

Time stability in human interaction networks: primitive typology of vertex, prominence of measures and time activity statistics (SUPPORTING INFORMATION)

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This document presents supporting information for the article describing interaction networks stability. The sections exhibit activity distribution in time and among participants; the fraction of vertices in the peripheral, intermediary and hub sectors; the combination of basic measures into principal components with greater variance; the analysis of interaction networks from Facebook, Twitter and Participa.br. There is a focus on email list interaction networks for benchmarking.

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This documents exemplifies the stability of interaction networks by the observation of 4 emails lists (LAD, LAU, MET and CPP), described in Section II of the main article. These results were produced with the Gmane public domain data and an open source python package designed for attaining these, and related, results. The interested reader should follow Appendix A of the main article to access both data and routines. Similar results can be reproduced for any number of (Gmane) email lists. To avoid repeating text of each table for each list, the text is given inline, with the tables following the explanation.

I. TIME TABLES IN DIFFERENT SCALES

Theory presented in Section III A and results exposed in Section IV A of the paper¹.

A. Time circular measures

The rescaled circular mean θ'_μ , the standard deviation $S(z)$, the variance $Var(z)$, the circular dispersion $\delta(z)$ and the relation of maximum and minimum incidence at each time unit $\frac{\max(incidence)}{\min(incidence)}$. Also, $\mu_{\frac{\max(incidence')}{\min(incidence')}}$ and $\sigma_{\frac{\max(incidence')}{\min(incidence')}}$ are given for 1000 uniform distribution simulations within the same number of bins and with the same number of samples. Section III A of the main document describes the theoretical background of directional (or circular) statistics.

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TABLE I. LAU circular measures

scale	θ'_μ	$S(z)$	$Var(z)$	$\delta(z)$	$\frac{\max(incidence)}{\min(incidence)}$	$\mu_{\frac{\max(incidence')}{\min(incidence')}}$	$\sigma_{\frac{\max(incidence')}{\min(incidence')}}$
seconds	--/--	3.31	1.00	29337.65	1.27	1.29	0.04
minutes	--/--	3.13	0.99	8879.19	1.32	1.29	0.04
hours	-8.76	1.56	0.71	4.92	8.38	1.14	0.03
weekdays	-0.21	2.14	0.90	45.41	1.62	1.05	0.02
month days	-0.64	2.76	0.98	1001.75	1.49	1.17	0.03
months	3.55	2.30	0.93	94.53	1.57	1.09	0.02

TABLE II. LAD circular measures

scale	θ'_μ	$S(z)$	$Var(z)$	$\delta(z)$	$\frac{\max(incidence)}{\min(incidence)}$	$\mu_{\frac{\max(incidence')}{\min(incidence')}}$	$\sigma_{\frac{\max(incidence')}{\min(incidence')}}$
seconds	--/--	3.13	0.99	9070.17	1.28	1.29	0.05
minutes	--/--	3.60	1.00	205489.40	1.22	1.29	0.05
hours	-9.61	1.52	0.68	4.36	9.77	1.14	0.03
weekdays	-0.03	2.03	0.87	29.28	1.72	1.05	0.02
month days	-2.65	2.93	0.99	2657.77	1.50	1.17	0.03
months	-0.56	2.14	0.90	44.00	2.25	1.09	0.02

TABLE III. MET circular measures

scale	θ'_μ	$S(z)$	$Var(z)$	$\delta(z)$	$\frac{\max(incidence)}{\min(incidence)}$	$\mu_{\frac{\max(incidence')}{\min(incidence')}}$	$\sigma_{\frac{\max(incidence')}{\min(incidence')}}$
seconds	--/--	3.06	0.99	5910.47	1.27	1.29	0.04
minutes	--/--	3.14	0.99	9696.29	1.34	1.29	0.04
hours	-9.20	1.35	0.60	2.76	19.26	1.14	0.03
weekdays	-0.27	1.86	0.82	13.82	2.89	1.05	0.02
month days	3.58	2.49	0.95	237.30	1.55	1.17	0.03
months	-2.92	1.73	0.78	9.20	3.04	1.09	0.02

TABLE IV. CPP circular measures

scale	θ'_μ	$S(z)$	$Var(z)$	$\delta(z)$	$\frac{\max(incidence)}{\min(incidence)}$	$\mu_{\frac{\max(incidence')}{\min(incidence')}}$	$\sigma_{\frac{\max(incidence')}{\min(incidence')}}$
seconds	--/--	3.31	1.00	28205.46	1.26	1.29	0.04
minutes	--/--	3.18	0.99	12275.59	1.27	1.29	0.04
hours	-9.39	1.48	0.67	3.91	11.18	1.15	0.03
weekdays	-0.17	1.83	0.81	12.66	2.59	1.05	0.02
month days	-10.12	3.16	0.99	10789.17	1.54	1.17	0.03
months	0.15	2.34	0.93	115.49	1.50	1.08	0.02

B. Time histograms**C. Histograms of activity along the hours of the day**

Activity percentages along the hours of the day. Higher activity was observed between noon and 6pm, followed by the time period between 6pm and midnight. Around 2/3 of the whole activity takes place from noon to midnight. Nevertheless, the activity peak occurs around midday, with a slight skew toward one hour before noon.

TABLE V. LAU activity along the hours of the day

	1h	2h	3h	4h	6h	12h
0h	3.58					
1h	2.22	5.80	7.43	8.49	10.14	
2h	1.63	2.69				
3h	1.06					
4h	0.84	1.66	2.72			
5h	0.82			5.20		36.88
6h	1.17	3.54				
7h	2.37		7.07			
8h	3.53	9.57			26.74	
9h	6.04			23.20		
10h	6.83		19.67			
11h	6.79	13.62				
12h	6.11	12.36	18.75	24.68	35.66	
13h	6.26					
14h	6.38	12.31				
15h	5.93		16.91			
16h	5.52	10.98		20.73		63.12
17h	5.46					
18h	5.23	9.75				
19h	4.52		14.30			
20h	4.55	8.97			27.46	
21h	4.42			17.71		
22h	4.51		13.16			
23h	4.23	8.74				

TABLE VI. LAD activity along the hours of the day

	1h	2h	3h	4h	6h	12h
0h	4.01					
1h	2.52	6.53	8.32	9.37	10.78	
2h	1.79	2.84				
3h	1.06					
4h	0.75	1.40	2.46			
5h	0.66			3.81		33.11
6h	0.85	2.41				
7h	1.56		5.36			
8h	2.95	7.61			22.33	
9h	4.66			19.93		
10h	5.92		16.98			
11h	6.40	12.32				
12h	6.41	12.53	18.85	24.82	37.24	
13h	6.12					
14h	6.32	12.29				
15h	5.97		18.39			
16h	6.40	12.42		23.44		66.89
17h	6.02					
18h	5.99	11.02		15.65		
19h	5.03				29.65	
20h	4.63	9.22				
21h	4.59			18.63		
22h	4.88		14.00			
23h	4.53	9.41				

TABLE VII. MET activity along the hours of the day

	1h	2h	3h	4h	6h	12h
0h	2.87					
1h	1.77	4.64	5.67	6.31	7.15	
2h	1.04	1.67				
3h	0.64					
4h	0.47	0.85	1.48			
5h	0.38			2.89		29.33
6h	0.72	2.04				
7h	1.33		4.71			
8h	2.67	7.07			22.18	
9h	4.40			20.14		
10h	6.29		17.47			
11h	6.78	13.07				
12h	7.33	14.41	21.50	28.65	42.22	
13h	7.08					
14h	7.09	14.24				
15h	7.14		20.72			
16h	6.68	13.58		24.79		70.67
17h	6.89					
18h	5.99	11.22	16.19			
19h	5.23				28.44	
20h	4.98	9.34				
21h	4.37			17.22		
22h	4.24		12.25			
23h	3.64	7.88				

TABLE VIII. CPP activity along the hours of the day

	1h	2h	3h	4h	6h	12h
0h	3.66	6.42	8.20	9.30	10.67	33.76
1h	2.76					
2h	1.79	2.88	2.47	3.44	23.09	
3h	1.10					
4h	0.68	1.37	4.35	21.03		
5h	0.69					
6h	0.83	2.07	18.75			
7h	1.24					
8h	2.28	6.80	25.05	37.63	66.24	
9h	4.52					
10h	6.62	14.23	18.68	23.60		
11h	7.61					
12h	6.44	12.48	15.88	17.59		
13h	6.04					
14h	6.47	12.57	12.73	8.36		
15h	6.10					
16h	6.22	12.58	8.36			
17h	6.36					
18h	6.01	11.02	8.36			
19h	5.02					
20h	4.85	9.23	8.36			
21h	4.38					
22h	4.06	8.36	8.36			
23h	4.30					

D. Histograms of activity along the days of the week

Activity percentages along the days of the week. Higher activity was observed during weekdays, with a decrease of activity on weekends of at least one third and two thirds in extreme cases.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
LAU	15.71	15.81	15.88	16.43	15.14	10.13	10.91
LAD	14.92	17.75	17.01	15.41	14.21	10.40	10.31
MET	17.53	17.54	16.43	17.06	17.46	7.92	6.06
CPP	17.06	17.43	17.61	17.13	16.30	6.81	7.67

E. Histograms of activity along the days of the month

Although slightly higher activity rates are found in the beginning of the month, the most important feature seems to be the homogeneity made explicit by the high circular dispersion in the tables of Section [IA](#).

TABLE IX. LAU activity along the days of the month.

	1 day	5	10	15 days
1	3.36	16.21	33.71	50.82
2	3.43			
3	3.31			
4	3.37			
5	2.75			
6	3.03	17.50		
7	3.93			
8	3.62			
9	3.84			
10	3.09			
11	3.20	17.11	34.02	49.18
12	3.40			
13	3.67			
14	3.71			
15	3.14			
16	3.08	16.91		
17	3.13			
18	3.43			
19	3.61			
20	3.67			
21	3.60	15.43	32.27	
22	3.42			
23	2.80			
24	2.64			
25	2.97			
26	3.06	16.85		
27	2.69			
28	3.79			
29	3.75			
30	3.57			

TABLE X. LAD activity along the days of the month.

	1 day	5	10	15 days
1	3.29	15.77	33.63	50.50
2	3.38			
3	2.85			
4	2.94			
5	3.31			
6	3.60	17.85		
7	2.68			
8	3.78			
9	3.88			
10	3.91			
11	3.22	16.87	33.41	49.50
12	2.79			
13	3.50			
14	3.95			
15	3.40			
16	3.32	16.54		
17	2.95			
18	3.50			
19	3.69			
20	3.07			
21	2.76	15.71	32.96	
22	3.35			
23	3.32			
24	3.15			
25	3.13			
26	3.68	17.25		
27	4.02			
28	3.49			
29	3.34			
30	2.72			

TABLE XI. MET activity along the days of the month.

	1 day	5	10	15 days			
1	3.05	18.25	35.24	50.96			
2	3.38						
3	3.62						
4	4.25						
5	3.94	16.98	35.24				
6	3.73						
7	3.17						
8	3.26						
9	3.56						
10	3.26						
11	3.81	15.73	31.98	49.04			
12	2.91						
13	3.30						
14	2.75						
15	2.95	16.25	31.98				
16	3.36						
17	3.16						
18	3.44						
19	3.36						
20	2.93	15.79	32.78				
21	3.20						
22	3.11						
23	3.60						
24	2.74	16.99					
25	3.13						
26	3.13						
27	3.07						
28	3.61						
29	3.60						
30	3.57						

TABLE XII. CPP activity along the days of the month.

	1 day	5	10	15 days
1	3.22	15.98	31.82	49.62
2	3.08			
3	3.19			
4	3.65			
5	2.84	15.84		
6	3.65			
7	3.53			
8	3.10			
9	2.49			
10	3.07			
11	3.47	17.80	34.22	
12	3.26			
13	3.55			
14	3.84			
15	3.68	16.42		
16	3.74			
17	3.40			
18	3.41			
19	2.95			
20	2.93	17.13		
21	3.15			
22	3.64			
23	3.51			
24	3.32	16.84		
25	3.51			
26	3.54			
27	3.21			
28	3.40			
29	3.83			
30	2.86			

F. Histograms of activity along months of the year

Activity percentages of the months along the year from LAD list messages. Activity is concentrated in Jun-Aug for MET and LAD, and in Dec-Mar for CPP, LAU and LAD. These observations fit academic calendars, vacations and end-of-year holidays.

TABLE XIII. LAU activity along the months of the year.

	m.	b.	t.	q.	s.
Jan	10.22	19.56	28.24	35.09	49.16
Fev	9.34				
Mar	8.67	15.53	20.93	30.36	
Apr	6.86				
Mai	7.28	14.07	24.47	34.55	50.84
Jun	6.80				
Jul	8.97	16.29	26.36	34.55	
Ago	7.32				
Set	8.18	16.25	26.36	34.55	
Out	8.06				
Nov	7.64	18.30	26.36	34.55	
Dez	10.66				

TABLE XIV. LAD activity along the months of the year.

	m.	b.	t.	q.	s.
Jan	11.24	18.51	26.46	36.07	57.96
Fev	7.26				
Mar	7.95	17.56	31.50	37.56	
Apr	9.61				
Mai	8.94	21.89	22.30	26.37	42.04
Jun	12.95				
Jul	9.03	15.67	19.74		
Ago	6.64				
Set	6.63	12.38			
Out	5.75				
Nov	7.61	13.99			
Dez	6.38				

TABLE XV. MET activity along the months of the year.

	m.	b.	t.	q.	s.
Jan	4.87	11.00	16.89	23.30	47.70
Fev	6.13				
Mar	5.89	12.30	30.81	47.87	
Apr	6.41				
Mai	10.45	24.40	31.21	28.83	52.30
Jun	13.95				
Jul	13.24	23.47	21.09	28.83	
Ago	10.22				
Set	7.75	16.79	21.09	28.83	
Out	9.04				
Nov	7.45	12.05	21.09	28.83	
Dez	4.59				

TABLE XVI. CPP activity along the months of the year.

	m.	b.	t.	q.	s.
Jan	8.70	17.00	27.23	36.49	54.27
Fev	8.29				
Mar	10.23	19.49	27.03	33.46	
Apr	9.26				
Mai	9.41	17.78	22.94	30.06	45.73
Jun	8.37				
Jul	8.70	15.68	22.80	30.06	
Ago	6.98				
Set	7.26	15.36	22.80	30.06	
Out	8.10				
Nov	7.89	14.69	22.80	30.06	
Dez	6.81				

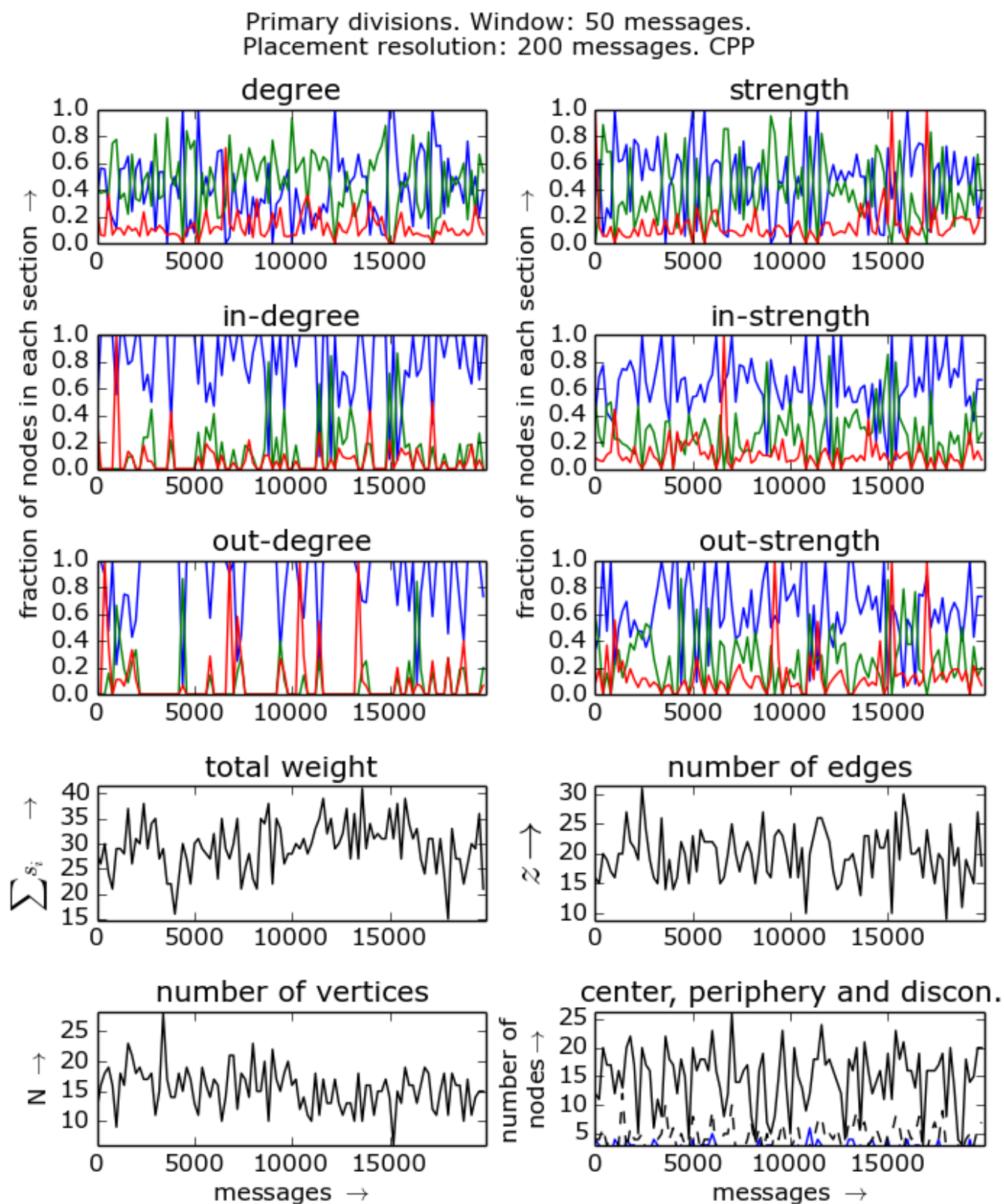
II. FRACTION OF PARTICIPANTS IN EACH ERDÖS SECTOR ALONG THE TIMELINE

Here we present the fraction of participants in each Erdős sector with respect to each criteria defined in Section III C of the mais document. Step sizes of 50, 100, 250, 500, 1000 and 5000 are shown bellow, first for CPP, than for LAD list.

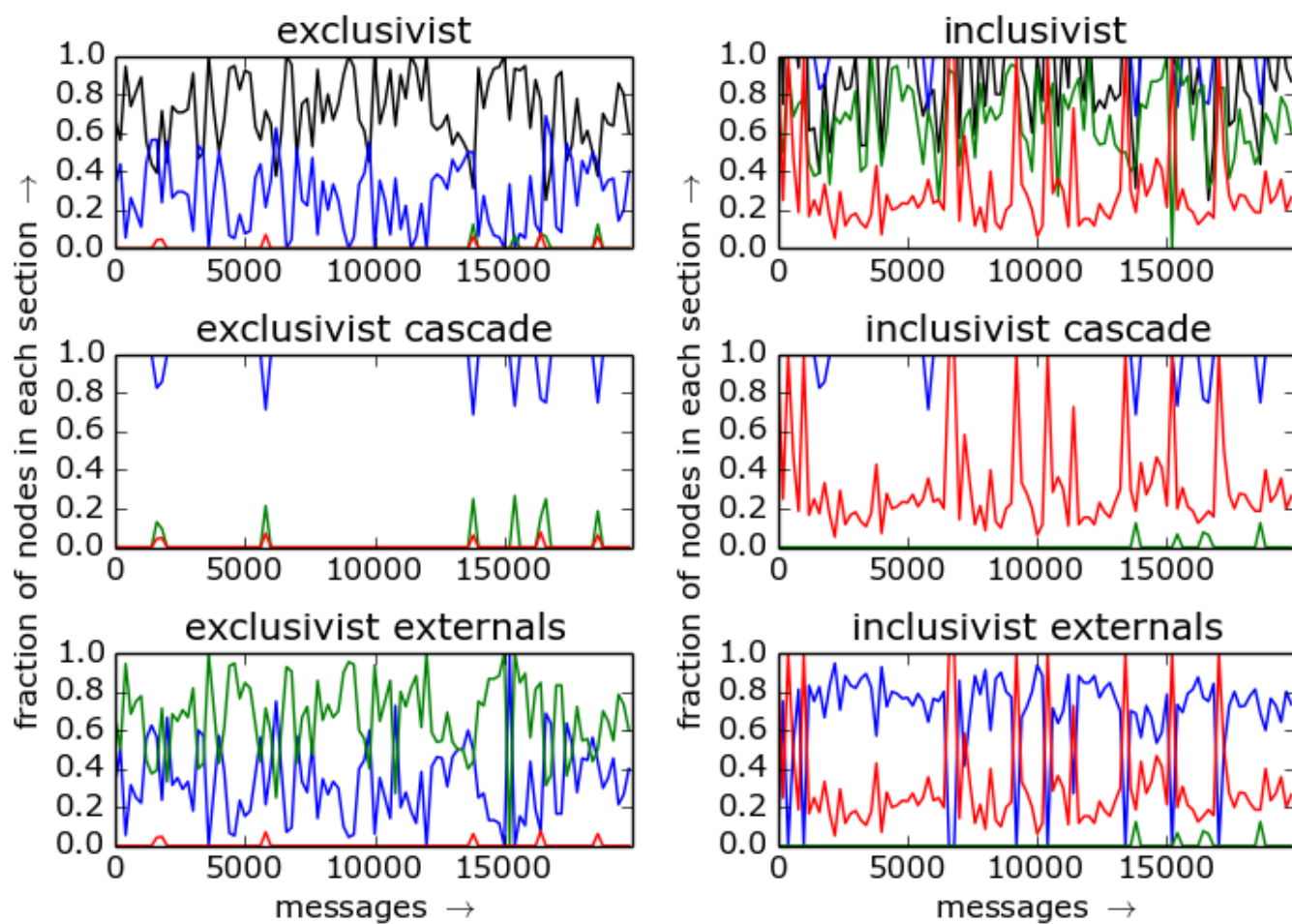
Each step size takes two pages of plot. On the first page, the criteria are each centrality measure observed separately: in, out and total degrees and strengths. In the first six plots, red is fraction of hubs, green is the fraction of intermediary and blue is for peripheral fraction. On the last plot, red is the center (maximum distance to another vertex is equal to radius), blue is periphery (maximum distance equals to diameter) of the greatest component. On the same graph, green counts the disconnected vertices.

On the second page we show the fractions of participants with respect to each compound criteria for the Erdős sectioning. In the first plot, the fraction of vertices with unique classification is plotted in black: $\frac{\text{number of nodes uniquely classified}}{\text{number of nodes}}$. On the second plot, black represents the exceeding classifications for the given vertices : $\frac{\text{number of classifications} - \text{number of nodes}}{\text{number of nodes}}$.

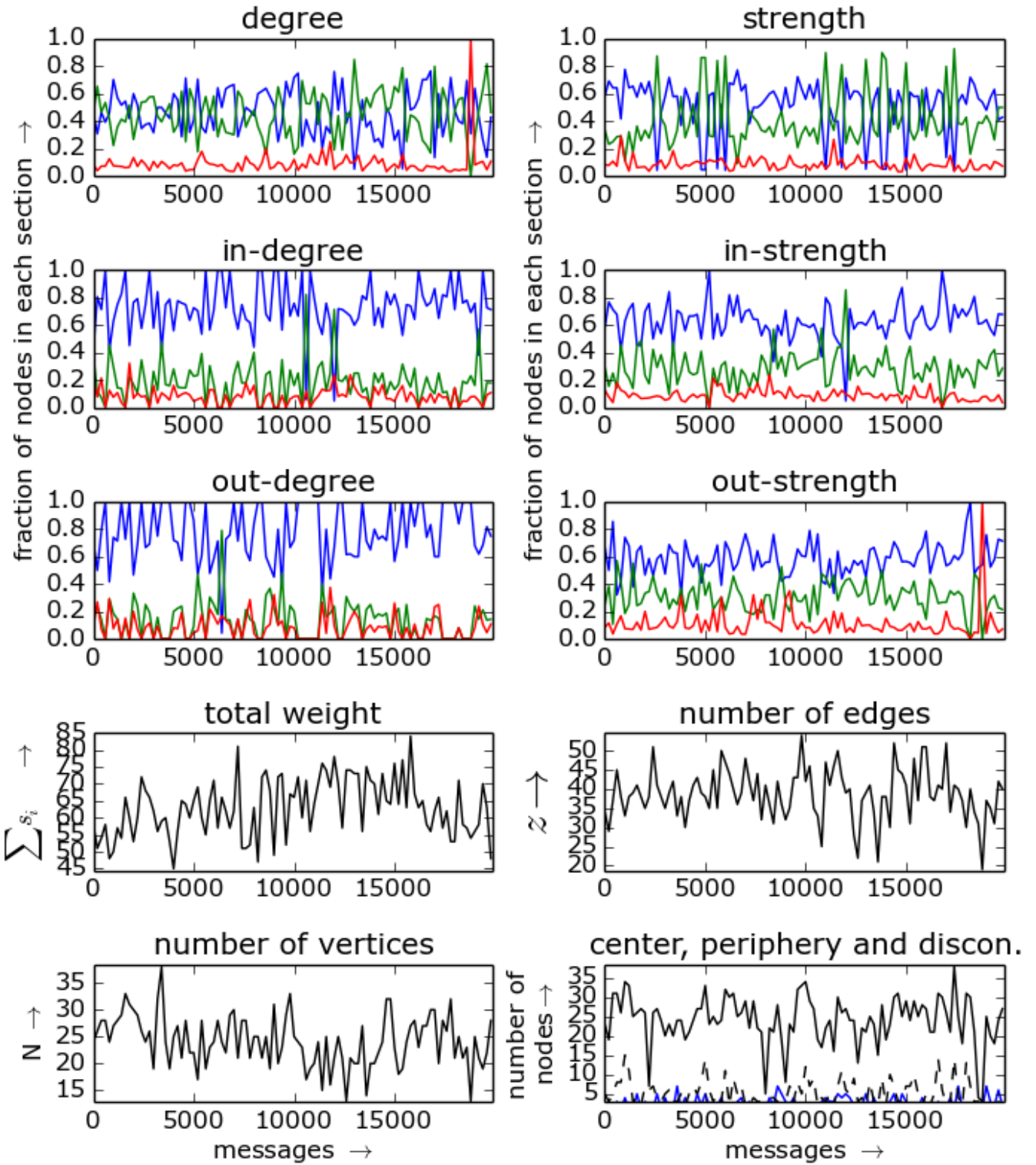
A. CPP list



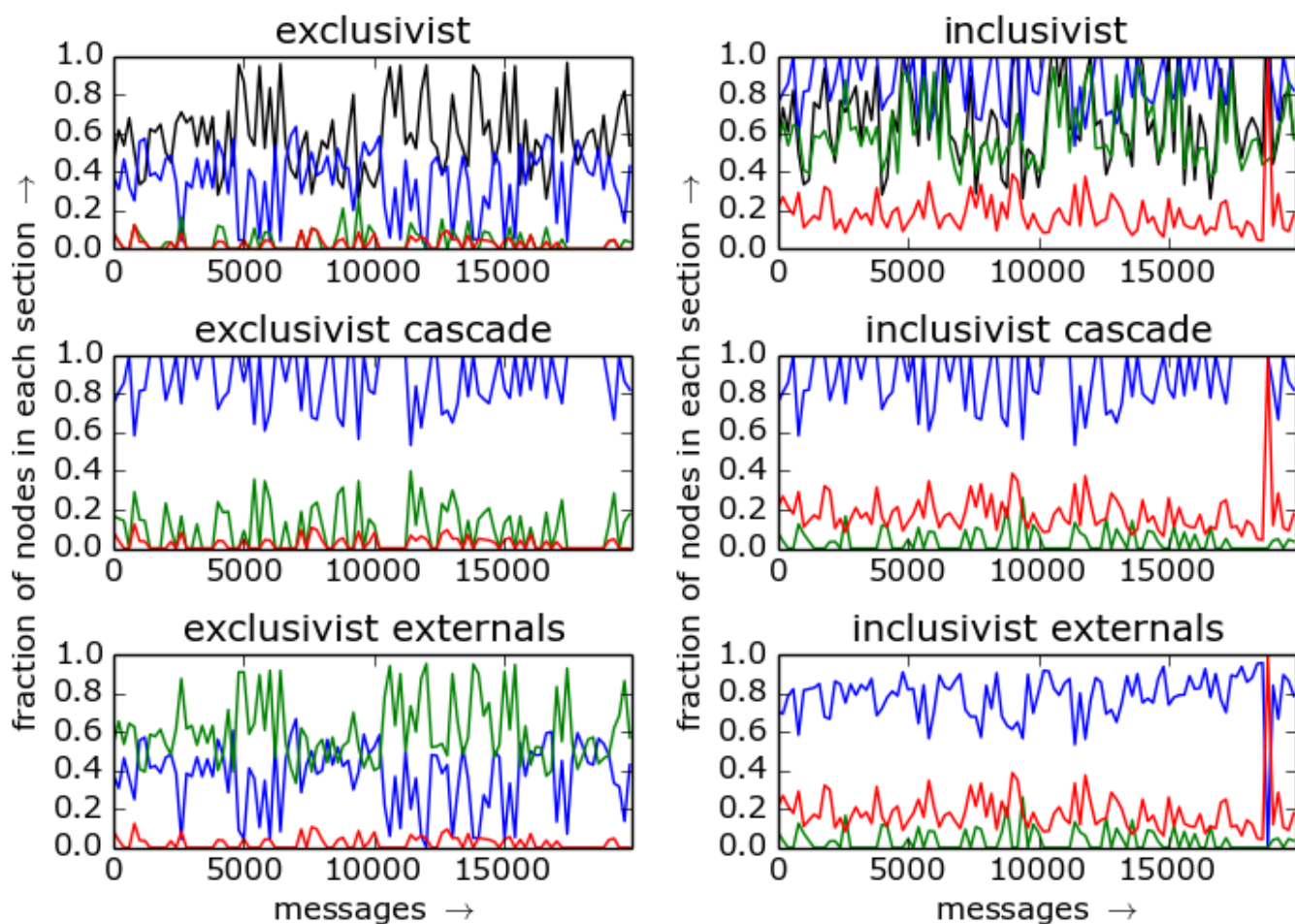
Compound divisions. Window: 50 messages.
Placement resolution: 200 messages. CPP



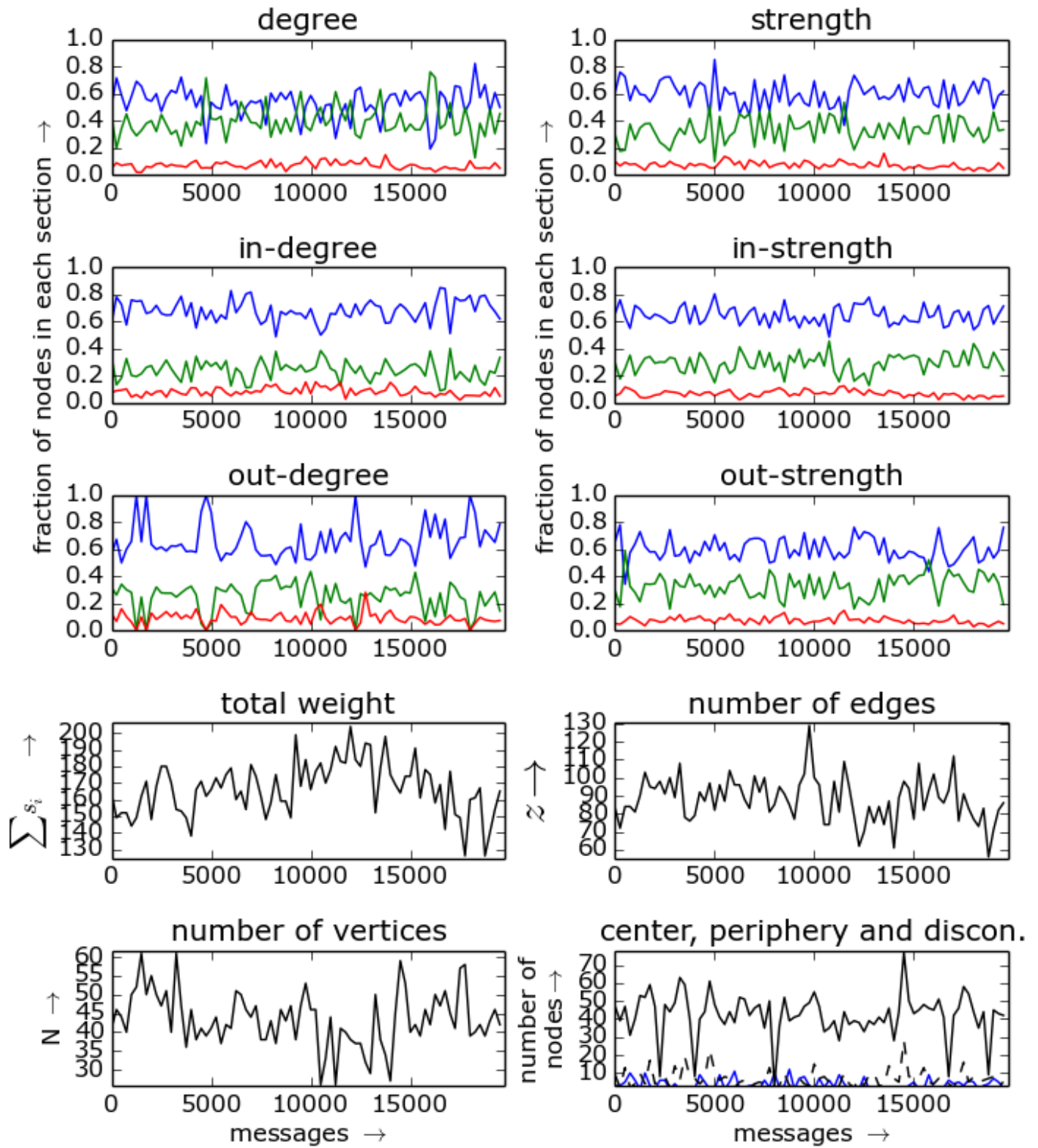
Primary divisions. Window: 100 messages.
Placement resolution: 200 messages. CPP



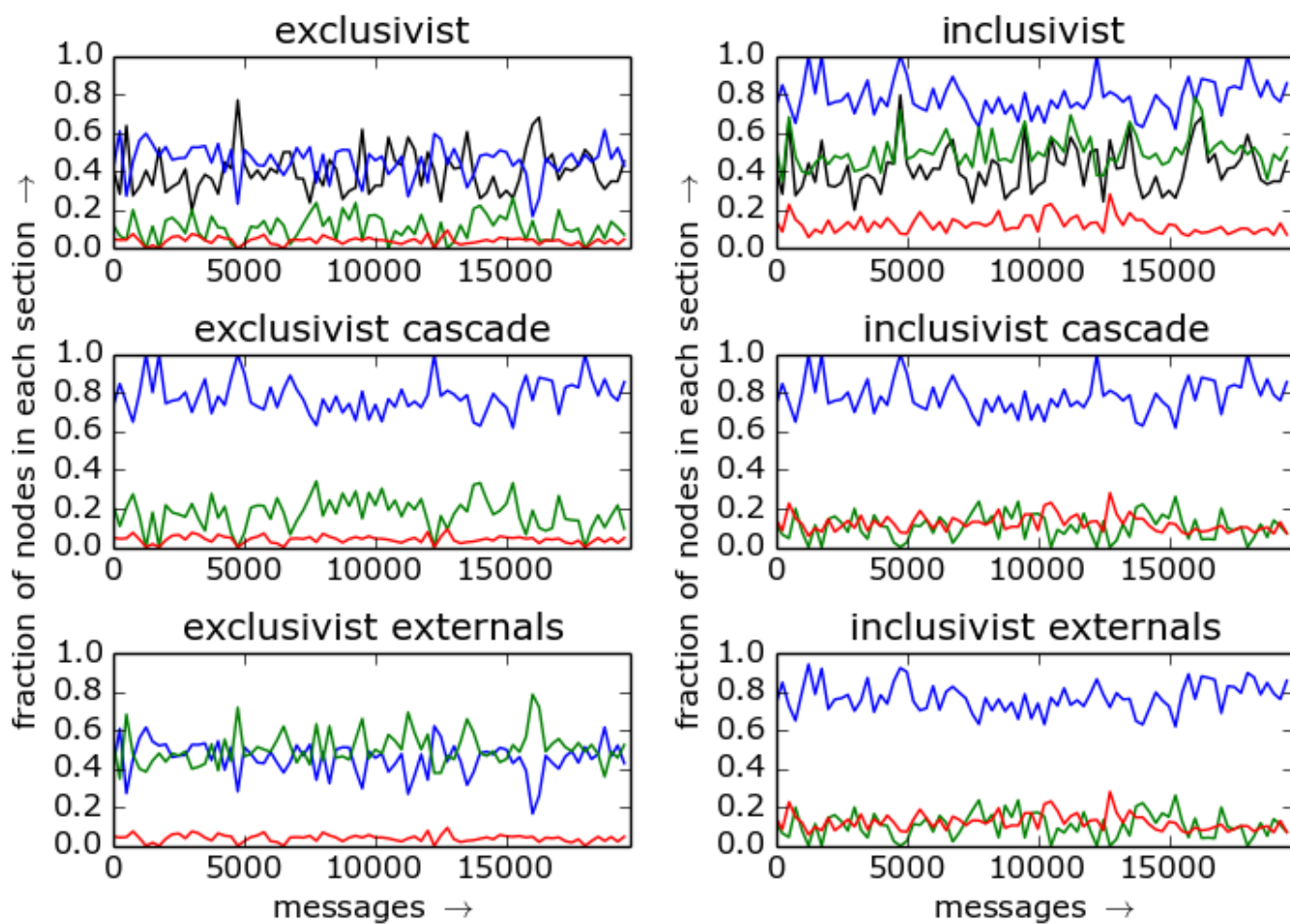
Compound divisions. Window: 100 messages.
Placement resolution: 200 messages. CPP



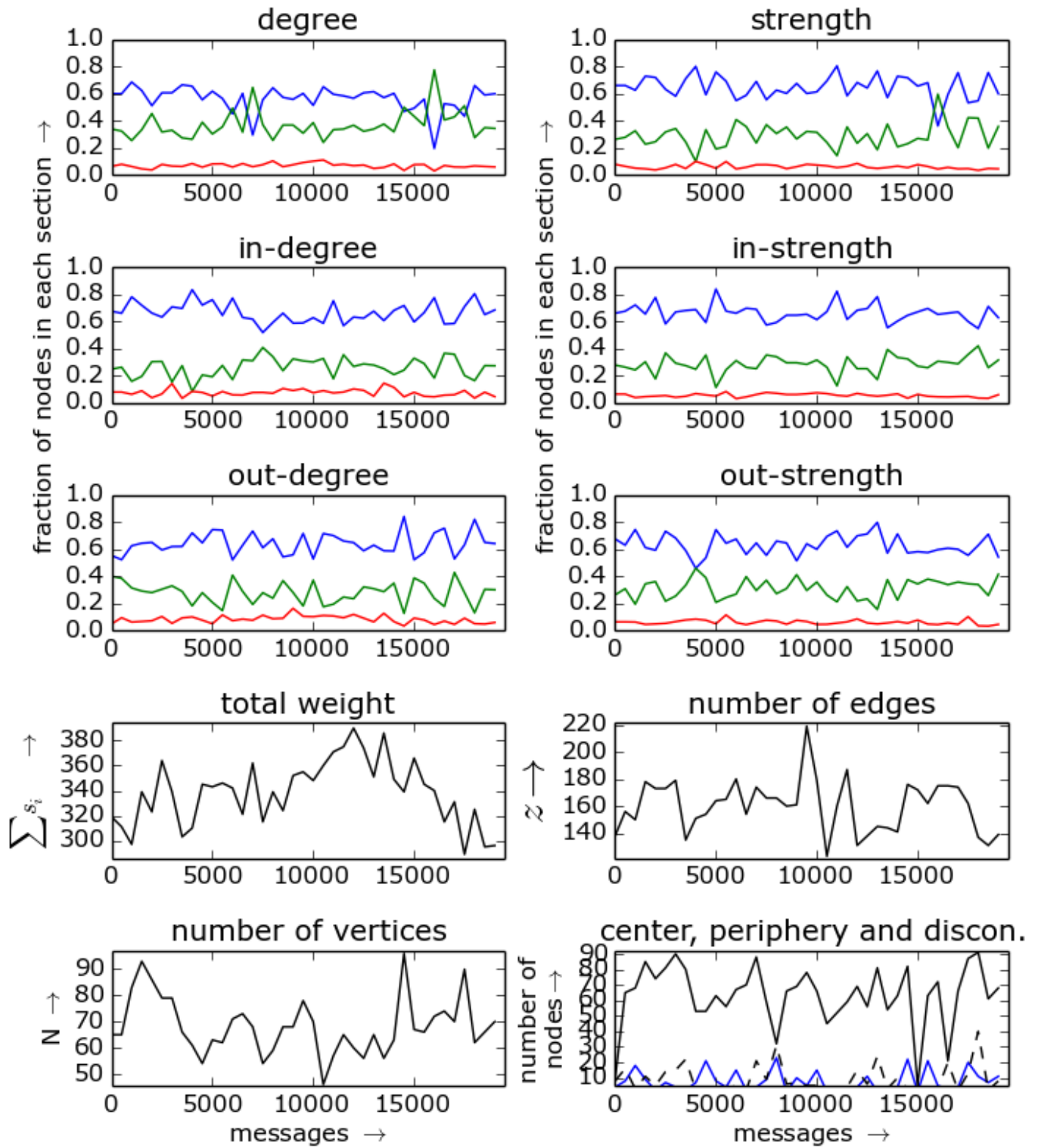
Primary divisions. Window: 250 messages.
Placement resolution: 250 messages. CPP



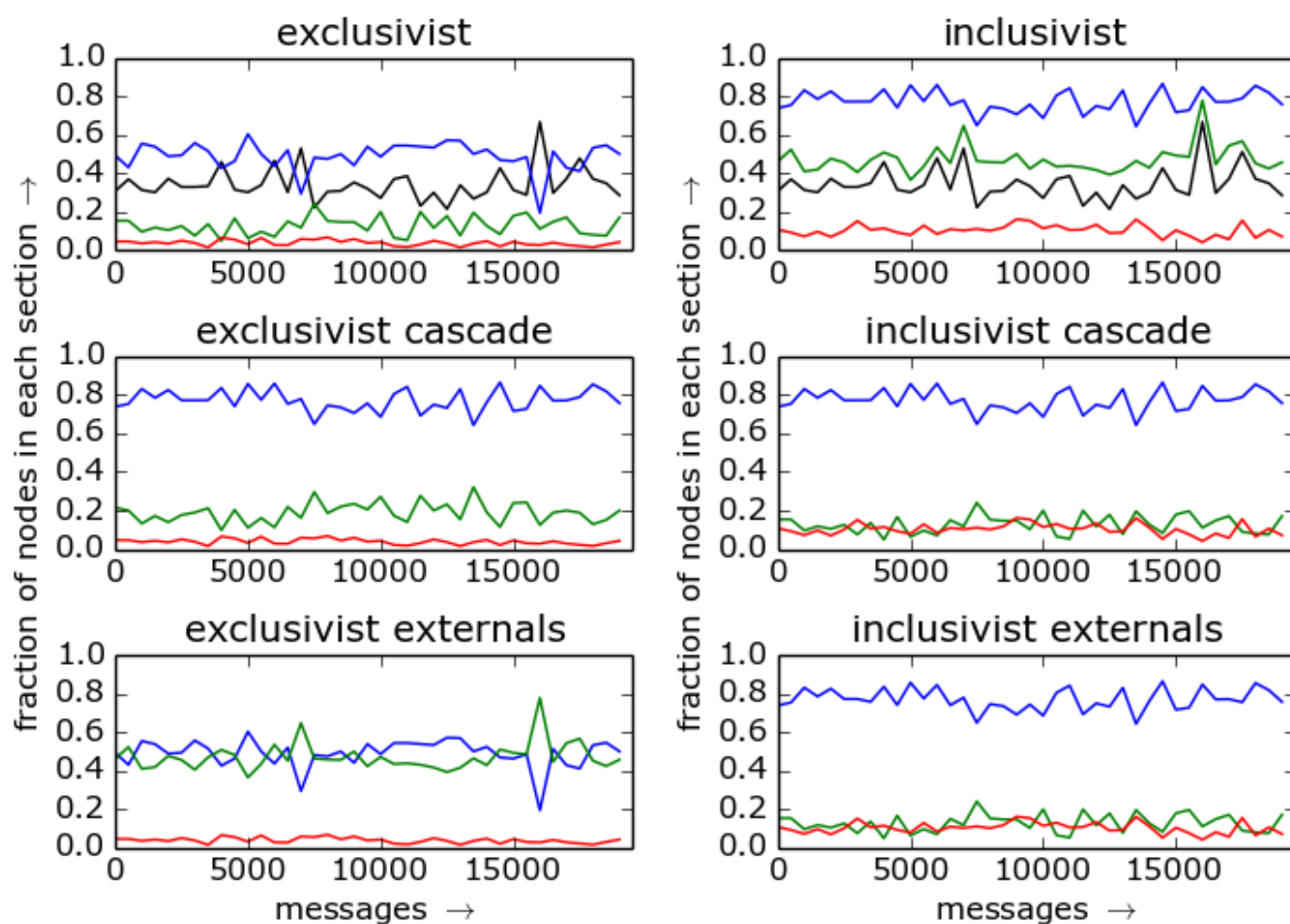
Compound divisions. Window: 250 messages.
Placement resolution: 250 messages. CPP



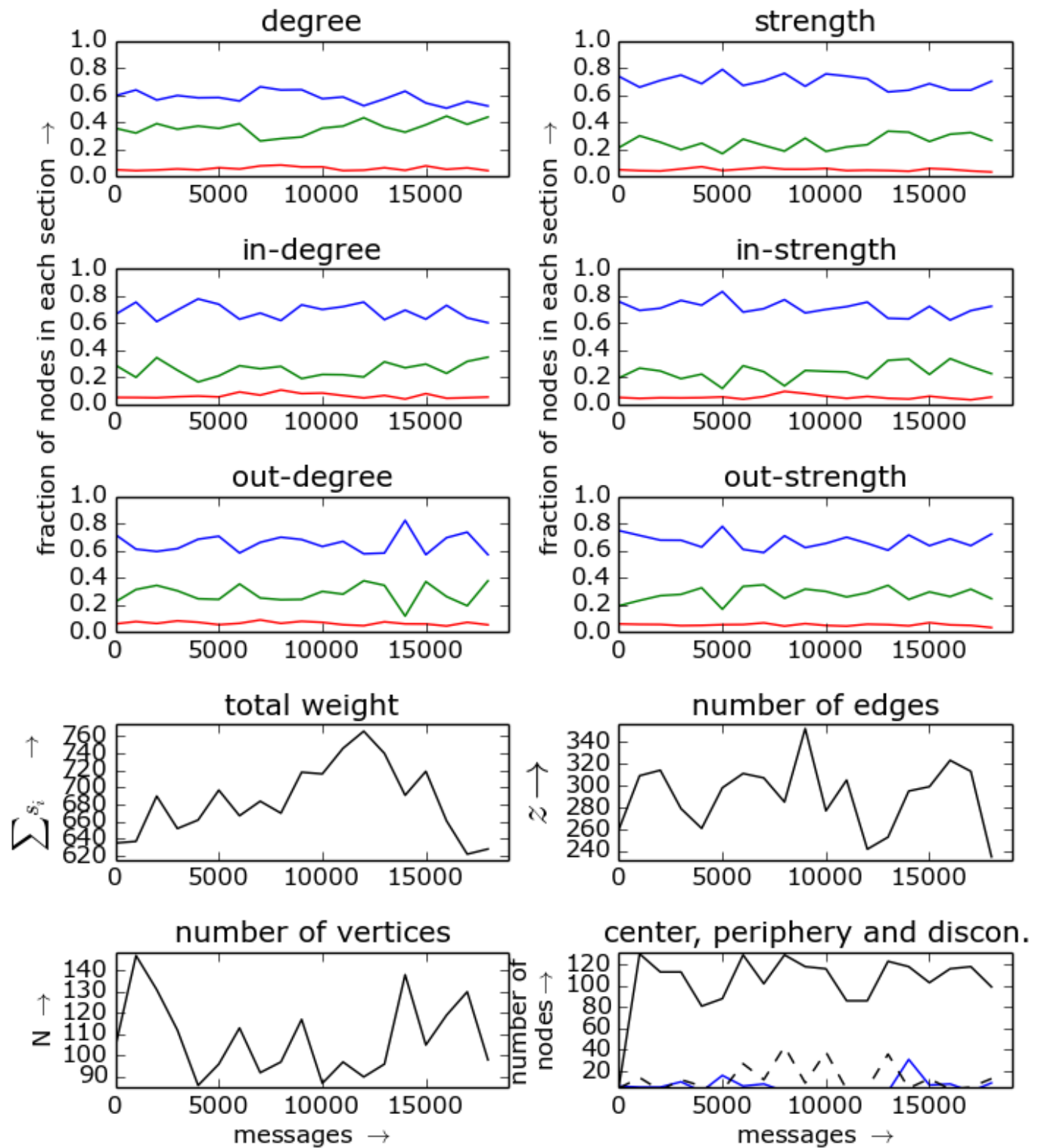
Primary divisions. Window: 500 messages.
Placement resolution: 500 messages. CPP



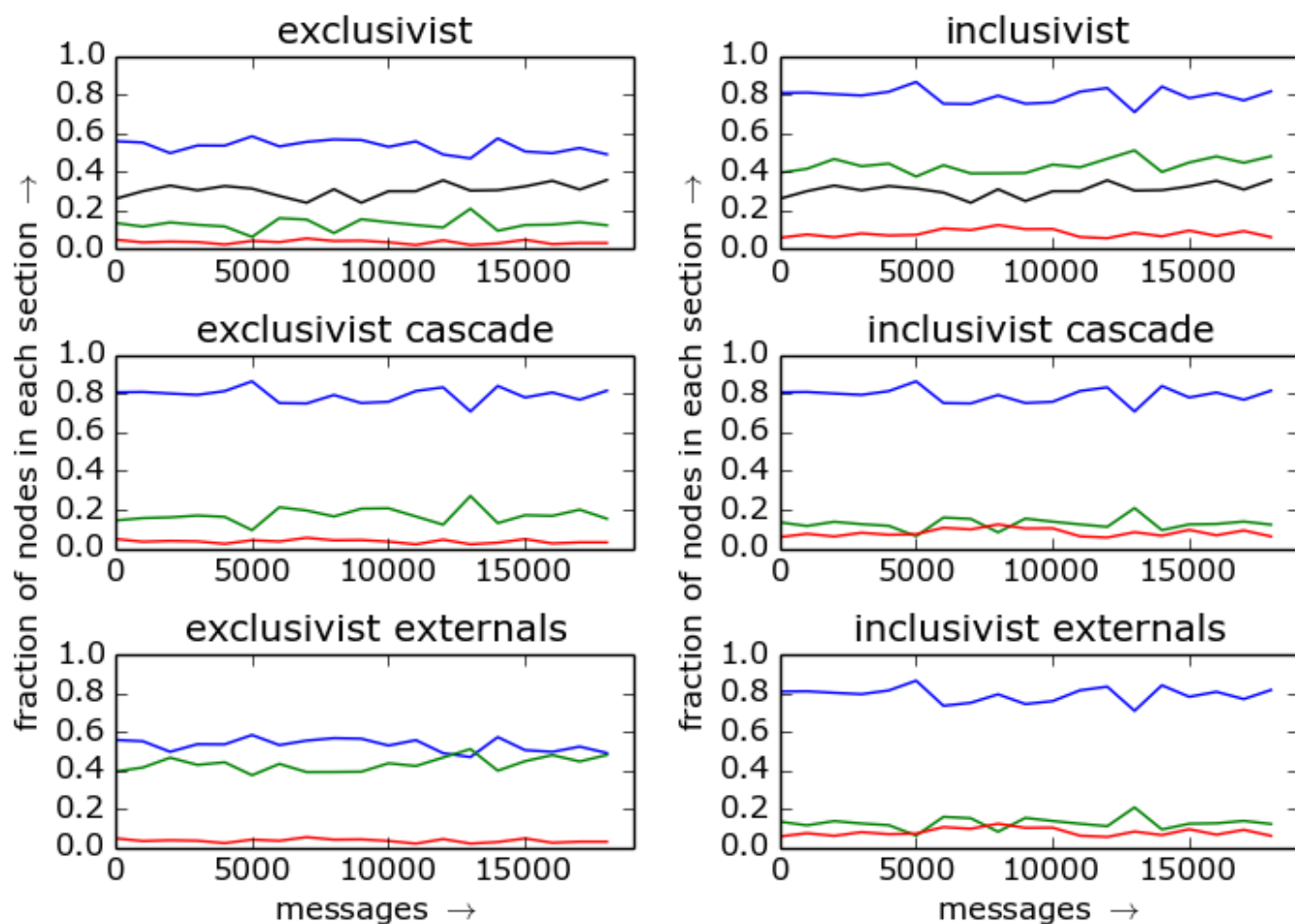
Compound divisions. Window: 500 messages.
Placement resolution: 500 messages. CPP



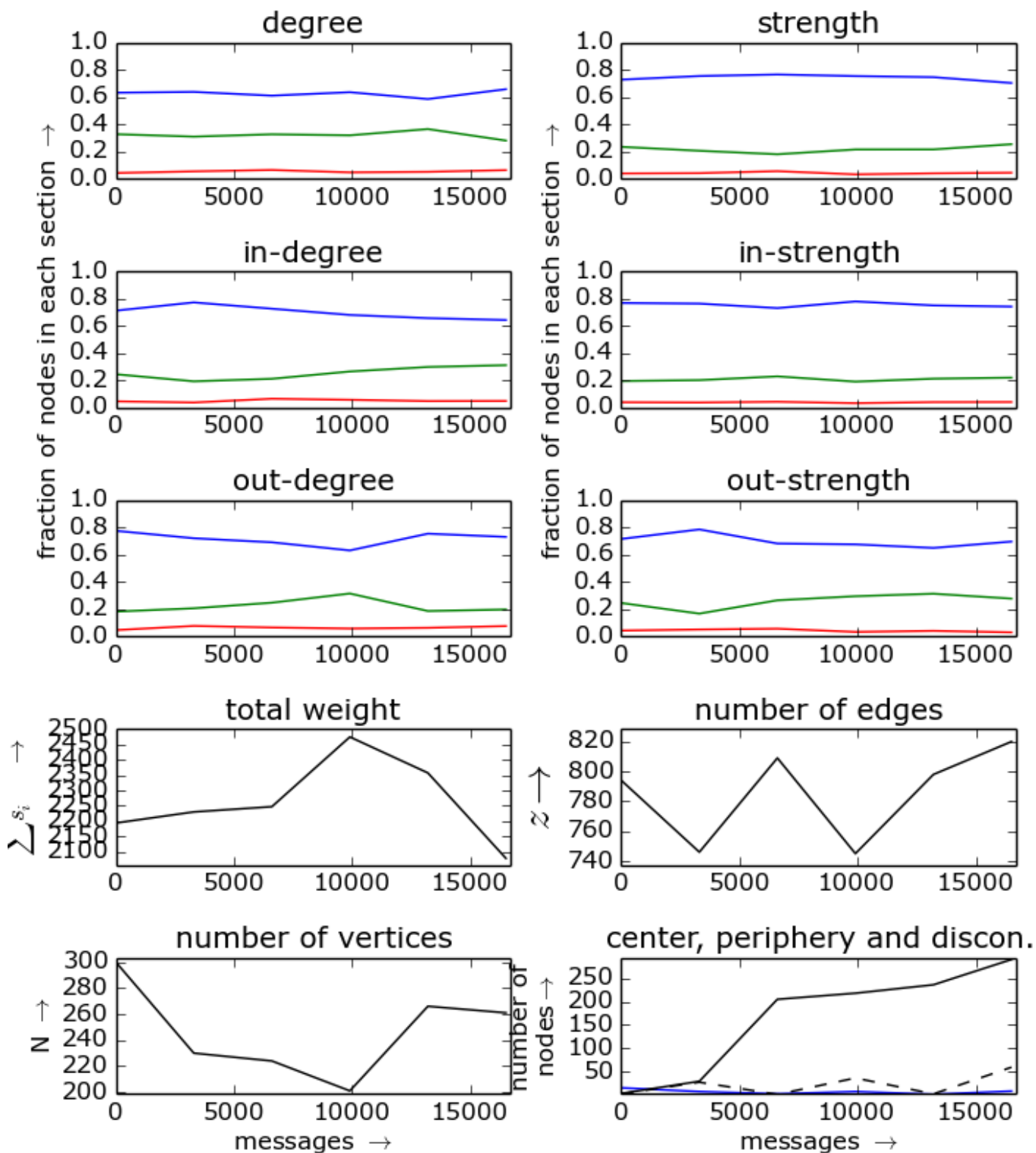
Primary divisions. Window: 1000 messages.
Placement resolution: 1000 messages. CPP



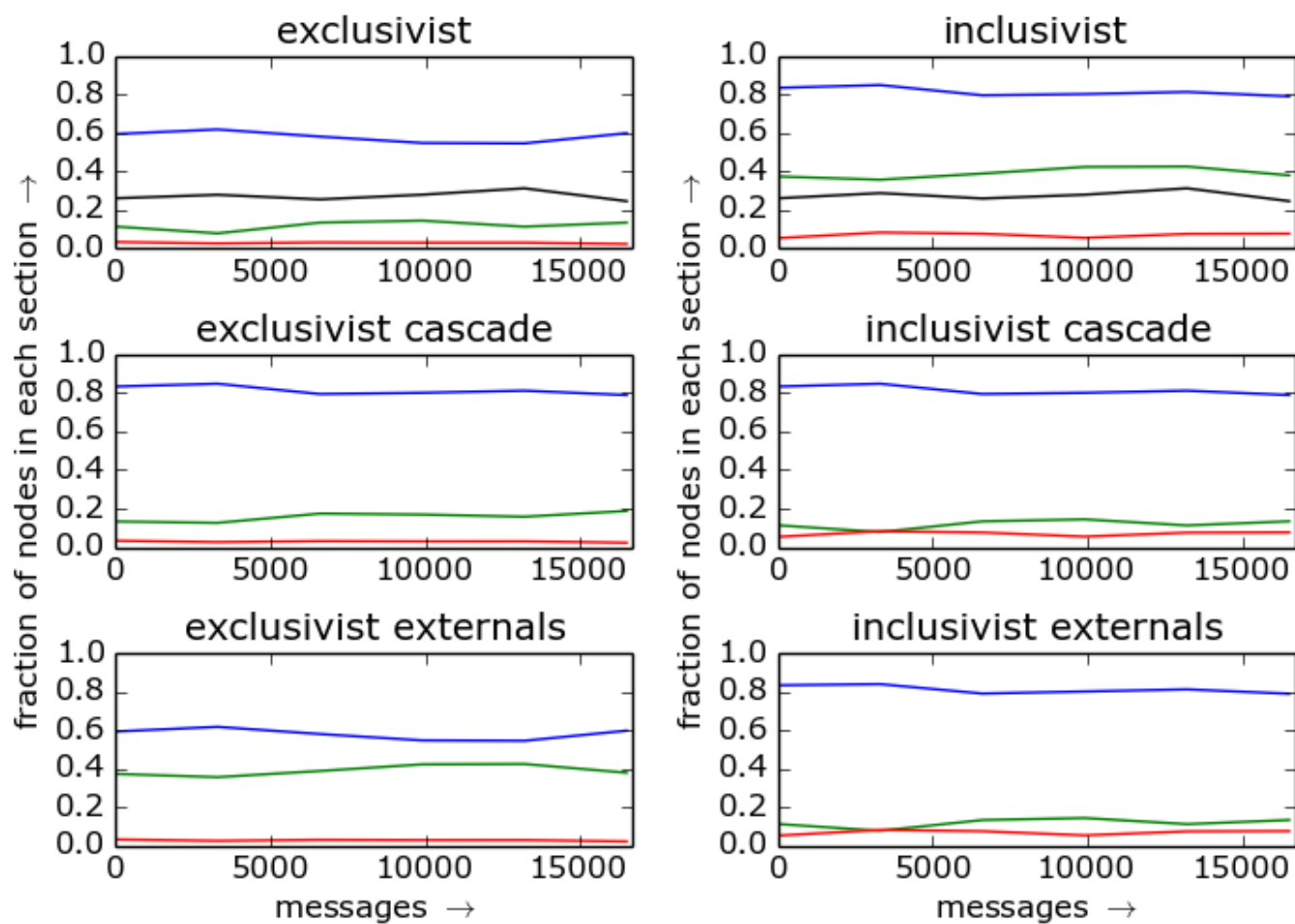
Compound divisions. Window: 1000 messages.
Placement resolution: 1000 messages. CPP



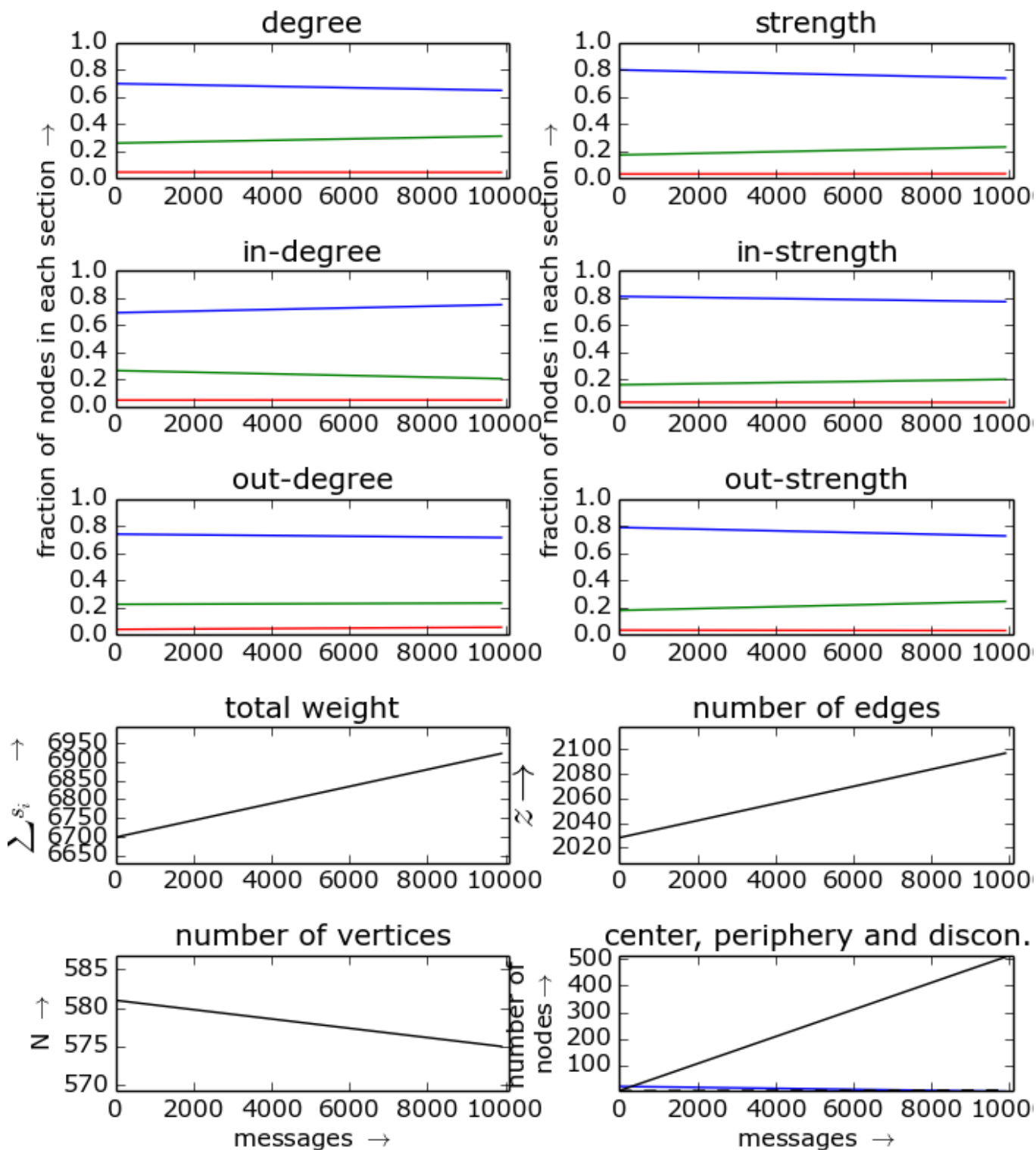
Primary divisions. Window: 3300 messages.
Placement resolution: 3300 messages. CPP



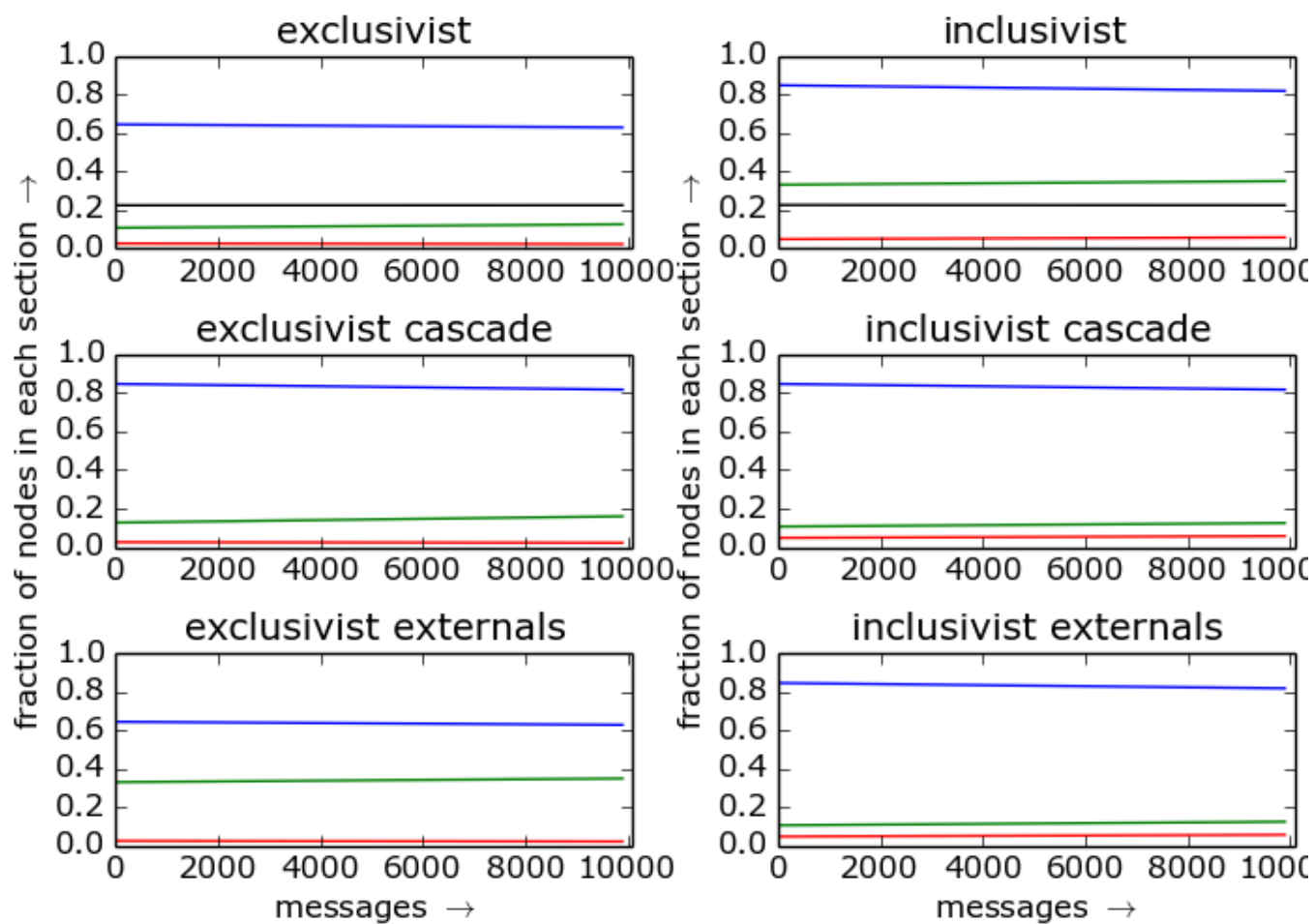
Compound divisions. Window: 3300 messages.
Placement resolution: 3300 messages. CPP



Primary divisions. Window: 9900 messages.
Placement resolution: 9900 messages. CPP

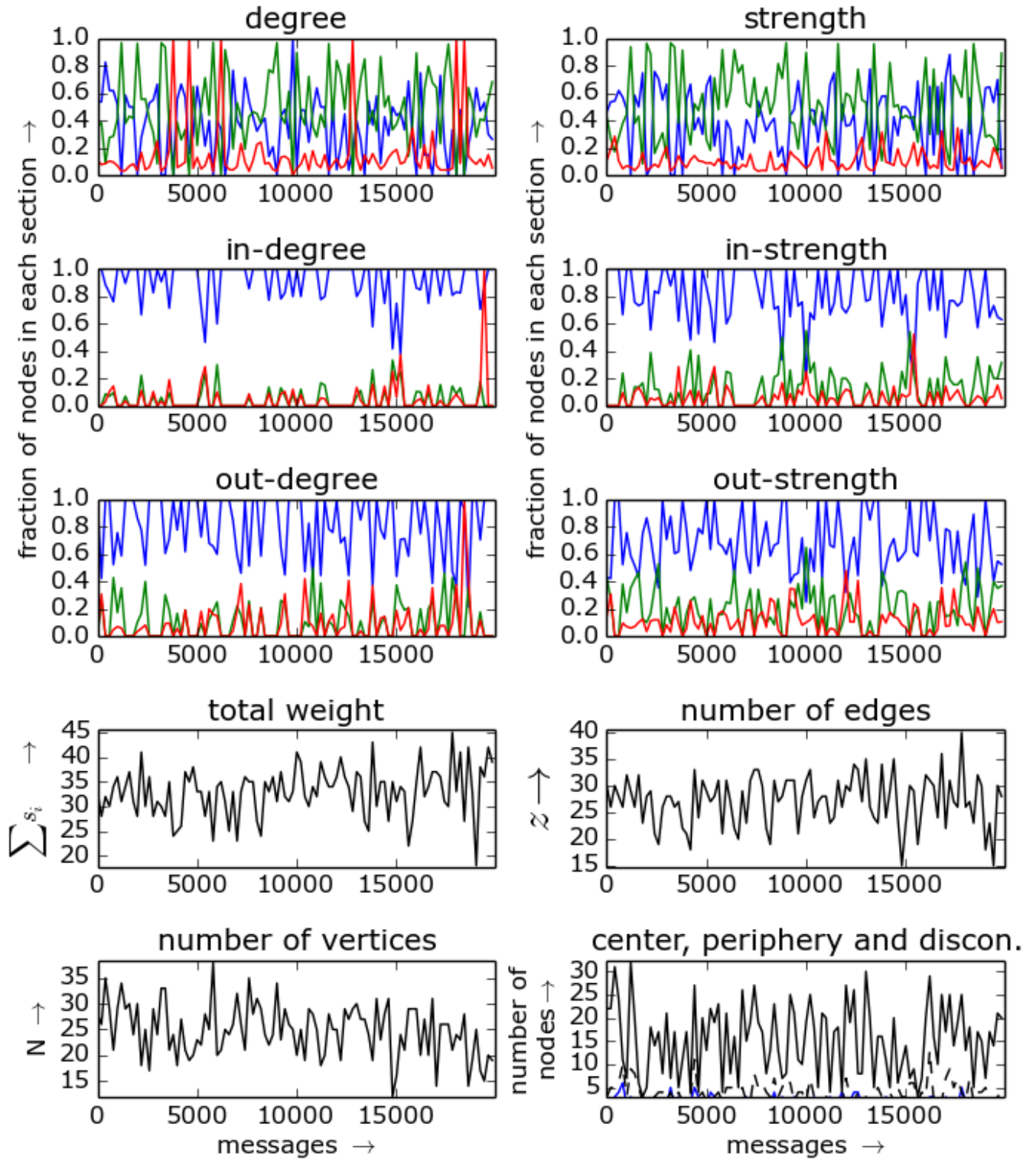


Compound divisions. Window: 9900 messages.
Placement resolution: 9900 messages. CPP

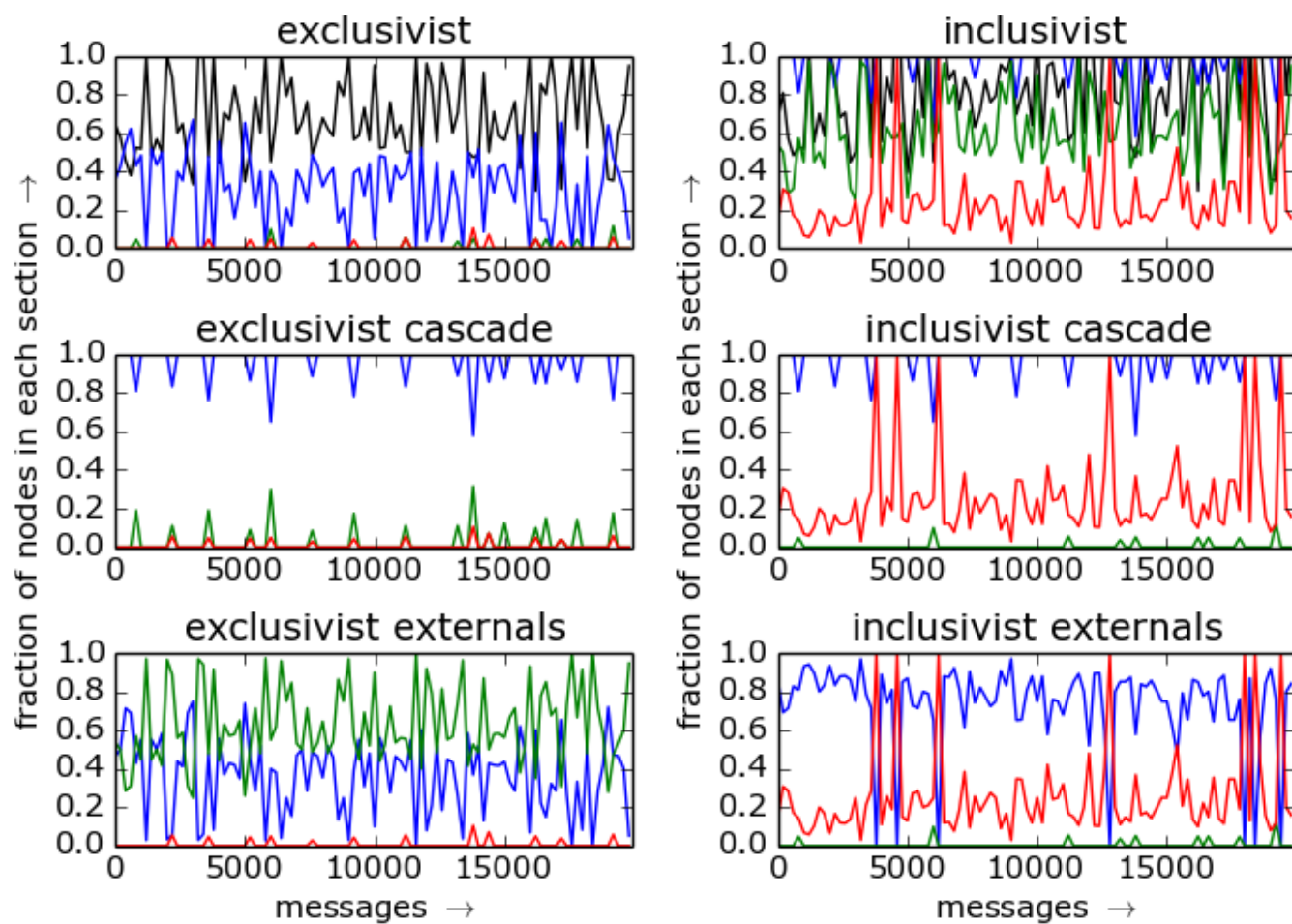


B. LAD list

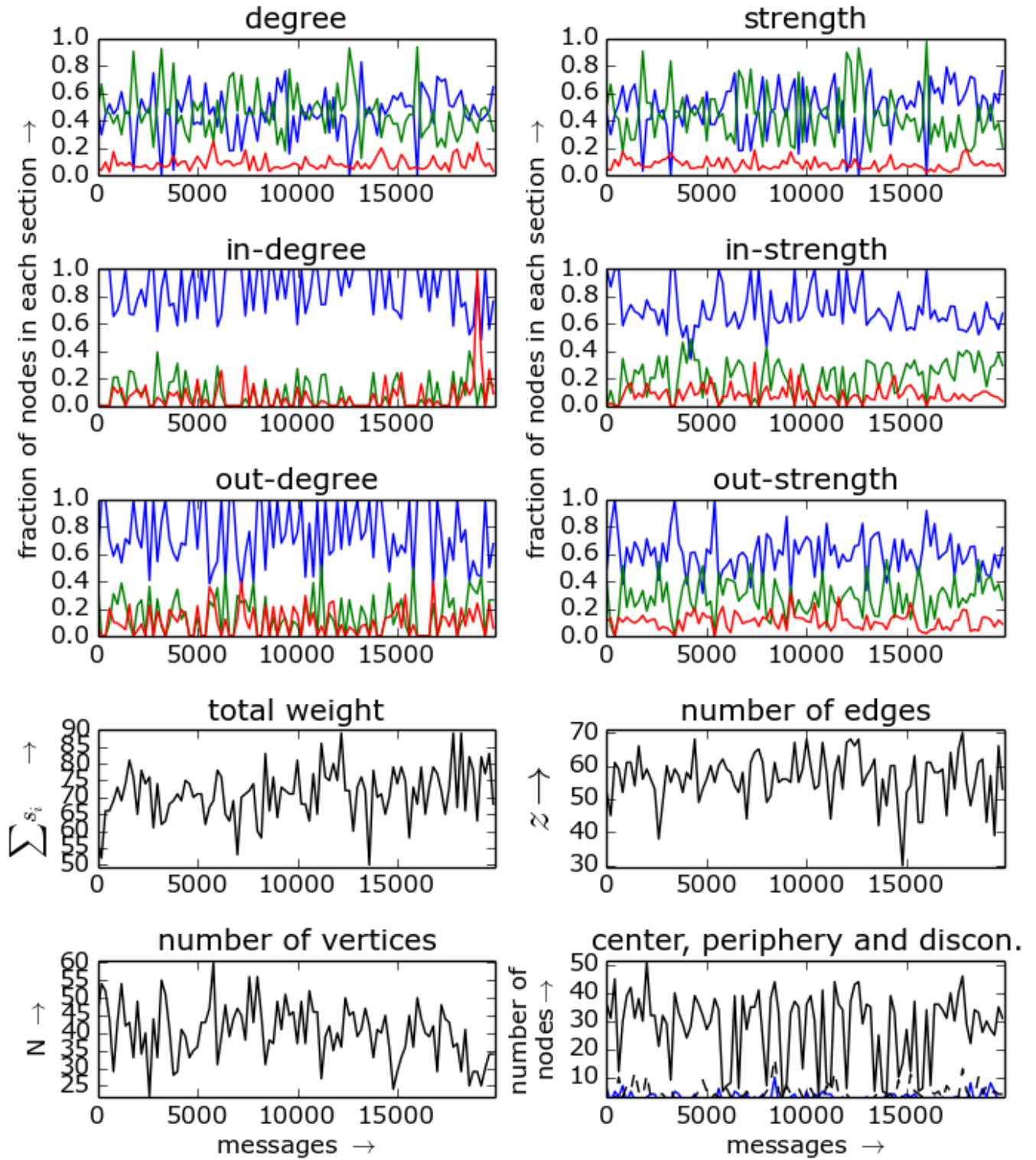
Primary divisions. Window: 50 messages.
Placement resolution: 200 messages. LAD



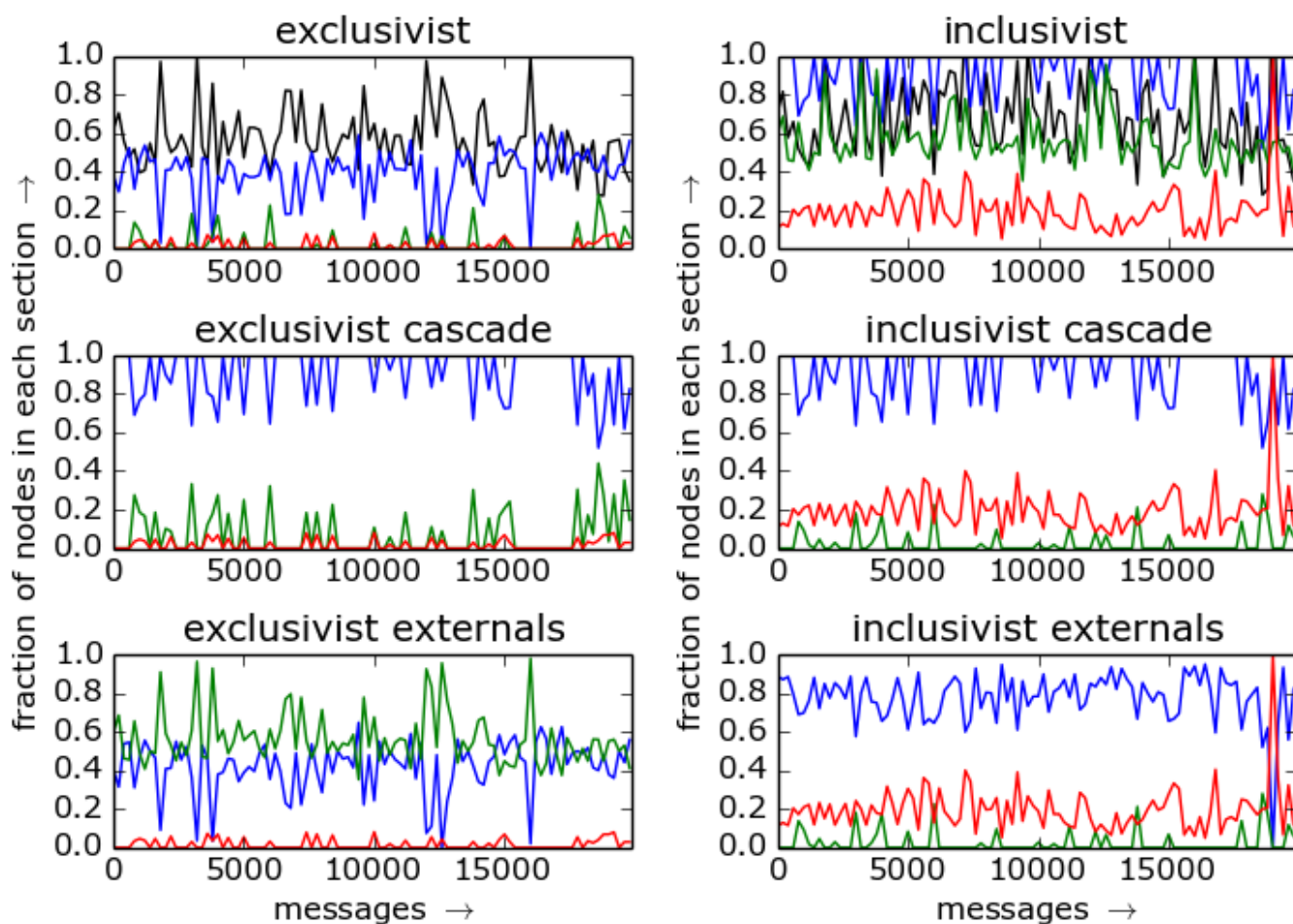
Compound divisions. Window: 50 messages.
Placement resolution: 200 messages. LAD

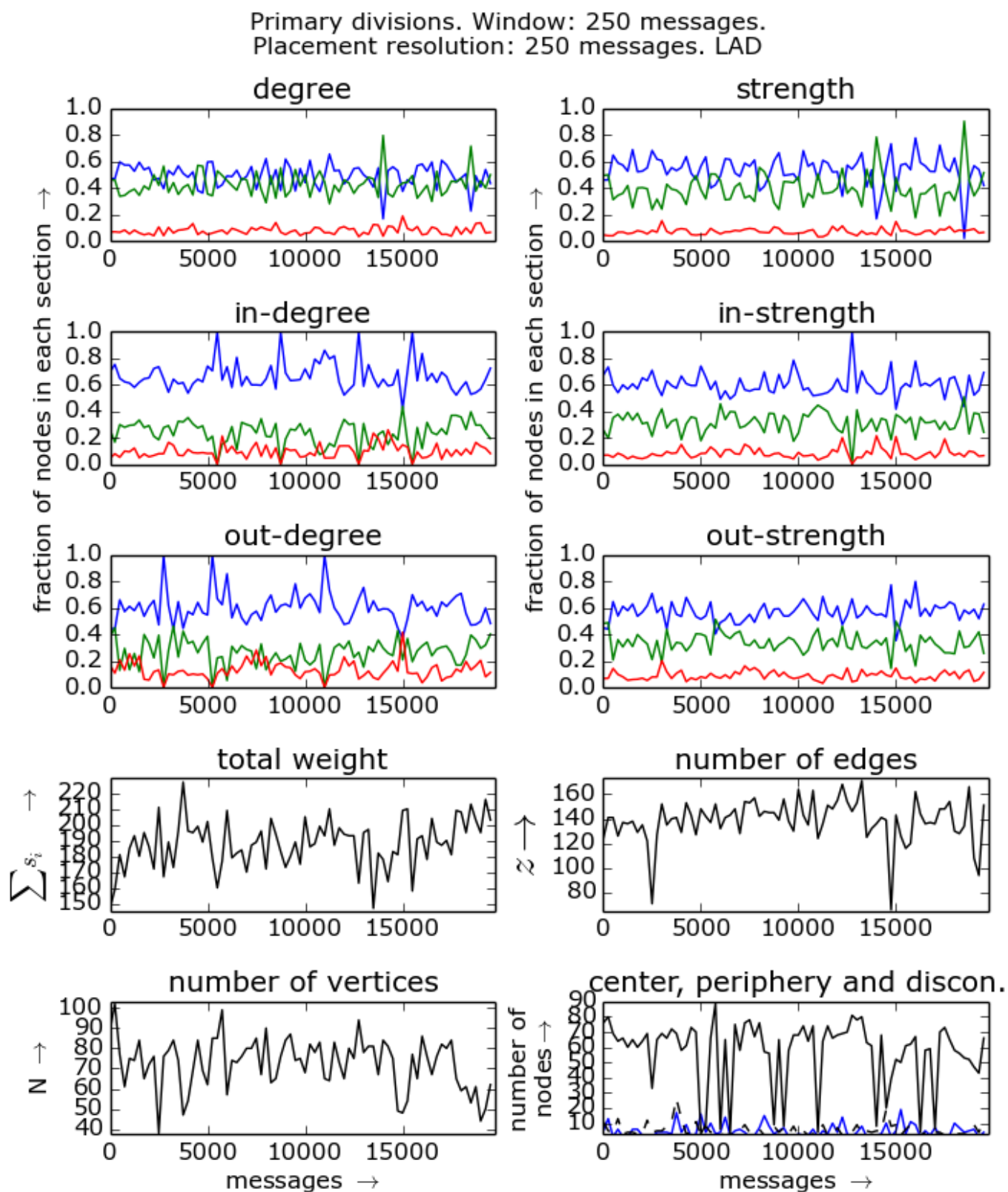


Primary divisions. Window: 100 messages.
Placement resolution: 200 messages. LAD

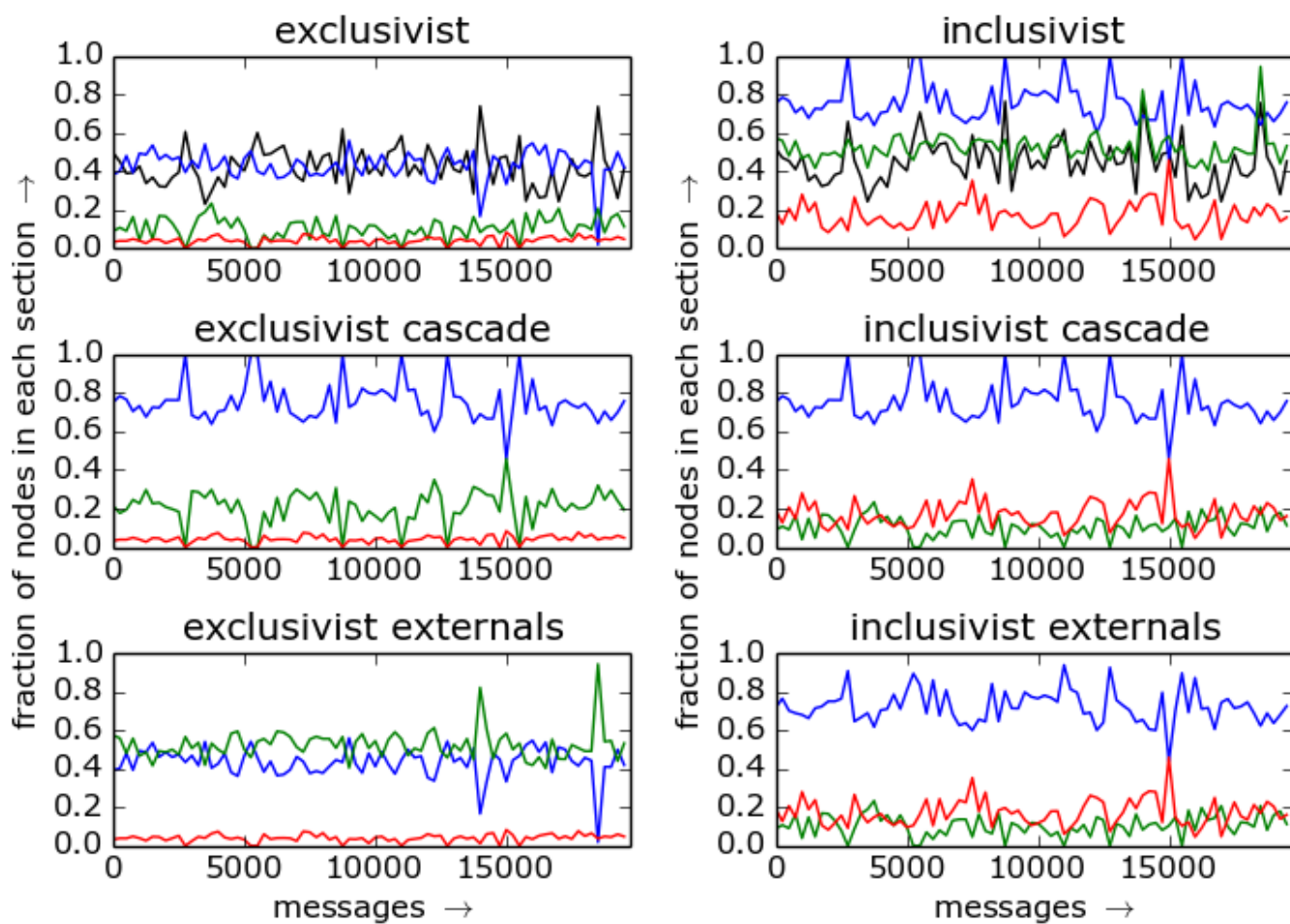


Compound divisions. Window: 100 messages.
Placement resolution: 200 messages. LAD

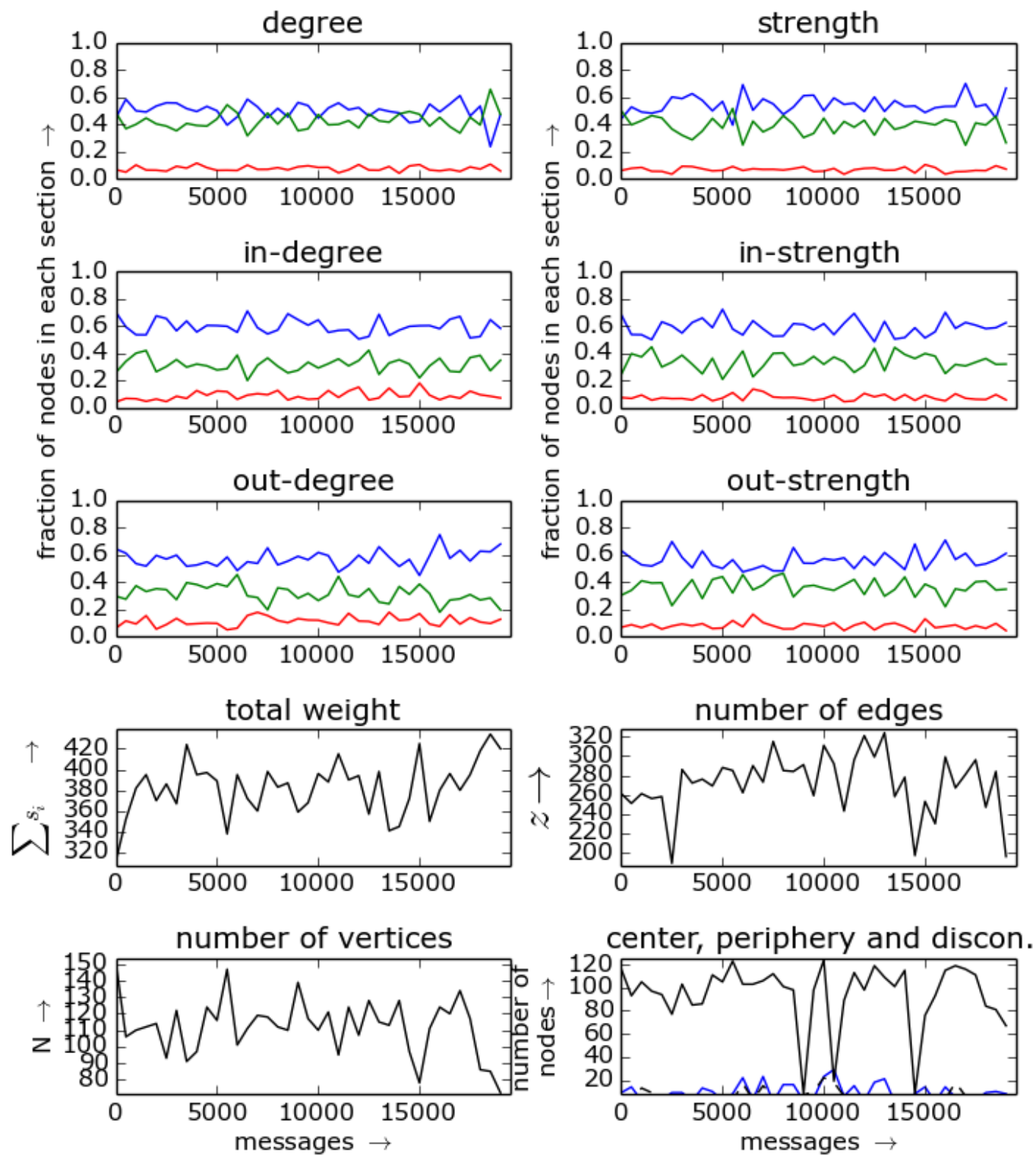




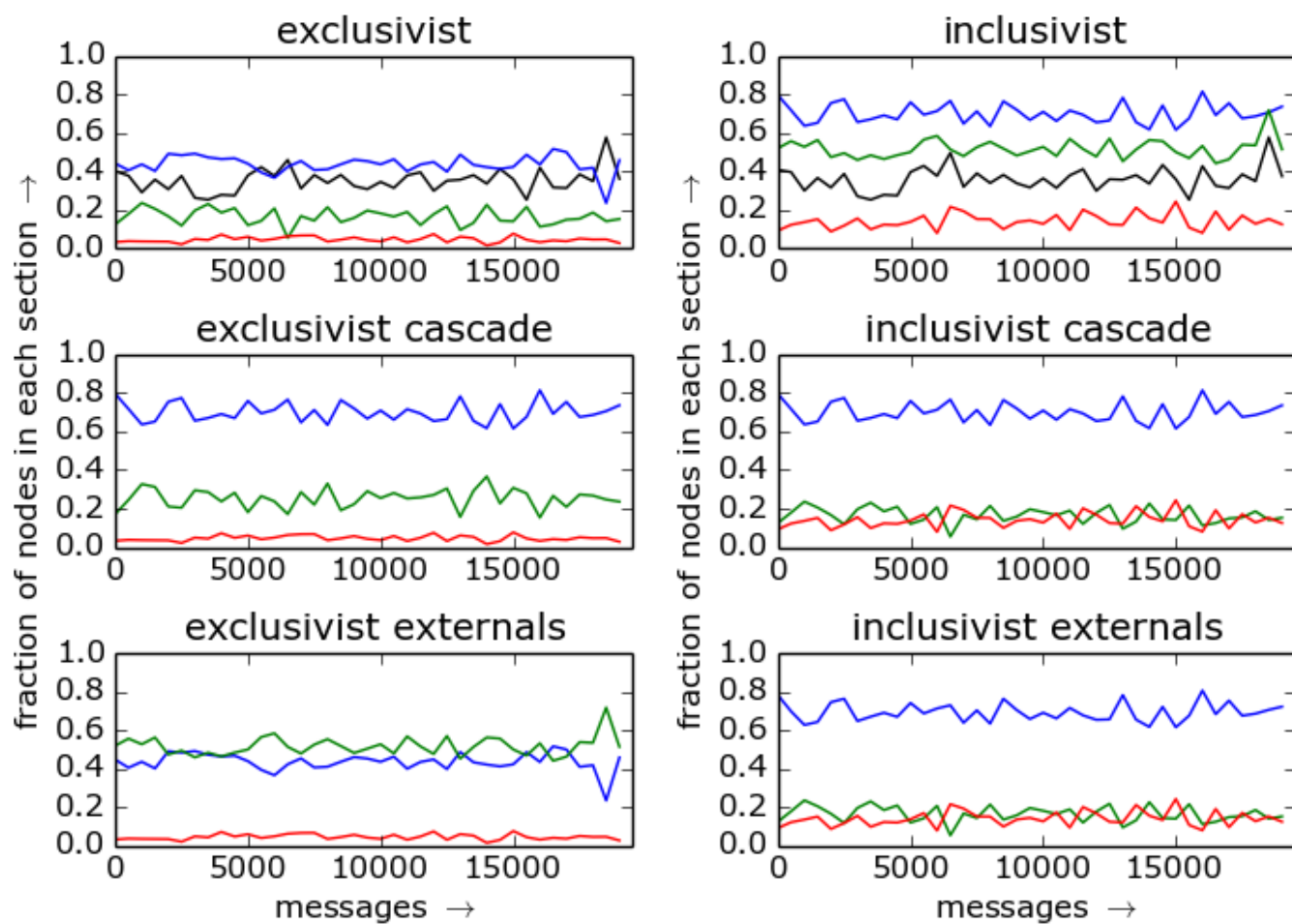
Compound divisions. Window: 250 messages.
Placement resolution: 250 messages. LAD



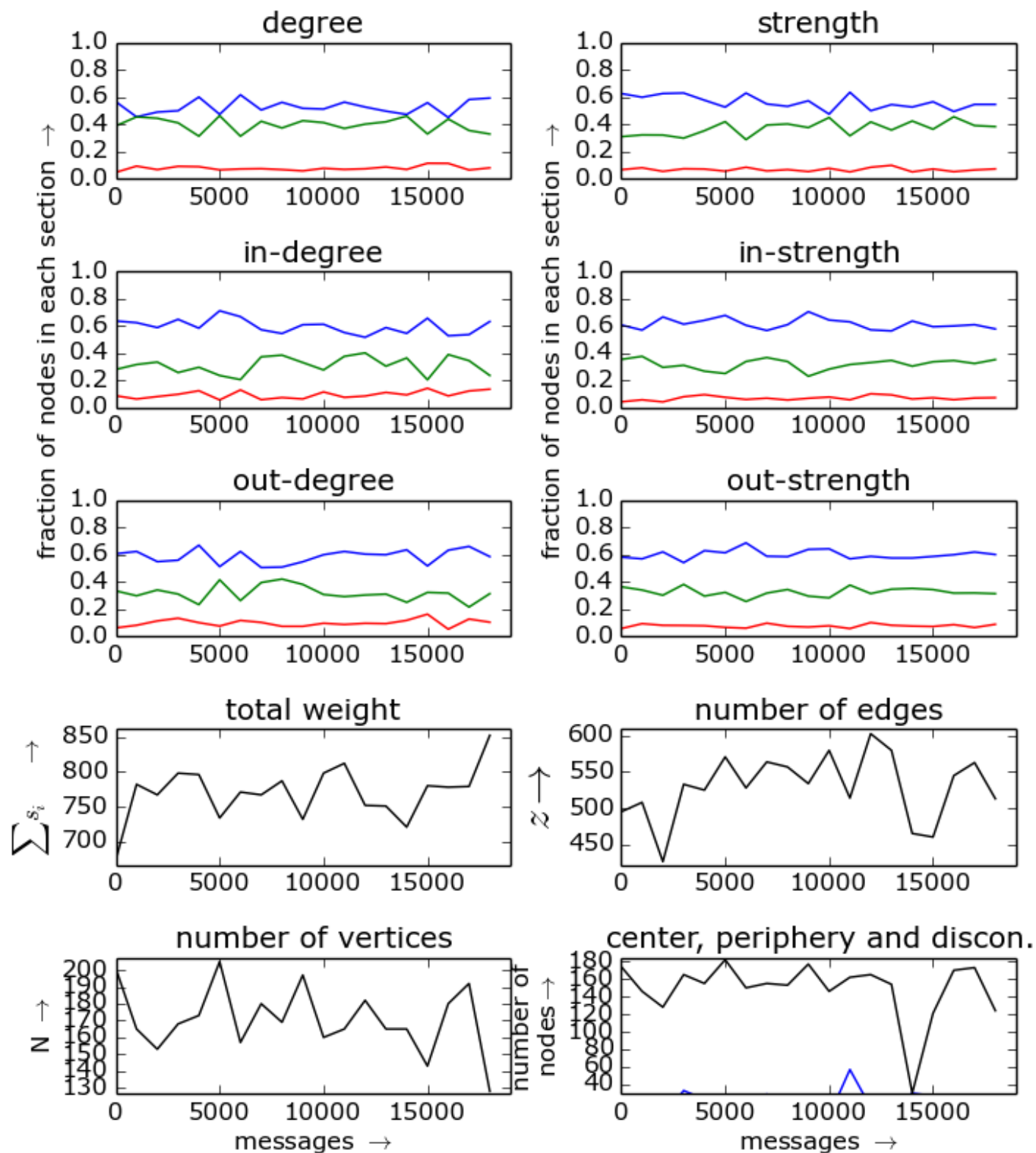
Primary divisions. Window: 500 messages.
Placement resolution: 500 messages. LAD



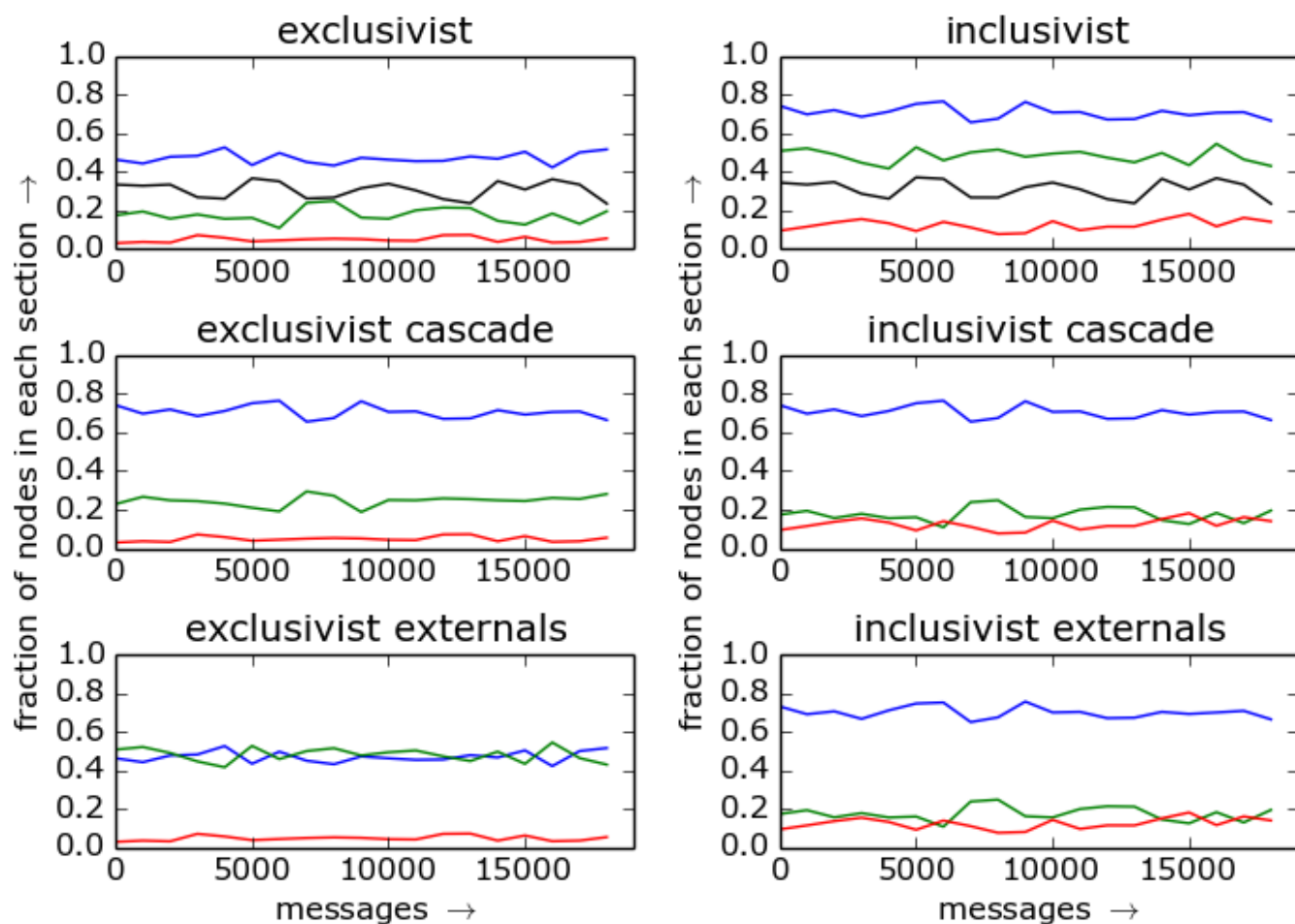
Compound divisions. Window: 500 messages.
Placement resolution: 500 messages. LAD



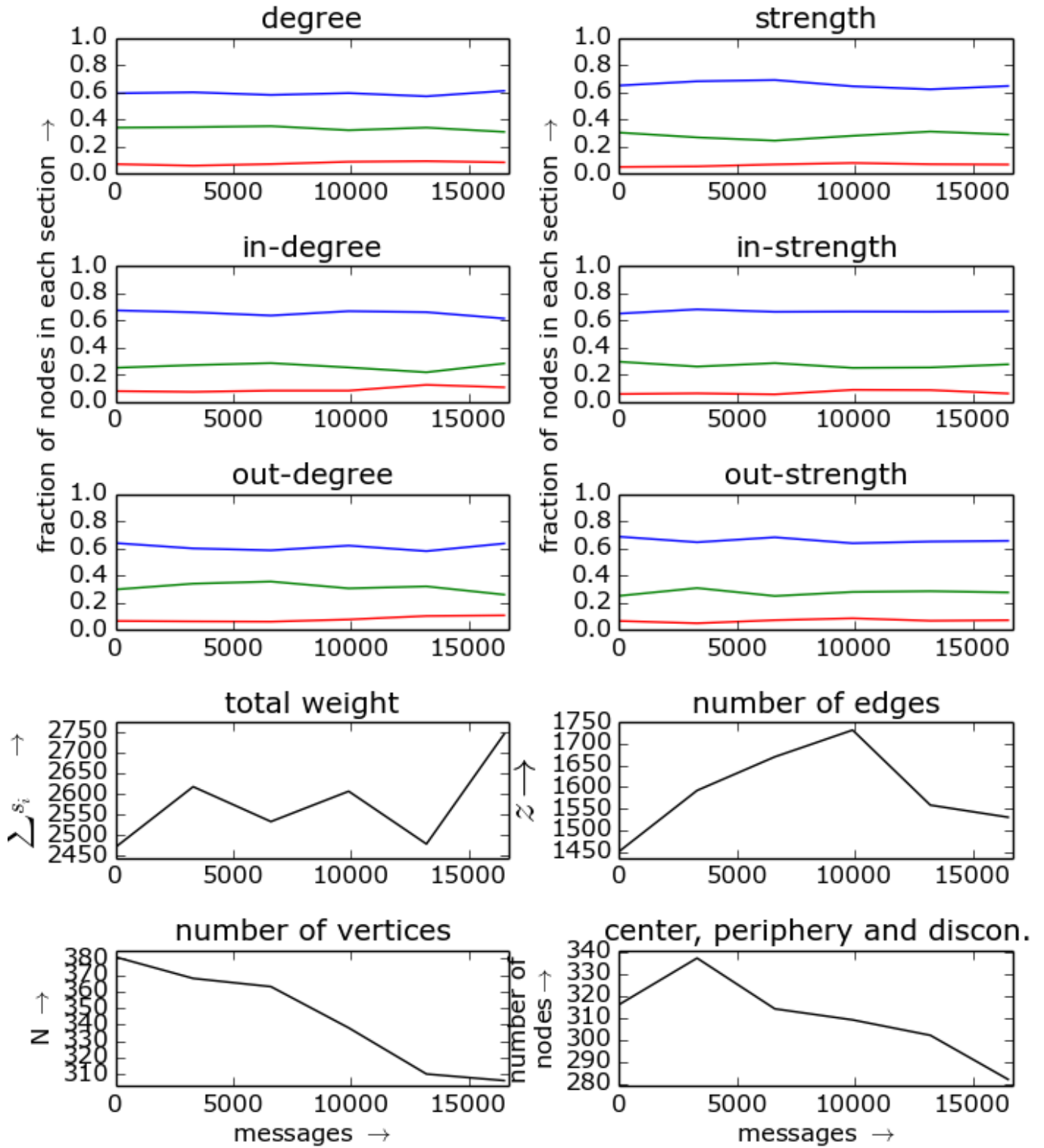
Primary divisions. Window: 1000 messages.
Placement resolution: 1000 messages. LAD



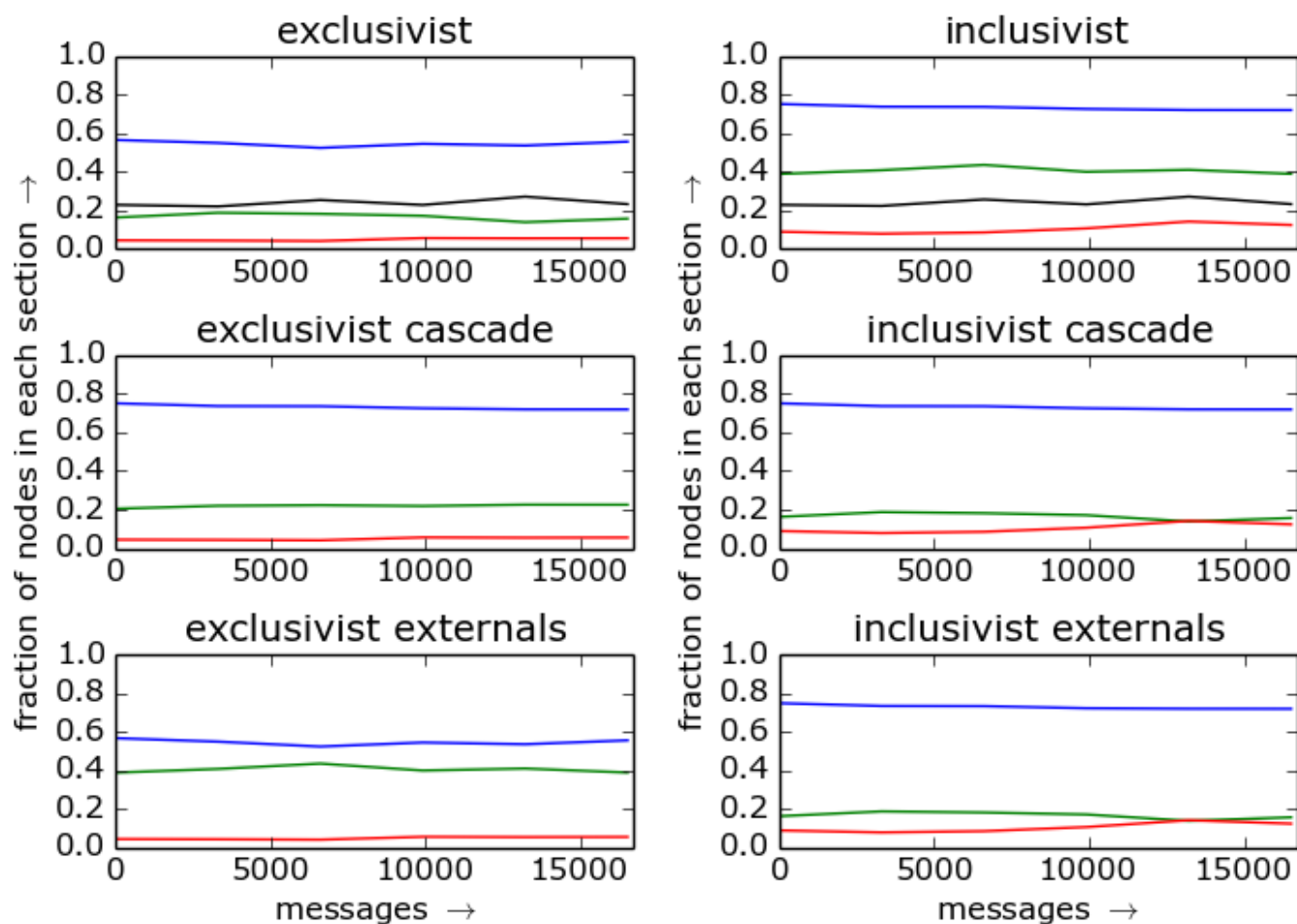
Compound divisions. Window: 1000 messages.
Placement resolution: 1000 messages. LAD



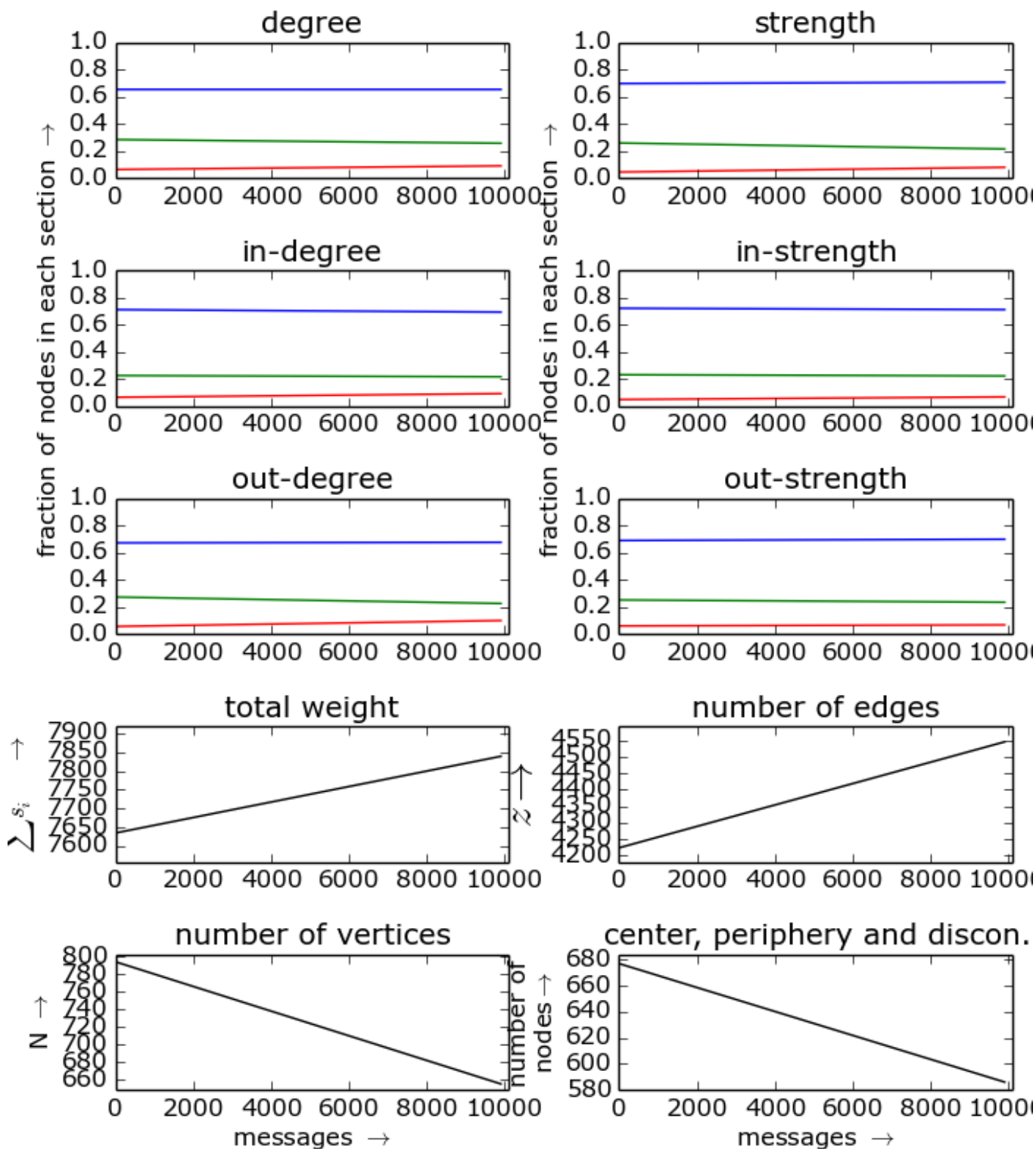
Primary divisions. Window: 3300 messages.
Placement resolution: 3300 messages. LAD



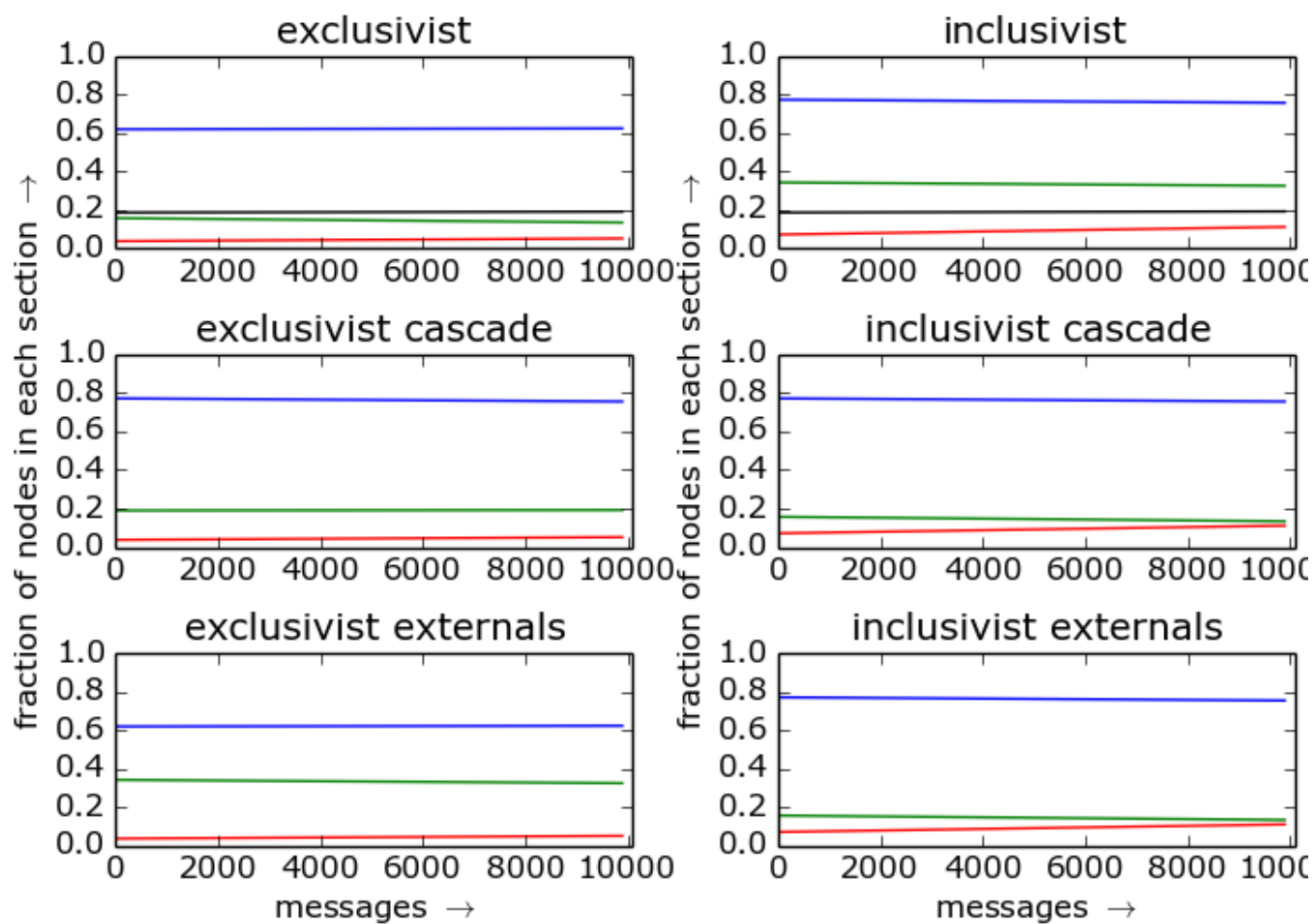
Compound divisions. Window: 3300 messages.
Placement resolution: 3300 messages. LAD



Primary divisions. Window: 9900 messages.
Placement resolution: 9900 messages. LAD



Compound divisions. Window: 9900 messages.
Placement resolution: 9900 messages. LAD



III. PCA OF MEASURES ALONG THE TIMELINE

Loadings for the 14 metrics into the principal components for all LAD, LAU, MET, CPP, lists, $ws = 1000$ messages in 20 disjoint positioning. The clustering coefficient (cc) appears as the first metric in the Table, followed by 7 centrality metrics and 6 symmetry-related metrics, in accordance with the notation made of Section IIID of the main document. Note that the centrality metrics, including degrees, strength and betweenness centrality, are the most important contributors for the first principal component, while the second component is dominated by symmetry metrics. The clustering coefficient is only relevant for the third principal component, coupled with standard deviations of strengths and degrees. The three components have in average 80.36% of the variance.

A. Betweenness, clustering and degree

TABLE XVII. LAU principal components formation and concentration of dispersion.

	PC1		PC2		PC3	
	μ	σ	μ	σ	μ	σ
cc	6.03	3.73	87.60	5.25	4.52	0.93
k	47.13	1.76	3.01	1.98	47.90	0.38
bt	46.84	1.97	9.39	4.31	47.58	0.57
λ	64.99	0.60	33.08	0.41	1.93	0.36

TABLE XVIII. LAD principal components formation and concentration of dispersion.

	PC1		PC2		PC3	
	μ	σ	μ	σ	μ	σ
cc	6.42	4.05	86.60	5.50	5.19	1.45
k	46.98	1.86	2.95	1.65	47.61	0.57
bt	46.59	2.18	10.45	4.72	47.20	0.90
λ	64.96	0.71	33.08	0.41	1.96	0.52

TABLE XIX. MET principal components formation and concentration of dispersion.

	PC1		PC2		PC3	
	μ	σ	μ	σ	μ	σ
cc	5.82	3.76	87.26	5.12	4.93	1.19
k	47.18	1.82	4.35	4.01	47.63	0.57
bt	47.01	1.96	8.40	4.22	47.44	0.67
λ	64.94	0.76	33.13	0.45	1.93	0.62

TABLE XX. CPP principal components formation and concentration of dispersion.

	PC1		PC2		PC3	
	μ	σ	μ	σ	μ	σ
cc	3.61	2.13	91.86	3.24	3.59	0.98
k	48.24	0.99	2.96	2.25	48.25	0.43
bt	48.15	1.14	5.18	3.89	48.16	0.56
λ	65.24	0.51	33.30	0.17	1.46	0.49

B. Betweenness, clustering, degrees and strengths

TABLE XXI. LAU principal components formation and concentration of dispersion.

	PC1		PC2		PC3	
	μ	σ	μ	σ	μ	σ
cc	1.59	0.81	80.37	5.18	3.09	1.89
s	14.40	0.15	0.81	0.68	4.75	4.43
s^{in}	14.00	0.14	2.32	1.49	18.98	4.93
s^{out}	13.96	0.14	2.72	1.44	18.25	6.36
k	14.49	0.15	0.54	0.35	1.37	0.98
k^{in}	14.01	0.13	2.72	1.35	18.69	5.01
k^{out}	13.85	0.13	2.37	1.73	22.63	3.79
bt	13.69	0.22	8.16	1.62	12.23	8.33
λ	81.87	0.88	12.48	0.15	3.33	0.70

TABLE XXII. LAD principal components formation and concentration of dispersion.

	PC1		PC2		PC3	
	μ	σ	μ	σ	μ	σ
cc	1.83	1.11	80.38	11.45	3.78	4.43
s	14.25	0.17	1.34	1.81	9.88	5.76
s^{in}	13.99	0.19	2.06	1.70	17.62	6.15
s^{out}	14.03	0.22	1.81	1.98	15.44	6.68
k	14.38	0.13	0.95	1.64	3.45	3.15
k^{in}	14.05	0.14	2.26	1.66	13.44	7.26
k^{out}	13.96	0.15	1.72	1.53	16.14	6.37
bt	13.51	0.35	9.48	2.86	20.26	9.87
λ	82.32	1.61	12.52	0.26	2.97	1.21

TABLE XXIII. MET principal components formation and concentration of dispersion.

	PC1		PC2		PC3	
	μ	σ	μ	σ	μ	σ
<i>cc</i>	1.16	0.76	81.72	3.00	1.61	1.78
<i>s</i>	14.32	0.16	1.76	1.12	11.39	5.50
<i>sⁱⁿ</i>	14.17	0.11	2.29	1.29	14.46	3.72
<i>s^{out}</i>	14.09	0.17	1.72	1.18	17.54	5.37
<i>k</i>	14.39	0.16	1.73	0.63	4.76	2.82
<i>kⁱⁿ</i>	14.12	0.13	1.02	0.71	11.69	6.93
<i>k^{out}</i>	14.06	0.13	3.11	1.58	12.18	9.24
<i>bt</i>	13.69	0.26	6.64	2.01	26.37	12.37
λ	83.41	1.53	12.53	0.11	2.34	1.16

TABLE XXIV. CPP principal components formation and concentration of dispersion.

	PC1		PC2		PC3	
	μ	σ	μ	σ	μ	σ
<i>cc</i>	0.84	0.61	80.59	6.89	2.30	2.19
<i>s</i>	14.28	0.07	0.97	1.03	15.89	1.15
<i>sⁱⁿ</i>	14.18	0.12	2.89	1.71	13.50	5.19
<i>s^{out}</i>	14.07	0.23	2.83	1.63	18.80	4.94
<i>k</i>	14.42	0.08	0.78	0.67	7.48	2.71
<i>kⁱⁿ</i>	14.29	0.10	2.36	1.41	7.21	4.49
<i>k^{out}</i>	14.16	0.12	3.62	1.83	8.79	4.58
<i>bt</i>	13.76	0.22	5.96	1.88	26.03	7.94
λ	83.32	1.42	12.60	0.08	2.61	1.15

C. Betweenness, clustering, degrees, strengths and symmetry measures

TABLE XXV. LAU principal components formation and concentration of dispersion.

	PC1		PC2		PC3	
	μ	σ	μ	σ	μ	σ
<i>cc</i>	1.64	0.77	2.42	1.71	19.20	3.96
<i>s</i>	12.80	0.46	0.89	0.82	2.53	0.63
<i>sⁱⁿ</i>	12.47	0.42	2.30	0.97	2.29	0.81
<i>s^{out}</i>	12.37	0.46	2.89	1.24	2.64	0.58
<i>k</i>	12.93	0.44	0.82	0.73	1.32	0.45
<i>kⁱⁿ</i>	12.54	0.37	2.88	1.13	1.02	0.56
<i>k^{out}</i>	12.32	0.46	3.82	1.14	1.57	0.68
<i>bt</i>	12.19	0.46	1.06	0.62	2.64	0.89
<i>asy</i>	0.93	0.81	20.38	0.82	1.66	1.09
μ_{asy}	0.96	0.83	20.26	0.82	1.66	1.04
σ_{asy}	6.18	0.71	1.24	0.92	27.98	1.74
<i>dis</i>	0.90	0.79	20.36	0.82	1.54	1.07
μ_{dis}	0.92	0.61	19.02	0.84	1.45	1.12
σ_{dis}	0.86	0.51	1.64	1.10	32.51	1.90
λ	48.41	0.52	27.95	0.36	12.81	0.79

TABLE XXVI. LAD principal components formation and concentration of dispersion.

	PC1		PC2		PC3	
	μ	σ	μ	σ	μ	σ
<i>cc</i>	1.96	0.95	3.07	1.46	17.94	5.38
<i>s</i>	12.34	0.57	1.72	0.99	2.43	0.93
<i>sⁱⁿ</i>	12.06	0.64	3.18	0.98	1.98	1.09
<i>s^{out}</i>	12.22	0.48	1.14	0.78	2.83	0.79
<i>k</i>	12.54	0.56	1.43	0.87	0.92	0.44
<i>kⁱⁿ</i>	12.15	0.61	3.81	0.79	0.61	0.42
<i>k^{out}</i>	12.27	0.45	1.51	1.08	1.56	0.39
<i>bt</i>	11.73	0.64	1.80	0.88	2.28	1.00
<i>asy</i>	1.51	0.97	19.66	1.63	3.02	1.66
μ_{asy}	1.41	0.99	19.53	1.62	3.00	1.69
σ_{asy}	5.62	0.68	2.01	1.23	27.46	3.31
<i>dis</i>	1.58	0.98	19.57	1.65	3.21	1.71
μ_{dis}	1.84	1.00	18.62	1.52	2.08	1.13
σ_{dis}	0.77	0.59	2.94	1.60	30.68	3.34
λ	48.65	1.03	27.84	0.31	13.00	0.77

TABLE XXVII. MET principal components formation and concentration of dispersion.

	PC1		PC2		PC3	
	μ	σ	μ	σ	μ	σ
<i>cc</i>	1.18	0.71	3.00	2.35	22.39	2.71
<i>s</i>	12.34	0.66	1.74	1.17	1.55	0.75
<i>sⁱⁿ</i>	12.25	0.62	1.74	0.96	1.45	0.77
<i>s^{out}</i>	12.11	0.72	2.42	1.35	1.78	0.78
<i>k</i>	12.48	0.63	1.46	0.91	0.54	0.48
<i>kⁱⁿ</i>	12.32	0.56	1.54	1.22	0.65	0.62
<i>k^{out}</i>	12.12	0.67	3.10	1.15	0.87	0.74
<i>bt</i>	11.85	0.62	1.46	0.87	1.16	0.70
<i>asy</i>	1.79	1.22	19.35	2.15	3.29	2.15
μ_{asy}	1.84	1.22	19.17	2.16	3.31	2.23
σ_{asy}	4.17	0.79	3.91	2.35	27.79	3.96
<i>dis</i>	1.78	1.18	19.26	2.15	3.38	2.29
μ_{dis}	1.53	1.10	18.23	2.12	3.32	1.71
σ_{dis}	2.23	0.93	3.61	2.38	28.54	3.23
λ	49.05	1.01	27.79	0.30	13.30	1.35

TABLE XXVIII. CPP principal components formation and concentration of dispersion.

	PC1		PC2		PC3	
	μ	σ	μ	σ	μ	σ
<i>cc</i>	0.89	0.59	1.93	1.33	21.22	2.97
<i>s</i>	11.71	0.57	2.97	0.82	2.45	0.72
<i>sⁱⁿ</i>	11.68	0.58	2.37	0.91	3.08	0.78
<i>s^{out}</i>	11.49	0.61	3.63	0.79	1.61	0.88
<i>k</i>	11.93	0.54	2.58	0.70	0.52	0.44
<i>kⁱⁿ</i>	11.93	0.52	1.19	0.88	1.41	0.71
<i>k^{out}</i>	11.57	0.61	4.34	0.70	0.98	0.66
<i>bt</i>	11.37	0.55	2.44	0.84	1.37	0.77
<i>asy</i>	3.14	0.98	18.52	1.97	2.46	1.69
μ_{asy}	3.32	0.99	18.23	2.01	2.80	1.82
σ_{asy}	4.91	0.59	2.44	1.47	26.84	3.06
<i>dis</i>	2.94	0.88	18.50	1.92	3.06	1.98
μ_{dis}	2.55	0.89	18.12	1.85	1.57	1.32
σ_{dis}	0.57	0.33	2.74	1.63	30.61	2.66
λ	49.56	1.16	27.14	0.54	13.25	0.95

IV. STABILITY IN OTHER NETWORKS: TWITTER, FACEBOOK, PARTICIPA.BR

To further verify the hypothesis that such stability is a general property of human social networks, we analyzed networks driven from Twitter, Facebook and Participa.br. Selected networks are summarized in Table [XXIX](#). Their Erdős sector relative sizes are given in Table [XXX](#). PCA formations are given in Tables [XXXI](#), [XXXII](#), [XXXIII](#) and [XXXIV](#). Friendship networks considered are undirected and unweighted, therefore all measures of strength, in- and out- centralities, asymmetry and disequilibrium have little or no meaning, which is why F1, F2, F3, F4 and F5 are only present in Table [XXXI](#). Most important results from the consideration herein of this analysis are:

- a further indicative that the stability reported with a focus on email interaction networks is valid for a broader class of phenomena.
- The greater stability of email interaction networks with the same number of participants. This is specially important for benchmarking and probing general properties.

TABLE XXIX. Overview of selected networks analyzed in addition to email interaction networks. Three social platforms were the sources of network structures: Facebook, Twitter and Participa.br. Both friendship and interaction networks were observed, yielding undirected and directed networks, respectively. The number of agents N and the number of edges z are given on the last columns. The acronyms, one for each network, are used throughout Tables XXX, XXXII, XXXI, XXXIII and XXXIV. All data collected in 2013 and 2014.

acronym	provenance	edge	directed	description	N	z
F1	Facebook	friendship	no	the friendship network of Renato Fabbri (author)	1367	28606
F2	Facebook	friendship	no	the friendship network of Massimo Canevacci (senior anthropologist)	4764	59995
F3	Facebook	friendship	no	the friendship network of a brazilian direct democracy group	3599	59471
F4	Facebook	friendship	no	the friendship network of the Silicon Valley Global Network group	2026	15586
F5	Participa.br	friendship	no	the friendship network of a brazilian federal social participation portal	443	910
I1	Facebook	interaction	yes	the interaction network of the Silicon Valley Global Network group	104	154
I2	Facebook	interaction	yes	the interaction network of a Solidarity Economy group	64	120
I3	Facebook	interaction	yes	the interaction network of a brazilian direct democracy group	214	310
I4	Facebook	interaction	yes	the interaction network of the 'Cience with Frontiers' group	530	1658
I5	Participa.br	interaction	yes	the interaction network of a brazilian federal social participation portal	222	300
TT1	Twitter	retweet	yes	the retweet network of $\approx 22k$ tweets with the hashtag #arenaNETmundial	2772	7222
TT2	Twitter	retweet	yes	same as TT1, but disconnected agents are not discarded	2975	7222

TABLE XXX. Percentage of agents in each Erdős sector in the friendship and interaction human networks of Table XXIX. The ratios found in email networks are preserved. I1 and I4 are outliers, to which a possible explanation is that they are better characterized as a superposition of networks, rather than one coherent network.

	periphery	intermediary	hubs
F1	53.11	43.31	3.58
F2	58.98	39.29	1.72
F3	65.41	31.87	2.72
F4	66.49	32.03	1.48
F5	62.98	36.12	0.90
I1	4.81	94.23	0.96
I2	53.12	45.31	1.56
I3	58.41	40.19	1.40
I4	39.06	59.43	1.51
I5	54.95	43.69	1.35
TT1	74.86	24.49	0.65
TT2	76.57	22.86	0.57

TABLE XXXI. First three principal components formation and variance concentration for each of the five friendship networks of Table XXIX in the simplest case: dimensions correspond to degree, clustering coefficient and betweenness centrality. Participa.br yields the networks that most resembles the email networks. Overall, the general characteristic is preserved: first component is an average of degree and betweenness, while clustering is the most relevant for second component. The friendship of Renato Fabbri (F1) is the only whose first component has more than 20% of clustering coefficient and second component has more than 40% of degree centrality.

	PC1					PC2					PC3				
	F1	F2	F3	F4	F5	F1	F2	F3	F4	F5	F1	F2	F3	F4	F5
cc	25.80	12.22	11.54	5.04	0.94	58.87	78.22	68.86	90.39	88.86	6.95	1.13	18.20	3.02	5.90
k	36.43	43.96	45.61	47.40	49.50	25.66	9.98	6.10	7.63	6.42	44.94	49.52	42.00	48.41	47.02
bt	37.77	43.82	42.85	47.56	49.56	15.46	11.80	25.04	1.98	4.72	48.11	49.35	39.80	48.57	47.08
λ	53.15	53.06	46.26	55.36	63.80	28.69	32.57	34.27	33.25	33.57	18.16	14.37	19.47	11.38	2.63

TABLE XXXII. First three principal components formation and variance concentration for each of the seven interaction networks of Table XXIX in the simplest case: dimensions correspond to degree, clustering coefficient and betweenness centrality. Twitter yields the networks that most resembles the email networks. Overall, the general characteristic is preserved: first component is an average of degree and betweenness, while clustering is the most relevant for second component.

	PC1							PC2							PC3						
	I1	I2	I3	I4	I5	TT1	TT2	I1	I2	I3	I4	I5	TT1	TT2	I1	I2	I3	I4	I5	TT1	TT2
<i>cc</i>	14.43	17.12	11.54	0.69	13.26	2.17	2.72	74.78	70.72	79.30	96.63	76.59	95.75	94.69	1.58	4.09	2.46	1.71	0.57	2.03	2.20
<i>k</i>	42.68	41.77	44.37	49.65	43.41	48.94	48.67	13.85	11.48	8.31	2.35	11.26	0.14	0.52	49.07	48.42	48.94	49.14	49.76	49.01	48.93
<i>bt</i>	42.89	41.11	44.09	49.66	43.34	48.89	48.61	11.37	17.80	12.39	1.02	12.15	4.12	4.79	49.35	47.49	48.60	49.15	49.67	48.96	48.87
λ	64.58	61.97	56.95	62.01	50.92	64.82	64.83	31.57	30.98	32.56	33.35	32.51	33.33	33.32	3.85	7.05	10.50	4.64	16.57	1.85	1.86

TABLE XXXIII. First three principal components formation and variance concentration for each of the seven interaction networks of Table XXIX considering dimensions of in- and out- degrees and strengths, clustering coefficient and betweenness centrality. Twitter yields the networks that most resembles email networks. Overall, the general characteristic is preserved: first component is an average of degree and betweenness, while clustering is the most relevant for second component. Important differences are: - the clustering coefficient was only important to the third component for two of the networks (*I2*, *I3*) and does not contribute significantly to any of the first three principal components in *I5*; - in the first component, *I5* exhibited less contribution from in-strength, in-degree and betweenness, *I4* exhibited less contribution from out-degree.

	PC1							PC2							PC3						
	I1	I2	I3	I4	I5	TT1	TT2	I1	I2	I3	I4	I5	TT1	TT2	I1	I2	I3	I4	I5	TT1	TT2
<i>cc</i>	2.79	4.34	2.57	0.82	1.29	0.66	0.76	28.44	9.46	3.29	21.95	6.95	29.82	30.04	32.24	60.89	80.24	43.85	3.81	33.84	33.54
<i>s</i>	15.28	15.84	16.46	16.01	16.70	15.49	15.47	3.78	4.95	2.90	3.26	17.78	1.95	2.05	1.95	0.34	0.87	4.84	11.15	0.52	0.43
<i>sⁱⁿ</i>	14.48	12.81	13.62	14.63	4.50	11.85	11.84	11.77	18.29	17.41	12.44	16.19	19.03	18.81	5.38	5.03	0.93	11.16	30.41	21.48	21.71
<i>s^{out}</i>	12.13	12.12	12.59	12.91	19.02	13.87	13.85	17.19	16.79	20.12	18.81	8.90	13.42	13.43	19.35	7.90	3.11	11.38	14.58	12.91	12.92
<i>k</i>	15.32	16.22	16.12	16.20	21.12	15.48	15.46	3.13	4.18	6.25	2.88	9.26	3.32	3.24	1.84	0.09	1.16	0.11	2.22	4.26	4.30
<i>kⁱⁿ</i>	14.49	13.56	12.90	15.34	7.29	12.99	12.98	10.45	16.50	19.68	11.13	20.75	17.89	17.86	8.78	4.07	1.26	6.07	15.41	14.67	14.65
<i>k^{out}</i>	11.70	11.25	11.80	9.24	21.09	14.20	14.19	19.14	20.50	21.19	26.13	0.19	12.36	12.28	18.80	7.50	4.68	20.44	10.57	12.14	12.20
<i>bt</i>	13.82	13.86	13.93	14.86	8.99	15.47	15.45	6.10	9.32	9.16	3.41	19.99	2.20	2.29	11.66	14.20	7.75	2.17	11.86	0.18	0.25
λ	71.73	60.58	60.35	64.53	41.28	70.06	70.08	15.23	21.53	20.13	16.42	22.83	13.83	13.86	9.95	11.37	12.25	11.19	15.71	11.43	11.38

TABLE XXXIV. First three principal components formation and variance concentration for each of the seven interaction networks of Table XXIX considering dimensions of in- and out- degrees and strengths, clustering coefficient, betweenness centrality and symmetry related metrics (see Section IIID). In comparison with the results found in email interaction networks, general characteristics are preserved: first component is an average of degree and betweenness, second component is mostly governed by symmetry related metrics, and clustering coefficient is mostly relevant for the third component. Standard deviation of asymmetry and disequilibrium metrics are again coupled to clustering coefficient in the third component. Important differences are: - the first component is a less regular average of centrality measures and has a greater contribution of symmetry metrics; - The first component of *I5* is formed mostly from symmetry, not centrality, metrics.

	PC1							PC2							PC3						
	I1	I2	I3	I4	I5	TT1	TT2	I1	I2	I3	I4	I5	TT1	TT2	I1	I2	I3	I4	I5	TT1	TT2
<i>cc</i>	3.46	4.19	2.44	0.36	2.18	1.28	1.17	3.06	1.61	1.23	1.19	2.57	3.03	2.17	17.36	16.88	21.68	17.00	10.00	18.65	19.13
<i>s</i>	10.05	9.21	9.60	9.31	3.54	10.27	10.59	5.81	5.74	7.33	8.47	9.24	6.26	5.96	4.58	8.02	4.91	2.21	13.10	0.92	1.53
<i>sⁱⁿ</i>	9.57	8.03	9.21	8.74	0.78	7.75	7.99	4.63	0.59	1.27	6.69	2.77	5.38	5.29	8.22	12.82	9.18	6.53	7.63	5.90	4.77
<i>s^{out}</i>	7.88	6.21	5.45	6.97	5.76	9.25	9.54	6.78	10.23	12.76	9.20	10.26	5.27	4.92	5.90	2.99	3.86	8.43	10.84	4.58	4.76
<i>k</i>	10.44	10.02	9.88	10.39	5.80	10.80	11.05	4.62	5.13	5.66	5.54	14.08	4.48	4.29	3.63	6.02	6.18	1.86	1.21	1.33	1.30
<i>kⁱⁿ</i>	10.12	9.30	9.50	9.98	4.43	8.64	8.86	2.69	0.70	0.88	4.49	9.61	5.40	5.50	7.12	10.55	8.70	6.17	8.24	7.27	6.54
<i>k^{out}</i>	7.27	5.29	4.43	5.43	9.11	10.10	10.33	8.36	12.52	13.63	5.65	11.61	3.38	3.08	7.82	5.77	2.07	13.52	5.68	5.00	4.65
<i>bt</i>	9.62	7.97	7.53	8.93	2.25	10.47	10.78	3.77	8.42	9.14	6.95	8.12	5.60	5.29	2.72	0.42	1.99	2.74	8.66	1.16	1.60
<i>asy</i>	5.42	7.05	7.97	8.48	15.47	6.16	5.79	14.17	12.88	11.78	11.02	4.67	12.48	13.39	2.95	1.03	0.58	2.71	0.87	6.54	5.80
μ_{asy}	5.48	6.99	7.99	8.47	15.44	6.18	5.80	14.12	13.04	11.78	11.01	4.72	12.46	13.37	2.92	0.76	0.75	2.77	0.76	6.58	5.83
σ_{asy}	6.53	7.39	7.63	7.15	2.37	5.59	5.48	1.69	3.80	1.75	8.46	7.49	5.94	5.45	11.32	8.91	11.14	3.04	15.54	13.70	15.31
<i>dis</i>	5.02	6.67	7.78	8.08	15.41	5.98	5.59	14.12	13.41	11.92	11.53	4.80	12.45	13.38	4.99	1.40	0.67	3.02	0.83	7.44	6.69
μ_{dis}	5.33	7.01	7.24	6.92	14.34	5.49	5.14	13.33	10.15	9.47	8.02	5.05	11.86	12.65	1.66	7.08	5.72	11.38	2.68	0.77	0.66
σ_{dis}	3.82	4.68	3.34	0.81	3.12	2.03	1.88	2.85	1.77	1.39	1.77	5.00	6.01	5.24	18.82	17.36	22.58	18.61	13.97	20.16	21.42
λ	46.11	43.48	44.29	46.95	30.34	44.12	43.52	26.42	24.97	24.76	19.99	23.91	25.98	26.13	14.90	14.72	11.82	13.16	17.32	11.62	12.15

¹R. Fabbri, “Time stability in human interaction networks,” arXiv preprint arXiv:1310.7769. <http://arxiv.org/abs/1310.7769>.