Math 3070/6070 Homework 2
Due: Sept 15th, 2022

1. (1.47 a - d) Prove that the following functions are cdfs.

    1. $\frac{1}{2} + \frac{1}{\pi}\tan^{-1}(x), x \in (-\infty, \infty)$

    2. $(1 + e^{-x})^{-1}, x \in (-\infty, \infty)$

    3. $e^{-e^{-x}}, x \in (-\infty, \infty)$

    4. $1 - e^{-x}, x \in (0, \infty)$

2. (1.51) An appliance store receives a shipment of 30 microwave ovens, 5 of which are (unknown to the manager) defective. The store manager selects 4 ovens at random, without replacement, and tests to see if they are defective. Let X = number of defectives found. Calculate the pmf and cdf of X and plot the cdf.

3. (1.52) Let X be a continuous random variable with pdf $f(x)$ and cdf $F(x)$. For a fixed number x_0, define the function
$$g(x) = \begin{cases} f(x)/[1 - F(x_0)] & x \geq x_0 \\ 0 & x < x_0. \end{cases}$$

    Prove that $g(x)$ is a pdf. (Assume that $F(x_0) < 1$.)

4. (1.53) A certain river floods every year. Suppose that the low-water mark is set at 1 and the high-water mark Y has distribution function
$$F_Y(y) = \Pr(Y \leq y) = 1 - \frac{1}{y^2}, \quad 1 \leq y < \infty.$$

    1. Verify that $F_Y(y)$ is a cdf.

    2. Find $f_Y(y)$, the pdf of Y.

    3. If the low-water mark is reset at 0 and we use a unit of measurement that is $\frac{1}{10}$ of that given previously, the high-water mark becomes $Z = 10(Y - 1)$. Find $F_Z(z)$.

5. (1.54) For each of the following, determine the value of c that makes $f(x)$ a pdf.

    1. $f(x) = c\sin x, \quad 0 < x < \pi/2$

    2. $f(x) = ce^{-|x|}, \quad -\infty < x < \infty$