CS542 Machine Learning

Homework 5

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8.3

$p(a,b) \neq p(a)p(b)$

But $p(a,b|c) = p(a|c)p(b|c)$ for $c = 0$ and $c = 1$

By the summation rule: $p(a,b) = \sum_{c\in\{0,1\}} p(a,b,c)$

$\Rightarrow p(a) = \sum_{b\in\{0,1\}} \sum_{c\in\{0,1\}} p(a,b,c),\ \ p(b) = \sum_{a\in\{0,1\}} \sum_{c\in\{0,1\}} p(a,b,c)$

The rule of conditional probability applying the product: $p(c,a,b) = p(a,b|c)p(c)$

$\Rightarrow p(a,b|c) = \frac{p(a,b,c)}{\sum_{a\in\{0,1\}} \sum_{b\in\{0,1\}} p(a,b,c)}$

Use similar technique:

$p(a|c) = \frac{\sum_{b\in\{0,1\}} p(a,b,c)}{\sum_{a\in\{0,1\}} \sum_{b\in\{0,1\}} p(a,b,c)}$ and $p(b|c) = \frac{\sum_{a\in\{0,1\}} p(a,b,c)}{\sum_{a\in\{0,1\}} \sum_{b\in\{0,1\}} p(a,b,c)}$

8.4

$p(a,b,c) = p(a)p(c|a)p(b|c)$

The value of $p(a)$ and $p(b|c)$ were computed as follow:

a	$p(a)$
0	0.600
1	0.400

b	c	$p(b\|c)$
0	0	0.800
0	1	0.400
1	0	0.200
1	1	0.600

Use the rule of product:

$p(c|a) = \frac{\sum_{b\in\{0,1\}} p(a,b,c)}{\sum_{b\in\{0,1\}} \sum_{c\in\{0,1\}} p(a,b,c)}$

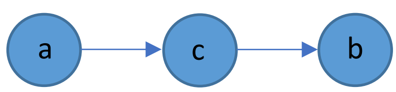
$\Rightarrow$

c	a	$p(c\|a)$
0	0	0.400
0	1	0.600
1	0	0.600
1	1	0.400

$\Rightarrow$ Multiply all the value

$\Rightarrow$ $p(a,b,c) = p(a)p(c|a)p(b|c)$

$\Rightarrow$ Corresponding directed graph:



8.11

$P(D=1|G=1)=0.900$ and $P(D=0|G=0)$

Use Bayse Theorem and marginalizing:

$P(F=0|D=0)=\frac{P(D=0|F=0)P(F=0)}{P(D=0)}$

$P(D=0|F=0)=\sum_{B,G} P(D=0|G)P(G|B,F=0)P(B)$

$\qquad\qquad\qquad\quad = 0.748\ (74.8\%)$

$P(D=0)=\sum_{B,G,F} P(D=0|G)P(G|B,F)P(B)P(F)$

$\qquad\qquad = 0.352\ (35.2\%)$

$\Rightarrow P(F=0|D=0)=0.213\ (21.3\%)$

$\Rightarrow P(F=0|D=0,B=0)=0.11\ (11\%)$

8.14

The energy function is $E(x,y)=h\sum_i x_i-\beta\sum_{ij} x_i x_j-v\sum_i x_i y_i$ where $n,h,\beta\geq 0$ and $x_i, y_i\in\{-1,1\}$

Set $h=\beta=0 \Rightarrow E(x,y)=-N\sum_i x_i y_i$

When the energy is the lowest, the probable configuration will be the most.

When the negative sign in form of N stays, can only happen if x_i and y_i are both 1 or both negative.

$\Rightarrow$ $x_i=y_i$ for all i