Errata for
“On Stability of a Class of Filters for Nonlinear Stochastic Systems”

Toni Karvonen, Silvère Bonnabel, Eric Moulines, and Simo Särkkä

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This is an errata for the article

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Bernstein’s concentration inequality in Theorem C.1 on p. 2043 is given in an erroneous form. The correct inequality (C.1), as given in Theorem 2.10 of Reference [10], is

$$\mathbb{P}\big[X - \mathbb{E}(X) \geq \alpha \mathrm{e}\big(\sqrt{2\delta} + \delta\big)\big] \leq \mathrm{e}^{-\delta}.$$

Because the proofs of Theorems 3.1 and 4.2 use the invalid Bernstein inequality on pp. 2046–2047, every concentration inequality in the article is erroneous. We thank Shihong Wei of Johns Hopkins University for bringing this error to our attention.

Concentration inequalities in the article can be fixed by replacing the use of Theorem C.1 in the proofs of Theorems 3.1 and 4.2 with the following result, the proof of which we provide for completeness. Equation (3.6) in Reference [19] is a slightly less optimal version of the concentration inequality in Equation (1) below.

Proposition. *Let X be a nonnegative random variable and suppose that there $\alpha > 0$ such that $\mathbb{E}(X^n) \leq \alpha^n n^n$ for every integer $n \geq 1$. Then*

$$\mathbb{P}\left[X \geq \alpha \mathrm{e}\left(\frac{1}{2} + \sqrt{\delta} + \delta\right)\right] \leq \mathrm{e}^{-\delta}, \tag{1}$$

for any $\delta > 0$.

Proof. Stirling’s approximation yields

$$b_n := \frac{(2n)!}{2^n n!} \geq \frac{\sqrt{4\pi n}\,(2n)^{2n}\mathrm{e}^n}{2^n \mathrm{e}\sqrt{n}\,n^n \mathrm{e}^{2n}} = \frac{\sqrt{4\pi}}{\mathrm{e}}\left(\frac{2}{\mathrm{e}}\right)^n n^n$$

for any $n \geq 1$. Therefore

$$\mathbb{E}(X^n) \leq \alpha^n n^n \leq \frac{\mathrm{e}}{\sqrt{4\pi}}\left(\frac{\mathrm{e}\alpha}{2}\right)^n \frac{(2n)!}{2^n n!} \leq \left(\frac{\mathrm{e}\alpha}{2}\right)^n \frac{(2n)!}{2^n n!}.$$

By Proposition 11.6.6 in Del Moral (2013) any random variable Z such that $\mathbb{E}(|Z|^n) \leq c^n b_n$ for a constant $c > 0$ and every $n \geq 1$ satisfies

$$\mathbb{P}\big[Z \geq c(1 + 2\delta + 2\sqrt{\delta}\,)\big] \geq \mathrm{e}^{-\delta}$$

for any $\delta > 0$. Setting $Z = X$ and $c = \alpha\mathrm{e}/2$ yields the claim. □

Using the above proposition in the place of Theorem C.1 results in the constant $\beta(\delta) = \mathrm{e}(\sqrt{2\delta} + \delta)$ being changed to

$$\beta(\delta) = \mathrm{e}\left(\frac{1}{2} + \sqrt{\delta} + \delta\right)$$

in Theorems 3.1 and 4.2 and Section 5.1.

References

Del Moral, P. (2013). *Mean Field Simulation for Monte Carlo Integration*. Number 153 in Chapman & Hall/CRC Monographs on Statistics & Applied Probability. Chapman and Hall/CRC, 1st edition.