Math 3070/6070 Homework 5
Due: Nov 11th, 2022

1. (3.15) In class, we showed that the $Poisson(\lambda)$ distribution is the limit of the negative binomial(r, p) distribution as $r \to \infty$, $p \to 1$, $r(1-p) \to \lambda$. Show that under these conditions the mgf of the negative binomial converges to that of the Poisson.
2. (3.28) Show that each of the following families is an exponential family
    1. normal family with either parameter μ or σ known.
    2. gamma family with either parameter a or b known or both unknown.
    3. beta family with either parameter a or b known or both unknown.
    4. Poisson family
    5. negative binomial family with r known, $0 < p < 1$.
3. (3.33) For each of the following families:
    1. Verify that it is an exponential family.
    2. Describe the curve on which the $\boldsymbol{\theta}$ parameter vector lies.
    3. Sketch a graph of the curved parameter space.

    (a) $n(\theta, \theta)$

    (b) $n(\theta, a\theta^2)$, a known
4. (3.37) Show that if $f(x)$ is a pdf, symmetric about 0, then μ is the median of the location-scale pdf
$$(1/\sigma) f\left((x-\mu)/\sigma\right), -\infty < x < \infty.$$