

# Temporal stability in human interaction networks (Supporting Information document)

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This Supporting Information document holds circular statistics and histograms of activity along time in Section **SI**, the fraction of vertices in the peripheral, intermediary and hub sectors in Section **SIII** and the combination of basic topological measures into principal components with greater variance in Section **SII**. There is a focus on email list interaction networks for benchmarking and Section **SIV** reinforces the results with the analysis of networks from Facebook, Twitter and Participabr. More context (e.g. methods, discussion, data and scripts) is given in the main document<sup>1</sup> to which this current document supplies supporting information.

## SI. TEMPORAL ACTIVITY IN DIFFERENT SCALES

This section presents information derived from the theory presented in Section III A for supporting the results in Section IV A.

### A. Temporal circular measures

The metrics with which we report measurements and results of activity along time are the rescaled circular mean  $\theta'_\mu$ , the standard deviation  $S(z)$ , the variance  $Var(z)$ , the circular dispersion  $\delta(z)$  and the ratio between the lowest  $b_l$  and the highest  $b_h$  incidences  $\frac{b_l}{b_h}$  at each time scale. Also, the mean  $\mu(\frac{b'_l}{b'_h})$  and the standard deviation  $\sigma(\frac{b'_l}{b'_h})$  of the relation between the minimum  $b'_l$  and the maximum  $b'_h$  incidences are given for 1000 uniform distribution simulations within the same number of bins and with the same number of samples<sup>2</sup>. Greater dispersion is found on the scales of seconds and minutes, followed by days of the month. Greater localization is found in the scale of hours of the day, followed by weekdays and months.

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TABLE S1. LAU circular measurements.

scale	$\theta'_\mu$	$S(z)$	$Var(z)$	$\delta(z)$	$\frac{b_L}{b_h}$	$\mu(\frac{b'_L}{b'_h})$	$\sigma(\frac{b'_L}{b'_h})$
seconds	-//-	3.31	1.00	29337.65	0.78	0.78	0.03
minutes	-//-	3.13	0.99	8879.19	0.76	0.78	0.03
hours	-8.76	1.56	0.71	4.92	0.12	0.87	0.02
weekdays	-0.21	2.14	0.90	45.41	0.62	0.95	0.01
month days	-0.64	2.76	0.98	1001.75	0.67	0.85	0.02
months	3.55	2.30	0.93	94.53	0.64	0.92	0.02

TABLE S2. LAD circular measurements.

scale	$\theta'_\mu$	$S(z)$	$Var(z)$	$\delta(z)$	$\frac{b_L}{b_h}$	$\mu(\frac{b'_L}{b'_h})$	$\sigma(\frac{b'_L}{b'_h})$
seconds	-//-	3.13	0.99	9070.17	0.78	0.78	0.03
minutes	-//-	3.60	1.00	205489.40	0.82	0.78	0.03
hours	-9.61	1.52	0.68	4.36	0.10	0.87	0.02
weekdays	-0.03	2.03	0.87	29.28	0.58	0.95	0.01
month days	-2.65	2.93	0.99	2657.77	0.67	0.85	0.02
months	-0.56	2.14	0.90	44.00	0.44	0.92	0.02

TABLE S3. MET circular measurements.

scale	$\theta'_\mu$	$S(z)$	$Var(z)$	$\delta(z)$	$\frac{b_L}{b_h}$	$\mu(\frac{b'_L}{b'_h})$	$\sigma(\frac{b'_L}{b'_h})$
seconds	-//-	3.06	0.99	5910.47	0.79	0.77	0.03
minutes	-//-	3.14	0.99	9696.29	0.75	0.77	0.03
hours	-9.20	1.35	0.60	2.76	0.05	0.87	0.02
weekdays	-0.27	1.86	0.82	13.82	0.35	0.95	0.02
month days	3.58	2.49	0.95	237.30	0.64	0.85	0.02
months	-2.92	1.73	0.78	9.20	0.33	0.92	0.02

TABLE S4. CPP circular measurements.

scale	$\theta'_\mu$	$S(z)$	$Var(z)$	$\delta(z)$	$\frac{b_L}{b_h}$	$\mu(\frac{b'_L}{b'_h})$	$\sigma(\frac{b'_L}{b'_h})$
seconds	-//-	3.31	1.00	28205.46	0.79	0.78	0.03
minutes	-//-	3.18	0.99	12275.59	0.79	0.78	0.03
hours	-9.39	1.48	0.67	3.91	0.09	0.87	0.02
weekdays	-0.17	1.83	0.81	12.66	0.39	0.95	0.01
month days	-10.12	3.16	0.99	10789.17	0.65	0.85	0.02
months	0.15	2.34	0.93	115.49	0.67	0.92	0.02

## B. Temporal histograms

### 1. Histograms of activity along the hours of the day

Higher activity was observed between noon and 6pm, followed by the time period between 6pm and midnight. Around 2/3 of the whole activity takes place from noon to midnight. The activity peak occurs around midday, with a slight skew toward one hour before noon.

TABLE S5. LAU activity along the hours of the day.

	1h	2h	3h	4h	6h	12h
0h	3.58	5.80	7.43	8.49	10.14	36.88
1h	2.22					
2h	1.63	2.69				
3h	1.06		2.72	5.20	26.74	63.12
4h	0.84	1.66				
5h	0.82					
6h	1.17	3.54	7.07	23.20	27.46	70.67
7h	2.37					
8h	3.53	9.57				
9h	6.04		19.67	17.71	12.25	28.44
10h	6.83	13.62				
11h	6.79					
12h	6.11	12.36	18.75	24.68	35.66	42.22
13h	6.26					
14h	6.38	12.31				
15h	5.93		16.91	20.73	14.41	21.50
16h	5.52	10.98				
17h	5.46					
18h	5.23	9.75	14.30	11.22	9.34	16.19
19h	4.52					
20h	4.55	8.97				
21h	4.42		13.16	7.88	4.24	3.64
22h	4.51	8.74				
23h	4.23					

TABLE S6. LAD activity along the hours of the day.

	1h	2h	3h	4h	6h	12h
0h	4.01	6.53	8.32	9.37	10.78	33.11
1h	2.52					
2h	1.79					
3h	1.06	2.84	2.46	3.81	22.33	66.89
4h	0.75					
5h	0.66					
6h	0.85	2.41	5.36	19.93	16.98	12.32
7h	1.56					
8h	2.95					
9h	4.66	7.61	16.98	24.82	18.85	18.39
10h	5.92					
11h	6.40					
12h	6.41	12.53	18.85	23.44	15.65	29.65
13h	6.12					
14h	6.32					
15h	5.97	12.29	18.39	14.00	9.22	9.41
16h	6.40					
17h	6.02					
18h	5.99	11.02	15.65	18.63	9.22	9.41
19h	5.03					
20h	4.63					
21h	4.59	9.22	14.00	18.63	9.22	9.41
22h	4.88					
23h	4.53					

TABLE S7. MET activity along the hours of the day.

	1h	2h	3h	4h	6h	12h
0h	2.87	4.64	5.67	6.31	7.15	29.33
1h	1.77					
2h	1.04					
3h	0.64	1.67	1.48	2.89	22.18	70.67
4h	0.47					
5h	0.38					
6h	0.72	2.04	4.71	20.14	17.47	13.07
7h	1.33					
8h	2.67					
9h	4.40	7.07	17.47	28.65	24.79	16.19
10h	6.29					
11h	6.78					
12h	7.33	14.41	21.50	42.22	12.25	7.88
13h	7.08					
14h	7.09					
15h	7.14	14.24	20.72	24.79	17.22	9.34
16h	6.68					
17h	6.89					
18h	5.99	11.22	16.19	12.25	7.88	4.24
19h	5.23					
20h	4.98					
21h	4.37	9.34	12.25	17.22	9.34	16.19
22h	4.24					
23h	3.64					

TABLE S8. CPP activity along the hours of the day.

	1h	2h	3h	4h	6h	12h
0h	3.66	6.42	8.20	9.30	10.67	33.76
1h	2.76					
2h	1.79	2.88	2.47	3.44	23.09	
3h	1.10					
4h	0.68	1.37	4.35	21.03		
5h	0.69					
6h	0.83	2.07	18.75	25.05		
7h	1.24					
8h	2.28	6.80	18.95	23.60		
9h	4.52					
10h	6.62	14.23	15.88	17.59	28.61	
11h	7.61					
12h	6.44	12.48	18.68	23.60	28.61	66.24
13h	6.04					
14h	6.47	12.57	12.73	17.59	28.61	
15h	6.10					
16h	6.22	12.58	8.36	17.59	28.61	
17h	6.36					
18h	6.01	11.02	15.88	17.59	28.61	
19h	5.02					
20h	4.85	9.23	12.73	17.59	28.61	
21h	4.38					
22h	4.06	8.36	12.73	17.59	28.61	
23h	4.30					

## 2. Histograms of activity along weekdays.

Most notably, the pattern of activity along weekdays presents a decrease of activity on weekend days of at least one third and at most two thirds compared against workweek days.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
LAU	15.71	15.81	15.88	16.43	15.14	<b>10.13</b>	<b>10.91</b>
LAD	14.92	17.75	17.01	15.41	14.21	<b>10.40</b>	<b>10.31</b>
MET	17.53	17.54	16.43	17.06	17.46	<b>7.92</b>	<b>6.06</b>
CPP	17.06	17.43	17.61	17.13	16.30	<b>6.81</b>	<b>7.67</b>

## 3. Histograms of activity along the days of the month

The most important feature seems to be the homogeneity made explicit by the high circular dispersion in the tables of Section [S1A](#). Slightly higher activity rates are found in the beginning of the month, although not statistically significant.

TABLE S9. LAU activity along the days of the month.

	1 day	5	10	15 days		
1	3.36	16.21	33.71	50.82		
2	3.43					
3	3.31					
4	3.37					
5	2.75	17.50	33.71		50.82	
6	3.03					
7	3.93					
8	3.62					
9	3.84	17.11	34.02			50.82
10	3.09					
11	3.20					
12	3.40					
13	3.67	16.91	34.02	50.82		
14	3.71					
15	3.14					
16	3.08					
17	3.13	16.91	34.02		50.82	
18	3.43					
19	3.61					
20	3.67					
21	3.60	15.43	32.27			49.18
22	3.42					
23	2.80					
24	2.64					
25	2.97	16.85	32.27	49.18		
26	3.06					
27	2.69					
28	3.79					
29	3.75	16.85	32.27		49.18	
30	3.57					

TABLE S10. LAD activity along the days of the month.

	1 day	5	10	15 days
1	3.29	15.77	33.63	50.50
2	3.38			
3	2.85			
4	2.94			
5	3.31			
6	3.60			
7	2.68			
8	3.78			
9	3.88	17.85		
10	3.91			
11	3.22			
12	2.79			
13	3.50	16.87	33.41	
14	3.95			
15	3.40			
16	3.32			
17	2.95	16.54		
18	3.50			
19	3.69			
20	3.07			
21	2.76	15.71	32.96	
22	3.35			
23	3.32			
24	3.15			
25	3.13	17.25		
26	3.68			
27	4.02			
28	3.49			
29	3.34	15.88		
30	2.72			

TABLE S11. MET activity along the days of the month.

	1 day	5	10	15 days
1	3.05	18.25	35.24	50.96
2	3.38			
3	3.62			
4	4.25			
5	3.94	16.98		
6	3.73			
7	3.17			
8	3.26			
9	3.56			
10	3.26			
11	3.81	15.73	31.98	49.04
12	2.91			
13	3.30			
14	2.75			
15	2.95	16.25		
16	3.36			
17	3.16			
18	3.44			
19	3.36			
20	2.93			
21	3.20	15.79	32.78	49.04
22	3.11			
23	3.60			
24	2.74			
25	3.13	16.99		
26	3.13			
27	3.07			
28	3.61			
29	3.60			
30	3.57			

TABLE S12. CPP activity along the days of the month.

	1 day	5	10	15 days
1	3.22	15.98	31.82	49.62
2	3.08			
3	3.19			
4	3.65			
5	2.84	15.84		
6	3.65			
7	3.53			
8	3.10			
9	2.49			
10	3.07			
11	3.47	17.80	34.22	
12	3.26			
13	3.55			
14	3.84			
15	3.68	16.42		
16	3.74			
17	3.40			
18	3.41			
19	2.95			
20	2.93			
21	3.15	17.13	33.96	50.38
22	3.64			
23	3.51			
24	3.32			
25	3.51	16.84		
26	3.54			
27	3.21			
28	3.40			
29	3.83			
30	2.86			

#### 4. Histograms of activity along months of the year

Activity is concentrated in Jun-Aug and/or in Dec-Mar. These observations mostly fit academic calendars, vacations and end-of-year holidays.

TABLE S13. LAU activity along the months of the year.

	m.	b.	t.	q.	s.
Jan	10.22	19.56	28.24	35.09	49.16
Fev	9.34				
Mar	8.67	15.53	20.93	30.36	50.84
Apr	6.86				
Mai	7.28	14.07	24.47	34.55	50.84
Jun	6.80				
Jul	8.97	16.29	26.36	34.55	50.84
Ago	7.32				
Set	8.18	16.25	26.36	34.55	50.84
Out	8.06				
Nov	7.64	18.30	26.36	34.55	50.84
Dez	10.66				

TABLE S14. LAD activity along the months of the year.

	m.	b.	t.	q.	s.
Jan	11.24	18.51	26.46	36.07	57.96
Fev	7.26				
Mar	7.95	17.56	31.50	37.56	42.04
Apr	9.61				
Mai	8.94	21.89	22.30	26.37	42.04
Jun	12.95				
Jul	9.03	15.67	19.74	26.37	42.04
Ago	6.64				
Set	6.63	12.38	19.74	26.37	42.04
Out	5.75				
Nov	7.61	13.99	19.74	26.37	42.04
Dez	6.38				

TABLE S15. MET activity along the months of the year.

	m.	b.	t.	q.	s.
Jan	4.87	11.00	16.89	23.30	47.70
Fev	6.13				
Mar	5.89	12.30	30.81	47.87	52.30
Apr	6.41				
Mai	10.45	24.40	31.21	28.83	52.30
Jun	13.95				
Jul	13.24	23.47	21.09	28.83	52.30
Ago	10.22				
Set	7.75	16.79	21.09	28.83	52.30
Out	9.04				
Nov	7.45	12.05	21.09	28.83	52.30
Dez	4.59				

TABLE S16. CPP activity along the months of the year.

	m.	b.	t.	q.	s.
Jan	8.70	17.00	27.23	36.49	54.27
Fev	8.29				
Mar	10.23	19.49	27.03	33.46	45.73
Apr	9.26				
Mai	9.41	17.78	22.94	30.06	45.73
Jun	8.37				
Jul	8.70	15.68	22.80	30.06	45.73
Ago	6.98				
Set	7.26	15.36	22.80	30.06	45.73
Out	8.10				
Nov	7.89	14.69	22.80	30.06	45.73
Dez	6.81				

## SII. PCA OF MEASURES ALONG THE TIMELINE

Loadings for topological metrics in principal components for LAD, LAU, MET, CPP, lists with the window size of  $ws = 1000$  messages 20 disjoint positioning. Further details are given in Sections IIIB 1 and IVC of the main document<sup>1</sup>.

### A. Betweenness, clustering and degree

This simplest PCA is characterized by the coupling of the centrality measures of degree  $k$  and betweenness  $bt$  in the first component. Second component is mostly the clustering coefficient  $cc$ . These three measures contribute almost equally to the dispersion of the system: first component holds two thirds of the dispersion and is resultant of two measures while the second component holds one third of the dispersion and results from one measure.

TABLE S17. LAU principal components formation and concentration of dispersion.

	PC1		PC2		PC3	
	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$
$cc$	6.03	3.73	87.60	5.25	4.52	0.93
$k$	47.13	1.76	3.01	1.98	47.90	0.38
$bt$	46.84	1.97	9.39	4.31	47.58	0.57
$\lambda$	64.99	0.60	33.08	0.41	1.93	0.36

TABLE S18. LAD principal components formation and concentration of dispersion.

	PC1		PC2		PC3	
	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$
$cc$	6.42	4.05	86.60	5.50	5.19	1.45
$k$	46.98	1.86	2.95	1.65	47.61	0.57
$bt$	46.59	2.18	10.45	4.72	47.20	0.90
$\lambda$	64.96	0.71	33.08	0.41	1.96	0.52

TABLE S19. MET principal components formation and concentration of dispersion.

	PC1		PC2		PC3	
	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$
<i>cc</i>	5.82	3.76	87.26	5.12	4.93	1.19
<i>k</i>	47.18	1.82	4.35	4.01	47.63	0.57
<i>bt</i>	47.01	1.96	8.40	4.22	47.44	0.67
$\lambda$	64.94	0.76	33.13	0.45	1.93	0.62

TABLE S20. CPP principal components formation and concentration of dispersion.

	PC1		PC2		PC3	
	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$
<i>cc</i>	3.61	2.13	91.86	3.24	3.59	0.98
<i>k</i>	48.24	0.99	2.96	2.25	48.25	0.43
<i>bt</i>	48.15	1.14	5.18	3.89	48.16	0.56
$\lambda$	65.24	0.51	33.30	0.17	1.46	0.49

## B. Betweenness, clustering, degrees and strengths

In this extension of the previous plot, the set of centrality metrics are extended to include strength  $s$  and the in- and out- degrees ( $k^{in}$ ,  $k^{out}$ ) and strengths ( $s^{in}$ ,  $s^{out}$ ). First component holds an average of the centrality metrics ( $k$ ,  $k^{in}$ ,  $k^{out}$ ,  $s$ ,  $s^{in}$ ,  $s^{out}$ ) while second component is mostly clustering coefficient  $cc$ . All metrics contribute the about same for total dispersion ( $\approx \frac{94}{8} = 11.75\%$ ).

TABLE S21. LAU principal components formation and concentration of dispersion.

	PC1		PC2		PC3	
	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$
<i>cc</i>	1.59	0.81	80.37	5.18	3.09	1.89
<i>s</i>	14.40	0.15	0.81	0.68	4.75	4.43
<i>s<sup>in</sup></i>	14.00	0.14	2.32	1.49	18.98	4.93
<i>s<sup>out</sup></i>	13.96	0.14	2.72	1.44	18.25	6.36
<i>k</i>	14.49	0.15	0.54	0.35	1.37	0.98
<i>k<sup>in</sup></i>	14.01	0.13	2.72	1.35	18.69	5.01
<i>k<sup>out</sup></i>	13.85	0.13	2.37	1.73	22.63	3.79
<i>bt</i>	13.69	0.22	8.16	1.62	12.23	8.33
$\lambda$	81.87	0.88	12.48	0.15	3.33	0.70

TABLE S22. LAD principal components formation and concentration of dispersion.

	PC1		PC2		PC3	
	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$
<i>cc</i>	1.83	1.11	80.38	11.45	3.78	4.43
<i>s</i>	14.25	0.17	1.34	1.81	9.88	5.76
<i>s<sup>in</sup></i>	13.99	0.19	2.06	1.70	17.62	6.15
<i>s<sup>out</sup></i>	14.03	0.22	1.81	1.98	15.44	6.68
<i>k</i>	14.38	0.13	0.95	1.64	3.45	3.15
<i>k<sup>in</sup></i>	14.05	0.14	2.26	1.66	13.44	7.26
<i>k<sup>out</sup></i>	13.96	0.15	1.72	1.53	16.14	6.37
<i>bt</i>	13.51	0.35	9.48	2.86	20.26	9.87
$\lambda$	82.32	1.61	12.52	0.26	2.97	1.21

TABLE S23. MET principal components formation and concentration of dispersion.

	PC1		PC2		PC3	
	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$
<i>cc</i>	1.16	0.76	81.72	3.00	1.61	1.78
<i>s</i>	14.32	0.16	1.76	1.12	11.39	5.50
<i>s<sup>in</sup></i>	14.17	0.11	2.29	1.29	14.46	3.72
<i>s<sup>out</sup></i>	14.09	0.17	1.72	1.18	17.54	5.37
<i>k</i>	14.39	0.16	1.73	0.63	4.76	2.82
<i>k<sup>in</sup></i>	14.12	0.13	1.02	0.71	11.69	6.93
<i>k<sup>out</sup></i>	14.06	0.13	3.11	1.58	12.18	9.24
<i>bt</i>	13.69	0.26	6.64	2.01	26.37	12.37
$\lambda$	83.41	1.53	12.53	0.11	2.34	1.16

TABLE S24. CPP principal components formation and concentration of dispersion.

	PC1		PC2		PC3	
	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$
<i>cc</i>	0.84	0.61	80.59	6.89	2.30	2.19
<i>s</i>	14.28	0.07	0.97	1.03	15.89	1.15
<i>s<sup>in</sup></i>	14.18	0.12	2.89	1.71	13.50	5.19
<i>s<sup>out</sup></i>	14.07	0.23	2.83	1.63	18.80	4.94
<i>k</i>	14.42	0.08	0.78	0.67	7.48	2.71
<i>k<sup>in</sup></i>	14.29	0.10	2.36	1.41	7.21	4.49
<i>k<sup>out</sup></i>	14.16	0.12	3.62	1.83	8.79	4.58
<i>bt</i>	13.76	0.22	5.96	1.88	26.03	7.94
$\lambda$	83.32	1.42	12.60	0.08	2.61	1.15

## C. Betweenness, clustering, degrees, strengths and symmetry measures

Loadings for 14 topological metrics in the first three principal components are given for LAD, LAU, MET, CPP, list. The clustering coefficient  $cc$  appears as the first metric in the tables, followed by 7 centrality metrics ( $k$ ,  $k^{in}$ ,  $k^{out}$ ,  $s$ ,  $s^{in}$ ,  $s^{ou}$ ,  $bt$ ) and 6 symmetry-related metrics ( $asy$ ,  $\mu^{asy}$ ,  $\sigma^{asy}$ ,  $dis$ ,  $\mu^{dis}$  and  $\sigma^{dis}$ ). The centrality metrics are most important for the first principal component, while the second component is predominantly the result of symmetry metrics. The clustering coefficient is only relevant for the third principal component, coupled



with standard deviations of asymmetry  $\sigma^{asy}$  and disequilibrium  $\sigma^{dis}$ . The three components sum up to  $\approx 90\%$  of the variance. In the PCA measures of the CPP list, the last table of these PCA-related tables, this general pattern is depicted in **boldface**. Further details are given in Sections IIIB 1 and IV C of the main document<sup>1</sup>.

TABLE S25. LAU principal components formation and concentration of dispersion.

	PC1		PC2		PC3	
	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$
<i>cc</i>	1.64	0.77	2.42	1.71	19.20	3.96
<i>s</i>	12.80	0.46	0.89	0.82	2.53	0.63
<i>s<sup>in</sup></i>	12.47	0.42	2.30	0.97	2.29	0.81
<i>s<sup>out</sup></i>	12.37	0.46	2.89	1.24	2.64	0.58
<i>k</i>	12.93	0.44	0.82	0.73	1.32	0.45
<i>k<sup>in</sup></i>	12.54	0.37	2.88	1.13	1.02	0.56
<i>k<sup>out</sup></i>	12.32	0.46	3.82	1.14	1.57	0.68
<i>bt</i>	12.19	0.46	1.06	0.62	2.64	0.89
<i>asy</i>	0.93	0.81	20.38	0.82	1.66	1.09
$\mu^{asy}$	0.96	0.83	20.26	0.82	1.66	1.04
$\sigma^{asy}$	6.18	0.71	1.24	0.92	27.98	1.74
<i>dis</i>	0.90	0.79	20.36	0.82	1.54	1.07
$\mu^{dis}$	0.92	0.61	19.02	0.84	1.45	1.12
$\sigma^{dis}$	0.86	0.51	1.64	1.10	32.51	1.90
$\lambda$	48.41	0.52	27.95	0.36	12.81	0.79

TABLE S26. LAD principal components formation and concentration of dispersion.

	PC1		PC2		PC3	
	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$
<i>cc</i>	1.96	0.95	3.07	1.46	17.94	5.38
<i>s</i>	12.34	0.57	1.72	0.99	2.43	0.93
<i>s<sup>in</sup></i>	12.06	0.64	3.18	0.98	1.98	1.09
<i>s<sup>out</sup></i>	12.22	0.48	1.14	0.78	2.83	0.79
<i>k</i>	12.54	0.56	1.43	0.87	0.92	0.44
<i>k<sup>in</sup></i>	12.15	0.61	3.81	0.79	0.61	0.42
<i>k<sup>out</sup></i>	12.27	0.45	1.51	1.08	1.56	0.39
<i>bt</i>	11.73	0.64	1.80	0.88	2.28	1.00
<i>asy</i>	1.51	0.97	19.66	1.63	3.02	1.66
$\mu^{asy}$	1.41	0.99	19.53	1.62	3.00	1.69
$\sigma^{asy}$	5.62	0.68	2.01	1.23	27.46	3.31
<i>dis</i>	1.58	0.98	19.57	1.65	3.21	1.71
$\mu^{dis}$	1.84	1.00	18.62	1.52	2.08	1.13
$\sigma^{dis}$	0.77	0.59	2.94	1.60	30.68	3.34
$\lambda$	48.65	1.03	27.84	0.31	13.00	0.77

TABLE S27. MET principal components formation and concentration of dispersion.

	PC1		PC2		PC3	
	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$
<i>cc</i>	1.18	0.71	3.00	2.35	22.39	2.71
<i>s</i>	12.34	0.66	1.74	1.17	1.55	0.75
<i>s<sup>in</sup></i>	12.25	0.62	1.74	0.96	1.45	0.77
<i>s<sup>out</sup></i>	12.11	0.72	2.42	1.35	1.78	0.78
<i>k</i>	12.48	0.63	1.46	0.91	0.54	0.48
<i>k<sup>in</sup></i>	12.32	0.56	1.54	1.22	0.65	0.62
<i>k<sup>out</sup></i>	12.12	0.67	3.10	1.15	0.87	0.74
<i>bt</i>	11.85	0.62	1.46	0.87	1.16	0.70
<i>asy</i>	1.79	1.22	19.35	2.15	3.29	2.15
$\mu^{asy}$	1.84	1.22	19.17	2.16	3.31	2.23
$\sigma^{asy}$	4.17	0.79	3.91	2.35	27.79	3.96
<i>dis</i>	1.78	1.18	19.26	2.15	3.38	2.29
$\mu^{dis}$	1.53	1.10	18.23	2.12	3.32	1.71
$\sigma^{dis}$	2.23	0.93	3.61	2.38	28.54	3.23
$\lambda$	49.05	1.01	27.79	0.30	13.30	1.35

TABLE S28. CPP principal components formation and concentration of dispersion.

	PC1		PC2		PC3	
	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$
<i>cc</i>	0.89	0.59	1.93	1.33	<b>21.22</b>	2.97
<i>s</i>	<b>11.71</b>	0.57	2.97	0.82	2.45	0.72
<i>s<sup>in</sup></i>	<b>11.68</b>	0.58	2.37	0.91	3.08	0.78
<i>s<sup>out</sup></i>	<b>11.49</b>	0.61	3.63	0.79	1.61	0.88
<i>k</i>	<b>11.93</b>	0.54	2.58	0.70	0.52	0.44
<i>k<sup>in</sup></i>	<b>11.93</b>	0.52	1.19	0.88	1.41	0.71
<i>k<sup>out</sup></i>	<b>11.57</b>	0.61	4.34	0.70	0.98	0.66
<i>bt</i>	<b>11.37</b>	0.55	2.44	0.84	1.37	0.77
<i>asy</i>	3.14	0.98	<b>18.52</b>	1.97	2.46	1.69
$\mu^{asy}$	3.32	0.99	<b>18.23</b>	2.01	2.80	1.82
$\sigma^{asy}$	4.91	0.59	2.44	1.47	<b>26.84</b>	3.06
<i>dis</i>	2.94	0.88	<b>18.50</b>	1.92	3.06	1.98
$\mu^{dis}$	2.55	0.89	<b>18.12</b>	1.85	1.57	1.32
$\sigma^{dis}$	0.57	0.33	2.74	1.63	<b>30.61</b>	2.66
$\lambda$	49.56	1.16	27.14	0.54	13.25	0.95

### SIII. FRACTION OF PARTICIPANTS IN EACH ERDÖS SECTOR ALONG THE TIMELINE

In this section, the figures present timelines with the fractions of participants in each Erdős sector with respect to each criterion defined in Section III C of the main document<sup>1</sup>. Step and window sizes of 50, 100, 250, 500, 1000 and 5000 messages are shown below, first for CPP, then for LAD list.

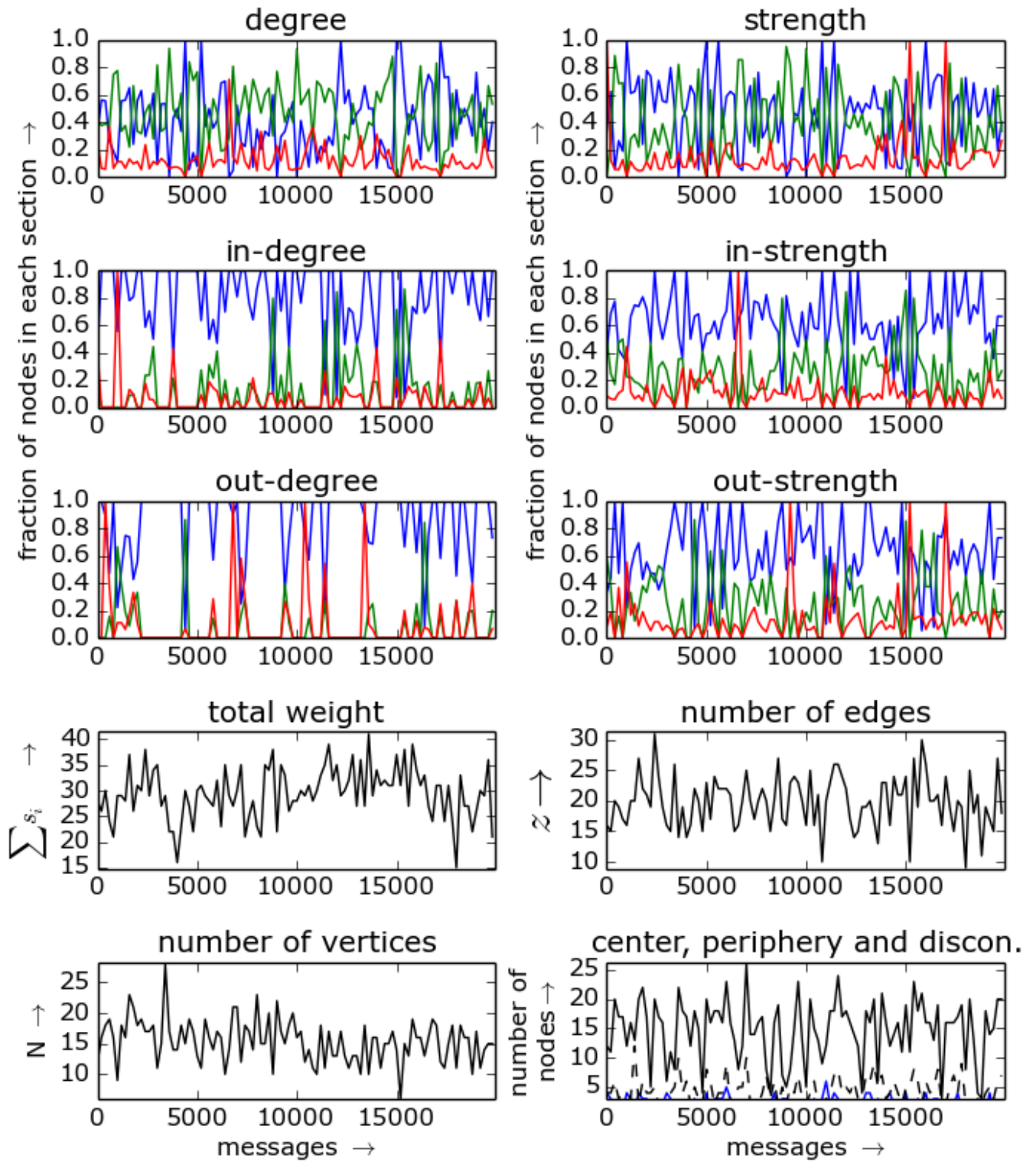
Each step size takes two pages of plot. On the first page, the criterion is based on each centrality metric observed separately: total, in and out degrees ( $k$ ,  $k^{in}$ ,  $k^{out}$ ) and strengths ( $s$ ,  $s^{in}$ ,  $s^{out}$ ). In the first six plots of every page, the colors have meanings as follows: **red for hubs**, **green for the intermediary** and **blue for the periphery**.

On the second page we show the fractions of participants with respect to each compound criterion for

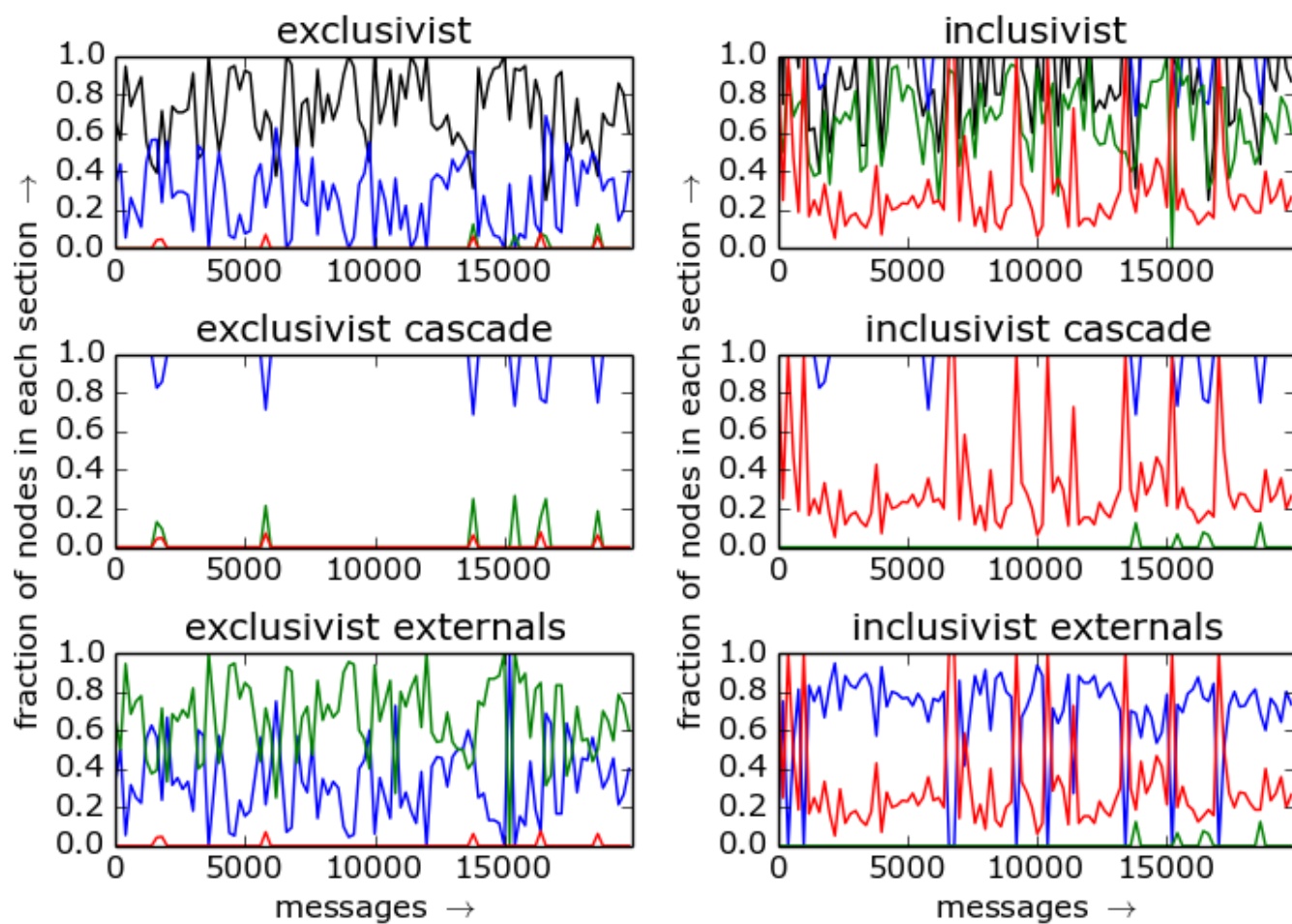
the Erdős sectioning. In the first plot, the fraction of vertices with unique classification is in **black**:  $\frac{\text{number of nodes uniquely classified}}{\text{number of nodes}}$ . On the second plot, **black** represents the fraction of classifications beyond the number of vertices:  $\frac{\text{number of classifications} - \text{number of nodes}}{\text{number of nodes}}$ .

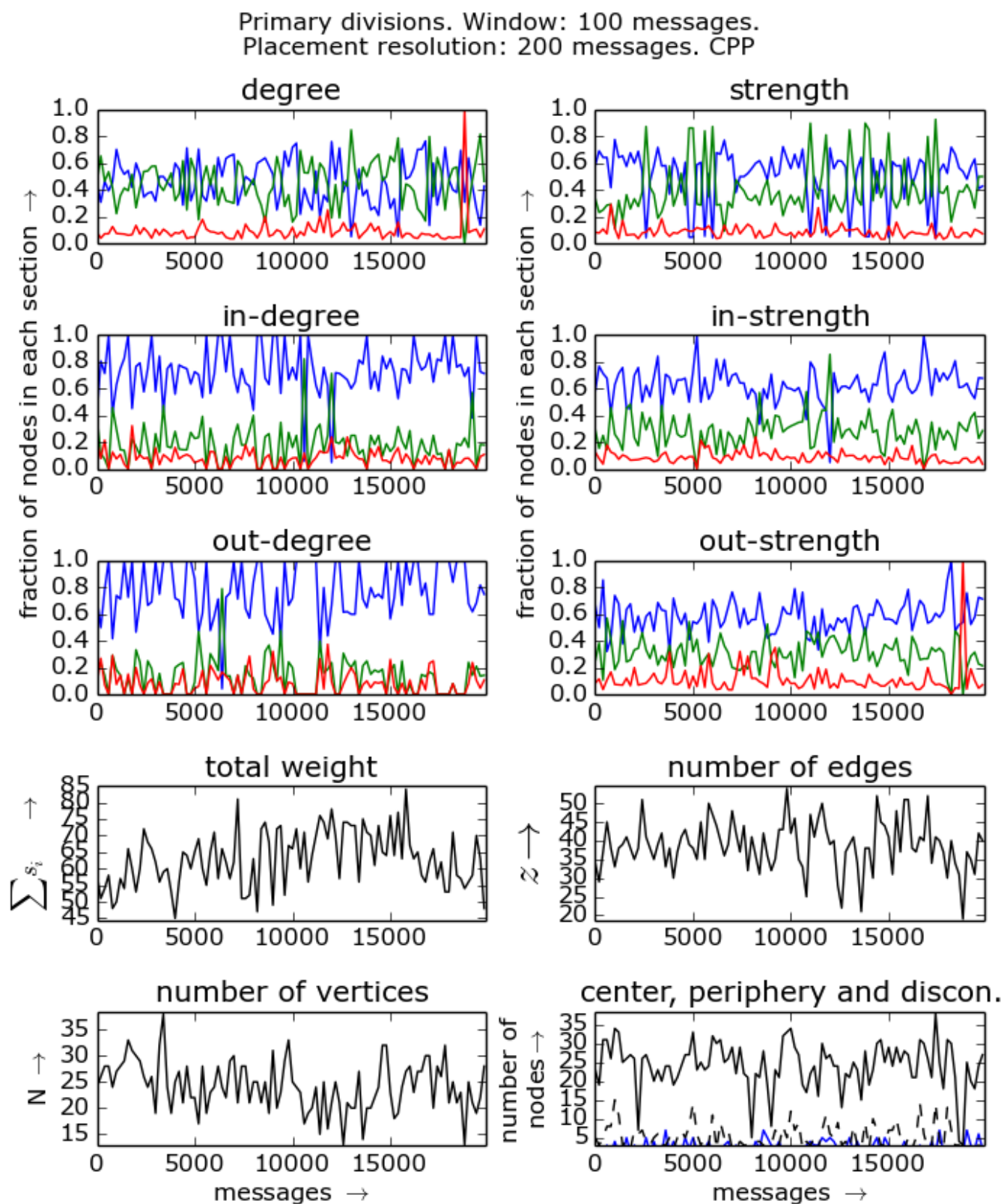
#### A. CPP list

Primary divisions. Window: 50 messages.  
Placement resolution: 200 messages. CPP



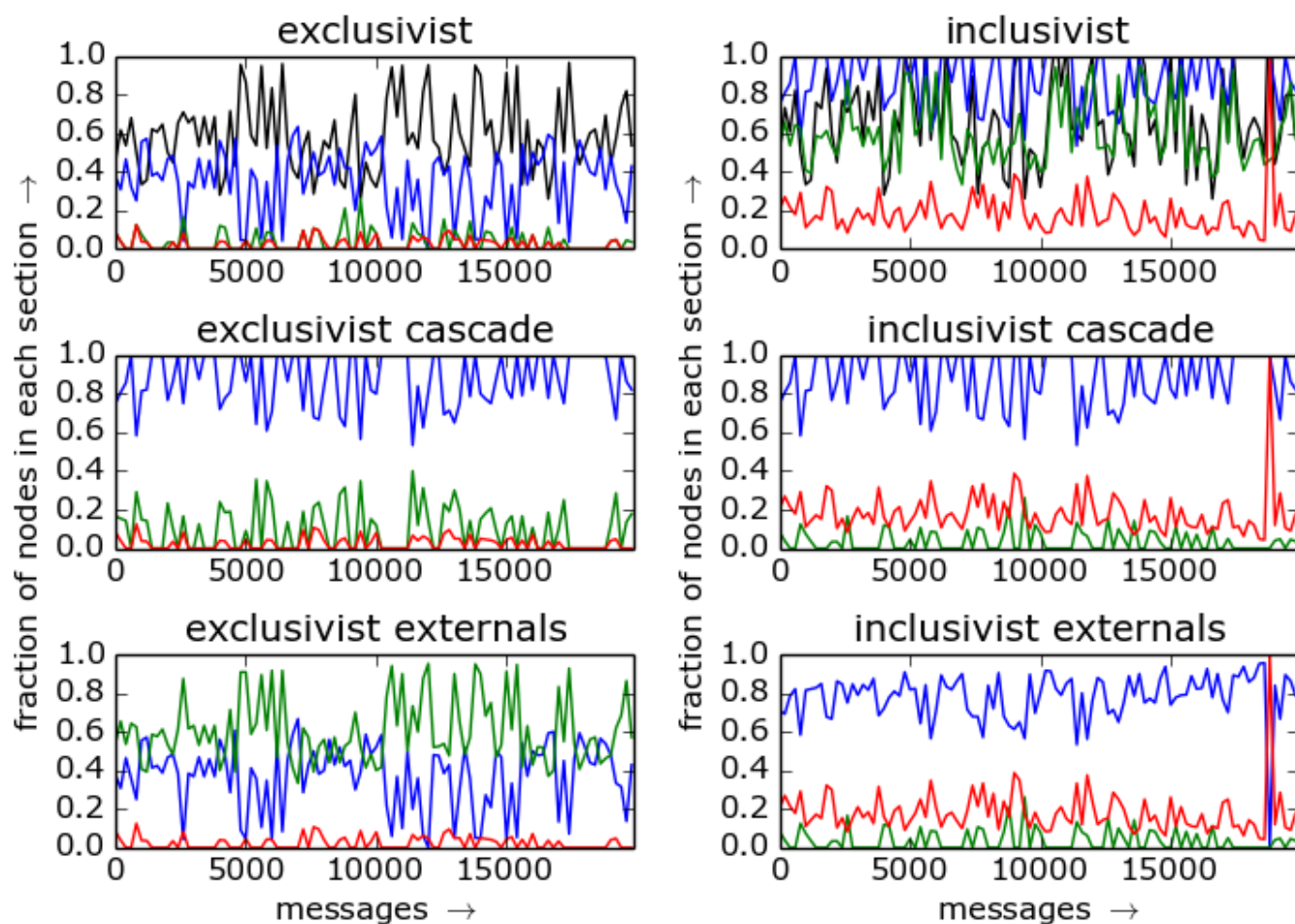
Compound divisions. Window: 50 messages.  
Placement resolution: 200 messages. CPP



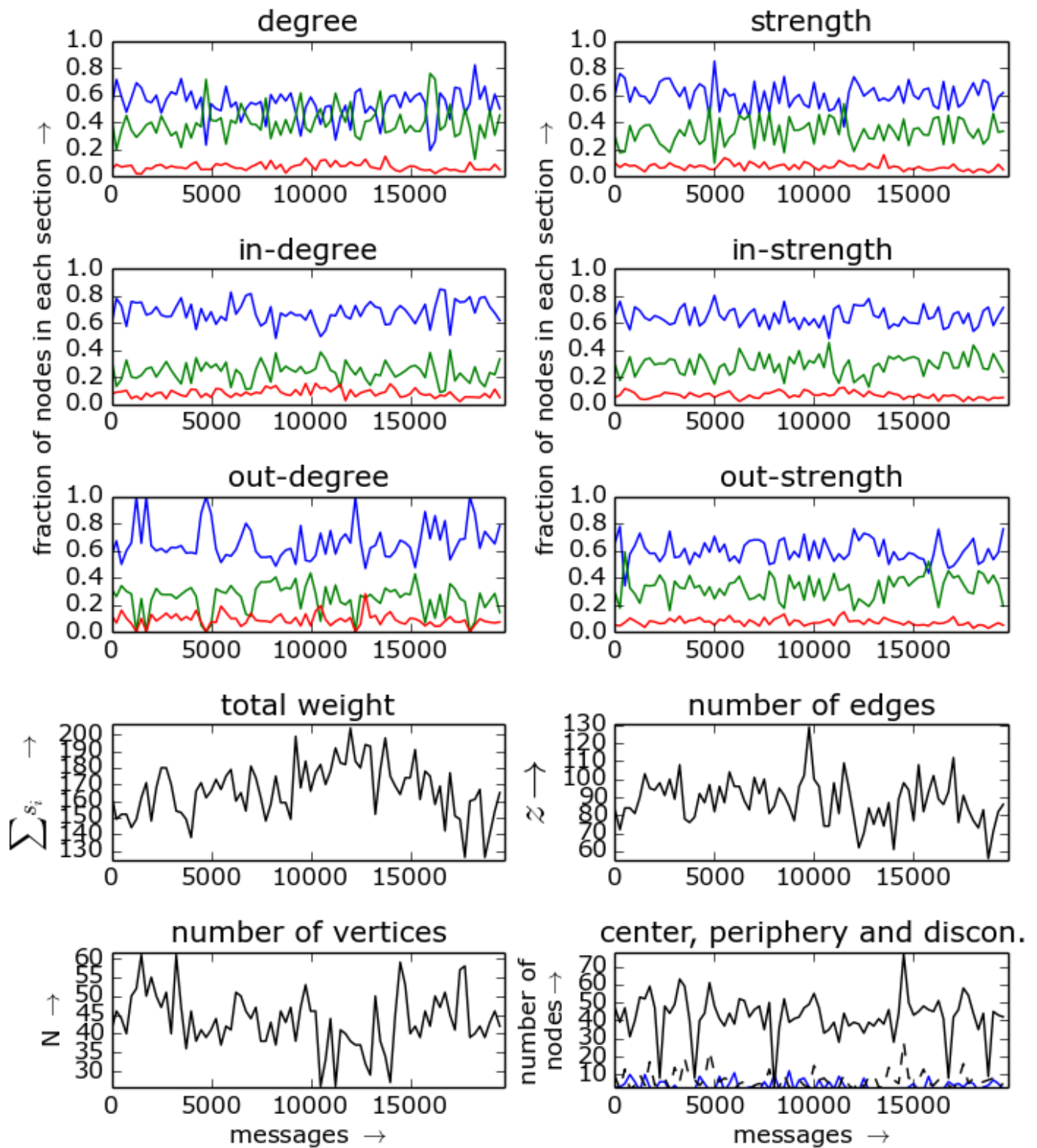




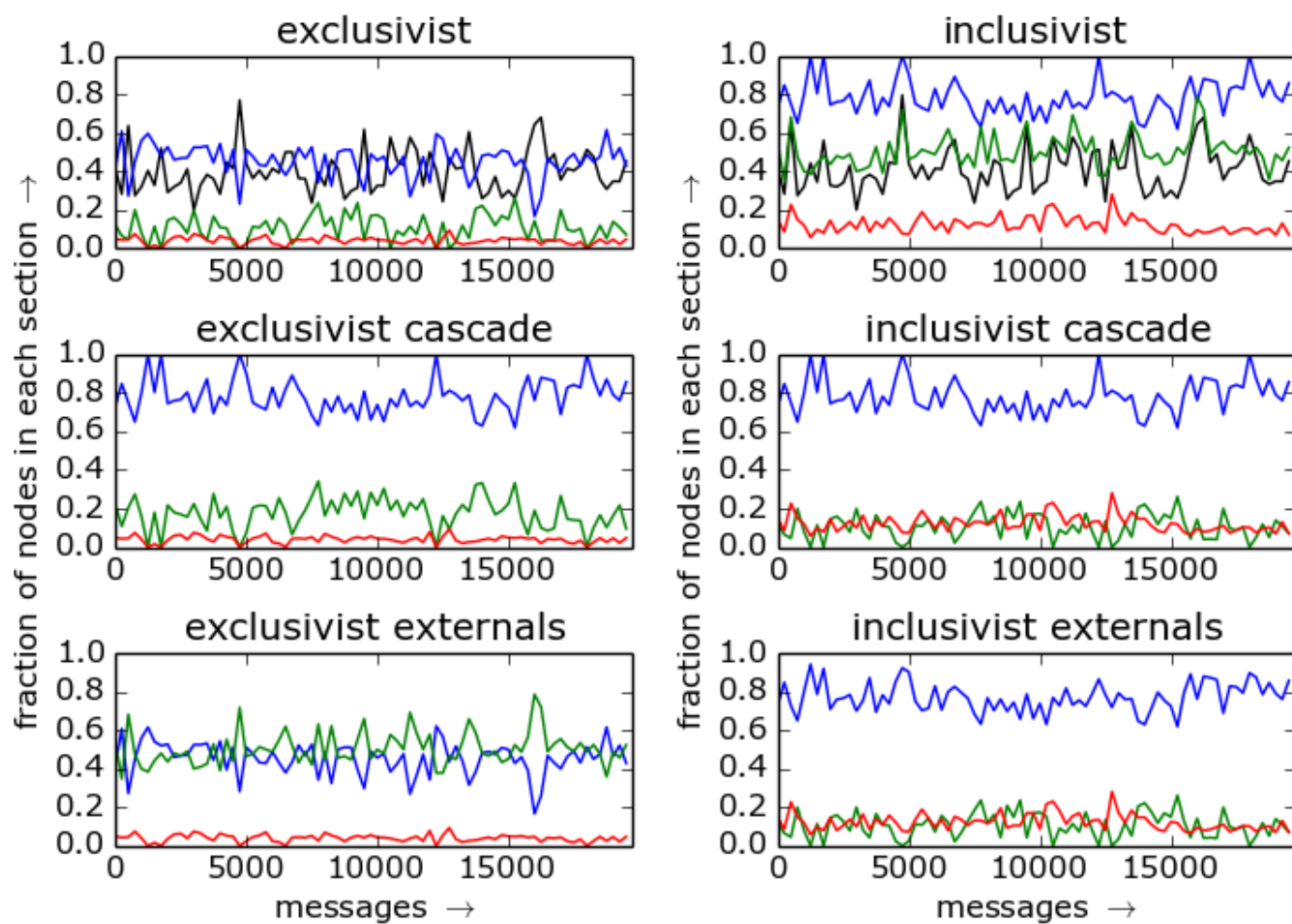
Compound divisions. Window: 100 messages.  
Placement resolution: 200 messages. CPP



Primary divisions. Window: 250 messages.  
Placement resolution: 250 messages. CPP

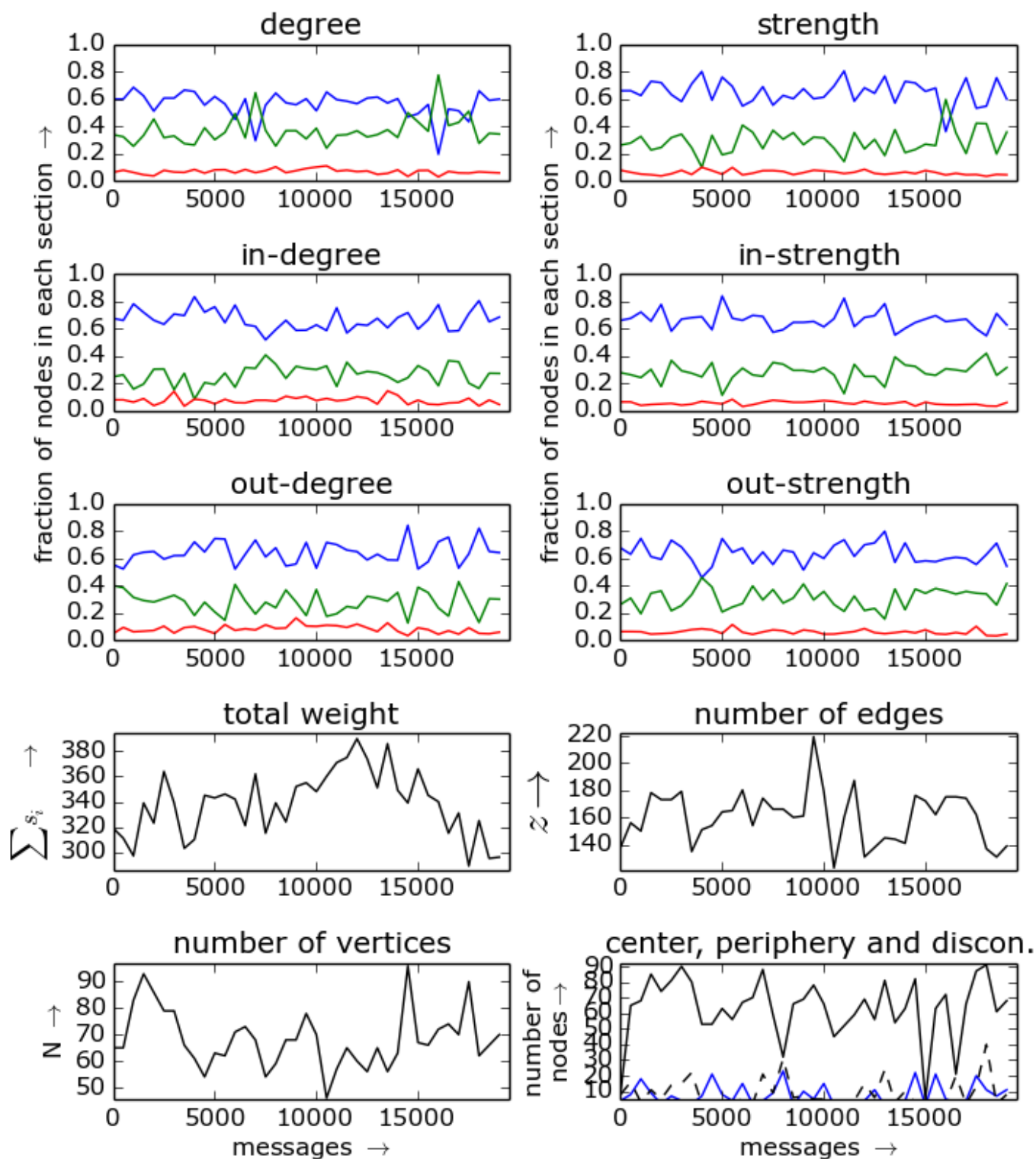


Compound divisions. Window: 250 messages.  
Placement resolution: 250 messages. CPP

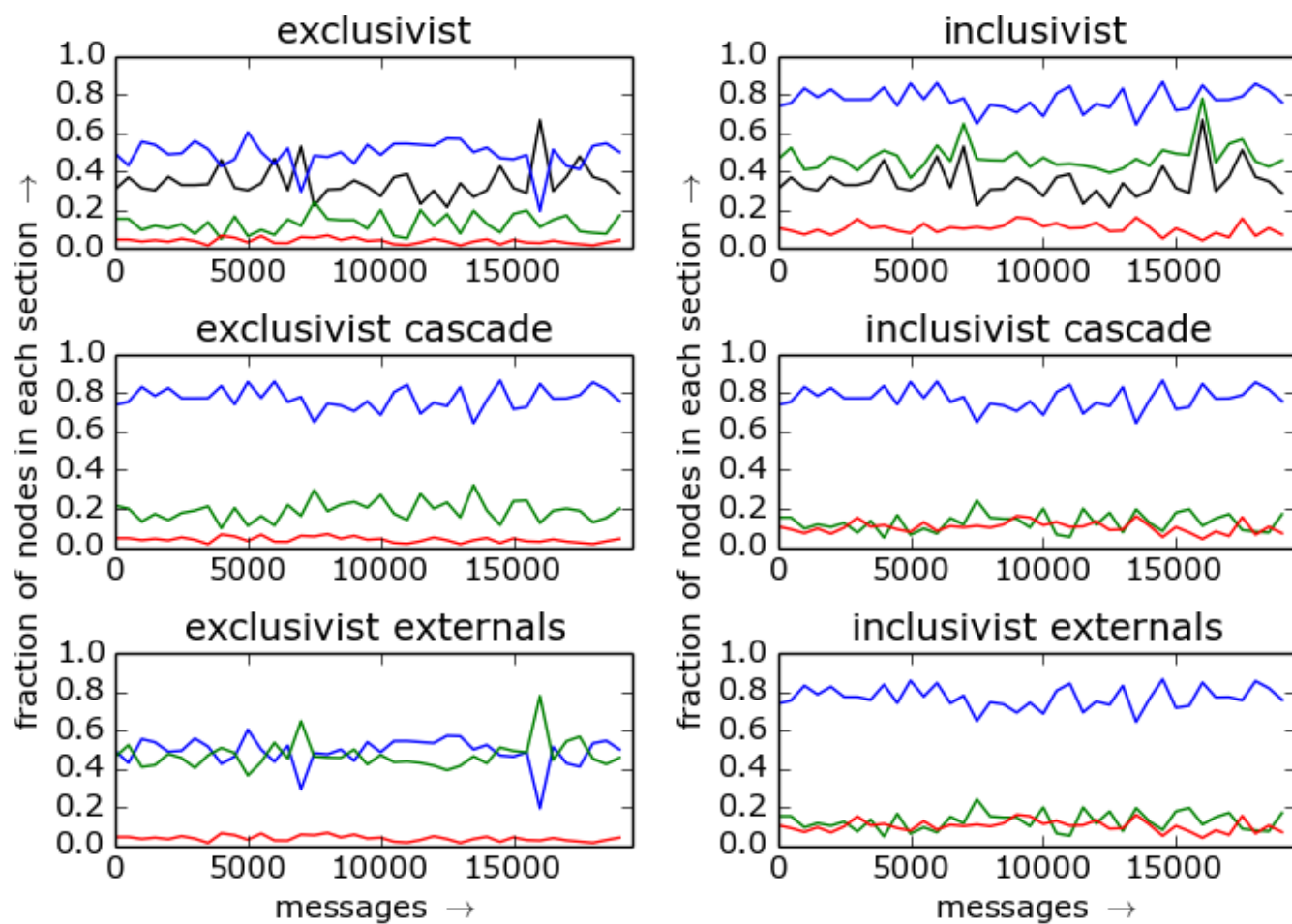




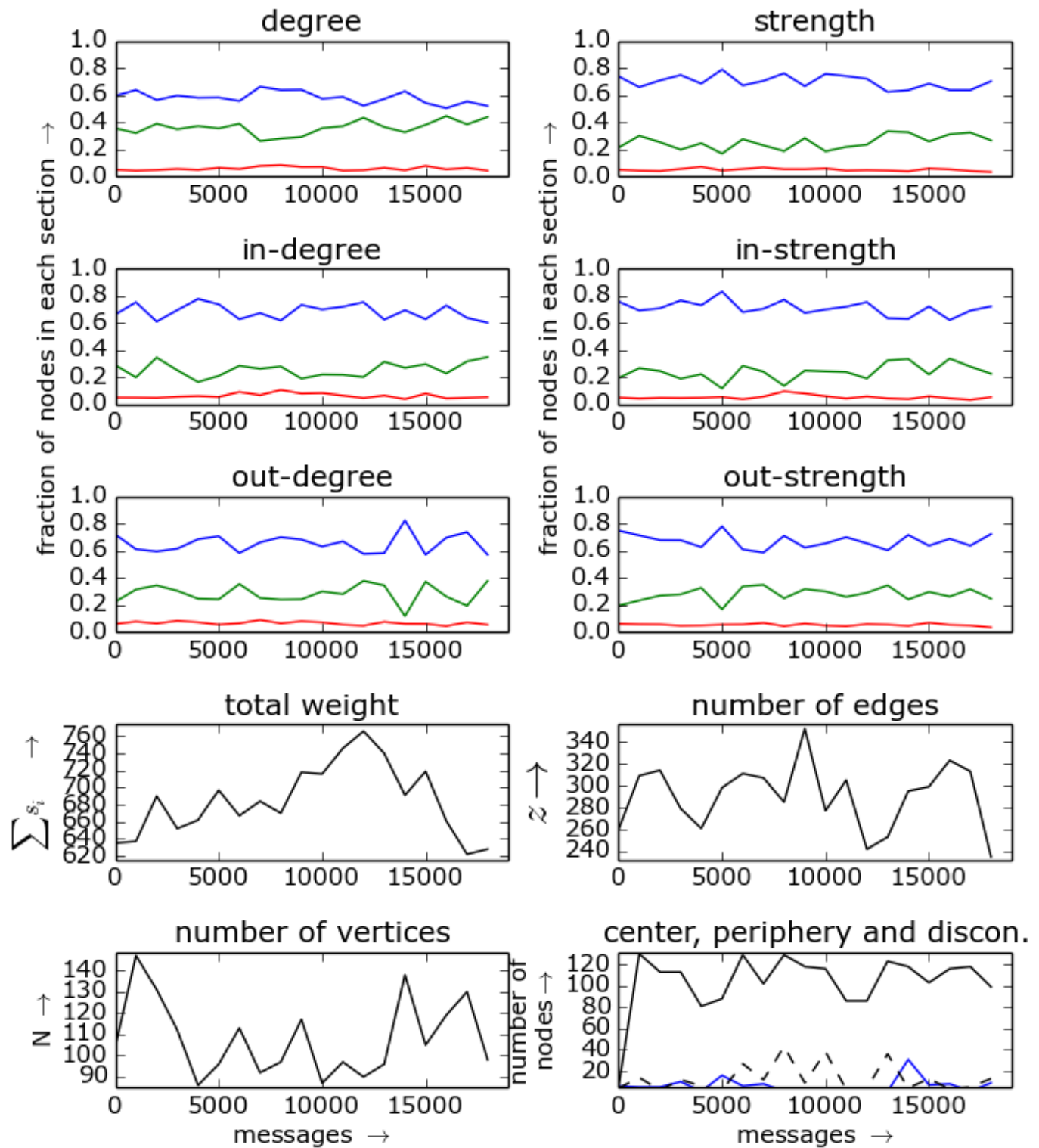
Primary divisions. Window: 500 messages.  
Placement resolution: 500 messages. CPP



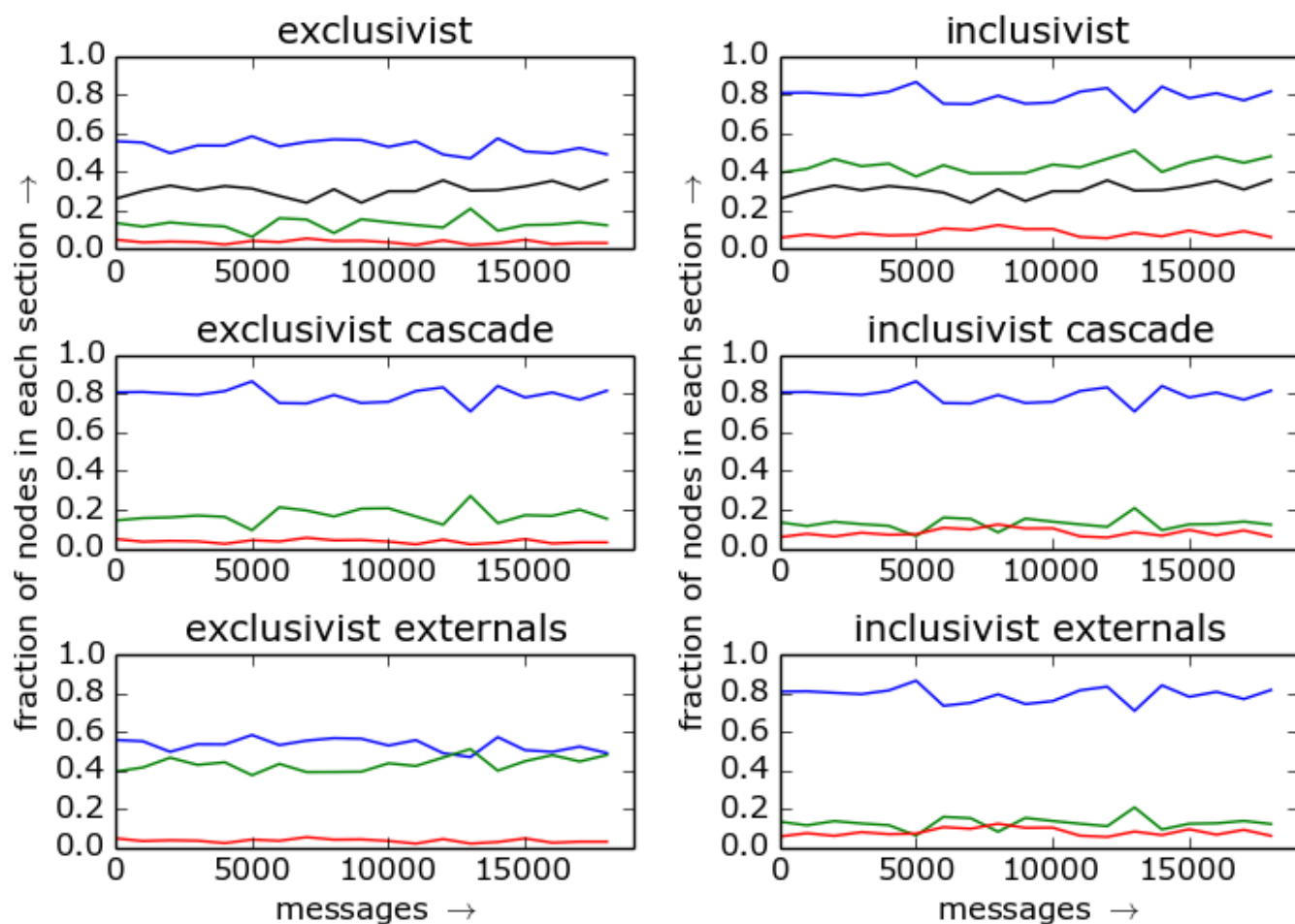
Compound divisions. Window: 500 messages.  
Placement resolution: 500 messages. CPP



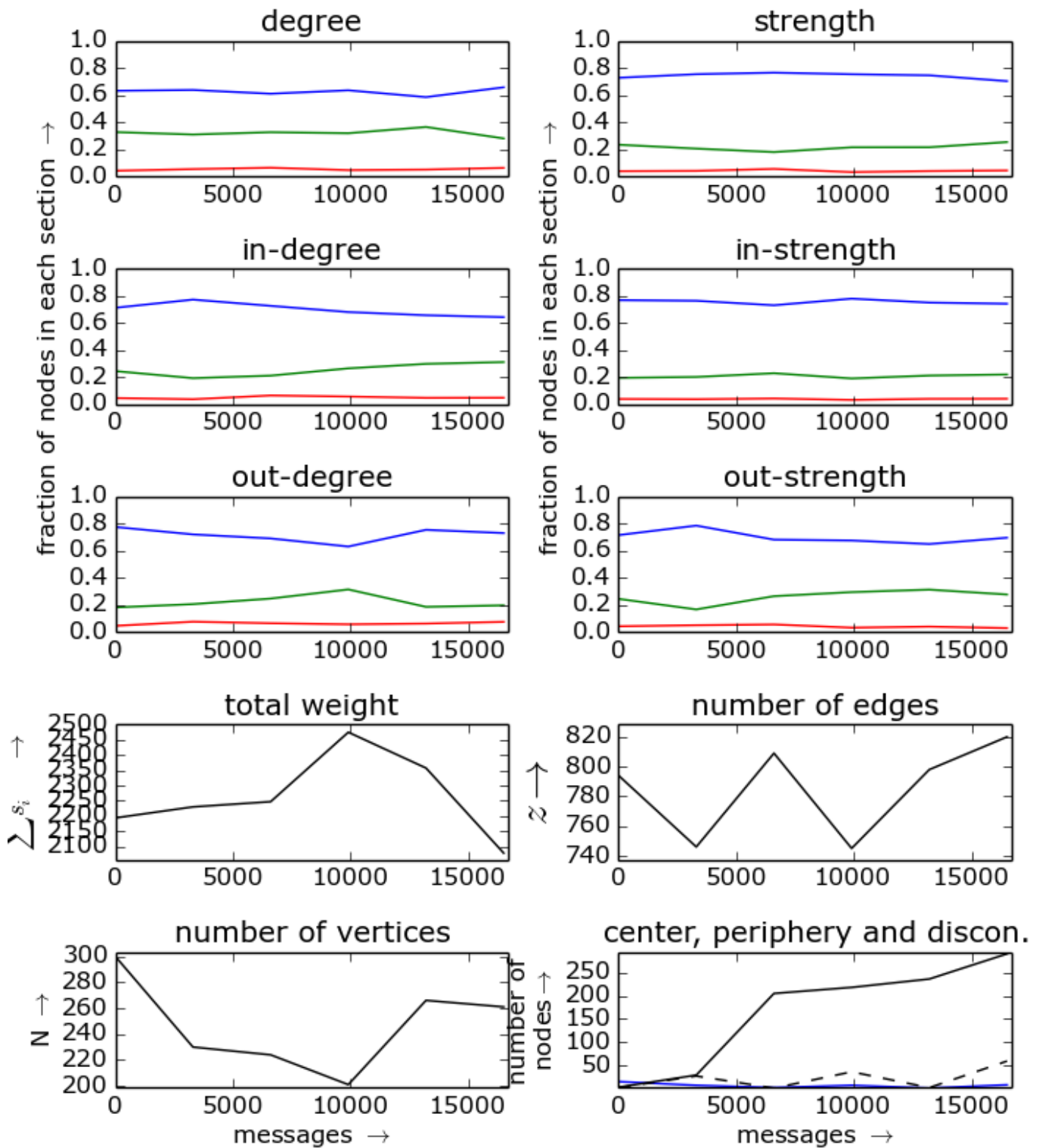
Primary divisions. Window: 1000 messages.  
Placement resolution: 1000 messages. CPP



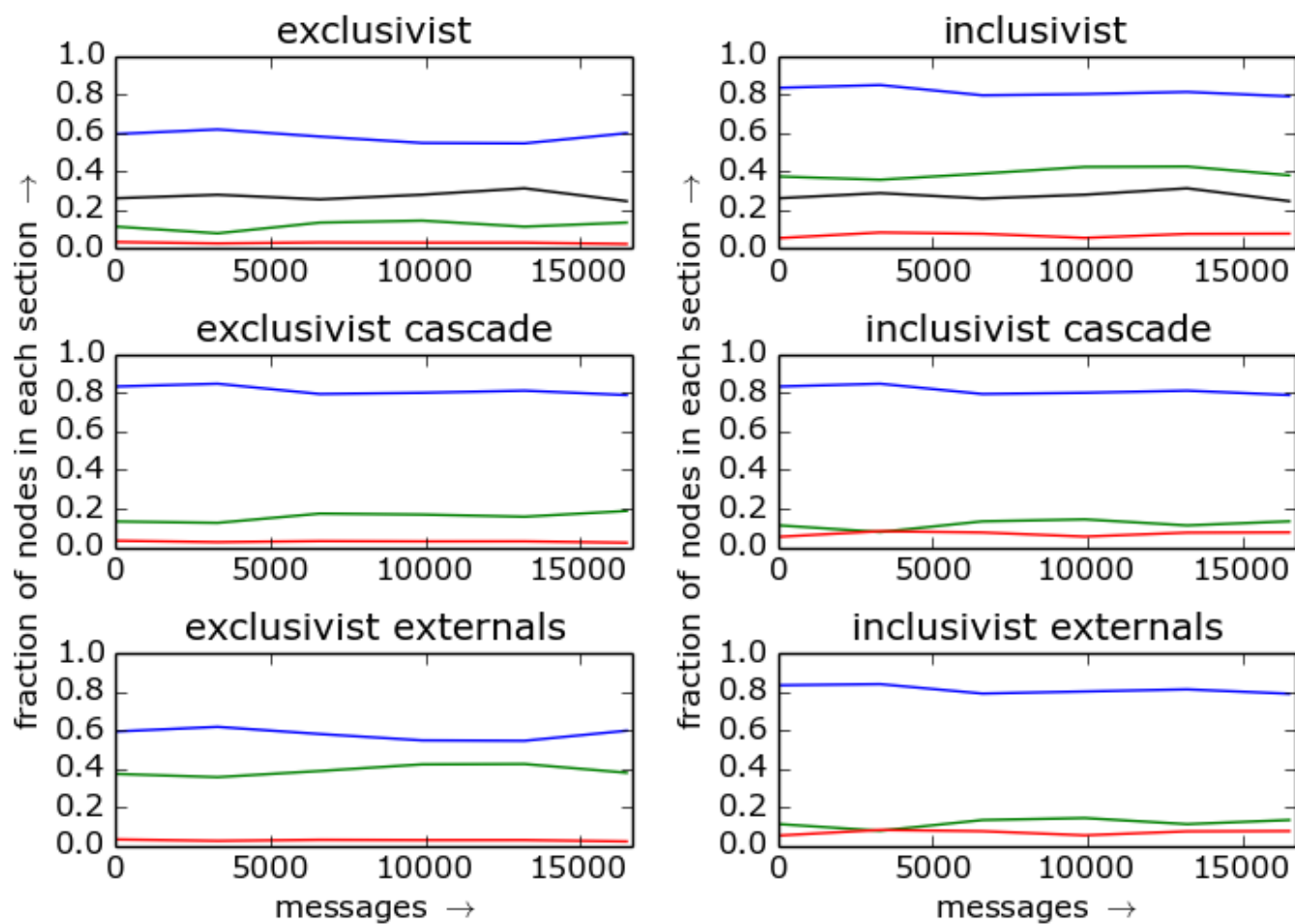
Compound divisions. Window: 1000 messages.  
Placement resolution: 1000 messages. CPP



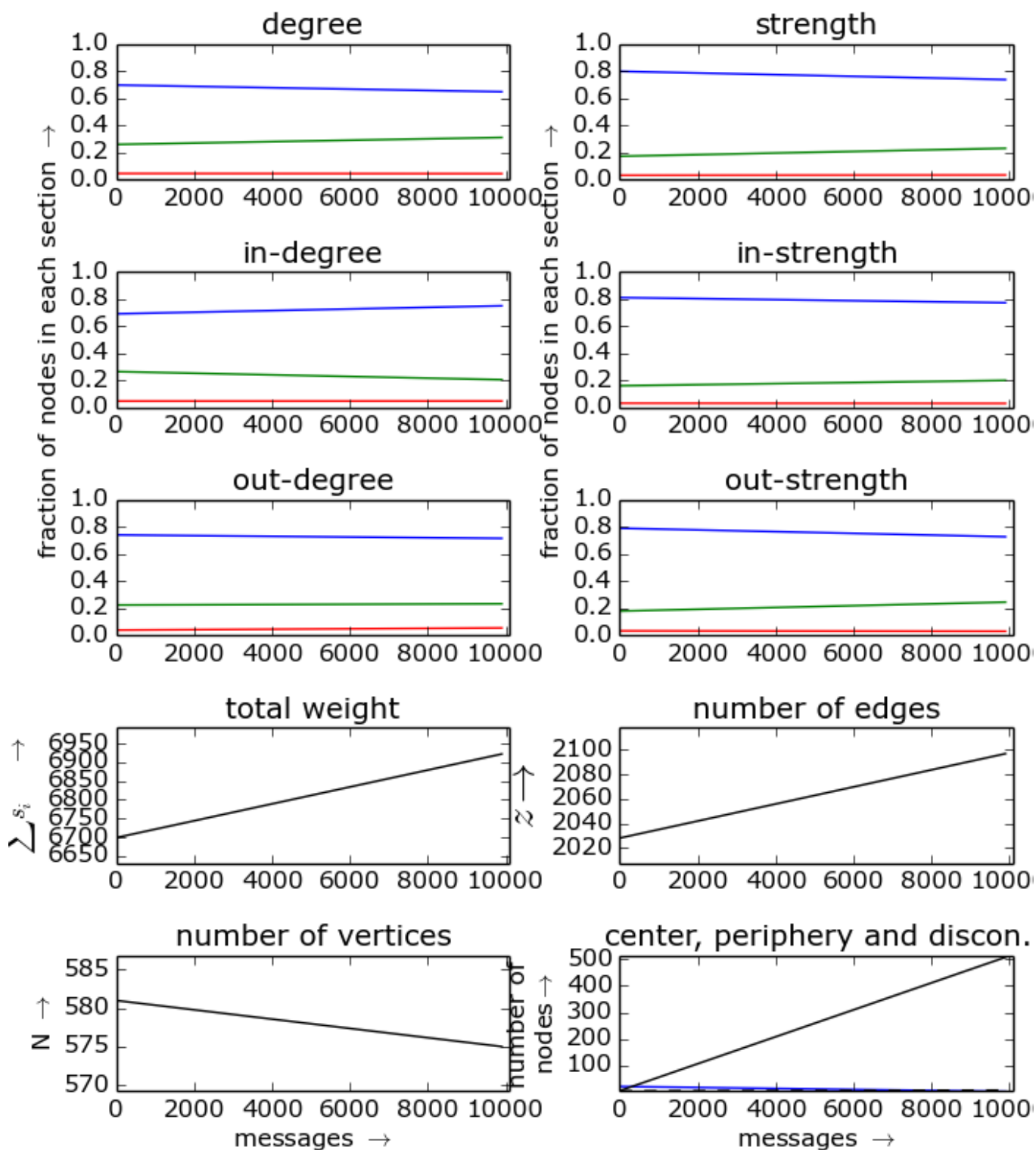
Primary divisions. Window: 3300 messages.  
Placement resolution: 3300 messages. CPP



Compound divisions. Window: 3300 messages.  
Placement resolution: 3300 messages. CPP

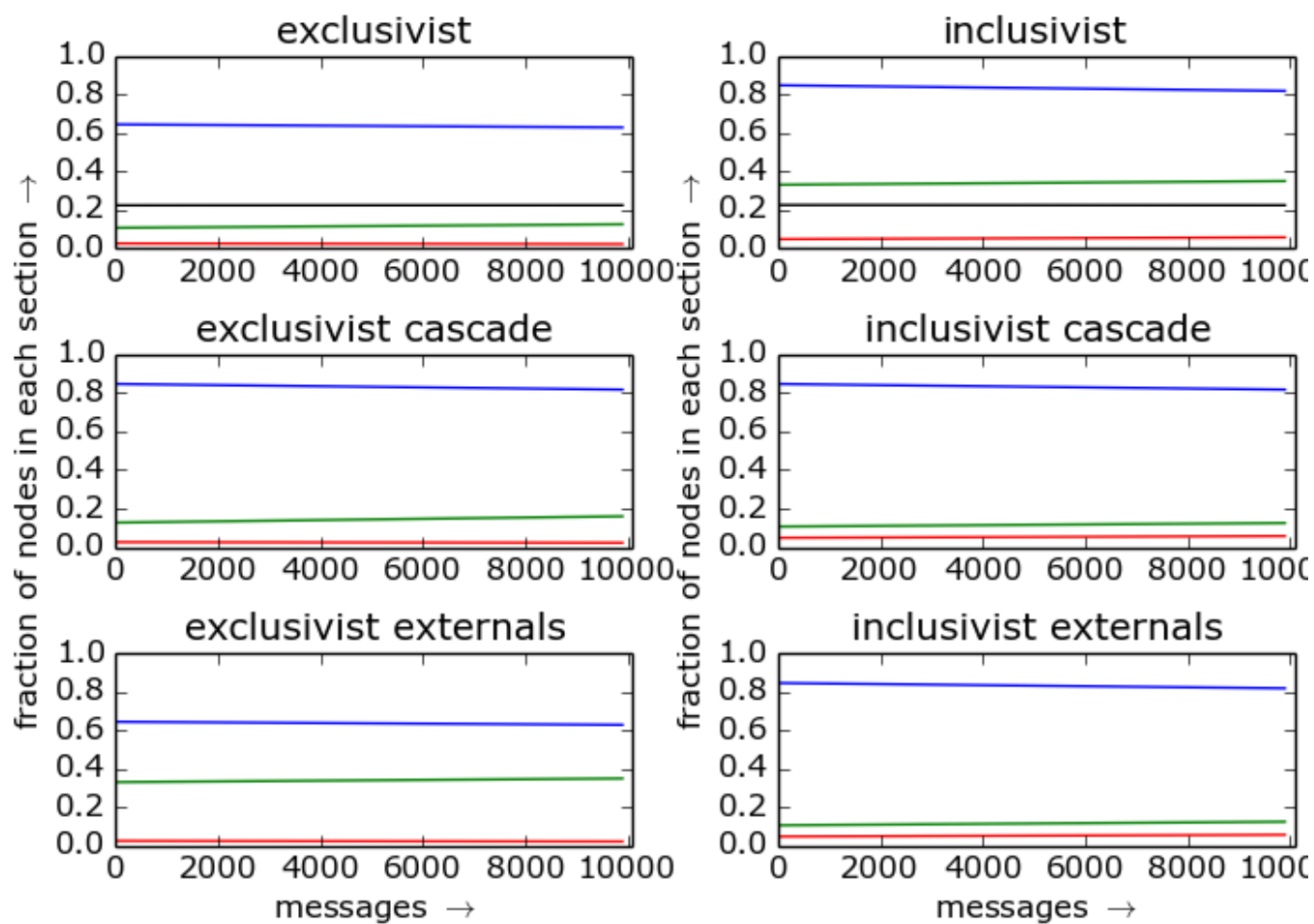


Primary divisions. Window: 9900 messages.  
Placement resolution: 9900 messages. CPP





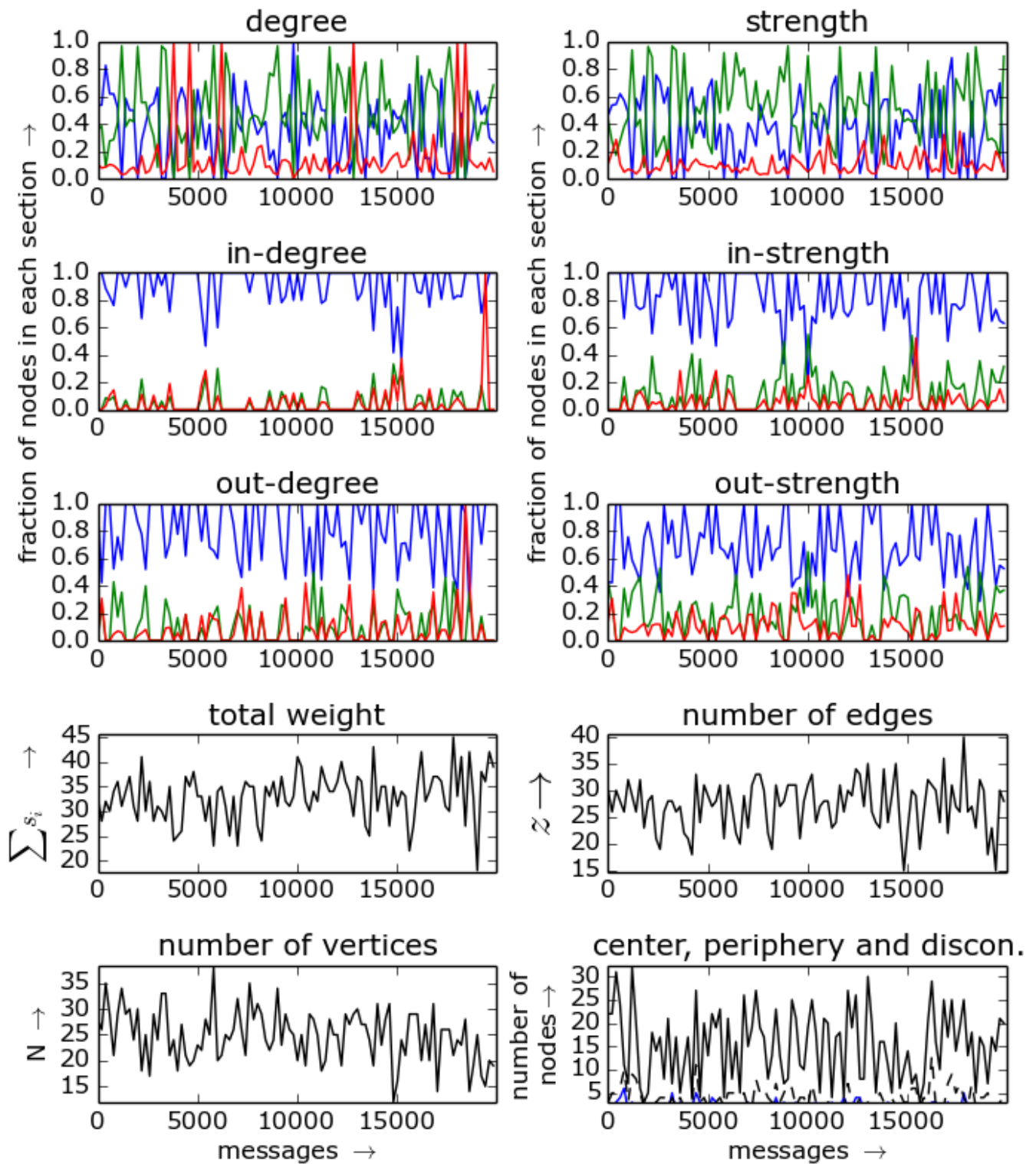
Compound divisions. Window: 9900 messages.  
Placement resolution: 9900 messages. CPP



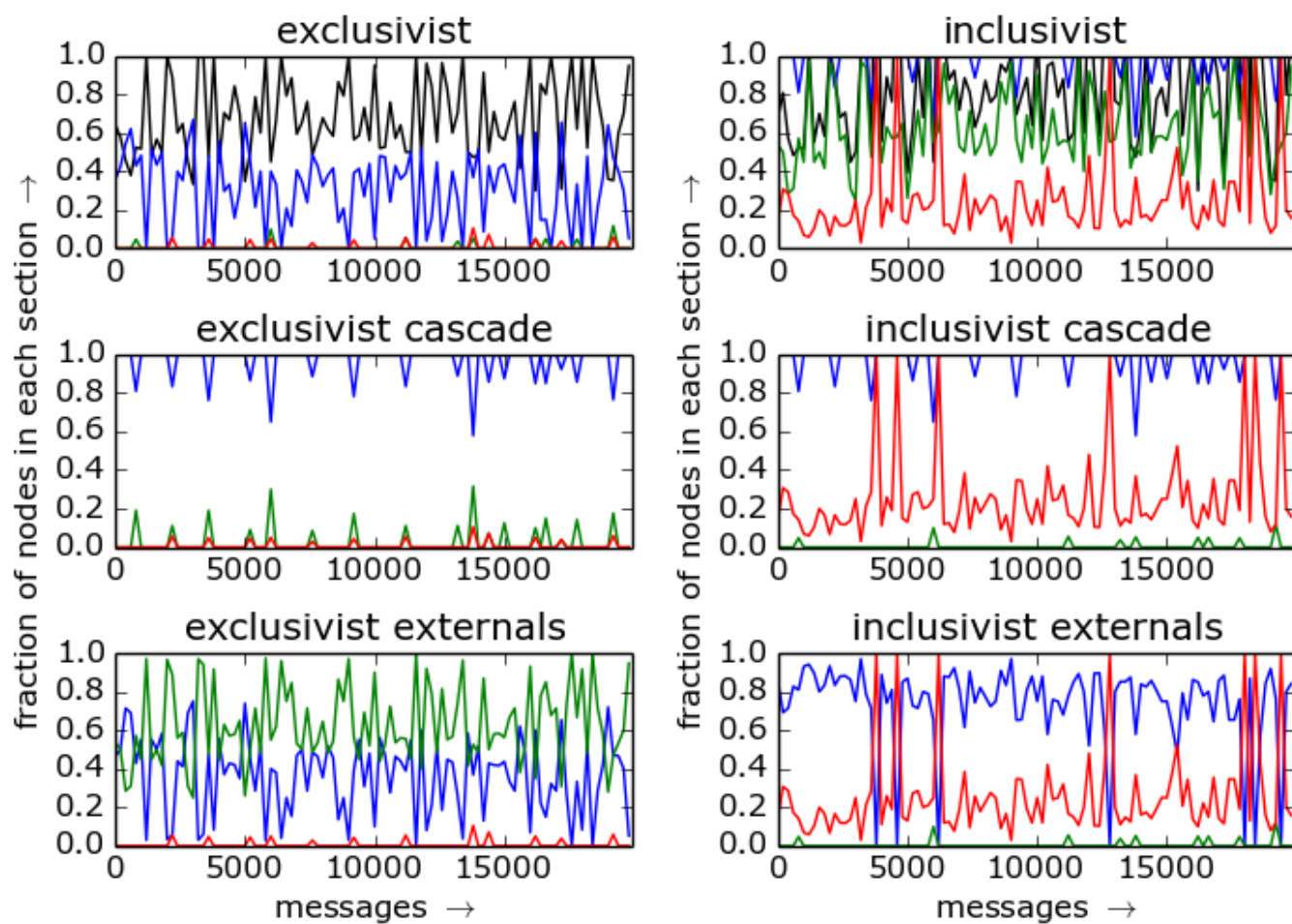


**B. LAD list**

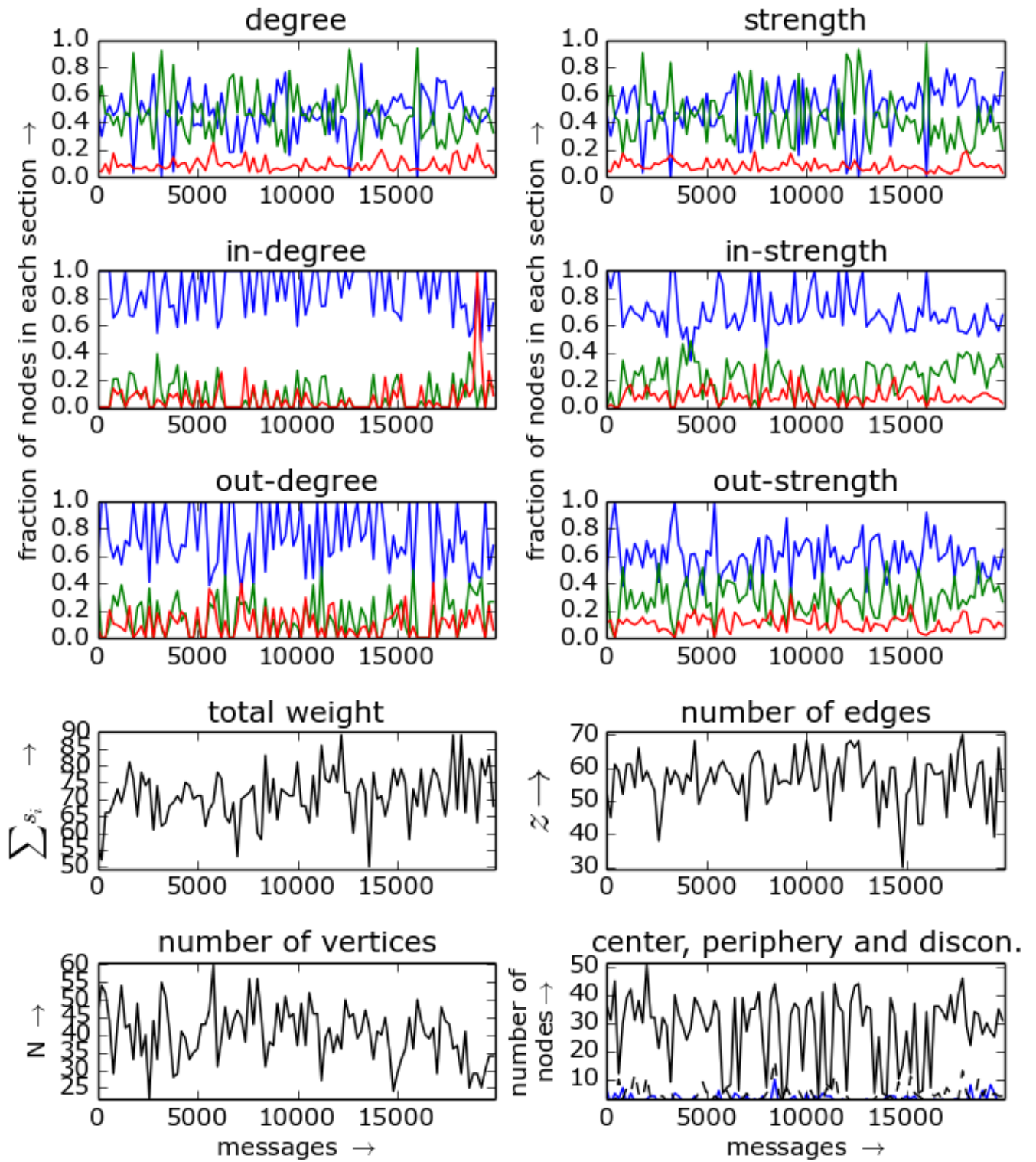
Primary divisions. Window: 50 messages.  
Placement resolution: 200 messages. LAD



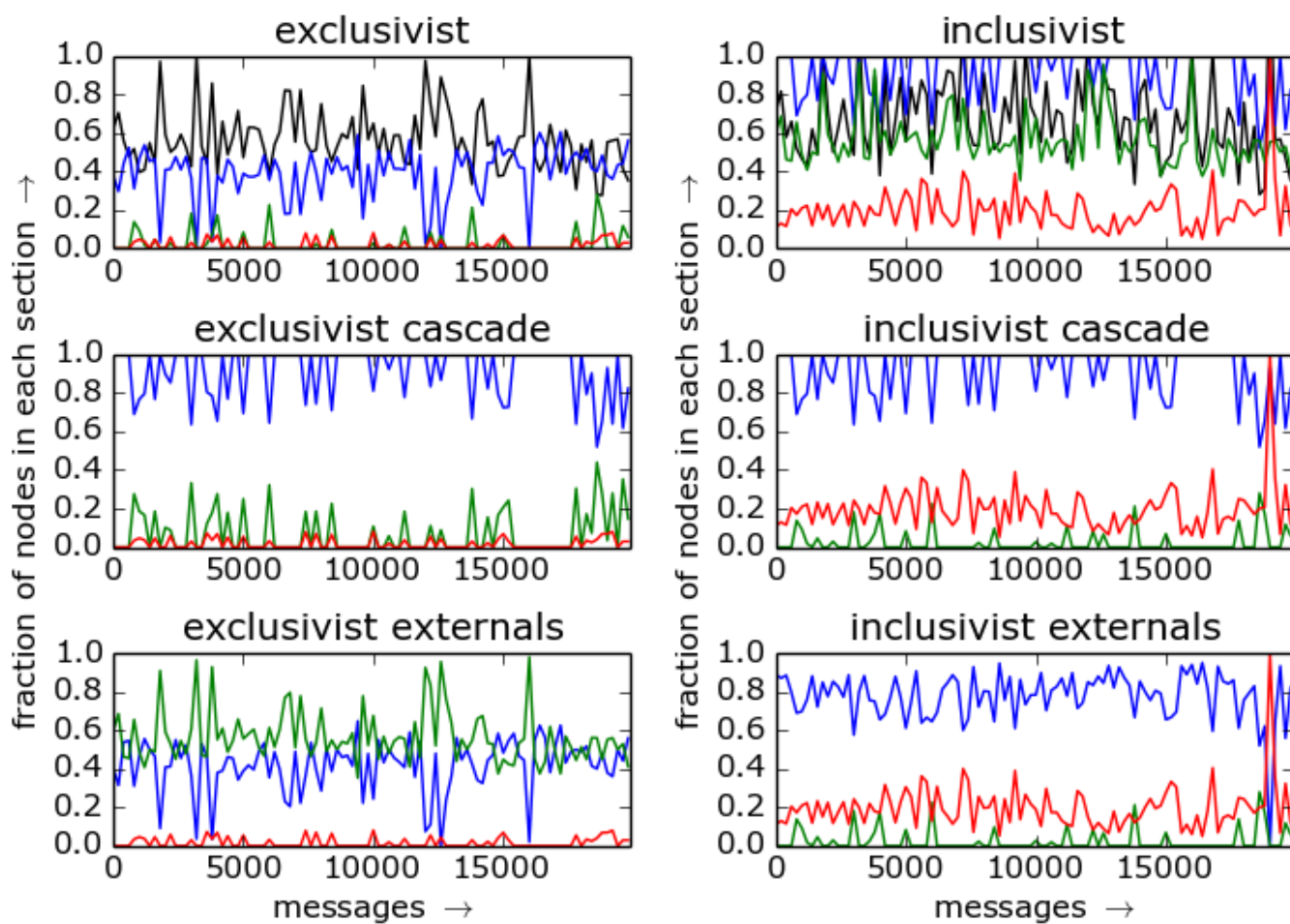
Compound divisions. Window: 50 messages.  
Placement resolution: 200 messages. LAD



Primary divisions. Window: 100 messages.  
Placement resolution: 200 messages. LAD

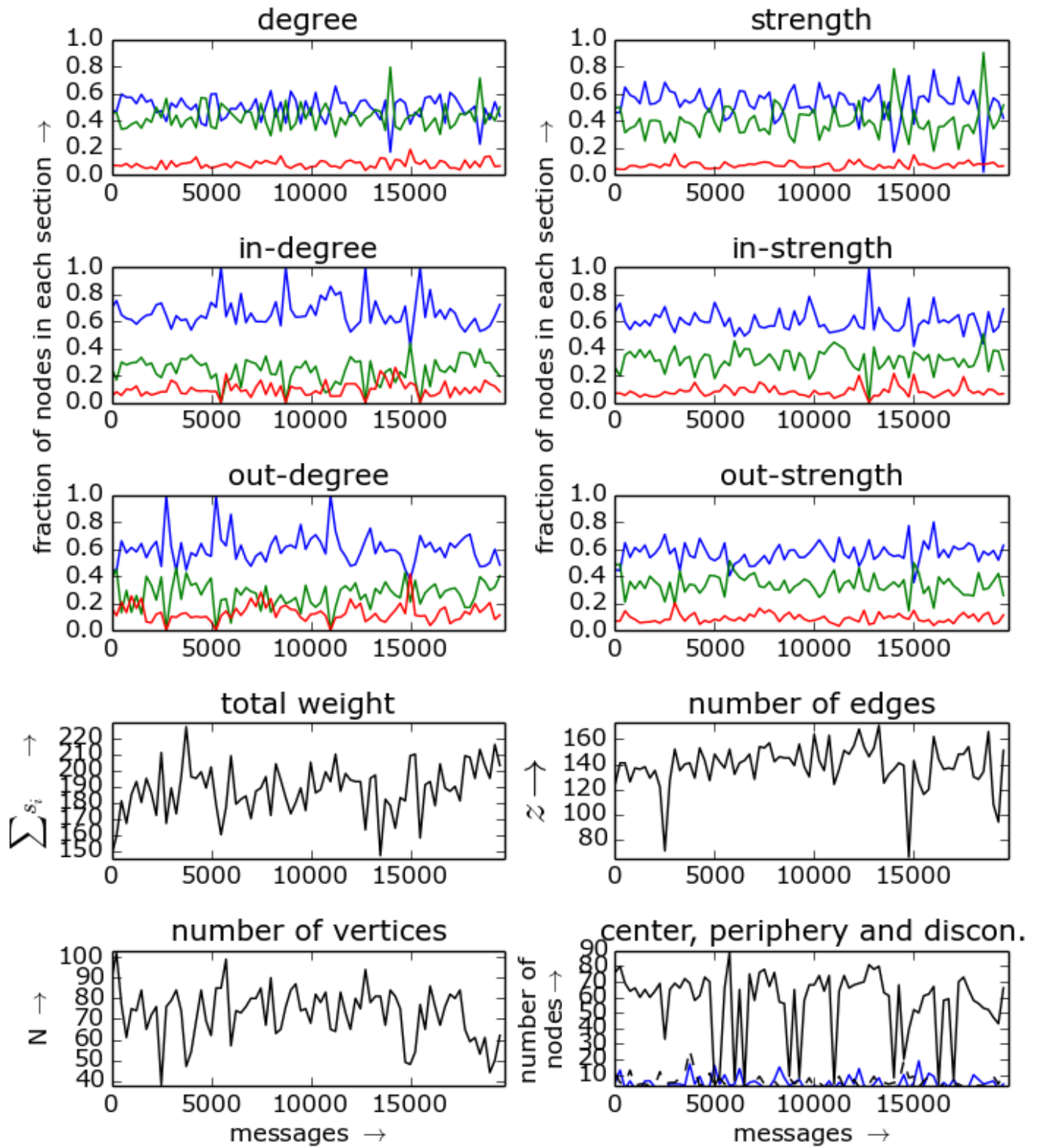


Compound divisions. Window: 100 messages.  
Placement resolution: 200 messages. LAD

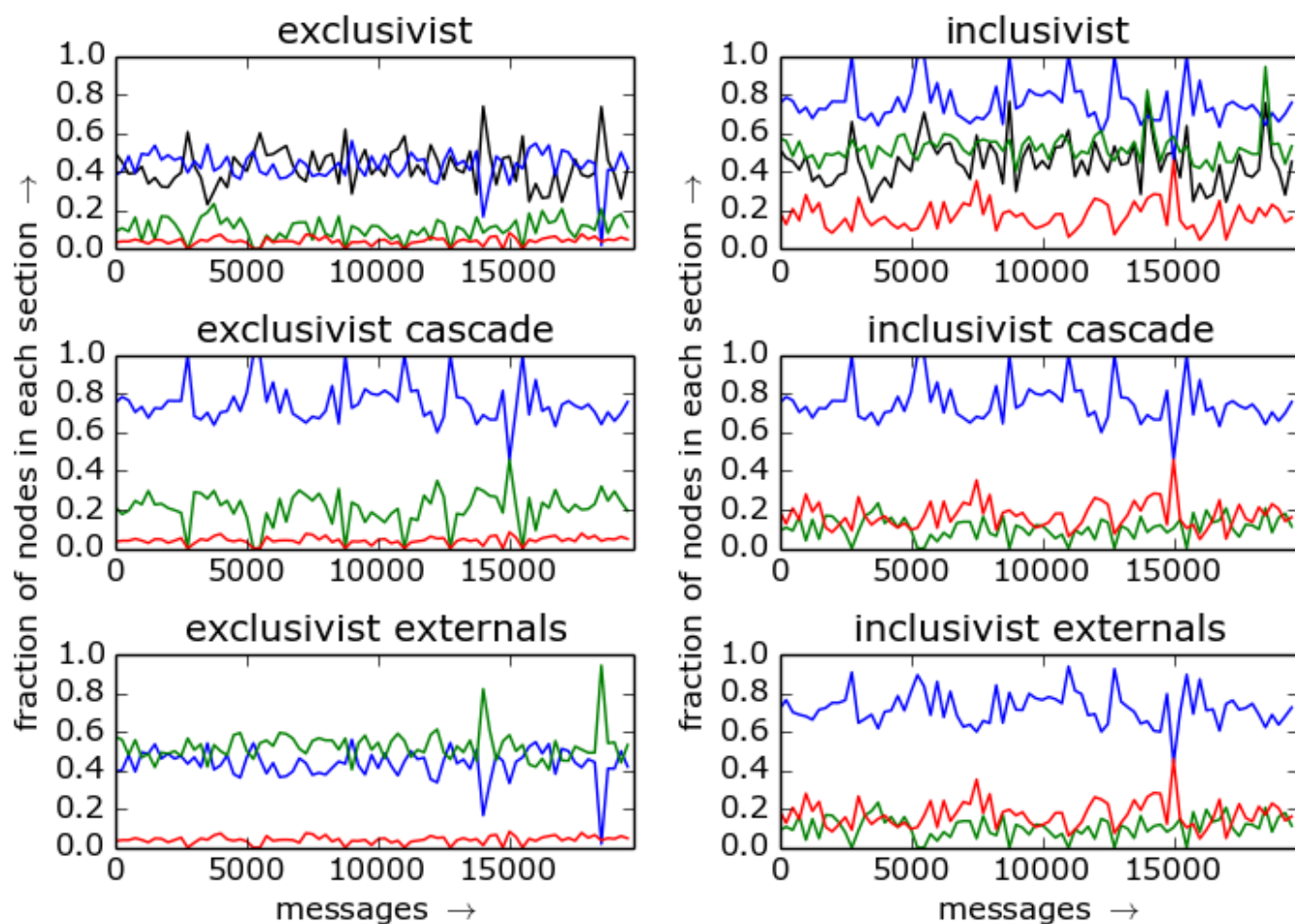




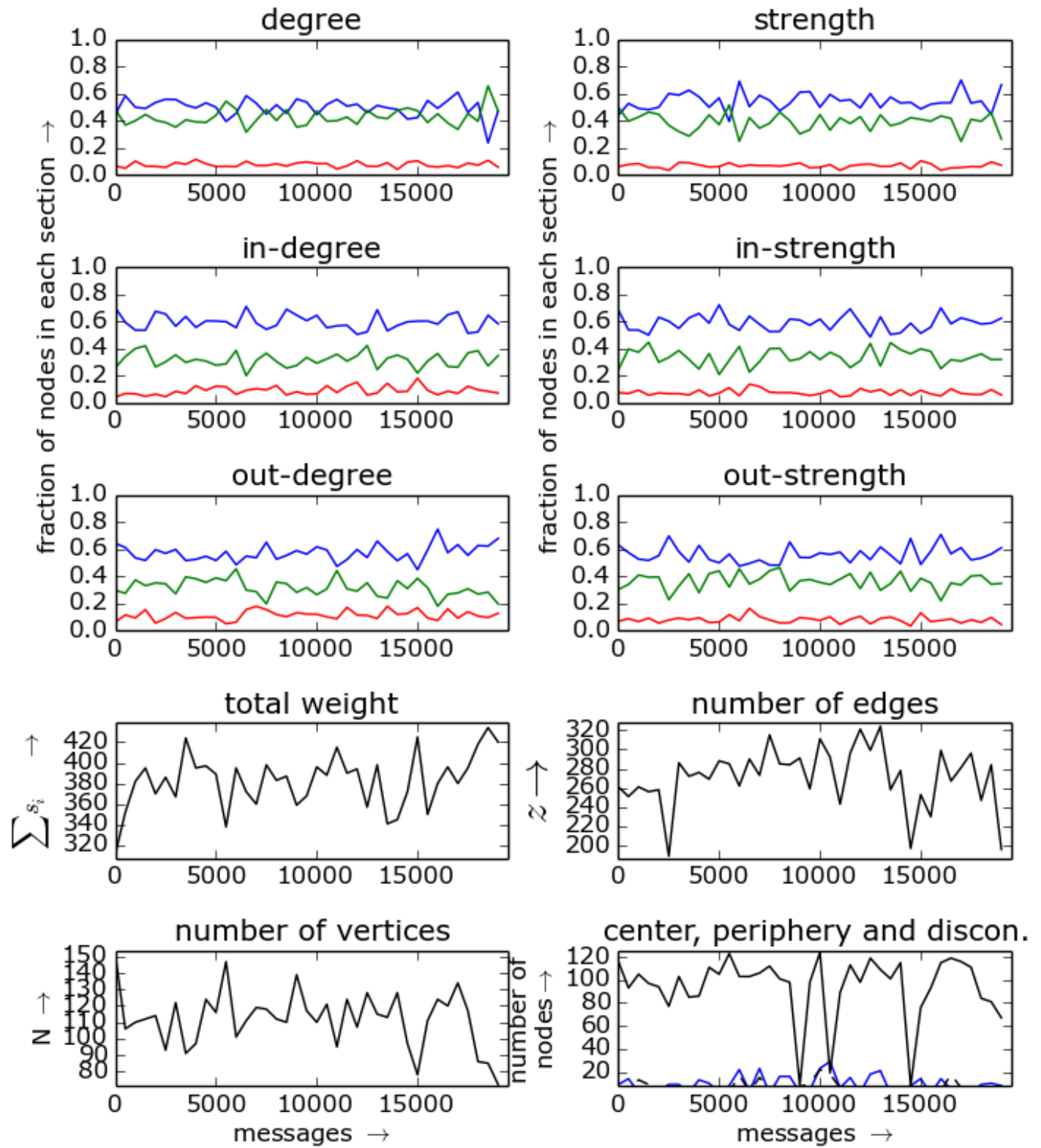
Primary divisions. Window: 250 messages.  
Placement resolution: 250 messages. LAD



Compound divisions. Window: 250 messages.  
Placement resolution: 250 messages. LAD

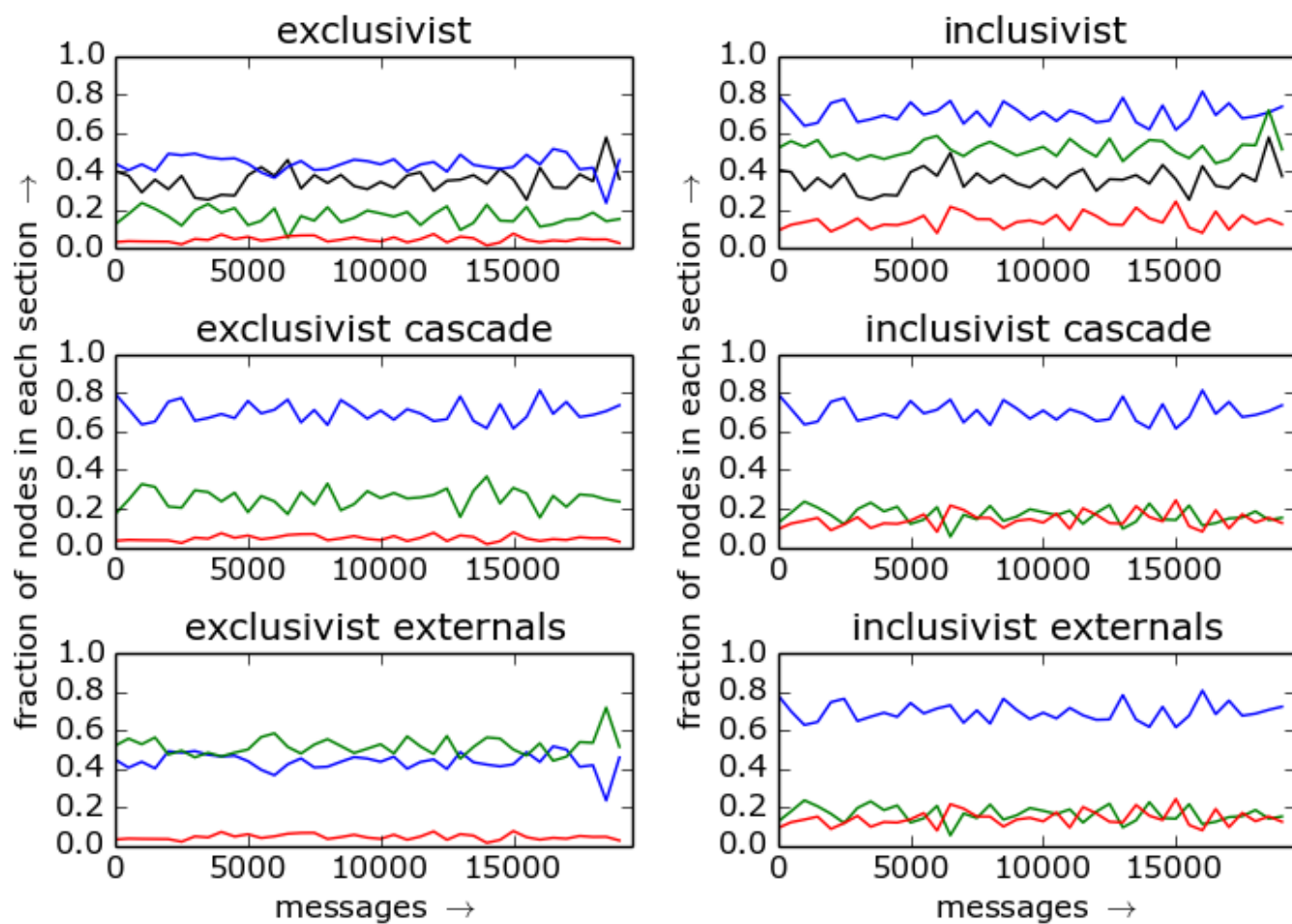


Primary divisions. Window: 500 messages.  
Placement resolution: 500 messages. LAD

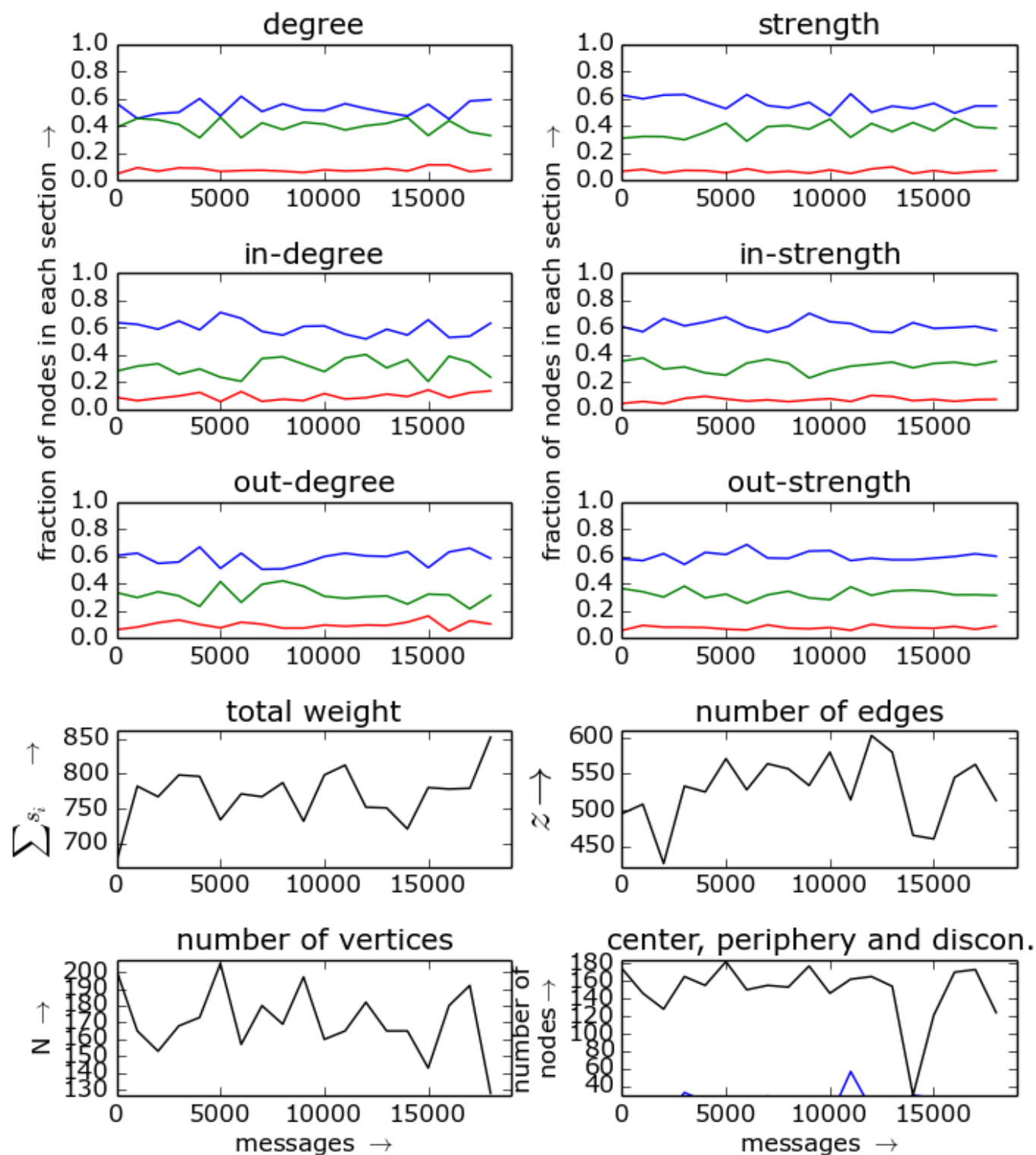




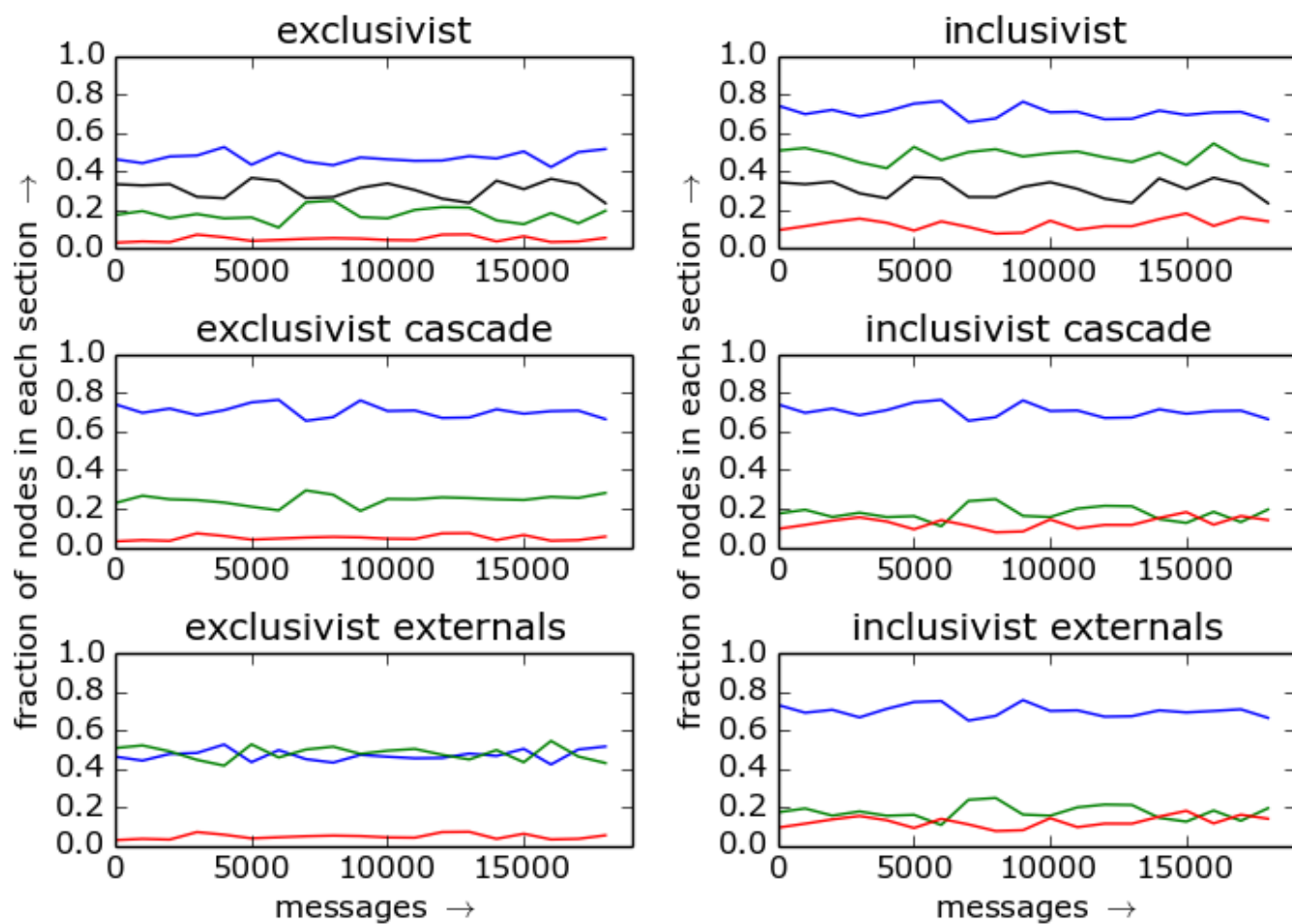
Compound divisions. Window: 500 messages.  
Placement resolution: 500 messages. LAD



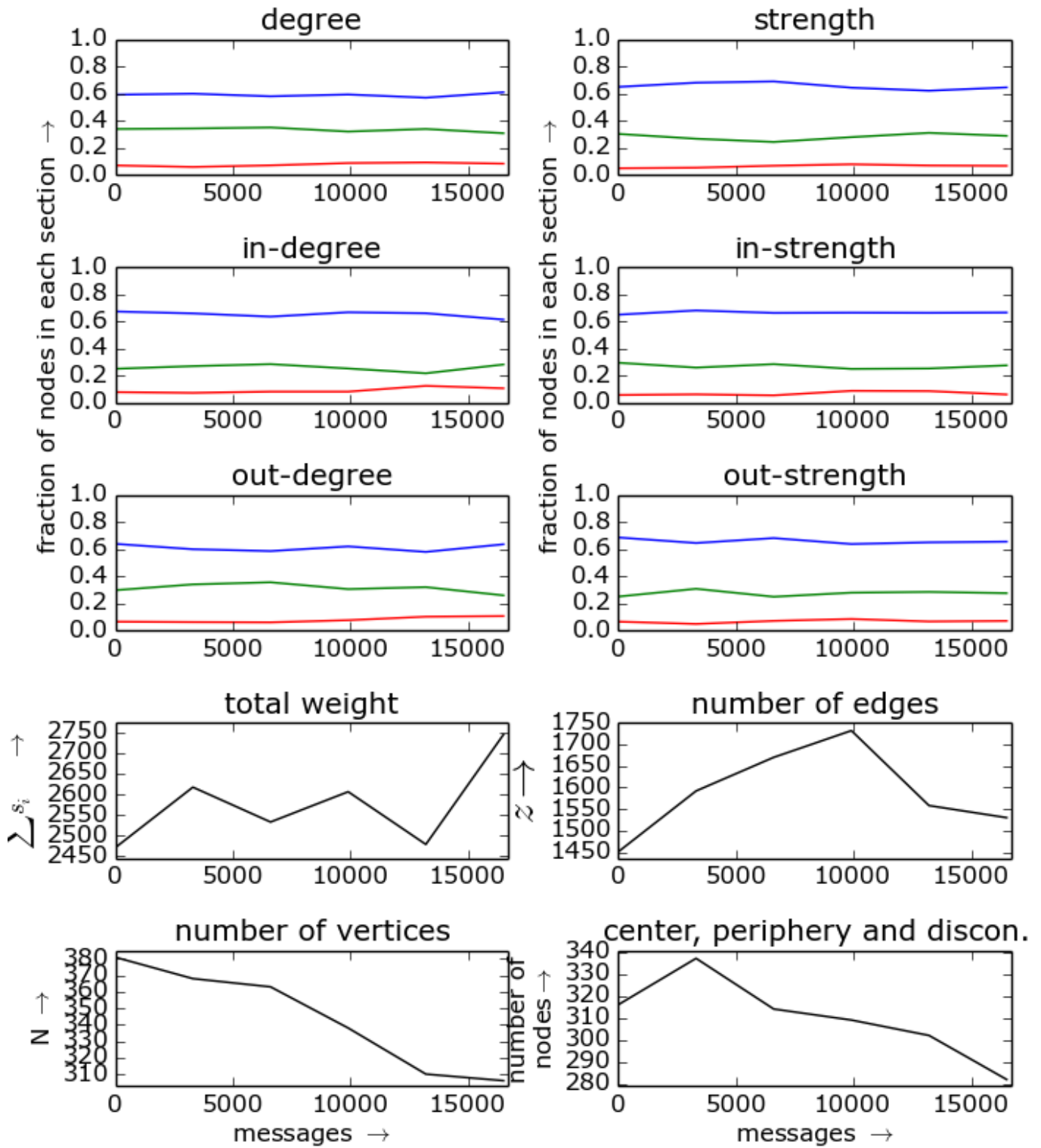
Primary divisions. Window: 1000 messages.  
Placement resolution: 1000 messages. LAD



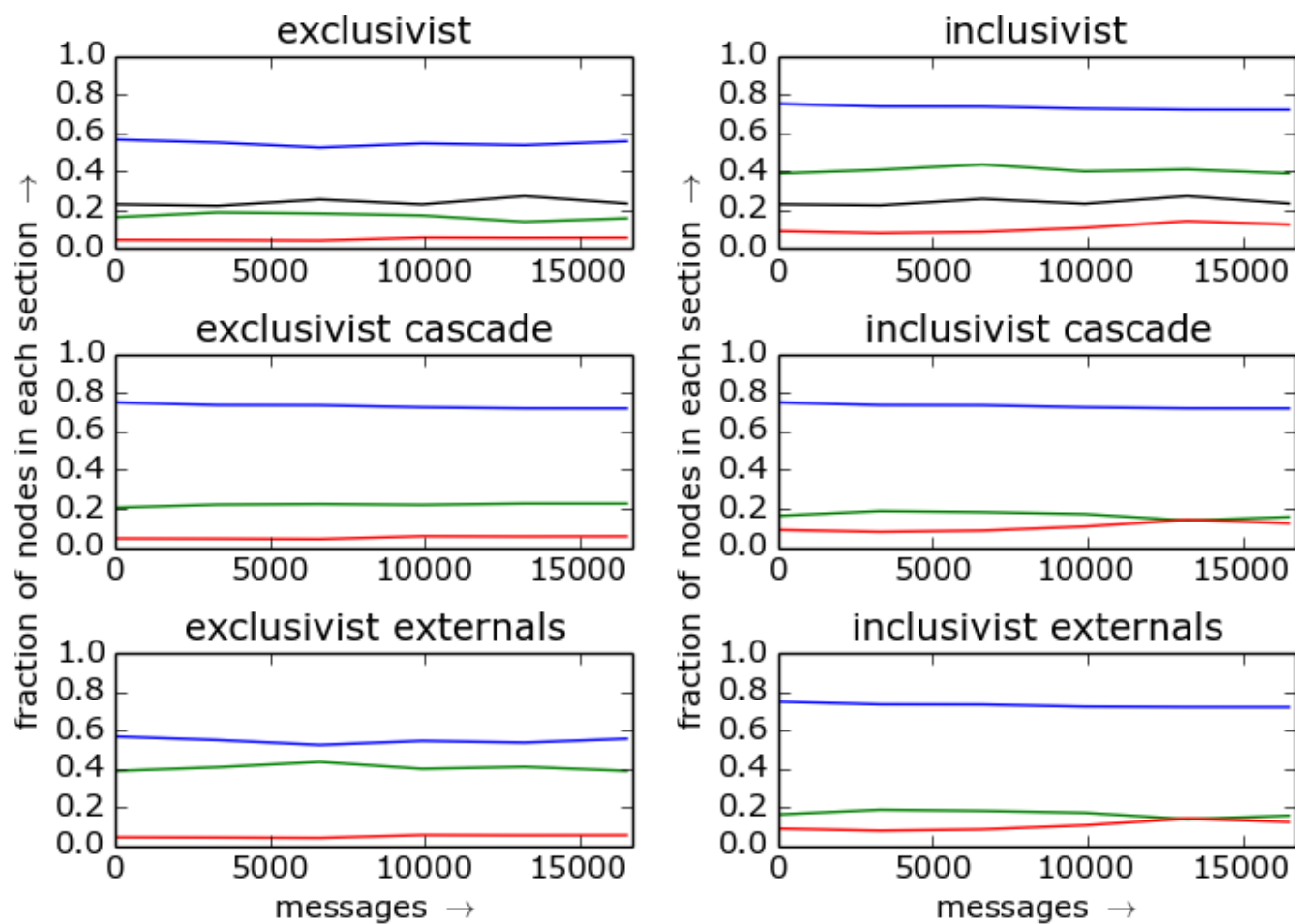
Compound divisions. Window: 1000 messages.  
Placement resolution: 1000 messages. LAD



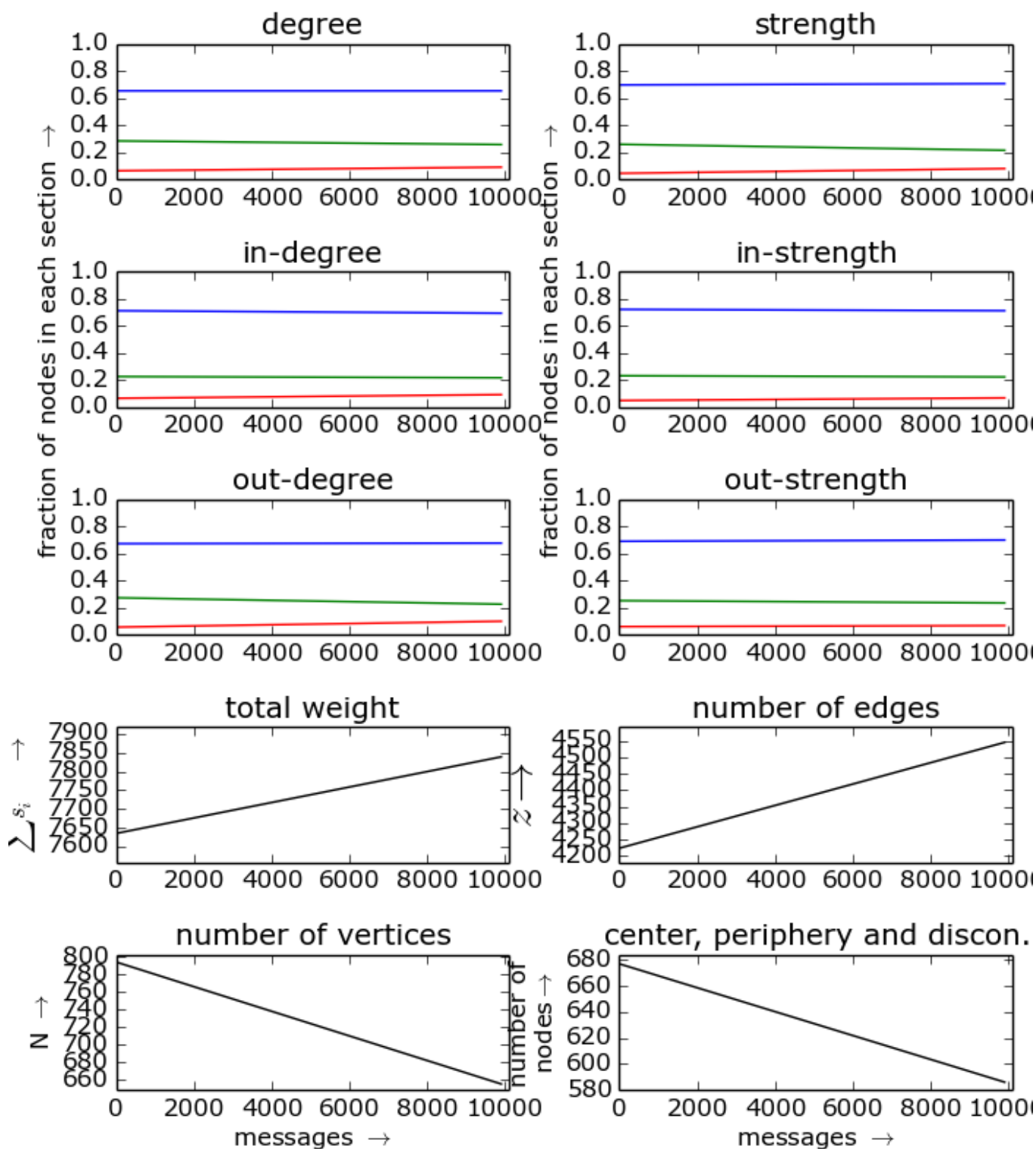
Primary divisions. Window: 3300 messages.  
Placement resolution: 3300 messages. LAD



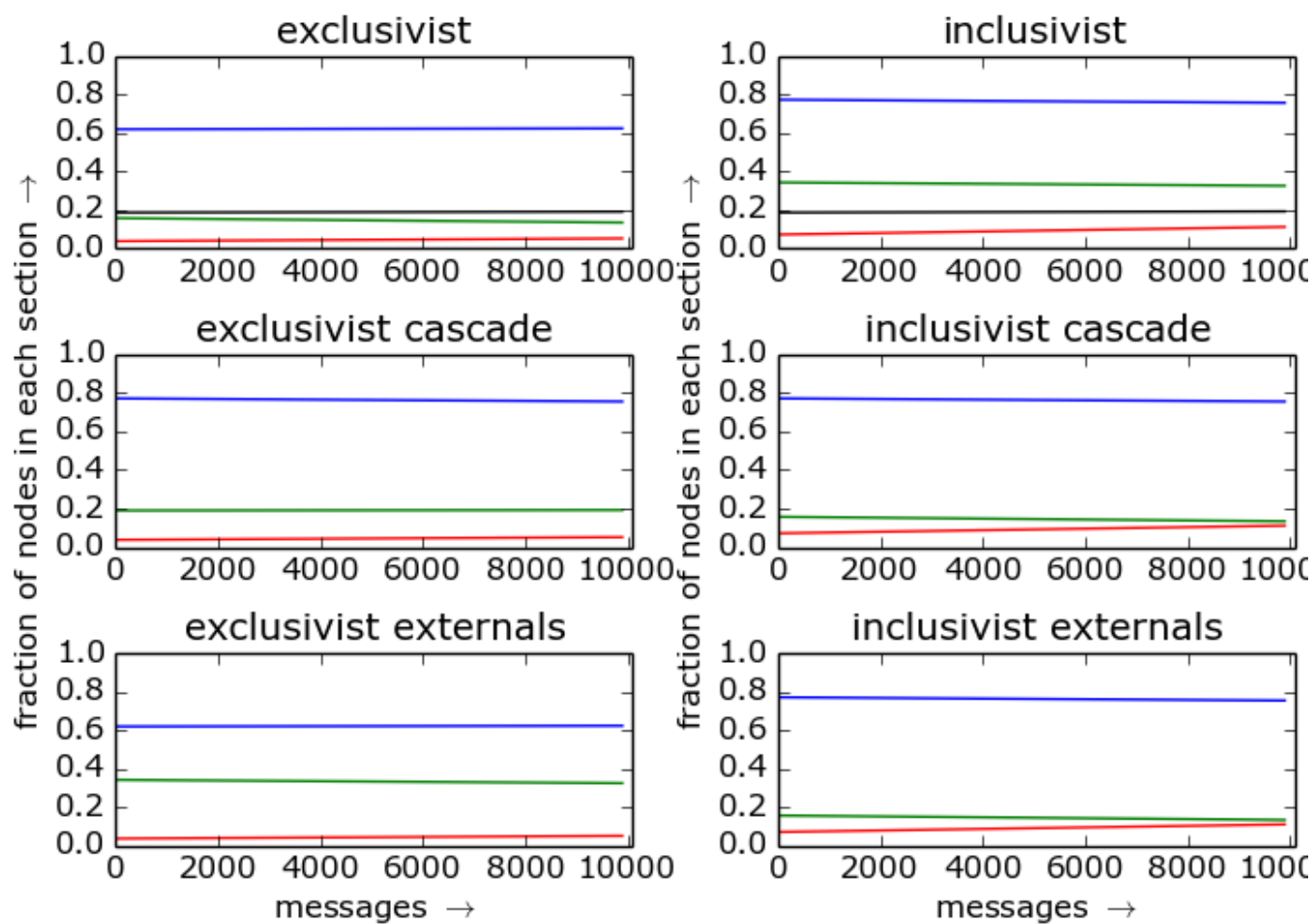
Compound divisions. Window: 3300 messages.  
Placement resolution: 3300 messages. LAD



Primary divisions. Window: 9900 messages.  
Placement resolution: 9900 messages. LAD



Compound divisions. Window: 9900 messages.  
Placement resolution: 9900 messages. LAD



#### SIV. STABILITY IN NETWORKS FROM TWITTER, FACEBOOK, PARTICIPABR

To ease hypothesizing about the generality of the reported stability of human interaction networks, this section presents the topological analysis of networks from Twitter, Facebook and Participabr. Selected networks are summarized in Table S29. Their Erdős sector relative sizes are given in Table S30. The formation of Principal components are given in Tables S31, S32, S33 and S34. The friendship networks considered are undirected and unweighted, therefore all measurements of strength, in- and out- centralities, asymmetry and disequilibrium have little or no meaning, which is why F1, F2, F3, F4 and F5 are only present in Table S31. The most important results from this analysis are:

- a further indicative that the stability reported with a focus on email interaction networks is valid for a broader class of phenomena.
- The stability in email interaction networks is higher than for the other networks, considering the same number of participants. This was especially important in choosing email networks for benchmarking and probing general properties.



TABLE S29. Selected networks from three social platforms: Facebook, Twitter and Participabr. Both friendship and interaction networks were observed, yielding undirected and directed networks, respectively. The number of agents  $N$  and the number of edges  $z$  are given on the last columns. The acronyms, one for each network, are used throughout Tables S30, S32, S31, S33 and S34. Data was collected in 2013 and 2014 within the anthropological physics framework<sup>3</sup>.

acronym	provenance	edge	directed	description	$N$	$z$
F1	Facebook	friendship	no	the friendship network of Renato Fabbri (author)	1367	28606
F2	Facebook	friendship	no	the friendship network of Massimo Canevacci (senior anthropologist)	4764	59995
F3	Facebook	friendship	no	the friendship network of a brazilian direct democracy group	3599	59471
F4	Facebook	friendship	no	the friendship network of the Silicon Valley Global Network group	2026	15586
F5	Participa.br	friendship	no	the friendship network of a brazilian federal social participation portal	443	910
I1	Facebook	interaction	yes	the interaction network of the Silicon Valley Global Network group	104	154
I2	Facebook	interaction	yes	the interaction network of a Solidarity Economy group	64	120
I3	Facebook	interaction	yes	the interaction network of a brazilian direct democracy group	214	310
I4	Facebook	interaction	yes	the interaction network of the 'Cience with Frontiers' group	530	1658
I5	Participa.br	interaction	yes	the interaction network of a brazilian federal social participation portal	222	300
TT1	Twitter	retweet	yes	the retweet network of $\approx 22k$ tweets with the hashtag #arenaNETmundial	2772	7222
TT2	Twitter	retweet	yes	same as TT1, but disconnected agents are not discarded	2975	7222

TABLE S30. Percentage of agents in each Erdős sector in the friendship and interaction networks of Table S29. The ratios found in email networks are preserved. I1 and I4 are outliers, probably because they should be better characterized as a superposition of networks, rather than one coherent network. The degree was used for establishing the sectors.

	periphery	intermediary	hubs
F1	53.11	43.31	3.58
F2	58.98	39.29	1.72
F3	65.41	31.87	2.72
F4	66.49	32.03	1.48
F5	62.98	36.12	0.90
I1	4.81	94.23	0.96
I2	53.12	45.31	1.56
I3	58.41	40.19	1.40
I4	39.06	59.43	1.51
I5	54.95	43.69	1.35
TT1	74.86	24.49	0.65
TT2	76.57	22.86	0.57

TABLE S31. Formation of first three principal components for each of the five friendship networks of Table S29 in the simplest case: dimensions correspond to degree  $k$ , clustering coefficient  $cc$  and betweenness centrality  $bt$ . Participabr yields the networks that most resemble the email networks. Overall, the general characteristic is preserved: first component is an average of degree and betweenness, while clustering is the most relevant for the second component. The friendship network of Renato Fabbri (F1) is the only network whose first component has more than 20% of clustering coefficient and second component has more than 20% of degree centrality.

	PC1					PC2					PC3				
	F1	F2	F3	F4	F5	F1	F2	F3	F4	F5	F1	F2	F3	F4	F5
$cc$	25.80	12.22	11.54	5.04	0.94	58.87	78.22	68.86	90.39	88.86	6.95	1.13	18.20	3.02	5.90
$k$	36.43	43.96	45.61	47.40	49.50	25.66	9.98	6.10	7.63	6.42	44.94	49.52	42.00	48.41	47.02
$bt$	37.77	43.82	42.85	47.56	49.56	15.46	11.80	25.04	1.98	4.72	48.11	49.35	39.80	48.57	47.08
$\lambda$	53.15	53.06	46.26	55.36	63.80	28.69	32.57	34.27	33.25	33.57	18.16	14.37	19.47	11.38	2.63

TABLE S32. Formation of the first three principal components for each of the seven interaction networks of Table S29 in the simplest case: dimensions correspond to degree  $k$ , clustering coefficient  $cc$  and betweenness centrality  $bt$ . Twitter yields the networks that most resemble the email networks. Overall, the general characteristic is preserved: first component is an average of degree and betweenness, while clustering is the most relevant for the second component.

	PC1							PC2							PC3						
	I1	I2	I3	I4	I5	TT1	TT2	I1	I2	I3	I4	I5	TT1	TT2	I1	I2	I3	I4	I5	TT1	TT2
$cc$	14.43	17.12	11.54	0.69	13.26	2.17	2.72	74.78	70.72	79.30	96.63	76.59	95.75	94.69	1.58	4.09	2.46	1.71	0.57	2.03	2.20
$k$	42.68	41.77	44.37	49.65	43.41	48.94	48.67	13.85	11.48	8.31	2.35	11.26	0.14	0.52	49.07	48.42	48.94	49.14	49.76	49.01	48.93
$bt$	42.89	41.11	44.09	49.66	43.34	48.89	48.61	11.37	17.80	12.39	1.02	12.15	4.12	4.79	49.35	47.49	48.60	49.15	49.67	48.96	48.87
$\lambda$	64.58	61.97	56.95	62.01	50.92	64.82	64.83	31.57	30.98	32.56	33.35	32.51	33.33	33.32	3.85	7.05	10.50	4.64	16.57	1.85	1.86

TABLE S33. Formation of the first three principal components for each of the seven interaction networks of Table S29 considering total, in- and out- degrees ( $k$ ,  $k^{in}$ ,  $k^{out}$ ) and strengths ( $s$ ,  $s^{in}$ ,  $s^{out}$ ), clustering coefficient  $cc$  and betweenness centrality  $bt$ . Twitter yields the networks that most resemble email networks. The general characteristic is preserved: first component is an average of degree and betweenness, while clustering coefficient is the most relevant for the second component. Important differences are: the clustering coefficient was only important to the third component for two of the networks ( $I2$ ,  $I3$ ) and does not contribute significantly to any of the first three principal components in  $I5$ ; in the first component,  $I5$  exhibited less contribution from in-strength, in-degree and betweenness,  $I4$  exhibited less contribution from out-degree.

	PC1							PC2							PC3						
	I1	I2	I3	I4	I5	TT1	TT2	I1	I2	I3	I4	I5	TT1	TT2	I1	I2	I3	I4	I5	TT1	TT2
$cc$	2.79	4.34	2.57	0.82	1.29	0.66	0.76	28.44	9.46	3.29	21.95	6.95	29.82	30.04	32.24	60.89	80.24	43.85	3.81	33.84	33.54
$s$	15.28	15.84	16.46	16.01	16.70	15.49	15.47	3.78	4.95	2.90	3.26	17.78	1.95	2.05	1.95	0.34	0.87	4.84	11.15	0.52	0.43
$s^{in}$	14.48	12.81	13.62	14.63	4.50	11.85	11.84	11.77	18.29	17.41	12.44	16.19	19.03	18.81	5.38	5.03	0.93	11.16	30.41	21.48	21.71
$s^{out}$	12.13	12.12	12.59	12.91	19.02	13.87	13.85	17.19	16.79	20.12	18.81	8.90	13.42	13.43	19.35	7.90	3.11	11.38	14.58	12.91	12.92
$k$	15.32	16.22	16.12	16.20	21.12	15.48	15.46	3.13	4.18	6.25	2.88	9.26	3.32	3.24	1.84	0.09	1.16	0.11	2.22	4.26	4.30
$k^{in}$	14.49	13.56	12.90	15.34	7.29	12.99	12.98	10.45	16.50	19.68	11.13	20.75	17.89	17.86	8.78	4.07	1.26	6.07	15.41	14.67	14.65
$k^{out}$	11.70	11.25	11.80	9.24	21.09	14.20	14.19	19.14	20.50	21.19	26.13	0.19	12.36	12.28	18.80	7.50	4.68	20.44	10.57	12.14	12.20
$bt$	13.82	13.86	13.93	14.86	8.99	15.47	15.45	6.10	9.32	9.16	3.41	19.99	2.20	2.29	11.66	14.20	7.75	2.17	11.86	0.18	0.25
$\lambda$	71.73	60.58	60.35	64.53	41.28	70.06	70.08	15.23	21.53	20.13	16.42	22.83	13.83	13.86	9.95	11.37	12.25	11.19	15.71	11.43	11.38

TABLE S34. Formation of the first three principal components for each of the seven interaction networks of Table S29 considering total, in- and out- degrees ( $k$ ,  $k^{in}$ ,  $k^{out}$ ) and strengths ( $s$ ,  $s^{in}$ ,  $s^{out}$ ), clustering coefficient  $cc$ , betweenness centrality  $bt$  and symmetry related metrics ( $asy$ ,  $\mu^{asy}$ ,  $\sigma^{asy}$ ,  $dis$ ,  $\mu^{dis}$  and  $\sigma^{dis}$  defined in Section IIIB 1). The characteristics found in email interaction networks are preserved: the first component is an average of degree and betweenness, the second component is mostly governed by symmetry related metrics, and clustering coefficient is mostly relevant for the third component. Standard deviation of asymmetry and disequilibrium metrics are again coupled to clustering coefficient in the third component. Important differences are: the first component is a less regular average of centrality measures and has a greater contribution of symmetry metrics; the first component of  $I5$  is formed mostly from symmetry, not centrality, metrics.

	PC1							PC2							PC3						
	I1	I2	I3	I4	I5	TT1	TT2	I1	I2	I3	I4	I5	TT1	TT2	I1	I2	I3	I4	I5	TT1	TT2
$cc$	3.46	4.19	2.44	0.36	2.18	1.28	1.17	3.06	1.61	1.23	1.19	2.57	3.03	2.17	17.36	16.88	21.68	17.00	10.00	18.65	19.13
$s$	10.05	9.21	9.60	9.31	3.54	10.27	10.59	5.81	5.74	7.33	8.47	9.24	6.26	5.96	4.58	8.02	4.91	2.21	13.10	0.92	1.53
$s^{in}$	9.57	8.03	9.21	8.74	0.78	7.75	7.99	4.63	0.59	1.27	6.69	2.77	5.38	5.29	8.22	12.82	9.18	6.53	7.63	5.90	4.77
$s^{out}$	7.88	6.21	5.45	6.97	5.76	9.25	9.54	6.78	10.23	12.76	9.20	10.26	5.27	4.92	5.90	2.99	3.86	8.43	10.84	4.58	4.76
$k$	10.44	10.02	9.88	10.39	5.80	10.80	11.05	4.62	5.13	5.66	5.54	14.08	4.48	4.29	3.63	6.02	6.18	1.86	1.21	1.33	1.30
$k^{in}$	10.12	9.30	9.50	9.98	4.43	8.64	8.86	2.69	0.70	0.88	4.49	9.61	5.40	5.50	7.12	10.55	8.70	6.17	8.24	7.27	6.54
$k^{out}$	7.27	5.29	4.43	5.43	9.11	10.10	10.33	8.36	12.52	13.63	5.65	11.61	3.38	3.08	7.82	5.77	2.07	13.52	5.68	5.00	4.65
$bt$	9.62	7.97	7.53	8.93	2.25	10.47	10.78	3.77	8.42	9.14	6.95	8.12	5.60	5.29	2.72	0.42	1.99	2.74	8.66	1.16	1.60
$asy$	5.42	7.05	7.97	8.48	15.47	6.16	5.79	14.17	12.88	11.78	11.02	4.67	12.48	13.39	2.95	1.03	0.58	2.71	0.87	6.54	5.80
$\mu^{asy}$	5.48	6.99	7.99	8.47	15.44	6.18	5.80	14.12	13.04	11.78	11.01	4.72	12.46	13.37	2.92	0.76	0.75	2.77	0.76	6.58	5.83
$\sigma^{asy}$	6.53	7.39	7.63	7.15	2.37	5.59	5.48	1.69	3.80	1.75	8.46	7.49	5.94	5.45	11.32	8.91	11.14	3.04	15.54	13.70	15.31
$dis$	5.02	6.67	7.78	8.08	15.41	5.98	5.59	14.12	13.41	11.92	11.53	4.80	12.45	13.38	4.99	1.40	0.67	3.02	0.83	7.44	6.69
$\mu^{dis}$	5.33	7.01	7.24	6.92	14.34	5.49	5.14	13.33	10.15	9.47	8.02	5.05	11.86	12.65	1.66	7.08	5.72	11.38	2.68	0.77	0.66
$\sigma^{dis}$	3.82	4.68	3.34	0.81	3.12	2.03	1.88	2.85	1.77	1.39	1.77	5.00	6.01	5.24	18.82	17.36	22.58	18.61	13.97	20.16	21.42
$\lambda$	46.11	43.48	44.29	46.95	30.34	44.12	43.52	26.42	24.97	24.76	19.99	23.91	25.98	26.13	14.90	14.72	11.82	13.16	17.32	11.62	12.15

<sup>1</sup>R. Fabbri, “Time stability in human interaction networks,” preprint arXiv:1310.7769. <http://arxiv.org/abs/1310.7769>.

<sup>2</sup>Numpy version 1.8.2, “random.randint” function, was used for simulations, algorithms in <https://pypi.python.org/pypi/gmane>.

<sup>3</sup>R. Fabbri, “What are you and i? [anthropological physics fundamentals],” (2015), [https://www.academia.edu/10356773/What\\_are\\_you\\_and\\_I\\_anthropological\\_physics\\_fundamentals\\_](https://www.academia.edu/10356773/What_are_you_and_I_anthropological_physics_fundamentals_).