Return your solutions on Moodle before **Sunday 8.3. at 23:59**. Your solutions must include intermediate steps and explain your reasoning.

Exercise 1.1. Convert the following numbers to decimal:

(a) $(170.03)_8$

(b) $(1022.02)_3$

(c) $(101010)_2$

(d) $(132.1)_5$

Convert the following decimal numbers to binary:

(e) 100

(f) 33

(g) 128.25

(h) 19.375

Exercise 1.2. Represent the following decimal numbers in the IEEE 754 half-precision (16-bit) format:

(a) 128.25

(b) -6.5

Your solutions must include intermediate steps.

Exercise 1.3. Which decimal numbers do the following IEEE 754 half-precision representations encode:

(a) `0 00100 1100000000`

(b) `1 01010 0110000000`

Exercise 1.4. Let $\text{ulp}(x)$ (unit in the last place) denote the distance between two consecutive representable numbers around x for 32-bit floats. That is,

$$\text{ulp}(x) = x_2 - x_1,$$

where $x_1 \neq x_2$ are floats such that $x_1 \leq x \leq x_2$ and the distance $x_2 - x_1$ is as small as possible.

(a) Determine $\text{ulp}(x)$ for $x \in [1, 2)$.

(b) Determine $\text{ulp}(x)$ for $x \in [4, 8)$.

(c) Determine $\text{ulp}(x)$ for $x \in [2^k, 2^{k+1})$ for all $k \in \mathbb{N}$.

Exercise 1.5. Consider the matrix

$$A_\varepsilon = \begin{bmatrix} 11 & 10 & 14 \\ 12 & 11 + 1/922 - \varepsilon & -13 \\ 14 & 13 & -66 \end{bmatrix},$$

where $\varepsilon > 0$. In exact arithmetic the matrix product $A_\varepsilon \cdot A_\varepsilon^{-1}$ equals the 3×3 identity matrix for any $\varepsilon > 0$. Use a programming language of your choosing to first compute A_ε^{-1} and then the product $A_\varepsilon \cdot A_\varepsilon^{-1}$. Examine what happens when you do this for *very small* values of ε (e.g., $\varepsilon = 10^{-12}$ and smaller). Report your findings for several such ε and try to explain why the phenomenon you observe occurs.