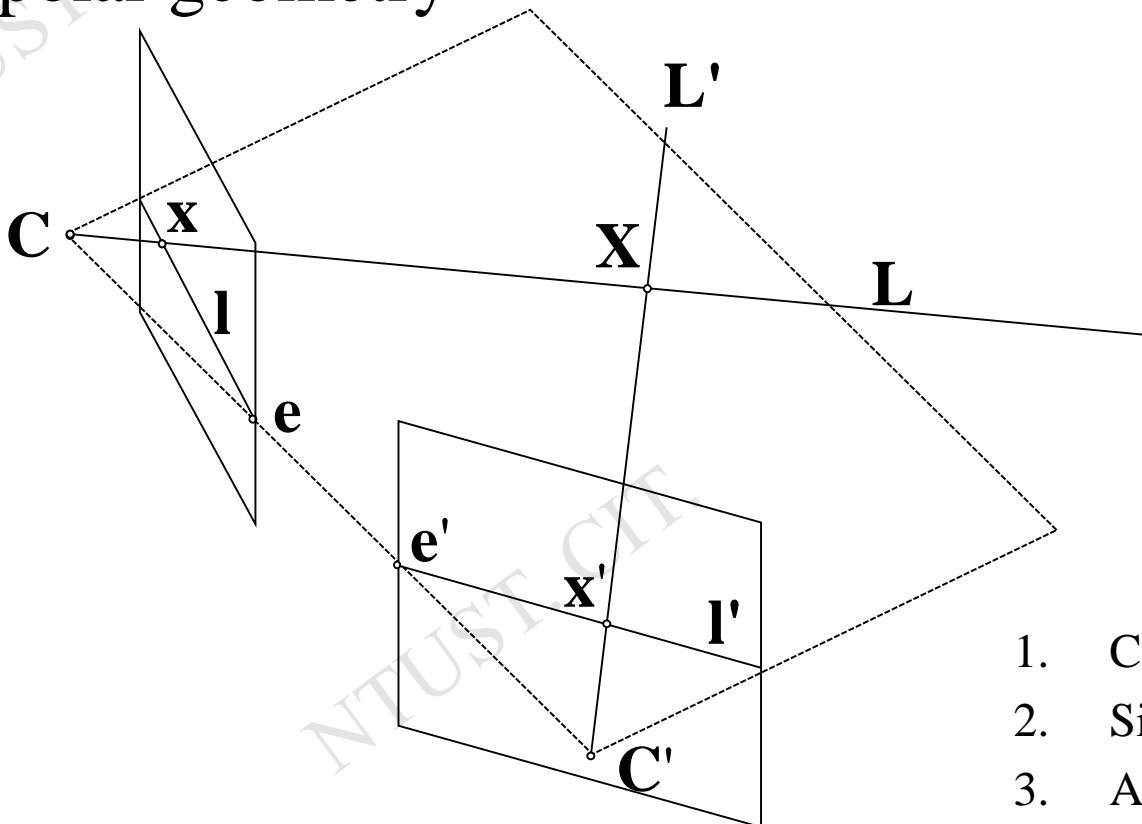
# 3D reconstruction

- The questions?
  - Correspondence geometry: Given an image point  $x$  in the first image, how does this constrain the position of the corresponding point  $x'$  in the second image?
  - Camera geometry (motion): Given a set of corresponding image points  $\{\mathbf{x}_i \leftrightarrow \mathbf{x}'_i\}$ ,  $i=1,\dots,n$ , what are the cameras  $\mathbf{P}$  and  $\mathbf{P}'$  for the two views?
  - Scene geometry (structure): Given corresponding image points  $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$  and cameras  $\mathbf{P}, \mathbf{P}'$ , what is the position of  $\mathbf{X}$  in space?



# 3D reconstruction

## ■ Epipolar geometry



$$\underbrace{\mathbf{x}^T \mathbf{F} \mathbf{x}}_{\mathbf{l}^T} = 0$$

1. Computable from corresponding points
2. Simplifies matching
3. Allows to detect outliers
4. Related to calibration



# 3D reconstruction of cameras and structure

- Reconstruction problem:

Given  $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$ , compute  $\mathbf{P}$ ,  $\mathbf{P}'$  and  $\mathbf{X}_i$

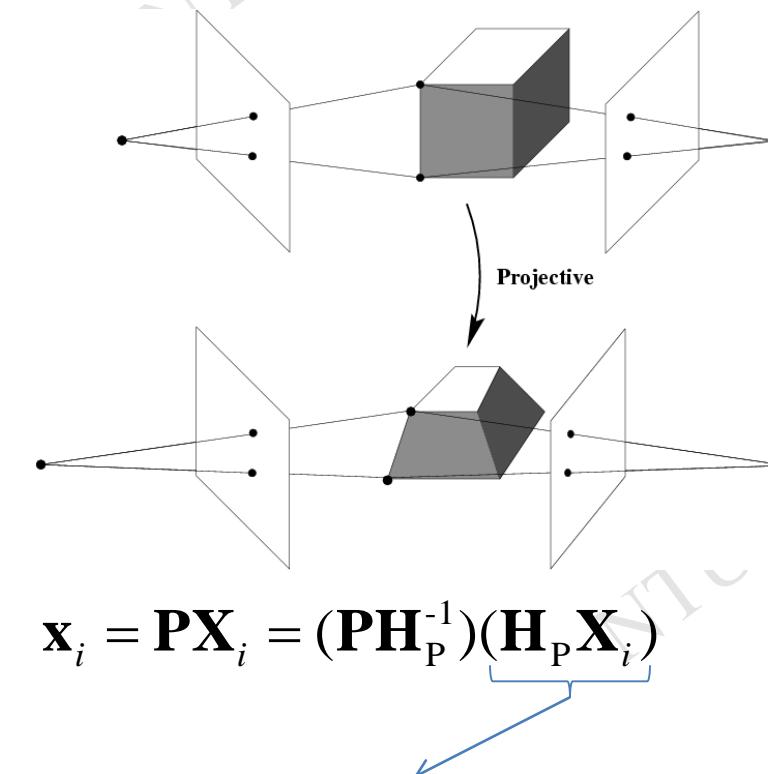
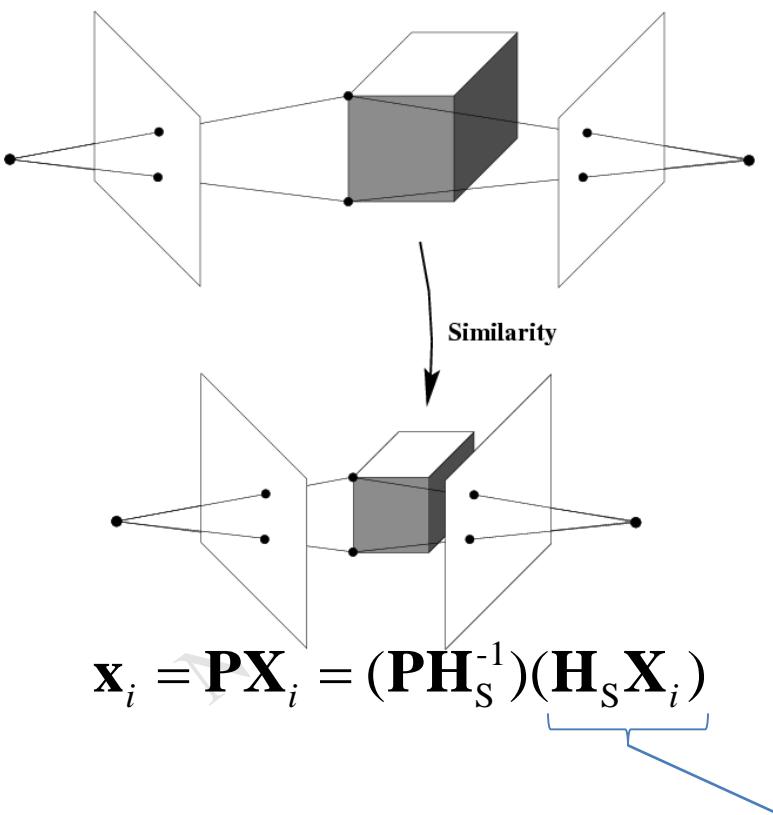
$$\mathbf{x}_i = \mathbf{P}\mathbf{X}_i \text{ and } \mathbf{x}'_i = \mathbf{P}'\mathbf{X}_i \text{ for all } i$$

without additional constraints possible up to projective ambiguity



# 3D reconstruction of cameras and structure

## ■ Reconstruction ambiguity (projective ambi.)



One solution which is  
projected to  $\mathbf{x}_i$ , as well.



# Outline of 3D reconstruction (from uncalibrated images)

- Compute  $F$  from correspondences
- Compute camera matrices from  $F$
- Compute 3D point for each pair of corresponding points

computation of  $F$

use  $\mathbf{x}'_i^T \mathbf{F} \mathbf{x}_i = 0$  equations, linear in coeff.  $F$

computation of camera matrices

use

$$\mathbf{P} = [\mathbf{I} \mid \mathbf{0}] \quad \mathbf{P}' = [[\mathbf{e}']_{\times} \mathbf{F} + \mathbf{e}' \mathbf{v}^T \mid \lambda \mathbf{e}'] \longrightarrow \text{General formula}$$

or

$$\mathbf{P} = [\mathbf{I} \mid \mathbf{0}] \quad \mathbf{P}' = [[\mathbf{e}']_{\times} \mathbf{F} \mid \mathbf{e}'] \leftarrow$$

triangulation

compute intersection of two back-projected rays

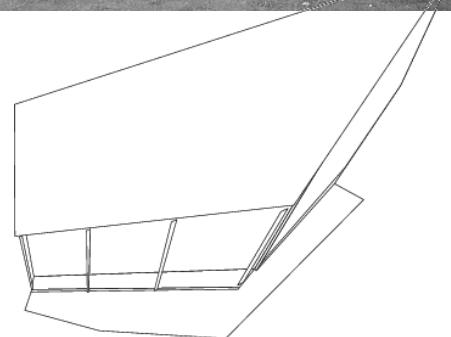
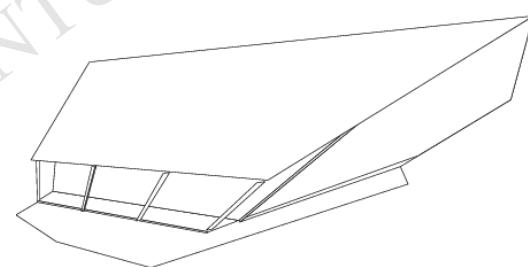
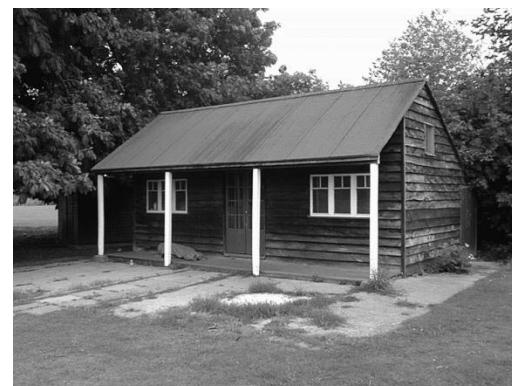
detail at

Q. Luong and T. Vieville, "Canonical representations for the geometries of multiple projective views," *Computer vision and image understanding*, vol. 64, no. 2, pp. 193-229, 1996.



# Projective reconstruction

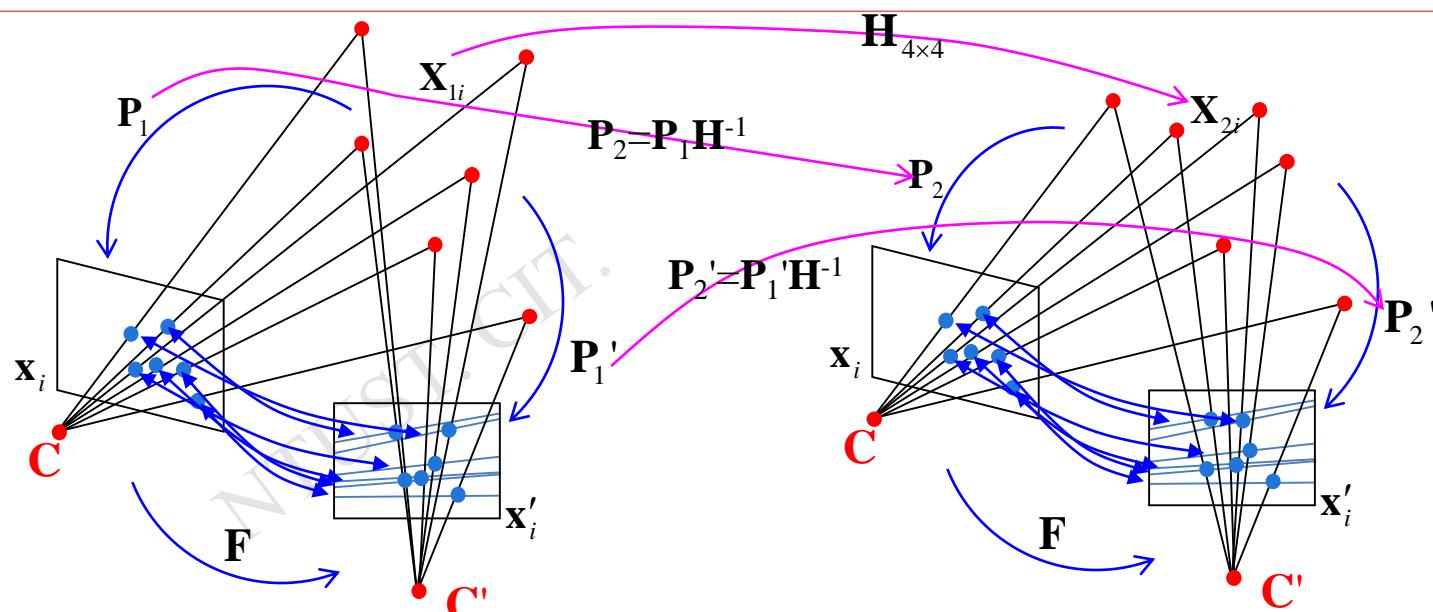
- The reconstruction required no information about camera matrices, or information about the scene geometry. The fundamental matrix  $F$  is computed from point correspondences between the images, camera matrices are retrieved from  $F$ , and then 3D points are computed by triangulation from the correspondences.





# Theorem for the projective reconstruction

Suppose that  $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$  is a set of correspondences between points in two images and that the fundamental matrix  $\mathbf{F}$  is uniquely determined by the condition  $\mathbf{x}'_i^T \mathbf{F} \mathbf{x}_i = 0$  for all  $i$ . Let  $(\mathbf{P}_1, \mathbf{P}_1', \{\mathbf{X}_{1i}\})$  and  $(\mathbf{P}_2, \mathbf{P}_2', \{\mathbf{X}_{2i}\})$  be two reconstructions of the correspondences  $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$ . Then there exists a non-singular matrix  $\mathbf{H}$  such that  $\mathbf{P}_2 = \mathbf{P}_1 \mathbf{H}^{-1}$ ,  $\mathbf{P}_2' = \mathbf{P}_1' \mathbf{H}^{-1}$  and  $\mathbf{X}_2 = \mathbf{H} \mathbf{X}_1$  for all  $i$ , except for those  $i$  such that  $\mathbf{F} \mathbf{x}_i = \mathbf{x}'_i \mathbf{F} = 0$



key result:

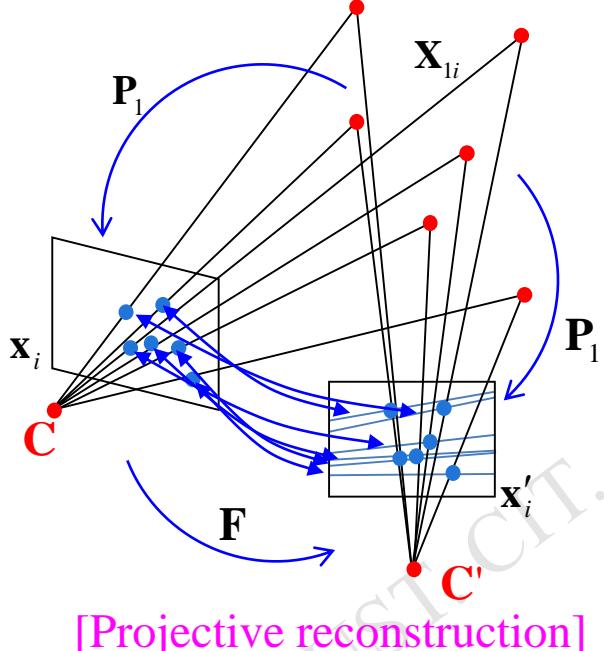
allows reconstruction from pair of uncalibrated images

$$\begin{aligned}\mathbf{P}_2 &= \mathbf{P}_1 \mathbf{H}^{-1} \\ \mathbf{P}_2' &= \mathbf{P}_1' \mathbf{H}^{-1} \\ \mathbf{X}_2 &= \mathbf{H} \mathbf{X}_1\end{aligned}$$

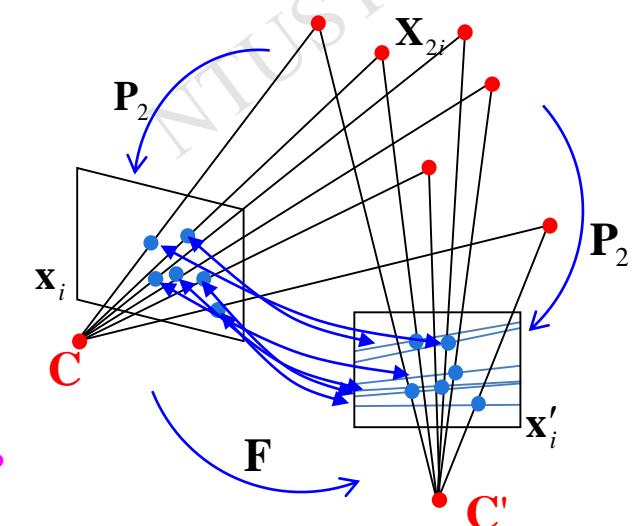


# Theorem for the projective reconstruction

## ■ In practice



- Step-1: solve epipolar geometry ( $F$ )
- Step-2: guess  $P$  &  $P'$  for initialization, then determine  $X$ .
- Step-3: determine  $H$ , then, finding new  $P$  &  $P'$  &  $X$ .



$$\begin{aligned} P_2 &= P_1 H^{-1} \\ P_2' &= P_1' H^{-1} \\ X_2 &= H X_1 \end{aligned}$$



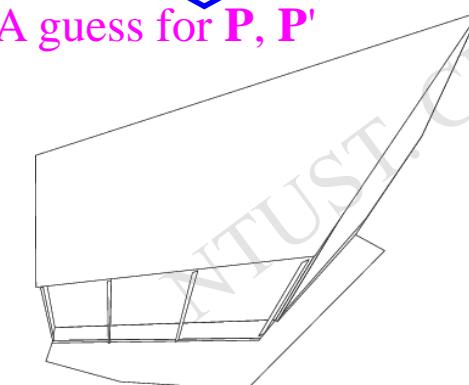
# 3D reconstruction for projective geometry

## ■ (in practice)

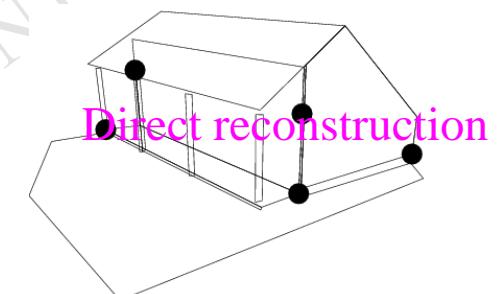
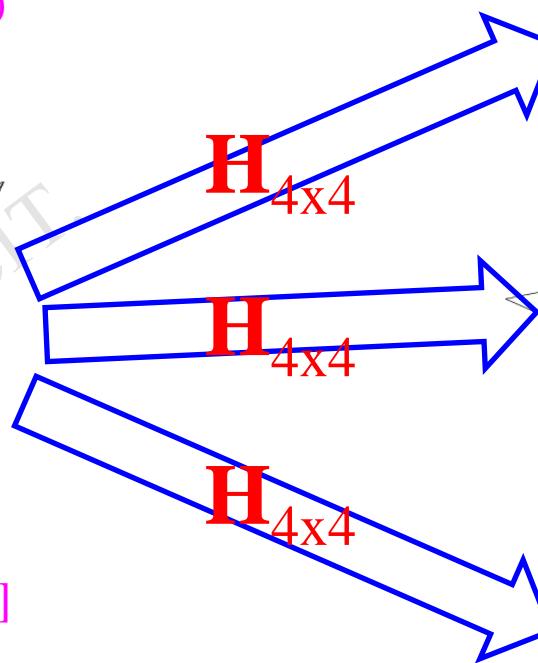


Two views geometry ( $\mathbf{F}$ )

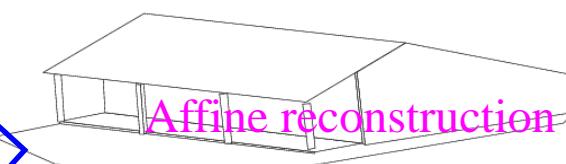
A guess for  $\mathbf{P}, \mathbf{P}'$



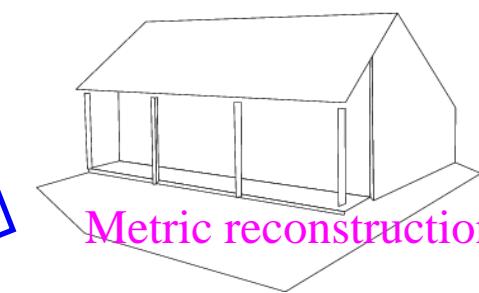
[Projective reconstruction]



Direct reconstruction



Affine reconstruction

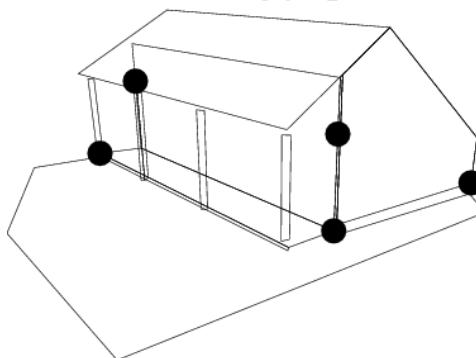


Metric reconstruction



# Direct reconstruction using ground truth

- use control points  $\mathbf{X}_{Ei}$  with known coordinates, to go from projective to metric

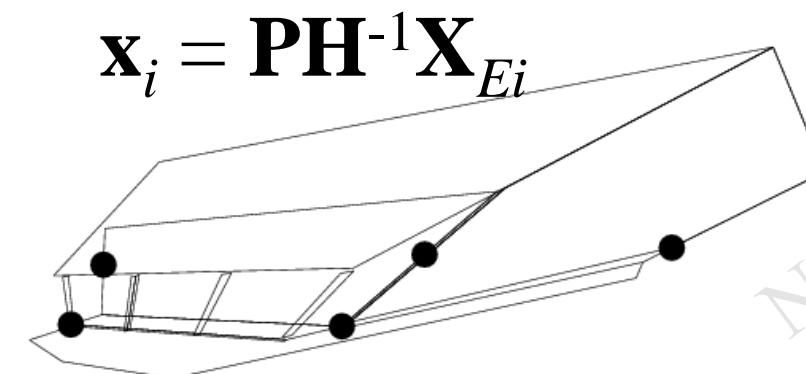


$$\mathbf{X}_{Ei} = \mathbf{H}\mathbf{X}_i$$

Ground truth  
(measurement in real environment)

Homography (4x4)

3D points for an estimated projective reconstruction.

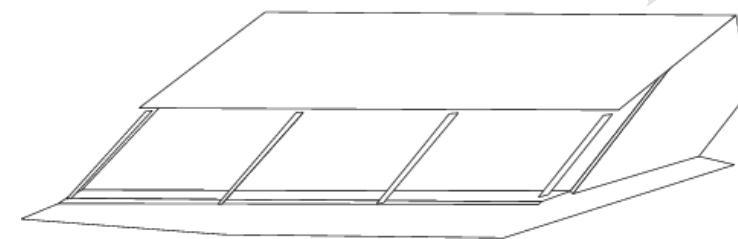
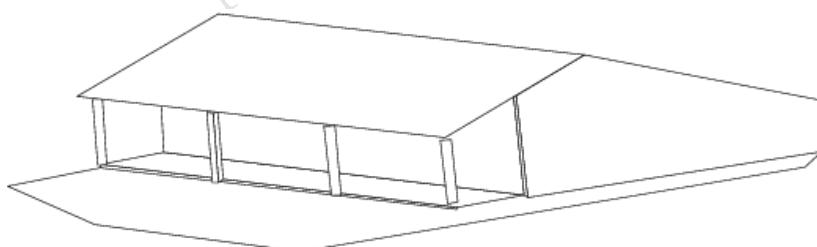
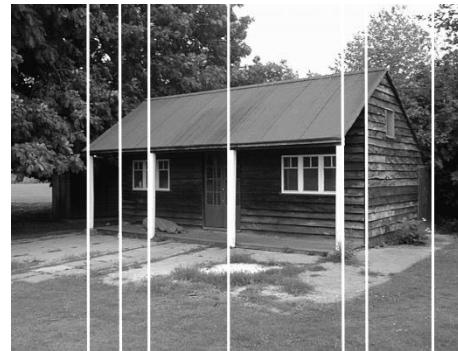


Note! do NOT select degenerated cases,  
Ex. 4 points on a plane, 3 points on a line.



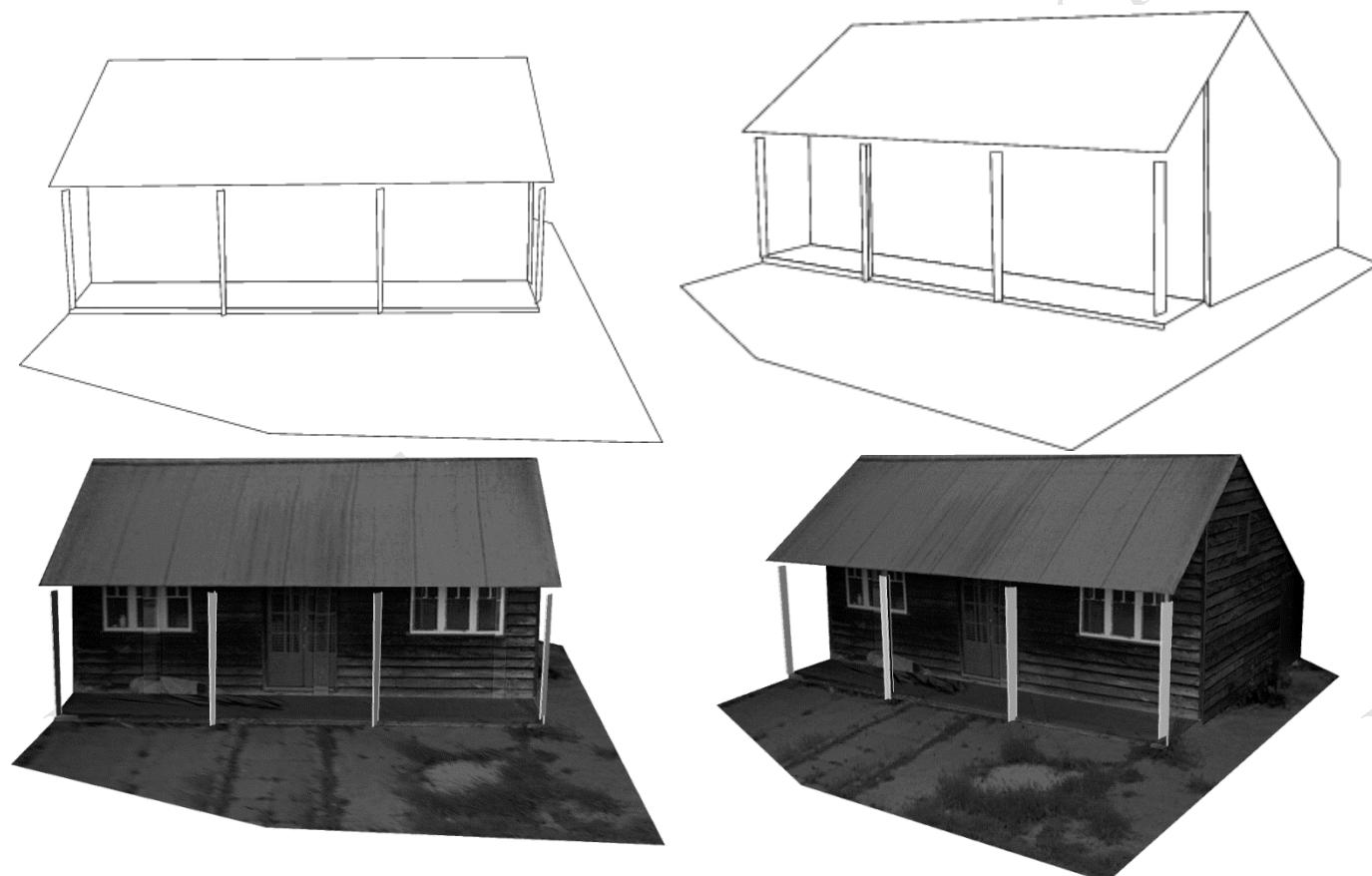
# Affine reconstruction

- There are 3 sets of parallel lines in the scene, each set with a different direction. These 3 sets enable the position of the planes at infinity to be computed in the projective reconstruction. Note that parallel scene lines are parallel in reconstruction, but lines are NOT perpendicular in the reconstruction.



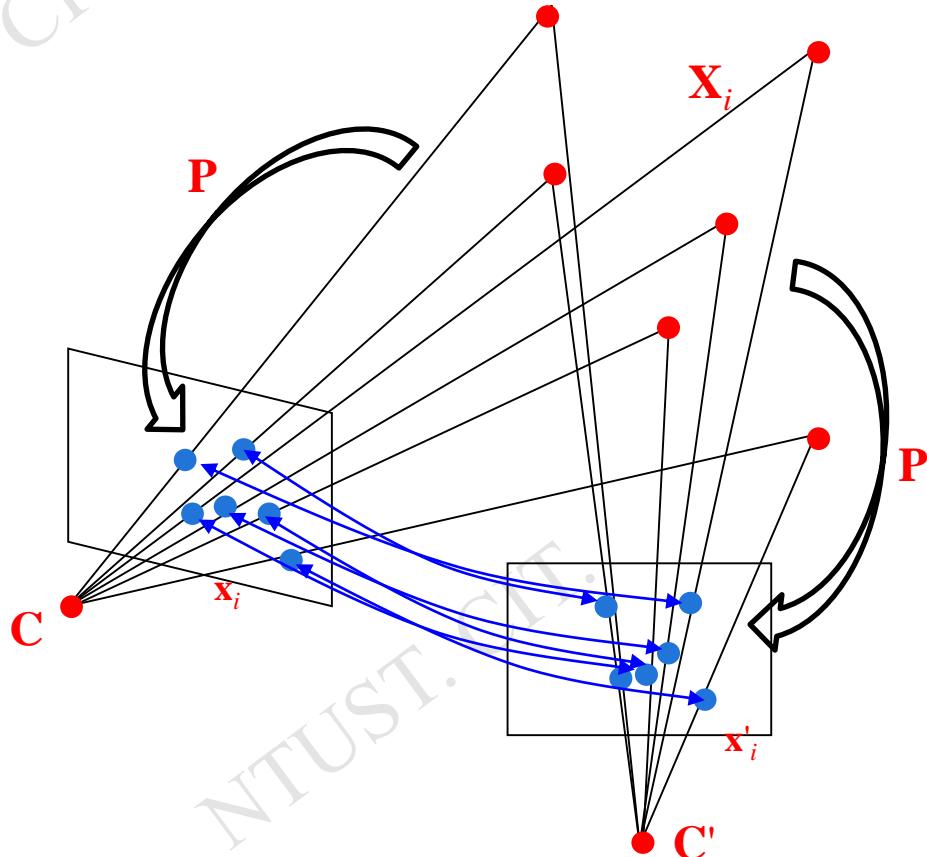


# Metric reconstruction





# 3D estimation Direct Triangulation method



- Condition-1

$P, P', x_i, x'_i$  : known

$X$  : unknown (to be solved)

- Condition-2

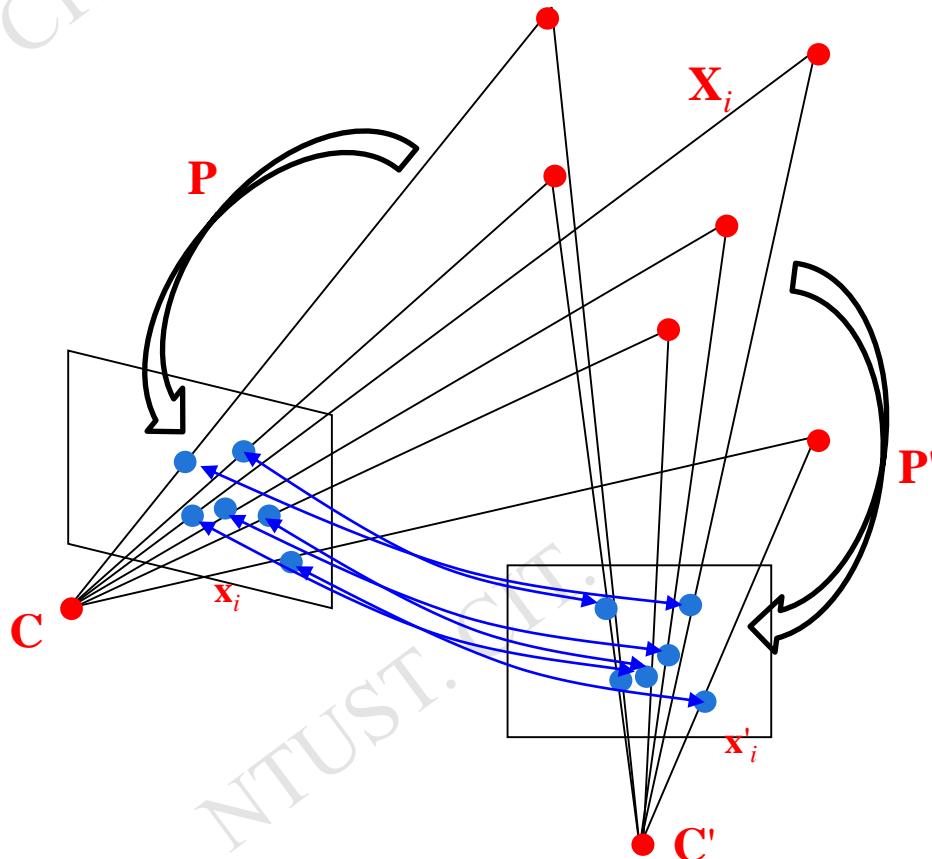
$x_i, x'_i, X$ : known

$P, P'$ : unknown (to be solved)

→ become a calibration problem



# 3D estimation Direct Triangulation method



- Condition-3(direct reconstruction)  
 $\mathbf{x}_i, \mathbf{x}'_i$  : known  
5 points  $\mathbf{X}$ : known  
 $\mathbf{P}, \mathbf{P}'$ : unknown (to be solved)
- Condition-4(affine reconstruction)  
 $\mathbf{x}_i, \mathbf{x}'_i$ , other cues: known  
 $\mathbf{P}, \mathbf{P}'$ : unknown (to be solved)
- Condition-5(metric reconstruction)  
 $\mathbf{x}_i, \mathbf{x}'_i, \mathbf{K}$ : known  
 $\mathbf{X}, \mathbf{P}, \mathbf{P}'$ : unknown (to be solved)



## Objective

Given two uncalibrated images compute  $(\mathbf{P}_M, \mathbf{P}'_M, \{\mathbf{X}_{Mi}\})$   
(i.e. within similarity of original scene and cameras)

Algorithm [Textbook: Hartley04, Algorithm 10.1]

- (i) Compute projective reconstruction  $(\mathbf{P}, \mathbf{P}', \{\mathbf{X}_i\})$ 
  - (a) Compute  $\mathbf{F}$  from  $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$
  - (b) Compute  $\mathbf{P}, \mathbf{P}'$  from  $\mathbf{F}$
  - (c) Triangulate  $\mathbf{X}_i$  from  $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$
- (ii) Rectify reconstruction from projective to metric

(a) **Direct method:** compute  $\mathbf{H}$  from control points →(condition-3 in the previous slide)

$$\mathbf{P}_M = \mathbf{P}\mathbf{H}^{-1}, \quad \mathbf{P}'_M = \mathbf{P}'\mathbf{H}^{-1}, \quad \mathbf{X}_{Mi} = \mathbf{H}\mathbf{X}_i$$

**Stratified method:**

(a) **Affine reconstruction:** →(condition-4 in the previous slide)

$$\mathbf{H} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \boldsymbol{\pi}_\infty^T & \end{bmatrix}$$

(b) **Metric reconstruction:** compute IAC  $\boldsymbol{\omega}$  →(condition-5 in the previous slide)

$$\mathbf{H} = \begin{bmatrix} \mathbf{A}^{-1} & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix} \quad \mathbf{A}\mathbf{A}^T = (\mathbf{M}^T \boldsymbol{\omega} \mathbf{M})^{-1}$$



# 3D reconstruction

Image information provided	View relations and projective objects	3-space objects	reconstruction ambiguity
point correspondences	$F$		projective
point correspondences including vanishing points	$F, H_\infty$	$p_\infty$	affine
Points correspondences and internal camera calibration	$F, H_\infty$ $\omega, \omega'$	$p_\infty$ $\Omega_\infty$	metric

The two-view relations, image entities, and their 3-space counterpart for various classes of reconstruction ambiguity.



# 3D estimation Direct Triangulation method

- If the projected point  $\mathbf{PX}$  is very close to  $\mathbf{x}$ , then

$$\mathbf{x} \times \mathbf{x} = \mathbf{0} \rightarrow \text{zero vector}$$

So, one constraint for  $\mathbf{X}$  is

$$\mathbf{x} \times (\mathbf{PX}) = \mathbf{0}$$

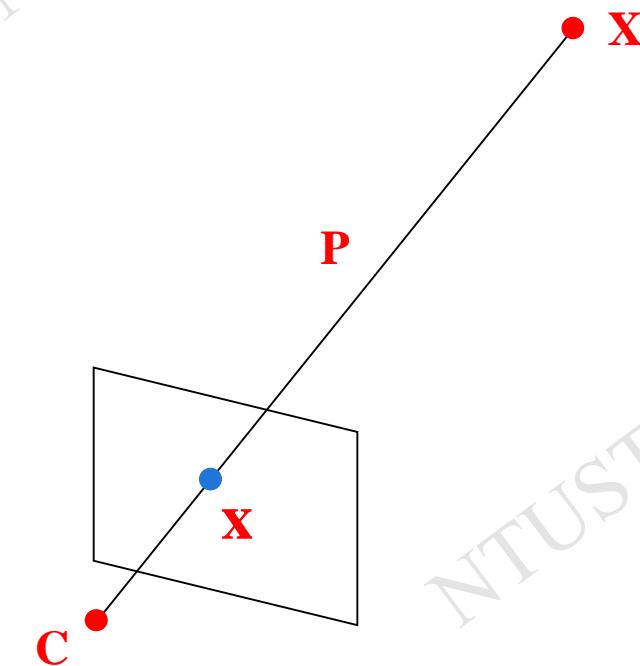
For convenience, rewrite  $\mathbf{P}$  as

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} = \begin{bmatrix} \mathbf{p}_1^T \\ \mathbf{p}_2^T \\ \mathbf{p}_3^T \end{bmatrix}_{3 \times 4} \quad \begin{array}{l} \text{1x4 vector} \\ \text{1x4 vector} \\ \text{1x4 vector} \end{array}$$

$$\mathbf{p}_1^T = [p_{11} \ p_{12} \ p_{13} \ p_{14}]$$

$$\mathbf{p}_2^T = [p_{21} \ p_{22} \ p_{23} \ p_{24}]$$

$$\mathbf{p}_3^T = [p_{31} \ p_{32} \ p_{33} \ p_{34}]$$





# 3D estimation Direct Triangulation method-co

- To solve the equation:

$$\mathbf{x} \times (\mathbf{P}\mathbf{X}) = \mathbf{0}$$

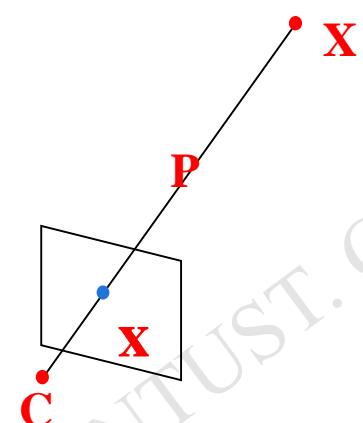
$$\rightarrow \begin{bmatrix} x \\ y \\ w \end{bmatrix} \times \begin{bmatrix} \mathbf{p}_1^T \mathbf{X} \\ \mathbf{p}_2^T \mathbf{X} \\ \mathbf{p}_3^T \mathbf{X} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{cases} y\mathbf{p}_3^T \mathbf{X} - w\mathbf{p}_2^T \mathbf{X} = 0 \\ -x\mathbf{p}_3^T \mathbf{X} + w\mathbf{p}_1^T \mathbf{X} = 0 \\ x\mathbf{p}_2^T \mathbf{X} - y\mathbf{p}_1^T \mathbf{X} = 0 \end{cases}$$

$$\rightarrow \begin{cases} -\mathbf{p}_2^T \mathbf{X} + \frac{y}{w} \mathbf{p}_3^T \mathbf{X} = 0 \\ -\mathbf{p}_1^T \mathbf{X} + \frac{x}{w} \mathbf{p}_3^T \mathbf{X} = 0 \\ y\mathbf{p}_1^T \mathbf{X} - x\mathbf{p}_2^T \mathbf{X} = 0 \end{cases}$$

Two independent eqs.  
only. (neglect 3rd eq.)

Let  $(u, v) = (x/w, y/w)$  as  
the image point

$$\begin{cases} (v\mathbf{p}_3^T - \mathbf{p}_2^T) \mathbf{X} = 0 \\ (u\mathbf{p}_3^T - \mathbf{p}_1^T) \mathbf{X} = 0 \end{cases}$$



The unknown  $\mathbf{X}$  must satisfy the  
above eqs. for this image (says C)



# 3D estimation Direct Triangulation method-co

- Of course, in the second image (says C'), the point X will have the same property as:

$$\begin{cases} (v' \mathbf{p}'_3^T - \mathbf{p}'_2^T) \mathbf{X} = 0 \\ (u' \mathbf{p}'_3^T - \mathbf{p}'_1^T) \mathbf{X} = 0 \end{cases}$$

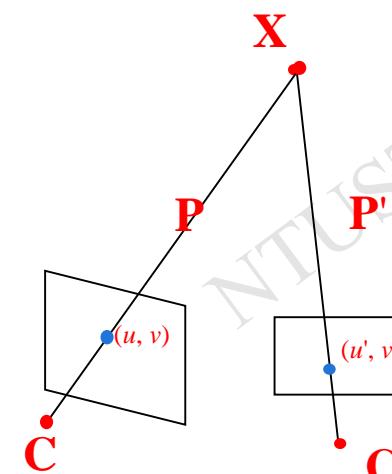
- Finally, from one correspondence  $(u, v)$  and  $(u', v')$ , we will have

$$\begin{cases} (v \mathbf{p}_3^T - \mathbf{p}_2^T) \mathbf{X} = 0 \\ (u \mathbf{p}_3^T - \mathbf{p}_1^T) \mathbf{X} = 0 \\ (v' \mathbf{p}'_3^T - \mathbf{p}'_2^T) \mathbf{X} = 0 \\ (u' \mathbf{p}'_3^T - \mathbf{p}'_1^T) \mathbf{X} = 0 \end{cases}$$

$$\begin{bmatrix} u \mathbf{p}_3^T - \mathbf{p}_1^T \\ v \mathbf{p}_3^T - \mathbf{p}_2^T \\ u' \mathbf{p}'_3^T - \mathbf{p}'_1^T \\ v' \mathbf{p}'_3^T - \mathbf{p}'_2^T \end{bmatrix} \mathbf{X}_{4 \times 1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve it by SVD

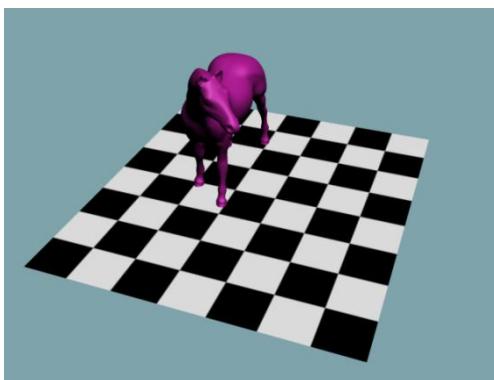
remember:  $\mathbf{p}_1^T = [p_{11} \quad p_{12} \quad p_{13} \quad p_{14}] \dots$



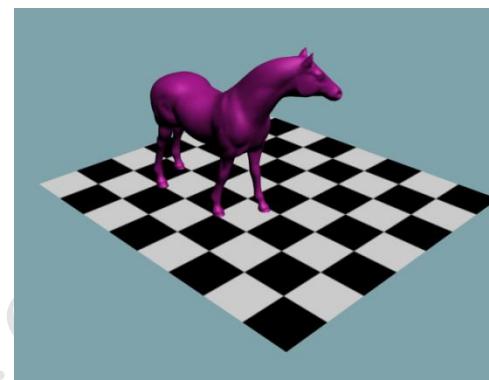


# 3D estimation Direct Triangulation method

- Example1 for Condition-1
  - $\mathbf{P}, \mathbf{P}', \mathbf{x}_i, \mathbf{x}'_i$  : known
  - $\mathbf{X}$  : unknown (to be solved)



$$\mathbf{P} = \begin{array}{cccc} 2.0179 & 1.5967 & -0.5695 & 113.8802 \\ 0.2820 & -0.7636 & -2.4258 & 305.7125 \\ -0.0009 & 0.0023 & -0.0018 & 1.0000 \end{array}$$



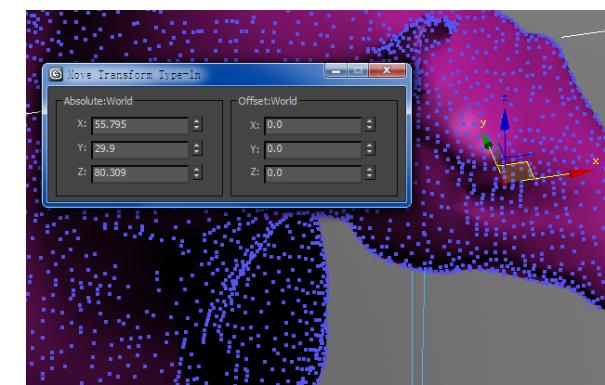
$$\mathbf{P}' = \begin{array}{cccc} 2.8143 & -1.3450 & -0.5673 & 347.4957 \\ -0.4439 & -0.4444 & -3.0134 & 371.1864 \\ 0.0023 & 0.0023 & -0.0018 & 1.0000 \end{array}$$

To determine the 3D coordinate of the horse's right eye, we pick up this feature on these two images, then we have

$$\mathbf{x} = [259, 120, 1]^T$$

$$\mathbf{x}' = [395, 89, 1]^T$$

$\rightarrow \mathbf{X}?$



Ground truth :

$$[55.795, 29.9, 80.309, 1]^T$$



# 3D estimation Direct Triangulation method

## ■ Example1 for Condition-1—cont.

$\mathbf{P}, \mathbf{P}', \mathbf{x}_i, \mathbf{x}'_i$  : known

$\mathbf{X}$  : unknown (to be solved)

$$\begin{bmatrix} u\mathbf{p}_3^T - \mathbf{p}_1^T \\ v\mathbf{p}_3^T - \mathbf{p}_2^T \\ u'\mathbf{p}'_3^T - \mathbf{p}'_1^T \\ v'\mathbf{p}'_3^T - \mathbf{p}'_2^T \end{bmatrix} \mathbf{X}_{4 \times 1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\mathbf{p}_1$	$\mathbf{p}_2$	$\mathbf{p}_3$	$[u \quad v]$
$p1 =$	$p2 =$	$p3 =$	$u =$
2.0179	0.2820	-0.0009	259
1.5967	-0.7636	0.0023	
-0.5695	-2.4258	-0.0018	$v =$
113.8802	305.7125	1.0000	120

$\mathbf{p}'_1$	$\mathbf{p}'_2$	$\mathbf{p}'_3$	$[u' \quad v']$
$pp1 =$	$pp2 =$	$pp3 =$	$up =$
2.8143	-0.4439	0.0023	395
-1.3450	-0.4444	0.0023	
-0.5673	-3.0134	-0.0018	$vp =$
347.4957	371.1864	1.0000	89

In Matlab (note the notation ' $\cdot$ ' in Matlab means transpose operation)

```
A=[u*p3'-p1';
v*p3'-p2';
up*pp3'-pp1';
vp*pp3'-pp2'];
[U,S,V]=svd(A)
```

...

```
v=
0.0038 0.7662 -0.3417 -0.5441
0.0030 -0.5582 -0.7735 -0.3002
0.0088 -0.3183 0.5337 -0.7834
-0.9999 -0.0015 0.0010 -0.0099
```

normalize

```
X =
55.1137
30.4065
79.3529
1.0000
```

Ground truth :  
 $[55.795, 29.9, 80.309, 1]^T$

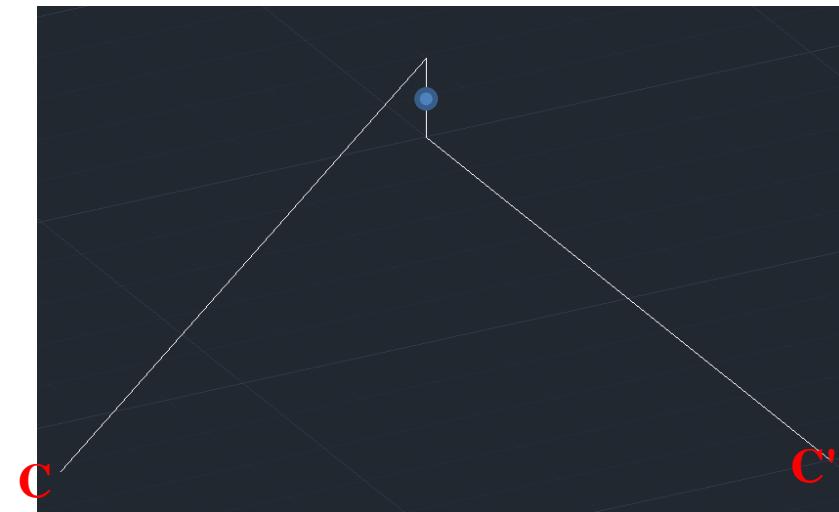
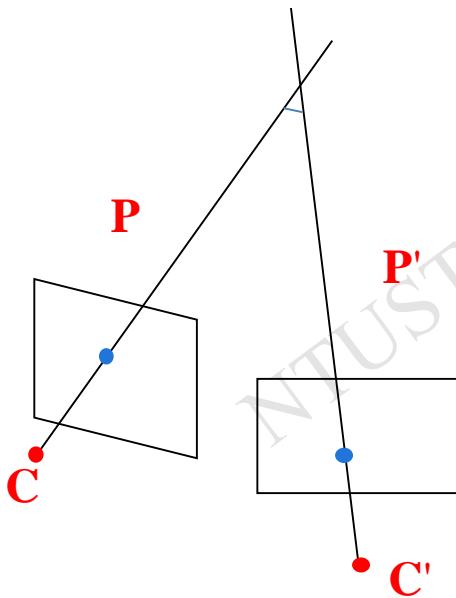


# 3D estimation Direct Triangulation method

- Example1 for Condition-1—cont.

## Note!

These two spatial lines are not necessary to intersect. The solution is the point whose residual error is smallest.





# 3D estimation Direct Triangulation method

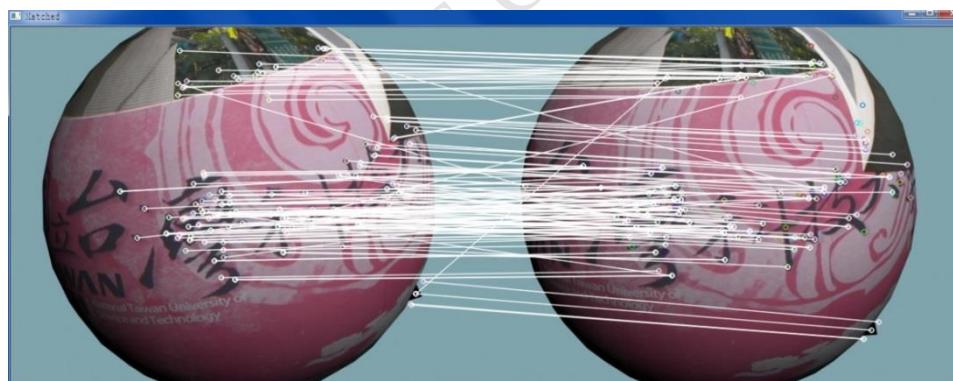
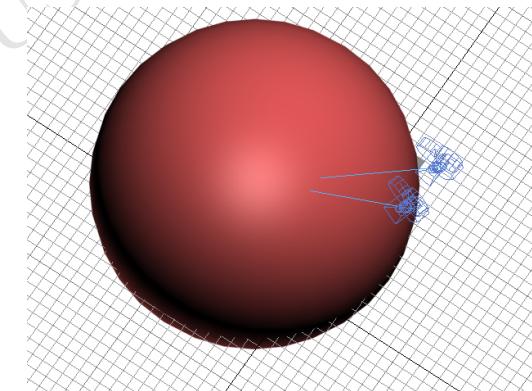
## ■ Example2 for Condition-1



$P =$   
2.5873 -0.3302 -0.9174 246.8139  
-0.5844 -1.8389 -1.8654 401.7092  
0.0004 0.0014 -0.0029 1.0000



$P_p =$   
2.5737 0.2628 -0.9172 207.1313  
-0.1437 -1.9363 -1.8245 382.3570  
0.0001 0.0014 -0.0029 1.0000



**The question is:**

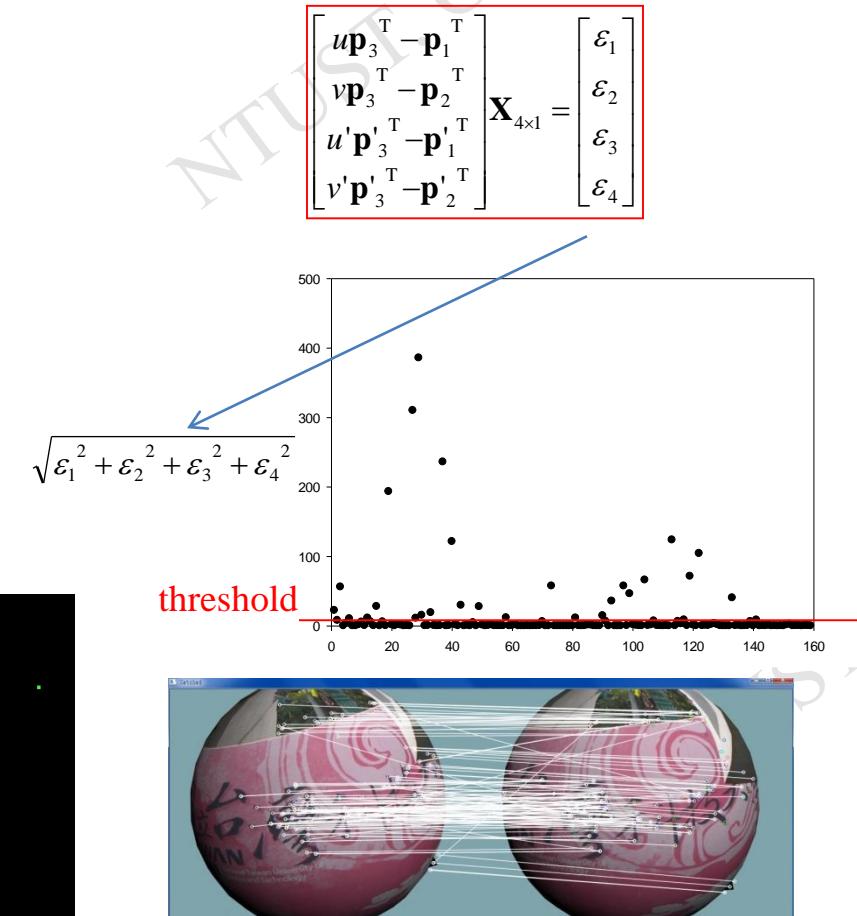
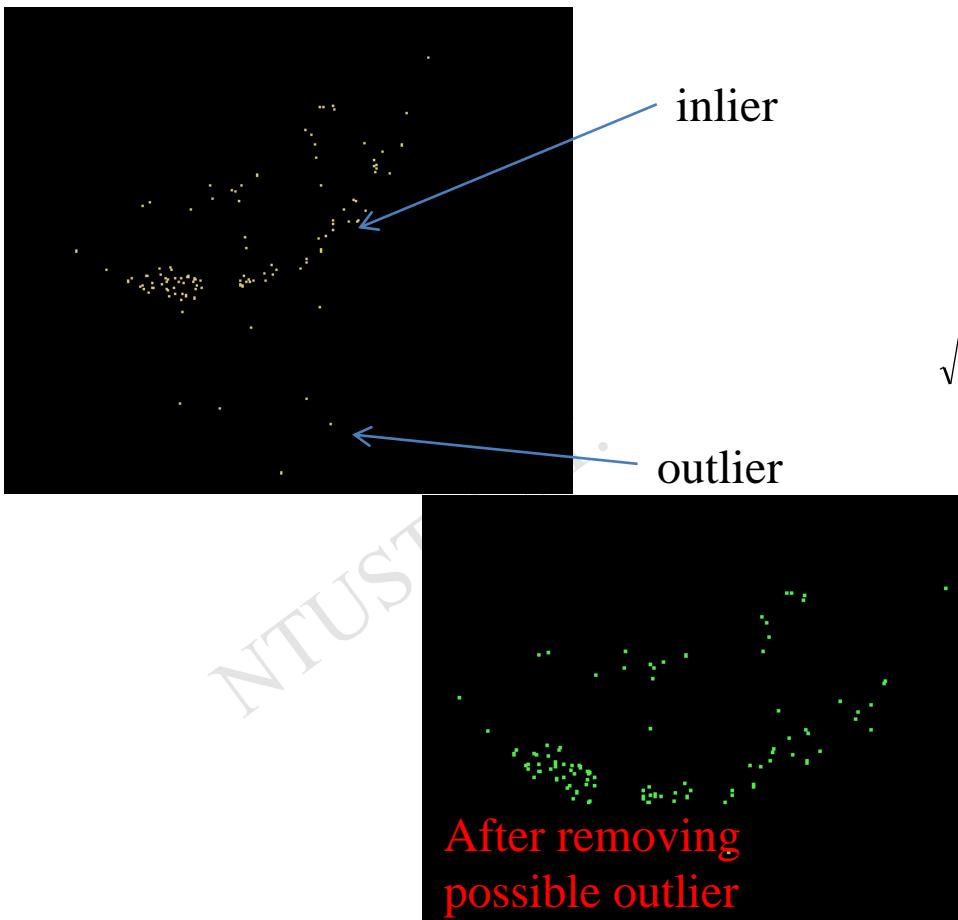
$P, P'$  : known

$\mathbf{x}_i, \mathbf{x}'_i$ : Generated by Feature  
Matching algorithm (including outlier)  
**X?**



# 3D estimation Direct Triangulation method

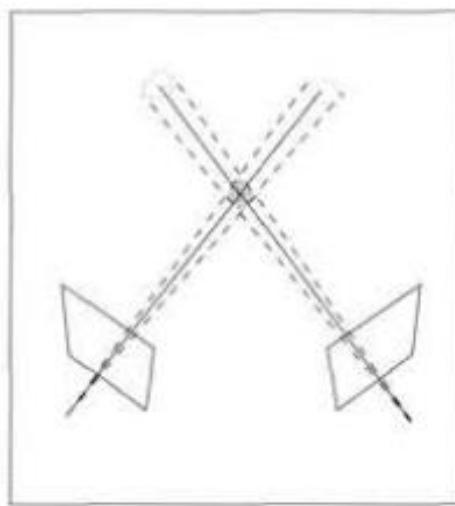
## ■ Example2 for Condition-1—cont.



In practice, the outliers are usually removed by Fundamental Matrix constraints.

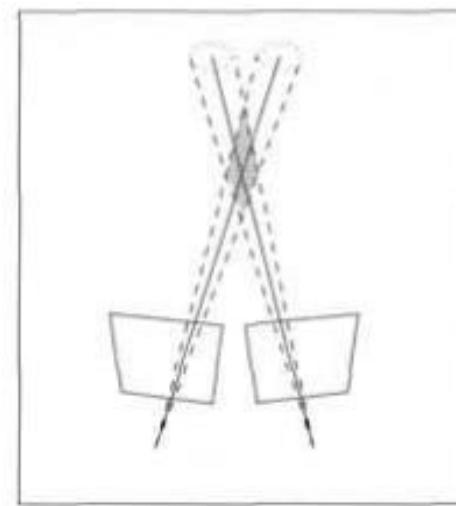


# 3D estimation Direct Triangulation method



More Difficult to find  
correspondences  
(feature matching)

Smaller uncertainty  
(Better 3D reconstruction)



Easier to find correspondences  
(feature matching)

Larger uncertainty  
(Poor 3D reconstruction)

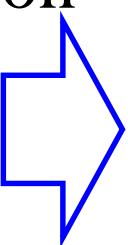


# 3D reconstruction for projective geometry

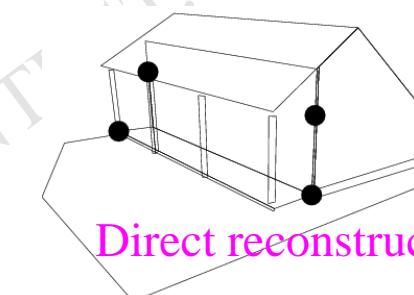
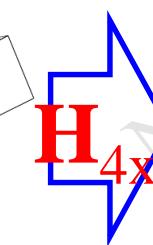
## ■ Direction reconstruction



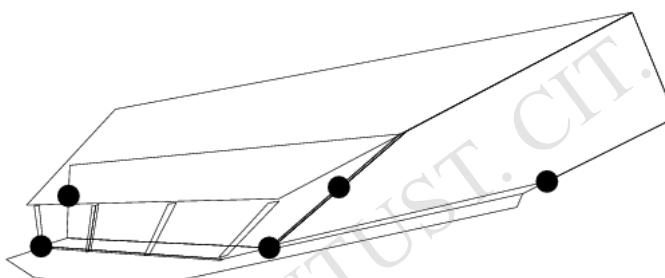
Two views geometry ( $\mathbf{F}$ )



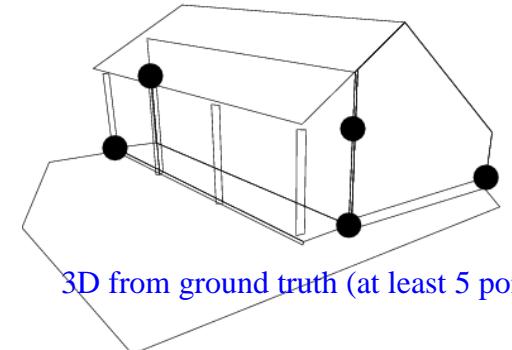
A guess for  $\mathbf{P}, \mathbf{P}'$   
[Projective reconstruction]



Direct reconstruction



3D from projective geometry  
(at least 5 points)



3D from ground truth (at least 5 points)

Select at least 5 correspondences → computer  $\mathbf{H}$   
(please review chapter "Projective 3D geometry" for detail)

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{13} & H_{14} \\ H_{21} & H_{22} & H_{23} & H_{24} \\ H_{31} & H_{32} & H_{33} & H_{34} \\ H_{41} & H_{42} & H_{43} & H_{44} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

$$\tilde{\mathbf{H}}_1^T = [H_{11} \ H_{12} \ H_{13} \ H_{14}]$$

$$\tilde{\mathbf{H}}_2^T = [H_{21} \ H_{22} \ H_{23} \ H_{24}]$$

$$\tilde{\mathbf{H}}_3^T = [H_{31} \ H_{32} \ H_{33} \ H_{34}]$$

$$\tilde{\mathbf{H}}_4^T = [H_{41} \ H_{42} \ H_{43} \ H_{44}]$$

$$\mathbf{X}^T = [X \ Y \ Z \ 1]$$

$$\begin{bmatrix} \mathbf{X}^T & 0^T & 0^T & -\mathbf{X}'\mathbf{X}^T \\ 0^T & \mathbf{X}^T & 0^T & -\mathbf{Y}'\mathbf{X}^T \\ 0^T & 0^T & \mathbf{X}^T & -\mathbf{Z}'\mathbf{X}^T \end{bmatrix}_{3 \times 16} \begin{bmatrix} \tilde{\mathbf{H}}_1 \\ \tilde{\mathbf{H}}_2 \\ \tilde{\mathbf{H}}_3 \\ \tilde{\mathbf{H}}_4 \end{bmatrix}_{16 \times 1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}_{3 \times 1}$$

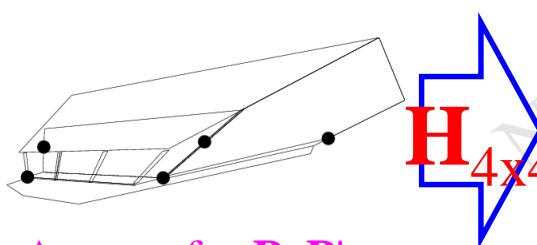
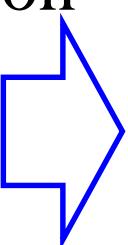


# 3D reconstruction for projective geometry

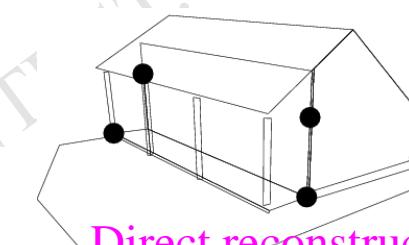
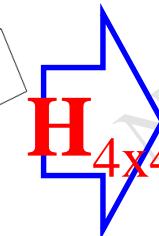
## ■ Direction reconstruction



Two views geometry ( $\mathbf{F}$ )



A guess for  $\mathbf{P}, \mathbf{P}'$   
[Projective reconstruction]



Direct reconstruction

Once having  $\mathbf{H}$ :

$$\mathbf{P}_2 = \mathbf{P}_1 \mathbf{H}^{-1}$$

$$\mathbf{P}_2' = \mathbf{P}_1' \mathbf{H}^{-1}$$

$$\mathbf{X}_2 = \mathbf{H} \mathbf{X}_1$$

Either

Apply homography  $\mathbf{H}$  to all old 3D points ( $\mathbf{X}_{1i}$ ), then get all new 3D points ( $\mathbf{X}_{2i}$ )

Or

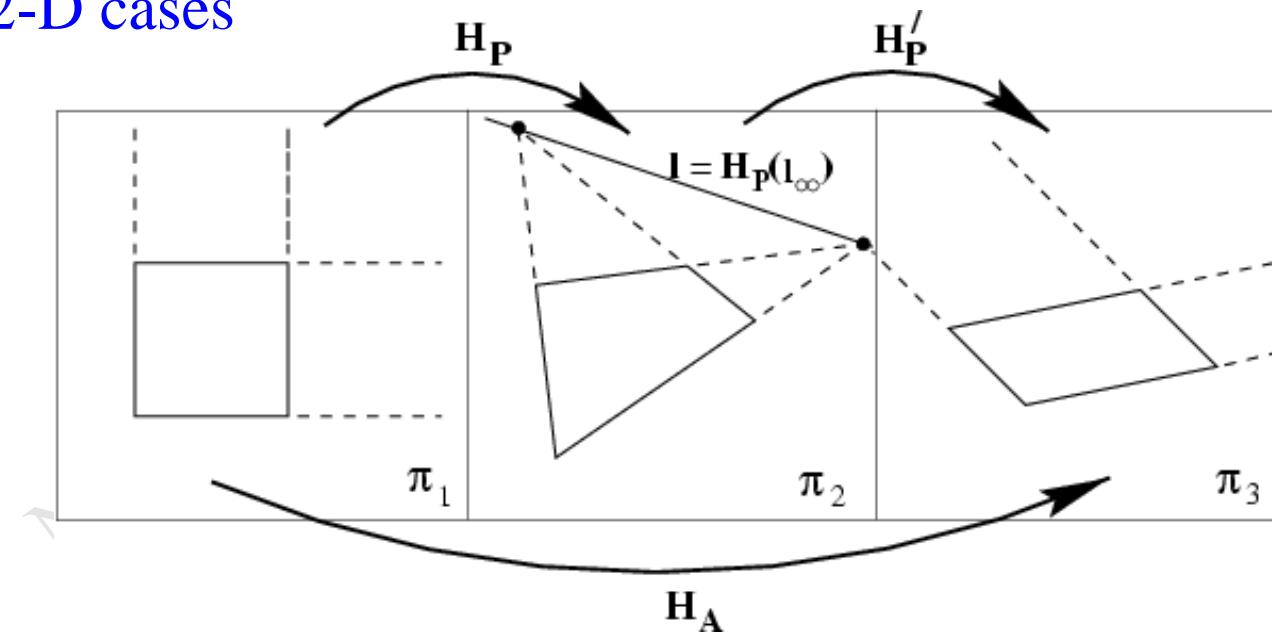
Compute new  $\mathbf{P}$  &  $\mathbf{P}'$ , and use triangulation (back-projection) from all 2D correspondences, then generate new 3D points ( $\mathbf{X}_{2i}$ )



# 3D reconstruction for projective geometry

- Affine reconstruction
  - Projective transformation, then affine trans..

2-D cases





# 3D reconstruction for projective geometry

- Affine reconstruction
  - Projective transformation, then affine trans..  
 $(\mathbf{P}, \mathbf{P}', \{\mathbf{X}_i\})$

$$\boldsymbol{\pi}_{\infty} = (A, B, C, D)^T \rightarrow (0, 0, 0, 1)^T$$

$\mathbf{H}^{-T} \boldsymbol{\pi}_{\infty} = (0, 0, 0, 1)^T$  → to find a  $\boldsymbol{\pi}_{\infty}$  under this constraint, but how?

$$\mathbf{H} = \begin{bmatrix} \mathbf{I} & 0 \\ \boldsymbol{\pi}_{\infty}^T & \end{bmatrix} \quad (\text{if determinant } \neq 0)$$

A desired solution if  $\boldsymbol{\pi}_{\infty}$  can be determined, the mapped 3D points will be an affine-mapping

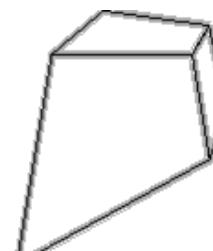


# 3D reconstruction for projective geometry

## ■ Hierarchy of transformations

Projective  
15dof

$$\begin{bmatrix} A & t \\ v^T & v \end{bmatrix}$$



Intersection and tangency

Affine  
12dof

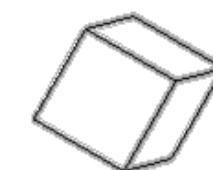
$$\begin{bmatrix} A & t \\ 0^T & 1 \end{bmatrix}$$



Parallelism of planes,  
Volume ratios, centroids,  
**The plane at infinity  $\pi_\infty$**

Similarity  
7dof

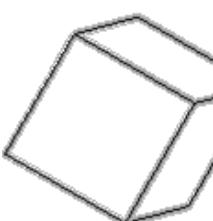
$$\begin{bmatrix} sR & t \\ 0^T & 1 \end{bmatrix}$$



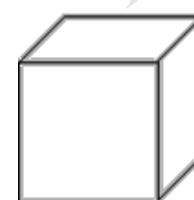
**The absolute conic  $\Omega_\infty$**

Euclidean  
6dof

$$\begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix}$$



Volume





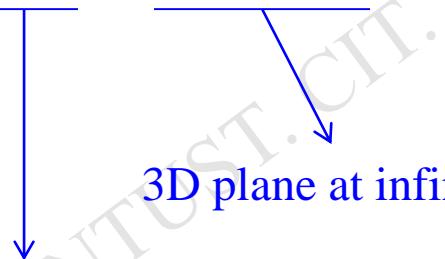
# 3D reconstruction for projective geometry

- Affine reconstruction
  - Projective transformation, then affine trans..

How to determine  $\pi_\infty$  ?

Here are various cases:

$$\mathbf{H}^{-T} \boldsymbol{\pi}_\infty = (0,0,0,1)^T$$



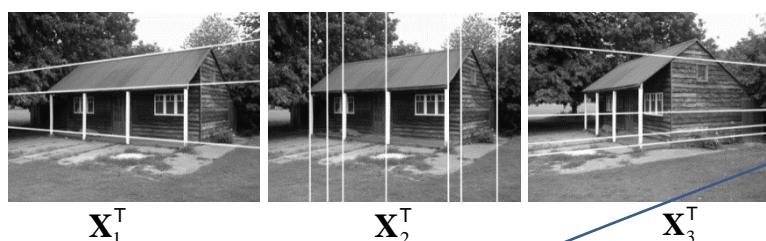
- Translation motion
- Parallel lines
- Distance ratios on a lines (angle)
- The infinite homography
- One of the cameras is affine

3D plane: the scene in the projective geometry  
(need to be determined, ex. use 3 points to find out)



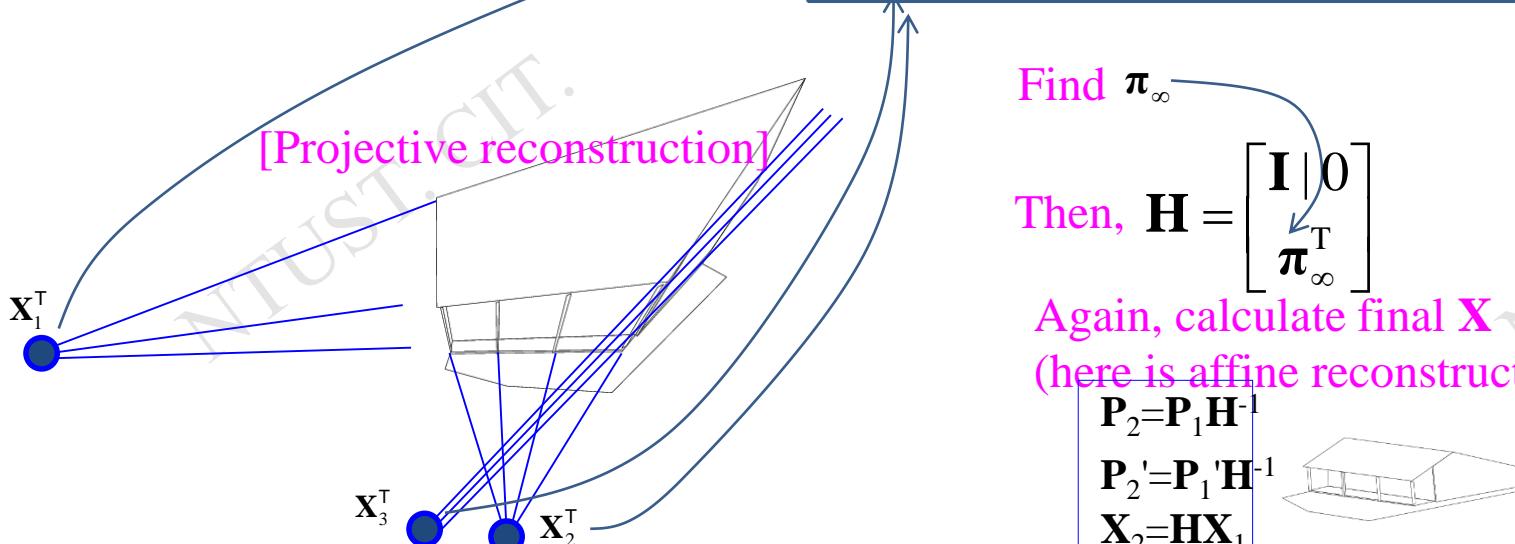
# 3D reconstruction for projective geometry

- Affine reconstruction
  - Projective transformation, then affine trans..



Recall slide: "Projective 3D geometry"  
Determine a 3D plane

$$\begin{bmatrix} \mathbf{x}_1^T \\ \mathbf{x}_2^T \\ \mathbf{x}_3^T \end{bmatrix} \boldsymbol{\pi} = 0 \quad \begin{bmatrix} (X_1)_1 & (X_1)_2 & (X_1)_3 & (X_1)_4 \\ (X_2)_1 & (X_2)_2 & (X_2)_3 & (X_2)_4 \\ (X_3)_1 & (X_3)_2 & (X_3)_3 & (X_3)_4 \end{bmatrix} \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \end{bmatrix} = 0$$





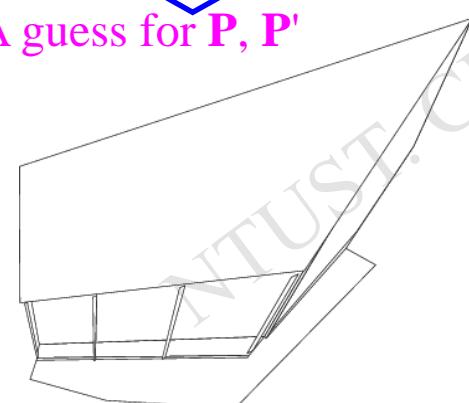
# 3D reconstruction for projective geometry

## ■ Metric reconstruction



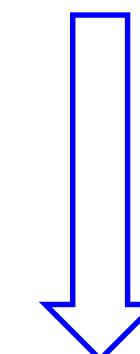
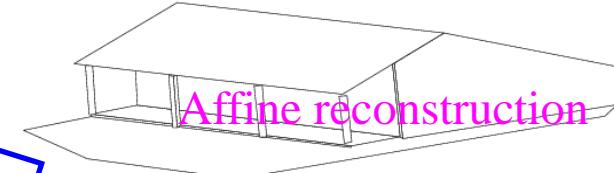
Two views geometry ( $F$ )

A guess for  $P, P'$

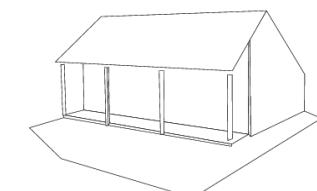


[Projective reconstruction]

$$\mathbf{H} = \begin{bmatrix} \mathbf{I} | 0 \\ \boldsymbol{\pi}_\infty^T \end{bmatrix}$$



$$\mathbf{H} = \begin{bmatrix} \mathbf{A}^{-1} & 0 \\ 0 & 1 \end{bmatrix}$$
$$\mathbf{A}\mathbf{A}^T = (\mathbf{M}^T \boldsymbol{\omega} \mathbf{M})^{-1}$$

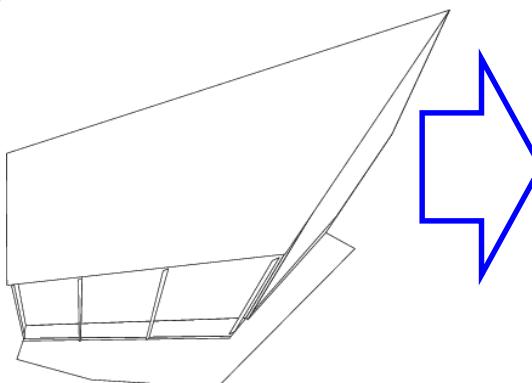


Metric reconstruction



# 3D reconstruction for projective geometry

## ■ Metric reconstruction—cont.



Either

$$\mathbf{P}_1 = [\mathbf{I} \mid 0] \quad \mathbf{P}_1' = [[\mathbf{e}']_x \mathbf{F} + \mathbf{e}' \mathbf{v}^T \mid \lambda \mathbf{e}']$$

Or

$$\mathbf{P}_1 = [\mathbf{I} \mid 0] \quad \mathbf{P}_1' = [[\mathbf{e}']_x \mathbf{F} \mid \mathbf{e}']$$

Triangulation for all 3D points ( $\mathbf{X}_{1i}$ ),

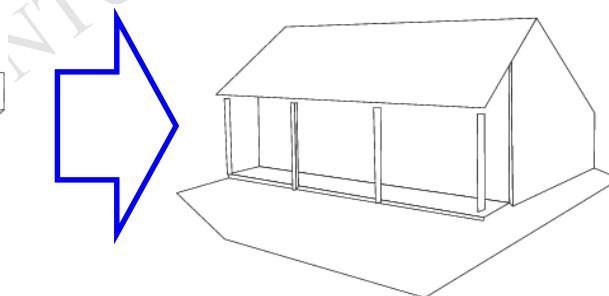
$$\text{ex } \begin{bmatrix} u\mathbf{p}_3^T - \mathbf{p}_1^T \\ v\mathbf{p}_3^T - \mathbf{p}_2^T \\ u'\mathbf{p}_3^T - \mathbf{p}_1^T \\ v'\mathbf{p}_3^T - \mathbf{p}_2^T \end{bmatrix} \mathbf{X}_{4 \times 1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{I} \mid 0 \\ \boldsymbol{\pi}_\infty^T \end{bmatrix}$$

$$\mathbf{P}_2 = \mathbf{P}_1 \mathbf{H}^{-1}$$

$$\mathbf{P}_2' = \mathbf{P}_1' \mathbf{H}^{-1}$$

$$\mathbf{X}_{2i} = \mathbf{H} \mathbf{X}_{1i}$$



$$\mathbf{H} = \begin{bmatrix} \mathbf{A}^{-1} & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{A} \mathbf{A}^T = (\mathbf{M}^T \boldsymbol{\omega} \mathbf{M})^{-1}$$

$$\mathbf{P}_3 = \mathbf{P}_2 \mathbf{H}^{-1}$$

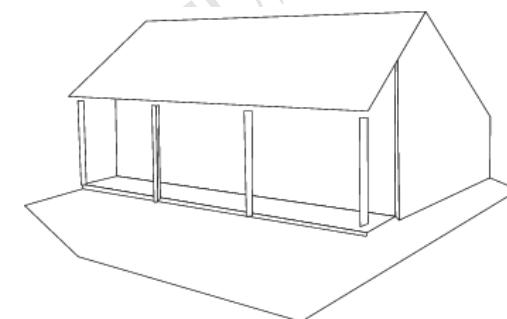
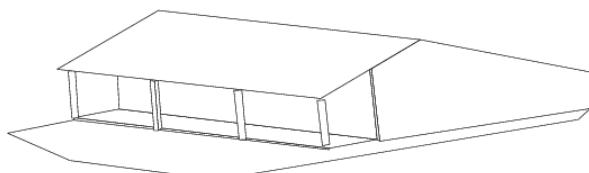
$$\mathbf{P}_3' = \mathbf{P}_2' \mathbf{H}^{-1}$$

$$\mathbf{X}_{3i} = \mathbf{H} \mathbf{X}_{2i}$$



# 3D reconstruction for projective geometry

## ■ Metric reconstruction—cont.



$$\mathbf{H} = \begin{bmatrix} \mathbf{A}^{-1} & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{A}\mathbf{A}^T = (\mathbf{M}^T \boldsymbol{\omega} \mathbf{M})^{-1}$$

Cholesky factorization

$$\mathbf{P}_3 = \mathbf{P}_2 \mathbf{H}^{-1}$$

$$\mathbf{P}_3' = \mathbf{P}_2' \mathbf{H}^{-1}$$

$$\mathbf{X}_{3i} = \mathbf{H} \mathbf{X}_{2i}$$

Here

$$\boldsymbol{\omega} = (\mathbf{K} \mathbf{K}^T)^{-1}$$

→ you may assume intrinsic parameter  $\mathbf{K}$  is known, or under some constraints.

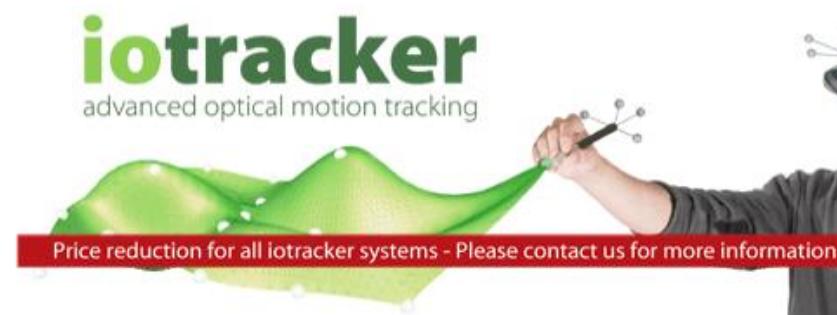
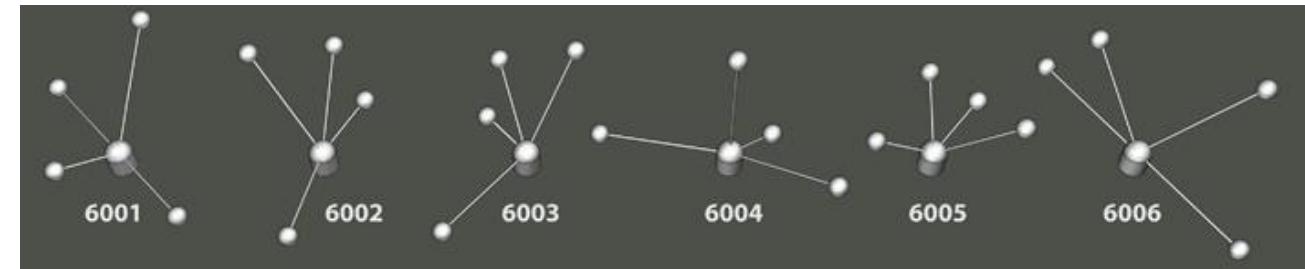
$$\mathbf{P}_2 = [\mathbf{M} \mid \mathbf{m}]$$

The projection matrix  
of affine reconstruction



# 3D reconstruction—Applications

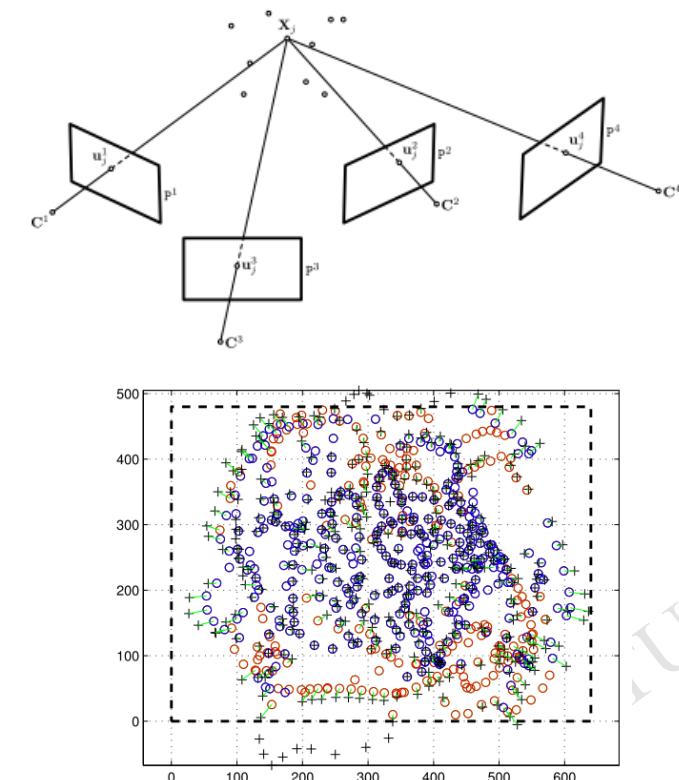
- Motion Tracking (other tracking issue)
- How to? 3D→2D (handling features)





# 3D reconstruction—Applications

## ■ Multi-camera calibration





# 3D reconstruction—Applications

- Other issues
  - 1. Bundle adjustment?
  - 2. Structure From motion?
  - 3. Factorization?

- Select paper:

M. Han and T. Kanade, “Creating 3D models with uncalibrated cameras,” in IEEE Workshop on Applications of Computer Vision, 2000, pp. 178-185.

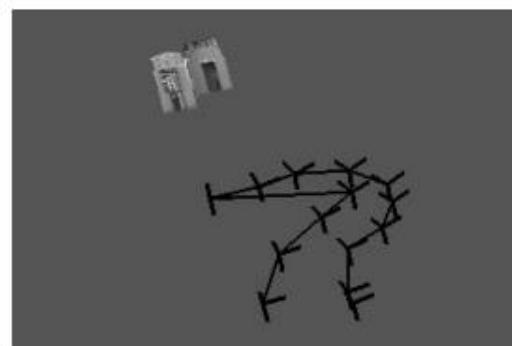


# 3D reconstruction—Applications

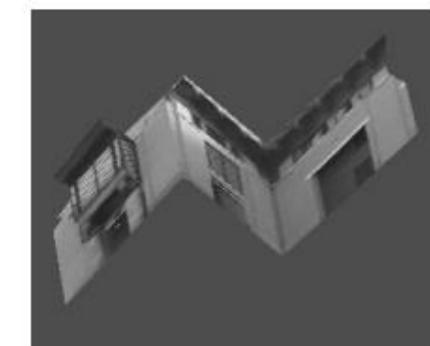
## ■ Factorization



(a)



(b)

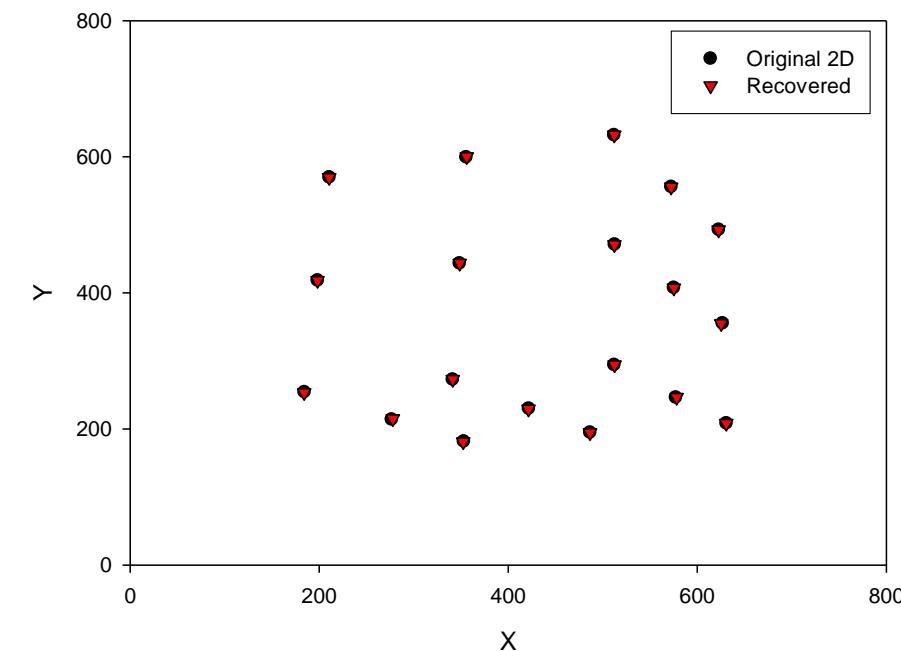
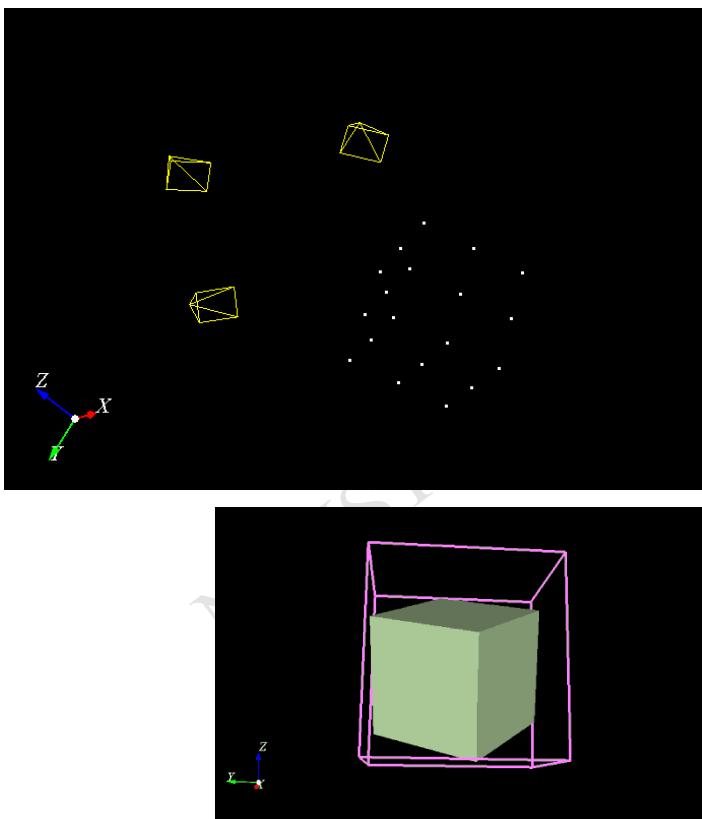


(c)



# 3D reconstruction—Applications

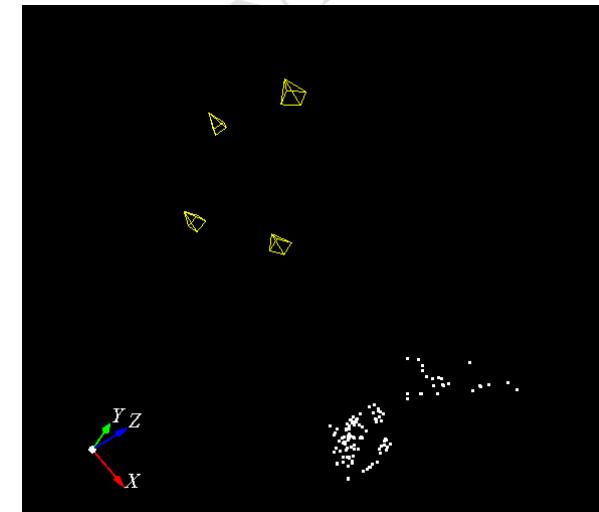
## ■ Factorization—Implementation result





# 3D reconstruction—Applications

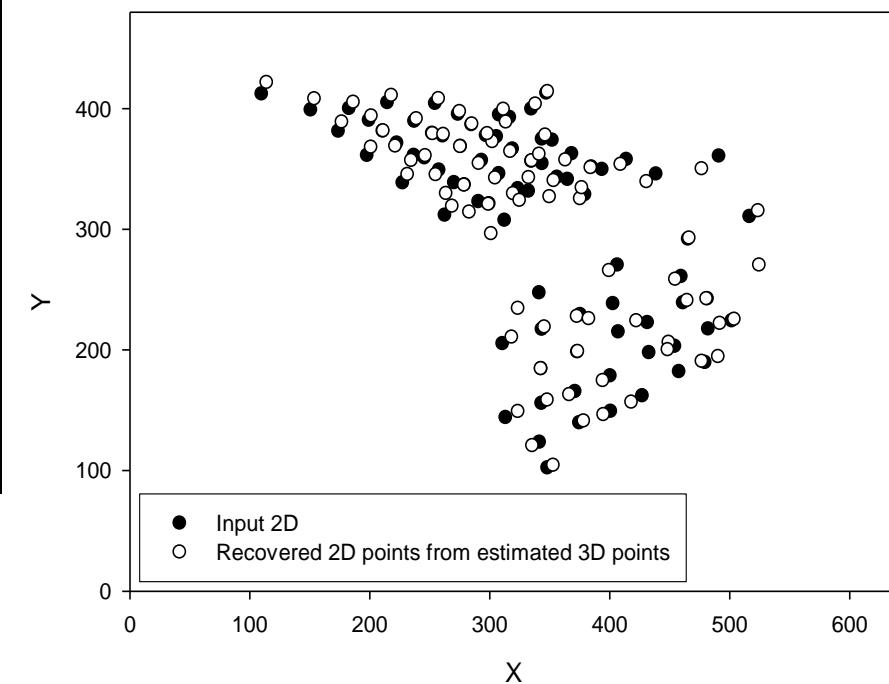
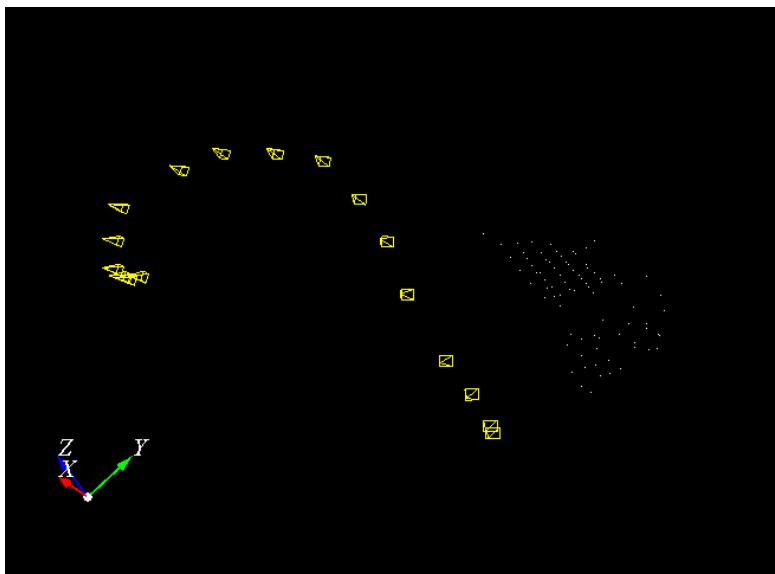
## ■ Factorization—Implementation result—cont.





# 3D reconstruction—Applications

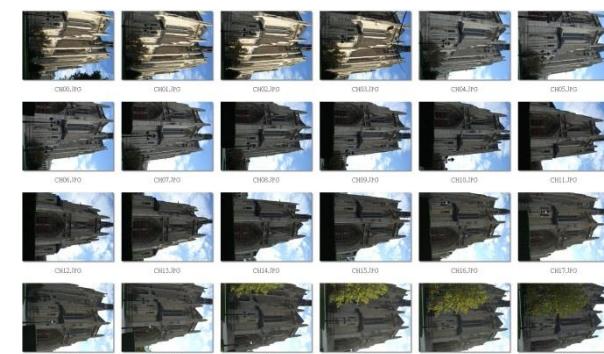
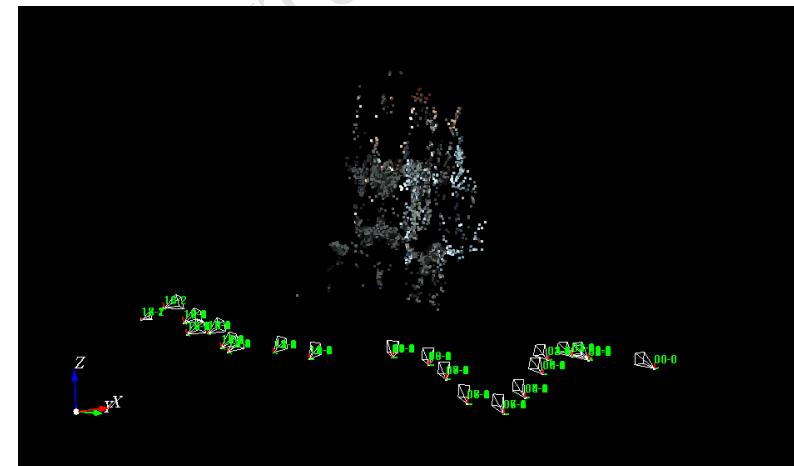
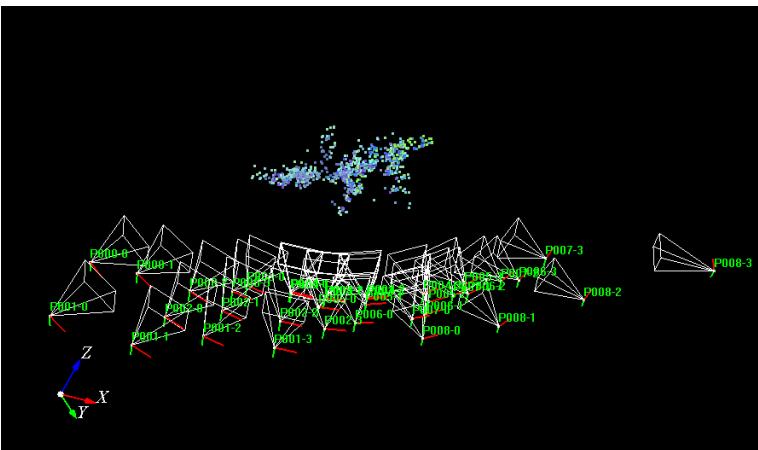
## ■ Factorization—Implementation result—cont.





# 3D reconstruction—Applications

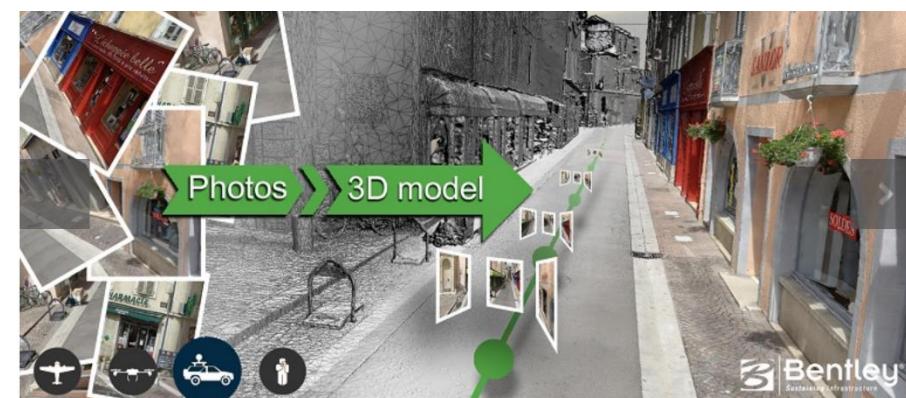
## ■ Factorization—Implementation result—cont.





# Commercial / Noncommercial Tools

- Agisoft PhotoScan
- RealityCapture
- PhotoModeler
- Autodesk Remake
- Strata Foto 3D CX2



<http://www.agisoft.com/>

<https://www.capturingreality.com/>

<http://www.photomodeler.com/index.html>

<https://remake.autodesk.com/about>

<https://www.strata.com/foto-3d-cx-create-textured-3d-models-from-your-digital-camera/>



# Commercial / Noncommercial Tools

- 3DF Zephyr Pro
- PIX4D
- DroneDeploy
- senseFly

...

<http://www.3dflow.net/3df-zephyr-pro-3d-models-from-photos/>

<https://pix4d.com/>

<https://www.dronedeploy.com/>

<https://www.sensefly.com/drones/ebee.html>



# Structure from motion

- CPMVVS
- Image-based 3D reconstruction (freeware)

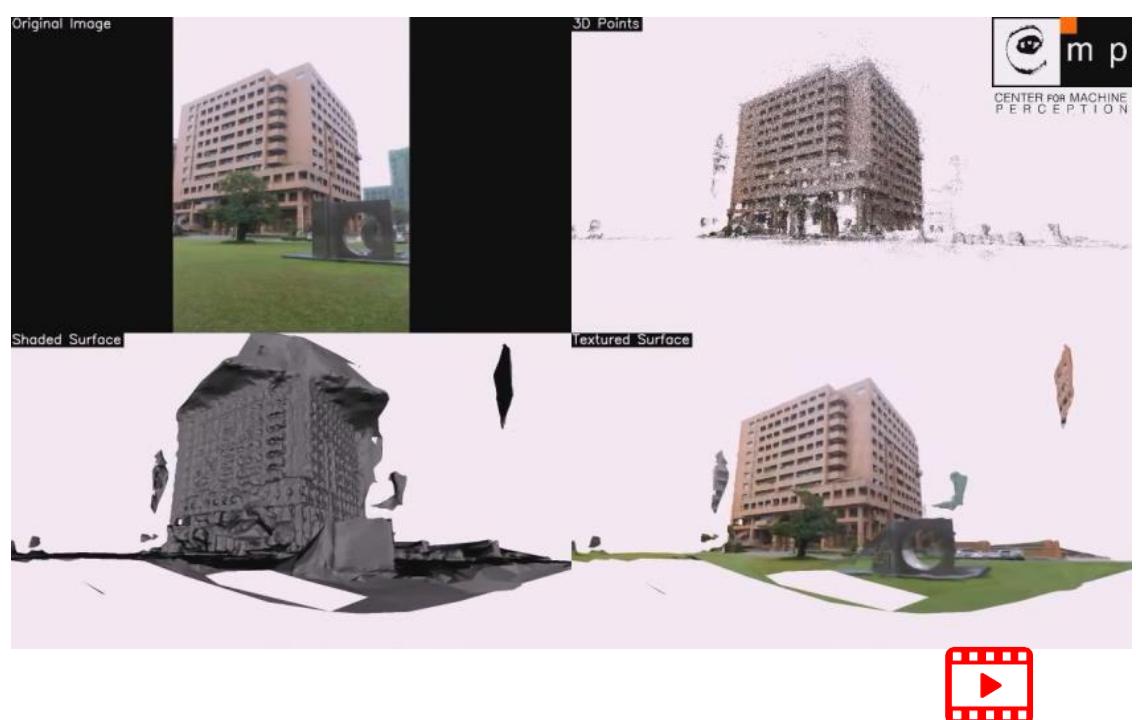
**CMPMVS - Multi-View Reconstruction Software**



**Authors:** Michal Jancosek & Tomas Pajdla  
**Software written by:** Michal Jancosek  
**Latest version:** 0.6.0  
**Release date:** September 28, 2012  
**Reference to cite:**  
[1] M. Jancosek, T. Pajdla. Multi-View Reconstruction Preserving Weakly-Supported Surfaces, CVPR 2011 - IEEE Conference on Computer Vision and Pattern Recognition 2011 (pdf).

**Introduction**  
CMPMVS is a multi-view reconstruction software. The input to our software is a set of perspective images and camera parameters (internal and external camera calibrations). The output is a textured mesh of the rigid scene visible in the images. Non-rigid objects are implicitly ignored.

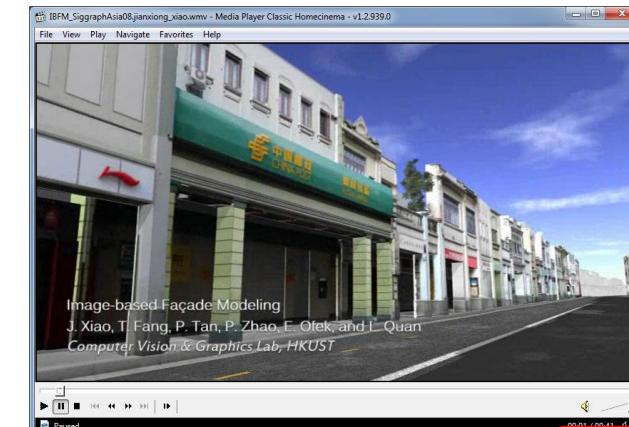
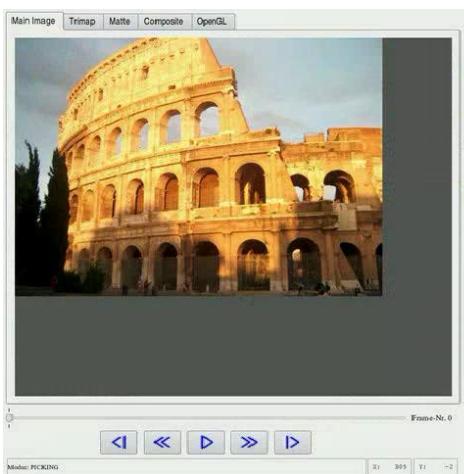
For discussion on the software please visit our Google group at  
<http://groups.google.com/group/cmpmvs>





# Structure from motion (software pre-product)

- Applications
- Video trace: image synthesis
- Authoring tools to 3D (static object)
- Reconstruct urban 3D: Deal with frontal textures



2006 - Pavić, Schönefeld, Kobbelt - Interactive image completion with perspective correction

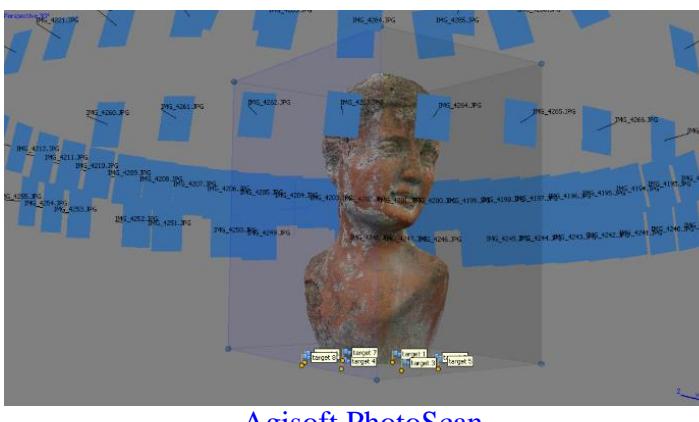
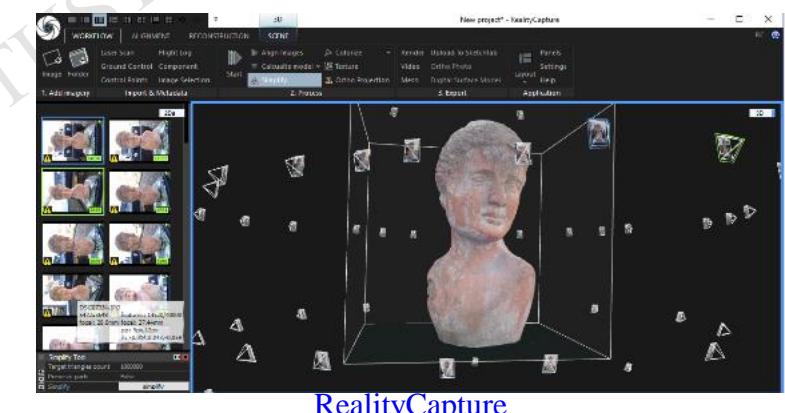
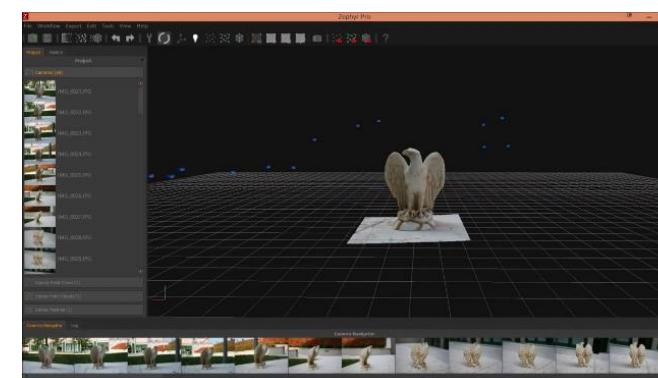
Siggraph 2007 Video trace

Image based façade modeling, SIGGRAPH ASIA 2008



# Multi-view in 3D reconstruction (photometric)

## ■ 3D software or services





色彩與照明科技研究所  
Graduate Institute of  
Color and Illumination Technology

