Programming assignment 2
Optical depth

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Optical depth simplified: We can describe optical depth, as follow. If we image that we have a block of a species/medium, and than send a beam of light through this medium. Some light beam will get absorbed, but for the light beams which is not absorbed. Will scatter randomly each time, and than maybe each of the light beams which have scattered randomly escapes at one optical depth example in 1. If we then think, how far can we see into this medium? The answer will be one optical depth (side note: light beam don't need to bounce it can also be absorbed). So optical depth is describing how much absorption we have, when a light beams travels through a absorbing medium. The light get absorbed, because of different atoms in the medium.

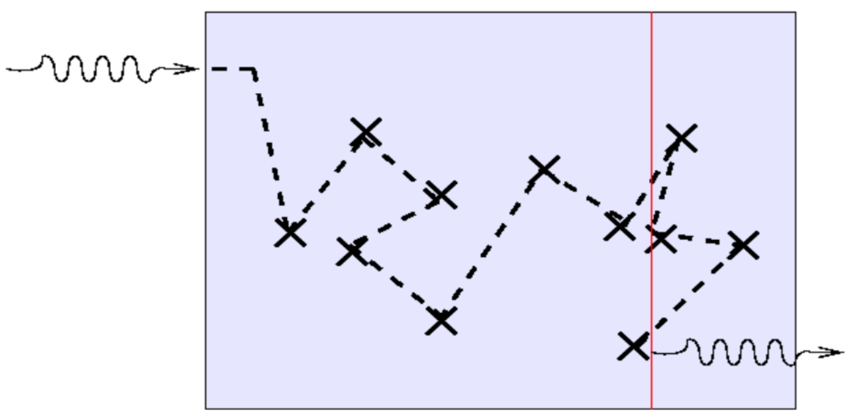


Figure 1: Example of random motion of a light beam, escaping at one optical depth. Figure from Michael Richmond.

Absorption cross section: the cross section, is the cross section of the absorbers with a radius r, and we can define the cross section as $\sigma = \pi r^2$ which is a geometric definition. But we work with atom/nuclear processes, hence we will have a more probabilistic definition in the real world. Than, absorption cross section describes the probability for a photon to be absorbed when traveling through a molecule at a specified wavelength multiplied with the average cross section of the molecule.

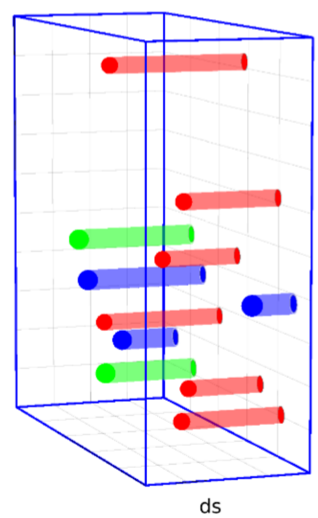
Theory

Optical depth

The optical depth (τ), with zero solar zenith angle we define as:

$$\tau(z_0, \lambda, \chi) = \sum_j \sigma_j^a(\lambda) \int_{z_0}^{\infty} n_j(z) dz \tag{1}$$

Where z_0 is a altitude which we start our measurement, λ is the wavelength and χ is solar zenith angle (illustrated in Fig. 2). In the expression we have that σ_j^a is the absorption cross section which is dependent on the wavelength of a species j, than we have n_j which is the height profiles of the concentration of the species j (number density).
We have a gain in optical depth, since $d\tau$ is simply the sum of absorption cross section of the atoms and molecules in dz (figure on the right). Because $d\tau$ is the sum of absorption cross section, and absorption cross section varies strongly with the wavelength the optical depth varies strongly with the wavelength. Hence each wavelength will have it's own optical depth.
As we can see from the function 1, it depends on altitude, wavelength and solar zenith angle (χ). This will make the function more complected, when we have that $\chi \neq 0$. Which we gone see later, when we only focus on the solar zenith angle, between 0° and 85°.



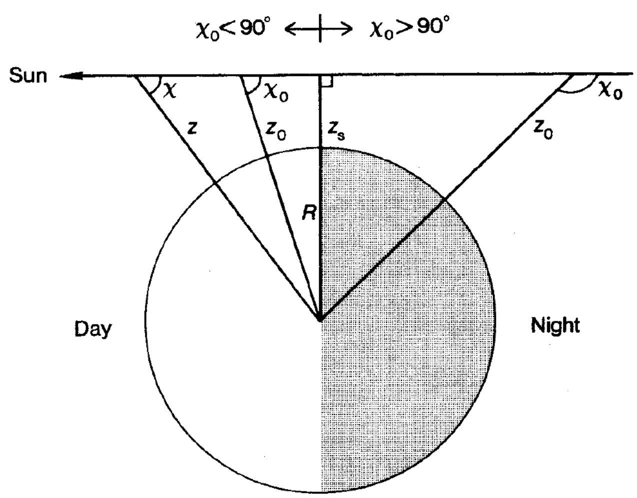


Figure 2: Geometry for solar zenith angles χ_0. Fig. 2.2.1 from Chapter 2 of Physics and Chemistry (M H Rees)

Photon-flux

The photon-flux at some given height and wavelength, is defined as:

$$I(z, \lambda) = I^{\infty}(\lambda) exp(-\tau(z, \lambda, \chi)) \tag{2}$$

Here $I^{\infty}(\lambda)$ is the photon-flux above the thermosphere at the distance ∞, and τ is optical depth. As the name states, photon-flux describes the number of photons fluxing through, some given area. In this case photons fluxing through some thermospheric altitudes and wavelengths.

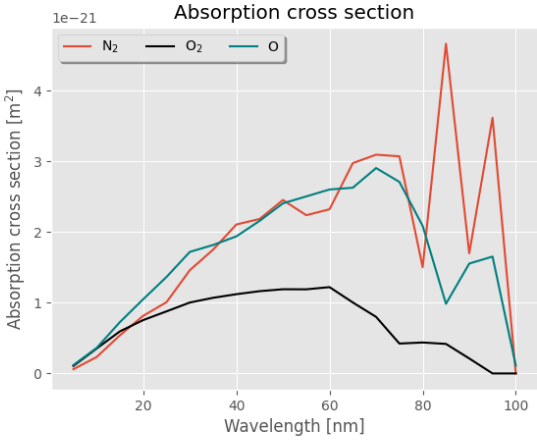
Task

We are asked to make functions that calculate the optical depth as a function of altitude and wavelength, for vertical incidence, and for variable zenith-angle of the incident light. And Calculate the EUV-photon flux as a function of wavelength and height.
This helps us to provide the following plots for:

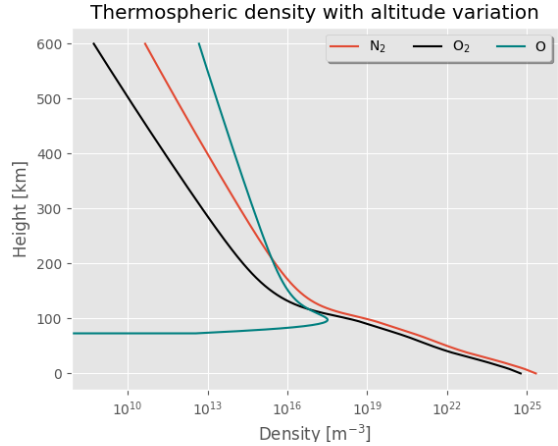
- Altitude-variation of the thermospheric densities
- Wavelength-variation of the photo-absorption cross-sections
- Optical depth
- Photon-fluxes as functions of wave-length and height

(I)

We know that the absorption cross section is strongly varying with the wavelength, which comes clear by looking at plot 3a for absorption cross section. This makes sense since the absorption cross section describes the probability for a photon to be absorbed when traveling through a molecule at a specified wavelength multiplied with the average cross section of the molecules.
In plot 3b we have a plot of the thermospheric density with altitude variation, we see that for N_2 and O_2 have a higher density at lower heights and density gets lower with the increasing height. Which makes since below 100 km, we have more pressure and the sun have a less effect on the molecules. This makes the molecules below 100 km, stick tighter tighter and at higher altitudes the molecules can be spread out more which makes the molecules not collide as often. So at increasing altitude, we have a reduction in mean molecular mass and vice versa.



(a) Wavelength-variation of the photo-absorption cross-sections



(b) Altitude-variation of the thermospheric densities

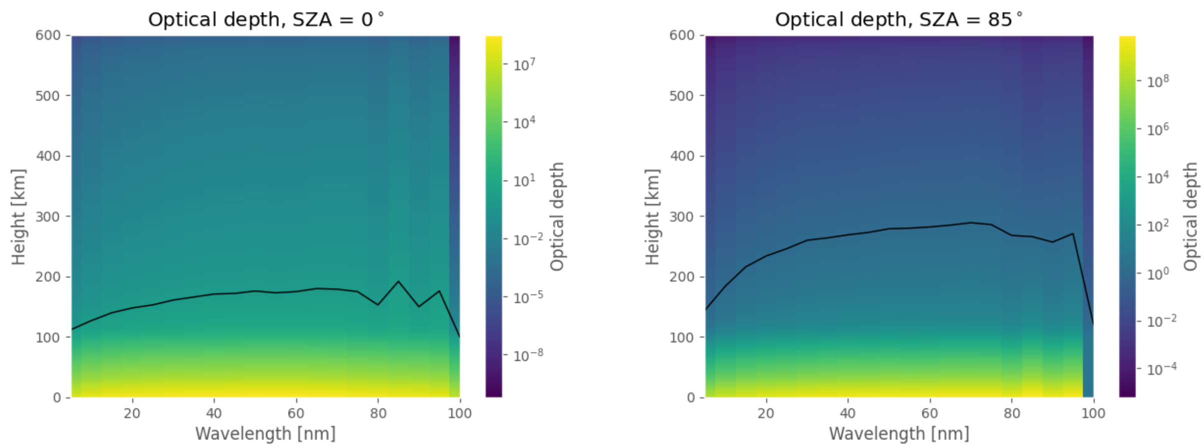
When plotting the optical depth, we want to adjust the solar zenith-angle (SZA) from 0° to 85° this means that we can not use formula 1, which is just for vertical incident light. Instead we use the following formula, that is valid for SZA from 0° to 90°:

$$\tau(z_0, \lambda, \chi) = \sum_j \sigma_j^a(\lambda) \int_{z_0}^{\infty} n_j(z) \Big[1 - \Big(\frac{R_E + z_0}{R_E + z'} \Big)^2 sin^2 \chi_0 \Big]^{-(1/2)} dz' \tag{3}$$

Where R_E is earth radius. Than plotting this formula, with wavelength on the x-axis, height/altitude on y-axis, the optical depth as a density on the plot and we also add a line where the optical depth is equal to one ($\tau = 1$). So that we a able to read of the plot, how to the optical depth is varying with the wavelength and altitude. By increasing SZA we should see a increase in optical depth, and increasing z_0 we will have a decrease in optical depth. This can be easily seen by noting that $sin\chi_0 = (R_E + z_s)/(R_E + z_0)$, and looking at the figure 2.

In figure 4, we have plotted the optical depth for $SZA = 0°$, $SZA = 85°$ and drawn a line for the unit optical depth $\tau = 1$. Figure 5 we have the optical depth with $SZA = 0°$ and $z_0 = 100km$.

As predicted, we see that by increasing the $SZA = 0°$ (Fig. 4a) to $SZA = 85°$ (Fig. 4b), the optical depth is increasing radically with SZA. As the prediction we stated earlier, also seams to be true. Which we see by look figure 5, it comes clear that when we increase z_0 the optical depth is lowered. I have only included the plots, which i see as relevant.



(a) Solar zenith angle at 0°

(b) Solar zenith angle at 85°

Figure 4: Optical depth as a function of altitude, wavelength and zenith-angle of the incident light. Line drawn at unit optical depth $\tau = 1$.

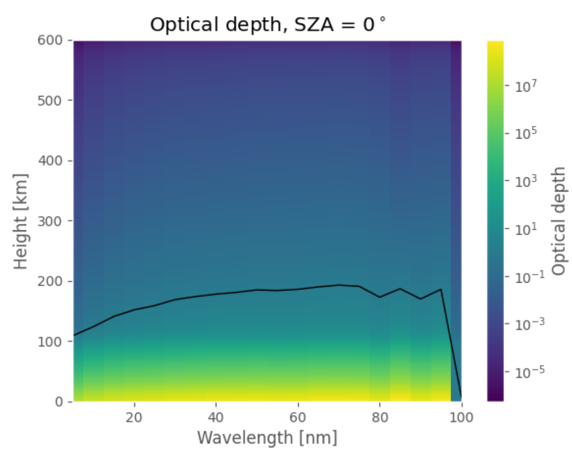
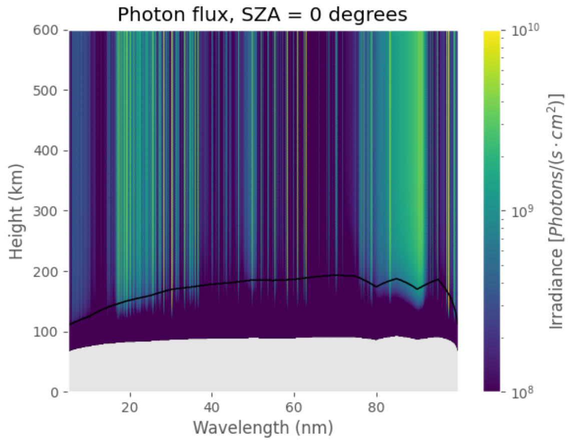


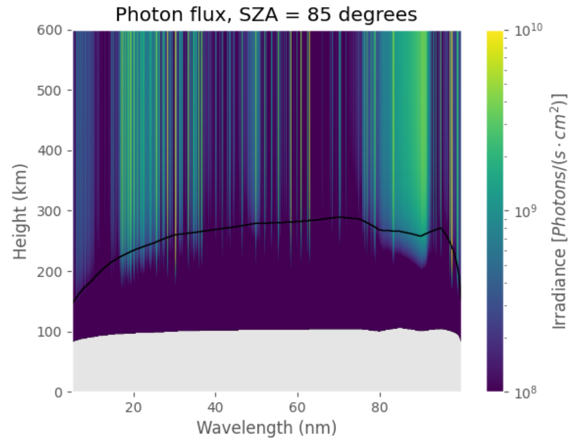
Figure 5: Solar zenith angle at 0° and $z_0 = 100km$

(II)

Now that we have calculated the optical depth τ, we can go forward to calculate the photon-flux. This is done by using formula 2, which gives us the following plots in figure 6 with a lot of emission lines, and a line is drawn for the unit optical depth ($\tau = 1$). We can see that the photon flux is varying strongly with wavelength, and by increasing $SZA = 0°$ (plot. 6a) to $SZA = 0°$ (plot. 6b) we get a strong increase in unit optical depth. But we see that the emission lines stops higher up than the $SZA = 0°$, this is because the photons goes through more molecules/particles, hence more incident light (photons) gets absorbed when the $SZA > 0$.



(a) Solar zenith angle at 0°



(b) Solar zenith angle at 85°

Figure 6: EUV-photon flux as a function of altitude, wavelength and zenith-angle. Line drawn at unit optical depth $\tau = 1$

Appendix