Lecture 25: Oct 28

Last time

- Common Continuous Distributions

Today

- Common Continuous Distributions

Normal Distribution Introduced by De Moivre (1667 - 1754) in 1733 as an approximation to the binomial. Later studied by Laplace and others as part of the Central Limit Theorem. Gauss derived the normal as a suitable distribution for outcomes that could be thought of as sums of many small deviations.

- Sample space: $\mathbb{R} = (-\infty, \infty)$
- pdf: For $Y \sim N(\mu, \sigma^2)$,

$$f(y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-\mu)^2}{2\sigma^2}} \qquad -\infty < y < \infty$$

- cdf: There is no closed form.
- When $\mu = 0$ and $\sigma = 1$, the distribution is called *standard normal*:

$$\Phi(y) = \Pr(Y \leqslant y), \quad \Phi(-y) = 1 - \Phi(y)$$

- Mean:

$$EY = \mu$$

- Variance:

$$Var(Y) = E(Y - \mu)^2 = \sigma^2$$

- Higher central moments:

$$E(Y-\mu)^m = \begin{cases} \frac{m!}{2^{m/2}(m/2)!}\sigma^m & m \text{ is even} \\ 0 & m \text{ is odd} \end{cases}$$

- In particular:

$$\mu_3 = E(Y-\mu)^3 = 0 (\text{Skewness})$$
$$\mu_4 = E(Y-\mu)^4 = 3\sigma^4$$

- Moment generating function:

$$M_Y(t) = \exp(\mu t + \sigma^2 t^2/2)$$

Standardization

$$Y \sim N(\mu, \sigma^2) \iff Z = \frac{Y - \mu}{\sigma} \sim N(0, 1)$$

Shifting and scaling:

$$Z \sim N(0, 1) \iff Y = \sigma Z + \mu \sim N(\mu, \sigma^2)$$

Notes

- Normal distribution is useful in many practical settings. E.g. measurement error.
- Plays an important role in *sampling distributions* in *large samples*, since the Central Limit Theorem syas that the sums of independent identically distributed random variables are approximately normal
- There are many important distributions that can be derived from functions of normal random variables (e.g. χ^2, t, F). We will briefly present the pdf's and sample spaces of these distributions.