

Contents

1	src/geom/farthestpoints.hpp	2
2	src/geom/convexhull.hpp	3
3	src/geom/closestpoints.hpp	4
4	src/geom/basic.hpp	6
5	src/numtheory/diophantine.hpp	8
6	src/numtheory/basic.hpp	10
7	src/graph/spanningtree.hpp	11
8	src/graph/stronglyconnected.hpp	12
9	src/graph/maxflow.hpp	14
10	src/graph/cutvertices.hpp	16
11	src/string/suffixarray.hpp	18
12	src/string/z.hpp	20
13	src/string/lcparray.hpp	21
14	src/logic/2sat.hpp	22
15	src/boilerplate.hpp	25
16	src/game/nim.hpp	27
17	src/datastruct/querysegtree.hpp	28
18	src/datastruct/querysegtree2d.hpp	30
19	src/datastruct/bindtree.hpp	32
20	src/datastruct/unionfind.hpp	33
21	src/datastruct/bindtree2d.hpp	34

1 src/geom/farthestpoints.hpp

```
#pragma once
#include "boilerplate.hpp"

#include "geom/convexhull.hpp"

/// Find the distance between farthest points in 'points'.
/// NOTE: may not work for collinear cases with floating point coordinates.
VC computeFarthestPoints(const vector<V>& points) {
    vector<V> hull = computeConvexHull(points);
    int n = hull.size();

    if(n < 2) fail();
    if(n == 2) return abs(hull[0] - hull[1]);

    auto prev = [n](int i) {
        if(i == 0) return n - 1;
        return i - 1;
    };
    auto next = [n](int i) {
        if(i == n - 1) return 0;
        return i + 1;
    };

    int f = 0;
    double d = 0;

    int a = 0;
    int b = 1;
    for(int i = 0; i <= n; ++i) {
        d = max(d, abs(hull[a] - hull[f]));

        auto dist = [&](int x) {
            return pointLineDistance(hull[x], hull[a], hull[b]);
        };
        while(dist(next(f)) > dist(f)) {
            f = next(f);
            d = max(d, abs(hull[a] - hull[f]));
        }
        while(dist(prev(f)) > dist(f)) {
            f = prev(f);
            d = max(d, abs(hull[a] - hull[f]));
        }

        a = next(a);
        b = next(b);
    }

    return d;
}
```

2 src/geom/convexhull.hpp

```
#pragma once
#include "boilerplate.hpp"

#include "geom/basic.hpp"

/// Compute the convex hull of given set of points. The points on the edges
/// of the hull are not listed in the result (if floating point numbers are
/// used, inaccuracy may cause them to be listed).
///
/// Runs in linear time, uses Andrew's monotone chain algorithm.
vector<V> computeConvexHull(vector<V> points) {
    auto cmp = [](V a, V b) {
        if(a.real() == b.real()) {
            return a.imag() < b.imag();
        } else {
            return a.real() < b.real();
        }
    };
    sort(points.begin(), points.end(), cmp);
    points.erase(unique(points.begin(), points.end()), points.end());

    int n = points.size();
    if(n <= 2) return points;

    vector<V> hull;
    hull.push_back(points[0]);
    for(int dir = 0; dir != 2; ++dir) {
        if(dir) reverse(points.begin(), points.end());

        int start = hull.size();
        for(int i = 1; i < n; ++i) {
            while(
                hull.size() > start &&
                !ccw(hull[hull.size() - 2], hull.back(), points[i])
            ) {
                hull.pop_back();
            }
            hull.push_back(points[i]);
        }
    }
    hull.pop_back();

    return hull;
}
```

3 src/geom/closestpoints.hpp

```
#pragma once
#include "boilerplate.hpp"

/// Find the distance between closest points in 'points'.
VC computeClosestPoints(vector<V> points) {
    int n = points.size();

    if(n < 2) fail();

    auto cmpx = [](V a, V b) { return a.real() < b.real(); };
    auto cmpy = [](V a, V b) { return a.imag() < b.imag(); };

    sort(points.begin(), points.end(), cmpx);

    vector<V> sorty = points;
    vector<V> sorty2(n);
    vector<V> tmp;

    VC d = abs(points[1] - points[0]);

    int x = 1;
    while(x < n) {
        for(int a = 0; a < n; a += 2 * x) {
            auto u = sorty.begin() + a;
            auto v = sorty.begin() + min(a + x, (int)sorty.size());
            auto w = sorty.begin() + min(a + 2 * x, (int)sorty.size());

            tmp.clear();
            for(auto i = v; i != w; ++i) {
                if(i->real() <= points[a + x - 1].real() + d) tmp.push_back(*i);
            }

            for(auto i = u; i != v; ++i) {
                auto f = lower_bound(tmp.begin(), tmp.end(), V(0, i->imag() - d),
cmpy);

                while(f != tmp.end() && f->imag() <= i->imag() + d) {
                    d = min(d, abs(*f - *i));
                    ++f;
                }
            }

            merge(u, v, v, w, sorty2.begin() + a, cmpy);
        }

        swap(sorty, sorty2);

        x <= 1;
    }

    return d;
}
```

}

4 src/geom/basic.hpp

```
#pragma once
#include "boilerplate.hpp"

// WARNING: be careful with overflows and accuracy. Check what the code does.

/// Return true iff points a, b, c are CCW oriented.
bool ccw(V a, V b, V c) {
    return ((c - a) * conj(b - a)).imag() > 0;
}

/// Return true iff points a, b, c are collinear.
/// NOTE: doesn't make much sense with non-integer COORD_TYPE.
bool collinear(V a, V b, V c) {
    return ((c - a) * conj(b - a)).imag() == 0;
}

/// Check whether segments [a, b] and [c, d] intersect.
/// The segments must not be collinear. Doesn't handle edge cases (endpoint of
/// a segment on the other segment) consistently.
bool intersects(V a, V b, V c, V d) {
    return ccw(a, d, b) != ccw(a, c, b) && ccw(c, a, d) != ccw(c, b, d);
}

/// Interpolate between points a and b with parameter t.
V interpolate(VC t, V a, V b) {
    return a + t * (b - a);
}

/// Return interpolation parameter between a and b of projection of v to the
/// line defined by a and b.
/// NOTE: no rounding behavior specified for integers.
VC projectionParam(V v, V a, V b) {
    return ((v - a) / (b - a)).real();
}

/// Compute the distance of point v from line a..b.
/// NOTE: Only for non-integers!
VC pointLineDistance(V p, V a, V b) {
    return abs(((p - a) / (b - a)).imag()) * abs(b - a);
}

/// Compute the distance of point v from segment a..b.
/// NOTE: Only for non-integers!
VC pointSegmentDistance(V p, V a, V b) {
    V z = (p - a) / (b - a);
    if(z.real() < 0) return abs(p - a);
    if(z.real() > 1) return abs(p - b);
    return abs(z.imag()) * abs(b - a);
}
```

```

/// Return interpolation parameter between a and b of the point that is also
/// on line c..d.
/// NOTE: Only for non-integers!
VC intersectionParam(V a, V b, V c, V d) {
    V u = (c - a) / (b - a);
    V v = (d - a) / (b - a);
    return (u.real() * v.imag() - u.imag() * v.real()) / (v.imag() - u.imag());
}

```

5 src/numtheory/diophantine.hpp

```
#pragma once
#include "boilerplate.hpp"

struct DiophantineSolution {
    int64_t x;
    int64_t y;
    int64_t dx;
    int64_t dy;
};

void assign(int64_t& a, int64_t& b, int64_t aval, int64_t bval) {
    a = aval;
    b = bval;
}

/// Solve linear diophantine equation  $ax + by = c$ . If there is no solution
/// (iff  $\gcd(a, b)$  does not divide  $c$ ), fails. Returns structure containing
/// a solution  $(x, y)$ , all solutions are  $(x + t * dx, y + t * dy)$ , for all
/// integers  $t$ .  $a$  and  $b$  must be nonzero.  $|a|$ ,  $|b|$  and  $|c|$  should have couple
/// of high bits zero so that computations do not overflow.
DiophantineSolution solveLinearDiophantine(int64_t a, int64_t b, int64_t c) {
    if(a == 0 || b == 0) fail();
    if(a < 0 || b < 0 || c < 0) {
        DiophantineSolution ret = solveLinearDiophantine(abs(a), abs(b), abs(c));

        if((a < 0) != (c < 0)) {
            ret.x *= -1;
            ret.dx *= -1;
        }
        if((b < 0) != (c < 0)) {
            ret.y *= -1;
            ret.dy *= -1;
        }

        return ret;
    }

    int64_t d = a;
    int64_t e = b;

    int64_t x = 0, y = 1, lx = 1, ly = 0;
    while(e != 0) {
        int64_t q = d / e;
        assign(d, e, e, d % e);
        assign(x, lx, lx - q * x, x);
        assign(y, ly, ly - q * y, y);
    }

    if(c % d != 0) fail();
    int64_t coef = c / d;
```



```

DiophantineSolution ret;
ret.x = 0;
ret.y = 0;
ret.dx = b / d;
ret.dy = -a / d;

int64_t B = int64_t(1) << 62;
for(int i = 0; i <= 62; ++i) {
    ret.x <= 1;
    ret.y <= 1;

    if(coef & B) {
        ret.x += lx;
        ret.y += ly;
    }

    int64_t t = (ret.y - ret.x) / (ret.dx - ret.dy);
    ret.x += t * ret.dx;
    ret.y += t * ret.dy;

    B >>= 1;
}

return ret;
}

```

6 src/numtheory/basic.hpp

```
#pragma once
#include "boilerplate.hpp"

Z gcd_(Z a, Z b) {
    if(a == 0) return b;
    return gcd_(b % a, a);
}

/// Return the greatest common divisor of two integers. Returns 0 if a and b
/// are zero, otherwise result is always positive.
Z gcd(Z a, Z b) {
    return gcd_(abs(a), abs(b));
}

/// Return lookup table of primes in [0, n[. (1 for prime, 0 for non-prime).
vector<char> genPrimeTable(int n) {
    vector<char> ret(n, 1);
    if(n > 0) ret[0] = 0;
    if(n > 1) ret[1] = 0;
    for(int i = 2; i * i <= n; ++i) {
        int x = i * i;
        while(x < n) {
            ret[x] = 0;
            x += i;
        }
    }
    return ret;
}
```

7 src/graph/spanningtree.hpp

```
#pragma once
#include "boilerplate.hpp"

#include "datastruct/unionfind.hpp"

/// Find the minimum spanning tree of a graph with vertices 0,...,n-1 and
/// edges given by (weight, (vertex1, vertex2)) using Kruskal algorithm.
/// If the input graph is not connected, the result consists of spanning trees
/// for each component. The return value is a pair of the resulting forest and
/// its total weight.
/// Weight type may be replaced with any number type.
pair<vector<vector<int> >, double> kruskal(
    int n,
    vector<pair<double, pair<int, int> > > edges
) {
    sort(edges.begin(), edges.end());

    vector<vector<int> > span(n);
    double weight = 0.0;
    UnionFind c(n);
    for(int i = 0; i < edges.size(); ++i) {
        int v1 = edges[i].second.first;
        int v2 = edges[i].second.second;
        if(c.find(v1) != c.find(v2)) {
            c.merge(v1, v2);
            span[v1].push_back(v2);
            span[v2].push_back(v1);
            weight += edges[i].first;
        }
    }

    return pair<vector<vector<int> >, double>(span, weight);
}
```

8 src/graph/stronglyconnected.hpp

```
#pragma once
#include "boilerplate.hpp"

/// Compute the strongly connected components of a directed graph given as adjacency
/// lists. The result is a vector of the strongly connected components, each
/// given as a vector of vertex indices.
///
/// Uses Tarjan's algorithm, runs in  $O(V + E)$  time.
vector<vector<int>> computeStronglyConnectedComponents(const vector<vector<int>>& G) {
    int n = G.size();

    vector<int> ind(n, -1);
    vector<int> low(n);
    vector<int> parent(n, -1);
    vector<int> pos(n, 0);
    vector<char> in_tarj(n, 0);

    stack<int> dfs, tarj;

    int cur_ind = 0;

    vector<vector<int>> ret;
    for(int s = 0; s < n; ++s) {
        if(ind[s] != -1) continue;

        dfs.push(s);
        while(!dfs.empty()) {
            int v = dfs.top();
            dfs.pop();

            if(pos[v] == 0) {
                if(ind[v] != -1) fail();
                ind[v] = cur_ind;
                low[v] = cur_ind;
                ++cur_ind;
                tarj.push(v);
                in_tarj[v] = 1;
            }

            if(pos[v] == G[v].size()) {
                if(parent[v] != -1) {
                    low[parent[v]] = min(low[parent[v]], low[v]);
                }
                if(low[v] == ind[v]) {
                    vector<int> comp;
                    int r;
                    do {
                        r = tarj.top();
                        tarj.pop();
                        in_tarj[r] = 0;
                    } while(r != v);
                    ret.push_back(comp);
                }
            }
        }
    }
}
```

```

        comp.push_back(r);
    } while(r != v);
    ret.push_back(move(comp));
} else {
    dfs.push(v);

    int x = G[v][pos[v]];
    if(ind[x] == -1) {
        parent[x] = v;
        dfs.push(x);
    } else if(in_tarj[x]) {
        low[v] = min(low[v], ind[x]);
    }

    ++pos[v];
}
}

return ret;
}

```

9 src/graph/maxflow.hpp

```
#pragma once
#include "boilerplate.hpp"

/// Edmonds-Karp algorithm for computing max flow in a graph. Edges are given
/// with addEdge(source, destination, capacity), and maxFlow computes the
/// maximum flow, populating the flow fields of the edges.
struct EdmondsKarp {
    struct Edge {
        int dest;
        Z cap; // Remaining capacity in the edge.
        Z flow; // Flow through the edge. >= 0 in normal edges, <= 0 in
                // backwards edges.
        int back; // Corresponding backwards edge.
    };

    EdmondsKarp(int n) : G(n) { }

    void addEdge(int src, int dest, Z cap) {
        if(src == dest) return;
        Edge e1 = {dest, cap, 0, (int)G[dest].size()};
        Edge e2 = {src, 0, 0, (int)G[src].size()};
        G[src].push_back(e1);
        G[dest].push_back(e2);
    }

    Z maxFlow(int s, int t) {
        if(s == t) fail();

        Z ret = 0;

        typedef pair<int, int> P;
        vector<P> parent;
        while(true) {
            parent.clear();
            parent.resize(G.size(), P(-1, -1));

            queue<int> Q;
            Q.push(s);
            parent[s] = P(s, -1);

            while(!Q.empty()) {
                int v = Q.front();
                Q.pop();

                if(v == t) break;

                for(int i = 0; i < G[v].size(); ++i) {
                    if(G[v][i].flow == G[v][i].cap) continue;
                    int x = G[v][i].dest;
                    if(parent[x].first != -1) continue;
```

```

        parent[x] = P(v, i);
        Q.push(x);
    }
}

if(parent[t].first == -1) break;

Z a = -1;
int v = t;
while(v != s) {
    P p = parent[v];
    Z rem = G[p.first][p.second].cap - G[p.first][p.second].flow;
    if(a < 0 || rem < a) a = rem;
    v = p.first;
}
ret += a;
v = t;
while(v != s) {
    P p = parent[v];
    G[p.first][p.second].flow += a;
    G[v][G[p.first][p.second].back].flow -= a;
    v = p.first;
}

return ret;
}

vector<vector<Edge> > G;
};

```

10 src/graph/cutvertices.hpp

```
#pragma once
#include "boilerplate.hpp"

/// Compute the sorted vector with value 1 for vertex iff it is a cutvertex.
/// The graph is given as adjacency lists.
///
/// Uses Hopcroft-Tarjan algorithm, runs in  $O(V + E)$  time.
vector<char> computeCutvertices(const vector<vector<int>>& G) {
    int n = G.size();

    if(n == 0) return vector<char>();

    vector<int> pos(n, 0);
    vector<int> parent(n);
    vector<int> depth(n, -1);
    vector<int> lowpoint(n);
    vector<int> prevchild(n, -1);
    vector<char> ret(n, 0);
    stack<int> S;

    for(int r = 0; r < n; ++r) {
        if(depth[r] != -1) continue;

        depth[r] = 0;
        parent[r] = -1;
        lowpoint[r] = 0;
        int rootchildren = 0;
        S.push(r);

        while(!S.empty()) {
            int v = S.top();

            if(prevchild[v] != -1) {
                if(v != r && lowpoint[prevchild[v]] >= depth[v]) {
                    ret[v] = 1;
                }

                lowpoint[v] = min(lowpoint[v], lowpoint[prevchild[v]]);

                prevchild[v] = -1;
            }

            if(pos[v] == G[v].size()) {
                if(v == r && rootchildren >= 2) ret[r] = 1;
                for(int x : G[v]) {
                    if(x == parent[v]) continue;
                    lowpoint[v] = min(lowpoint[v], depth[x]);
                }
            }

            S.pop();
        }
    }
}
```



```

    } else {
        int x = G[v][pos[v]++];

        if(depth[x] == -1) {
            if(v == r) ++rootchildren;

            prevchild[v] = x;

            depth[x] = depth[v] + 1;
            parent[x] = v;
            lowpoint[x] = depth[x];
            S.push(x);
        }
    }
}

return ret;
}

```

11 src/string/suffixarray.hpp

```
#pragma once
#include "boilerplate.hpp"

namespace suffixarray_ {
struct Elem {
    int start;
    int parts[2];
};
}

/// Return the start indices of the suffices of string S sorted in
/// lexicographical order. Characters past the end come before other characters
/// in the order. The empty suffix is included.
vector<int> constructSuffixArray(const vector<int>& S) {
    using suffixarray_::Elem;

    int n = S.size();
    vector<pair<int, int> > C(n + 1);
    vector<int> tmp(n + 1);
    vector<Elem> T(n + 1);
    vector<Elem> T2(n + 1);

    for(int i = 0; i <= n; ++i) {
        C[i].first = (i == n) ? INT_MIN : S[i];
        C[i].second = -i;
    }
    sort(C.begin(), C.end());

    for(int i = 0; i <= n; ++i) {
        int start = -C[i].second;
        T[i].start = start;
        if(start == n) {
            T[i].parts[0] = 0;
        } else {
            T[i].parts[0] = 1;
            T[i].parts[1] = S[start];
        }
    }

    int t = 1;
    while(t <= n) {
        for(int i = 0; i <= n; ++i) {
            if(i != 0 &&
                T[i].parts[0] == T[i - 1].parts[0] &&
                T[i].parts[1] == T[i - 1].parts[1]
            ) {
                tmp[T[i].start] = tmp[T[i - 1].start];
            } else {
                tmp[T[i].start] = i;
            }
        }
    }
}
```

```

    }

    for(int i = 0; i <= n; ++i) {
        T[i].start = i;
        T[i].parts[0] = (i + t > n) ? 0 : tmp[i + t];
        T[i].parts[1] = tmp[i];
    }

    for(int s = 0; s < 2; ++s) {
        fill(tmp.begin(), tmp.end(), 0);
        for(int i = 0; i <= n; ++i) {
            ++tmp[T[i].parts[s]];
        }

        int x = 0;
        for(int i = 0; i <= n; ++i) {
            int y = tmp[i];
            tmp[i] = x;
            x += y;
        }

        for(int i = 0; i <= n; ++i) {
            T2[tmp[T[i].parts[s]]++] = T[i];
        }

        swap(T, T2);
    }

    t *= 2;
}

for(int i = 0; i <= n; ++i) {
    tmp[i] = T[i].start;
}

return tmp;
}

```

12 src/string/z.hpp

```
#pragma once
#include "boilerplate.hpp"

/// Return vector containing for each position i the length of the longest
/// substring that is also a prefix of the string.
vector<int> zAlgorithm(const vector<int>& S) {
    int n = S.size();
    vector<int> ret(n, 0);

    int L = 0;
    int R = 0;
    for(int i = 1; i < n; ++i) {
        if(i > R) {
            L = i;
            R = i;
            while(R < n && S[R - L] == S[R]) ++R;
            ret[i] = R - L;
            --R;
        } else if(ret[i - L] < R - i + 1) {
            ret[i] = ret[i - L];
        } else {
            L = i;
            while(R < n && S[R - L] == S[R]) ++R;
            ret[i] = R - L;
            --R;
        }
    }

    return ret;
}
```

13 src/string/lcparray.hpp

```
#pragma once
#include "boilerplate.hpp"

/// Construct the LCP array for string S when the suffix array is A.
/// LCP-array[i] = LCP(S[i - 1], S[i]), LCP-array[0] = 0.
vector<int> constructLCPArray(const vector<int>& S, const vector<int>& A) {
    int n = S.size();

    vector<int> prev(n + 1);
    prev[A[0]] = -1;
    for(int i = 1; i <= n; ++i) {
        prev[A[i]] = A[i - 1];
    }

    int p = 0;
    vector<int> lcp(n + 1);
    for(int i = 0; i < n; ++i) {
        if(prev[i] == -1) {
            lcp[i] = 0;
        } else {
            while(i + p < n && prev[i] + p < n && S[i + p] == S[prev[i] + p]) ++p;
            lcp[i] = p;
            if(p != 0) --p;
        }
    }

    vector<int> ret(n + 1);
    ret[0] = 0;
    for(int i = 1; i <= n; ++i) {
        ret[i] = lcp[A[i]];
    }

    return ret;
}
```

14 src/logic/2sat.hpp

```
#pragma once
#include "boilerplate.hpp"

typedef pair<int, bool> PIB;

/// Solve 2SAT-problem, i.e. find assignments of  $x_0 \dots x_{(n-1)}$  to true or false
/// such that all clauses in 'clauses' hold. The clauses are given as
///  $((i_1, v_1), (i_2, v_2))$ , and a clause holds if  $x_{i_1} == v_1$  or  $x_{i_2} == v_2$ .
/// The function returns true if there is such an assignment and replaces the
/// content of 'solution' to one solution, otherwise returns false.
bool solve2SAT(int n, const vector<pair<PIB, PIB>>& clauses, vector<char>& solution) {
    vector<vector<PIB>> imp[2];
    imp[0].resize(n);
    imp[1].resize(n);

    for(pair<PIB, PIB> clause : clauses) {
        PIB a = clause.first;
        PIB b = clause.second;
        imp[!a.second][a.first].push_back(b);
        imp[!b.second][b.first].push_back(a);
    }

    vector<char> isset[2];
    vector<char> val[2];
    vector<int> log[2];
    vector<int> stack[2];
    vector<int> pos[2];

    for(int b = 0; b < 2; ++b) {
        isset[b].resize(n, false);
        val[b].resize(n);
        pos[b].resize(n);
    }

    int win = 0;
    for(int i = 0; i < n; ++i) {
        if(issset[0][i]) continue;

        bool failed[2] = {false, false};

        for(int b = 0; b < 2; ++b) {
            isset[b][i] = true;
            val[b][i] = b;
            pos[b][i] = 0;

            log[b].clear();
            stack[b].clear();

            log[b].push_back(i);
            stack[b].push_back(i);
        }
    }
}
```

```

    }

    win = -1;
    while(win == -1 && !(failed[0] && failed[1])) {
        for(int b = 0; b < 2; ++b) {
            if(failed[b]) continue;
            if(stack[b].empty()) {
                win = b;
                break;
            }

            int x = stack[b].back();
            bool v = val[b][x];

            if(pos[b][x] == imp[v][x].size()) {
                stack[b].pop_back();
            } else {
                PIB p = imp[v][x][pos[b][x]++];

                int y = p.first;
                if(isset[b][y]) {
                    if(val[b][y] != p.second) {
                        failed[b] = true;
                        continue;
                    }
                } else {
                    isset[b][y] = true;
                    val[b][y] = p.second;
                    log[b].push_back(y);

                    pos[b][y] = 0;
                    stack[b].push_back(y);
                }
            }
        }
    }

    if(win == -1) break;

    int lose = 1 - win;
    for(int v : log[lose]) {
        isset[lose][v] = false;
    }
    for(int v : log[win]) {
        isset[lose][v] = true;
        val[lose][v] = val[win][v];
    }
}

if(win == -1) return false;

solution = move(val[0]);
return true;
}

```


15 src/boilerplate.hpp

```
#pragma once

// Boilerplate required by other modules.

using namespace std;

#include <algorithm> // abs, binary_search, copy, equal_range, lower_bound, max,
                  // merge, min, upper_bound, sort, swap, ...
#include <bitset>    // bitset
#include <complex>   // complex
#include <climits>   // INT_MAX, INT_MIN, ...
#include <cstdlib>   // exit
#include <deque>     // deque
#include <iostream>  // cin, cout, cerr
#include <list>      // list
#include <map>       // map
#include <queue>     // queue, priority_queue
#include <random>    // mt19937, uniform_int_distribution, uniform_real_distribution, ...
#include <set>       // set
#include <stack>     // stack
#include <string>    // string
#include <sstream>   // stringstream
#include <tuple>     // tuple
#include <vector>    // vector

// Complex number/vector type for geometry.
#ifdef COORD_TYPE
    typedef COORD_TYPE VC;
#else
    typedef double VC;
#endif
typedef complex<VC> V;

// Signed integer type used for integers apart from indices.
#ifdef INT_TYPE
    typedef INT_TYPE Z;
#else
    typedef int Z;
#endif

const double PI = 4 * atan(1);

void fail(string msg) {
    cerr << "FAIL: " << msg << "\n";
    abort();
}

void fail() {
    cerr << "FAIL\n";
    abort();
}
```


16 src/game/nim.hpp

```
#pragma once
#include "boilerplate.hpp"

/// In the Nim game, there are heaps of coins with  $h[i]$  coins in  $i$ :th heap.
/// The players remove any positive number of coins from one heap in turns.
/// The player who removes the last coin wins. The function returns a pair
/// consisting of the index of the heap and the number of coins to remove
/// to win, or pair (-1, -1) if there is no winning strategy.
pair<int, Z> solveNim(const vector<Z>& h) {
    Z nim = 0;
    for(int i = 0; i < h.size(); ++i) {
        nim ^= h[i];
    }

    if(nim == 0) return pair<int, Z>(-1, -1);

    for(int i = 0; i < h.size(); ++i) {
        if((nim ^ h[i]) < h[i]) return pair<int, Z>(i, h[i] - (nim ^ h[i]));
    }

    fail();
    return pair<int, Z>(-2, -2);
}
```

17 src/datastruct/querysegtree.hpp

```
#pragma once
#include "boilerplate.hpp"

/// Array [0, n[ -> Z with fast queries of A[i] (x) A[i + 1] (x) ... (x) A[j - 1]
/// for given i, j where (x) is the associative operator 'oper' (implement).
struct QuerySegmentTree {
    /// The associative operator to use. To be implemented by the user.
    static Z oper(Z a, Z b); // { return <result>; }

    QuerySegmentTree(vector<Z> src) {
        N = 1;
        while(N < src.size()) N *= 2;
        tree.resize(2 * N);
        copy(src.begin(), src.end(), tree.begin() + N);
        for(int x = N - 1; x != 0; --x) {
            tree[x] = oper(tree[2 * x], tree[2 * x + 1]);
        }
    }

    /// Query A[x] (x) A[x + 1] (x) ... (x) A[y - 1].
    Z query(int x, int y) {
        x += N;
        y += N;

        if(x == y) fail();
        if(x + 1 == y) return tree[x];

        Z a = tree[x];
        Z b = tree[--y];
        while(x / 2 != y / 2) {
            if(x % 2 == 0) a = oper(a, tree[x + 1]);
            if(y % 2 == 1) b = oper(tree[y - 1], b);

            x /= 2;
            y /= 2;
        }
        return oper(a, b);
    }

    /// Set A[x] to val.
    void set(int x, Z val) {
        x += N;
        tree[x] = val;
        x /= 2;
        while(x != 0) {
            tree[x] = oper(tree[2 * x], tree[2 * x + 1]);
            x /= 2;
        }
    }
}
```

```
int N;  
vector<Z> tree;  
};
```

18 src/datastruct/querysegtree2d.hpp

```
#pragma once
#include "boilerplate.hpp"

/// Array [0, w[ x [0, h[ -> Z with fast queries of given rectangles.
struct QuerySegmentTree2D {
    /// The associative and commutative operator to use. To be implemented by
    /// the user. Should output (0, 0) -> 0.
    static Z oper(Z a, Z b); // { return <result>; }

    /// Initially set to 0.
    QuerySegmentTree2D(int w, int h) {
        ++w;
        ++h;
        W = 1;
        while(W < w) W *= 2;
        H = 1;
        while(H < h) H *= 2;
        tree.resize(2 * W);
        for(int i = 1; i < 2 * W; ++i) {
            tree[i].resize(2 * H, 0);
        }
    }

    /// Query from rectangle [x1, x2[ x [y1, y2[. Returns 0 if the set is empty.
    Z query(int x1, int y1, int x2, int y2) {
        if(x1 >= x2 || y1 >= y2) return 0;

        x1 += W;
        x2 += W + 1;
        y1 += H;
        y2 += H + 1;

        bool first = true;
        Z ret = 0;

        auto addCol = [&](int x) {
            auto add = [&](int y) {
                int u1 = x;
                int u2 = x;
                int v1 = y;
                int v2 = y;
                while(u1 < W) u1 *= 2;
                while(u2 < W) u2 = 2 * u2 + 1;
                while(v1 < H) v1 *= 2;
                while(v2 < H) v2 = 2 * v2 + 1;

                if(first) {
                    ret = tree[x][y];
                    first = false;
                } else {

```

```

        ret = oper(ret, tree[x][y]);
    }
};

int t1 = y1;
int t2 = y2;
while(t1 / 2 != t2 / 2) {
    if(t1 % 2 == 0) add(t1 + 1);
    if(t2 % 2 == 1) add(t2 - 1);

    t1 /= 2;
    t2 /= 2;
}

};

while(x1 / 2 != x2 / 2) {
    if(x1 % 2 == 0) addCol(x1 + 1);
    if(x2 % 2 == 1) addCol(x2 - 1);

    x1 /= 2;
    x2 /= 2;
}

return ret;
}

/// Set A[x][y] to val.
void set(int x, int y, Z val) {
    x += W + 1;
    y += H + 1;

    tree[x][y] = val;

    while(x != 0) {
        if(x < W) tree[x][y] = oper(tree[2 * x][y], tree[2 * x + 1][y]);

        int t = y;
        t /= 2;
        while(t != 0) {
            tree[x][t] = oper(tree[x][2 * t], tree[x][2 * t + 1]);

            t /= 2;
        }
        x /= 2;
    }
}

int W;
int H;
vector<vector<Z>> tree;
};

```

19 src/datastruct/bindtree.hpp

```
#pragma once
#include "boilerplate.hpp"

/// Array [0, n[ -> Z with fast query of sums [0, x[.
/// Initially set to zero.
struct BinIndexedTree {
    BinIndexedTree(int n) : n(n), tree(n + 1, 0) { }

    /// Add x to i:th value, 0 <= i < n.
    void change(int i, Z x) {
        ++i;
        while(i <= n) {
            tree[i] += x;
            i += (i & -i);
        }
    }

    /// Get sum of elements [0, i[, 0 <= i <= n.
    Z sum(int i) {
        Z ret = 0;
        while(i != 0) {
            ret += tree[i];
            i -= (i & -i);
        }
        return ret;
    }

    int n;
    vector<Z> tree;
};
```


20 src/datastruct/unionfind.hpp

```
#pragma once
#include "boilerplate.hpp"

/// Data structure containing a subdivision of set  $0, \dots, n-1$ . Each set has a
/// representative element of the set, and two sets can be merged fast.
/// Initially each element is in its own set.
struct UnionFind {
    UnionFind(int n) : parent(n) {
        for(int i = 0; i < n; ++i) {
            parent[i] = i;
        }
    }

    /// Find the representative element for set containing x.
    int find(int x) {
        if(parent[x] != x) {
            parent[x] = find(parent[x]);
        }
        return parent[x];
    }

    /// Merge sets containing x and y.
    void merge(int x, int y) {
        x = find(x);
        y = find(y);
        parent[x] = y;
    }

    vector<int> parent;
};
```

21 src/datastruct/bindtree2d.hpp

```
#pragma once
#include "boilerplate.hpp"

/// Array [0, w[ x [0, h[ -> Z with fast query of sums [0, x[ x [0, y[.
/// Initially set to zero.
struct BinIndexedTree2D {
    BinIndexedTree2D(int w, int h) : w(w), h(h) {
        tree.resize(w + 1);
        for(vector<Z>& v : tree) v.resize(h + 1, 0);
    }

    /// Add v to i:th value, 0 <= x < w, 0 <= y < h.
    void change(int x, int y, Z v) {
        ++x;
        ++y;
        while(x <= w) {
            int t = y;
            while(t <= h) {
                tree[x][t] += v;
                t += (t & -t);
            }
            x += (x & -x);
        }
    }

    /// Get sum of elements [0, x[ x [0, y[, 0 <= x, y <= n.
    Z sum(int x, int y) {
        Z ret = 0;
        while(x != 0) {
            int t = y;
            while(t != 0) {
                ret += tree[x][t];
                t -= (t & -t);
            }
            x -= (x & -x);
        }
        return ret;
    }

    int w;
    int h;
    vector<vector<Z>> tree;
};
```