Set $B = \{ b_1, b_2, b_3, \dots \}$ is an indexed set of beats. $|B|$ is the length of the song / score.

Set P is a set of hand positions.

$\forall\, p, p' \in P\,, T(p, p') =$ energy required to transition from p to p'

$\forall p \in P, \forall b \in B, E(p, b) =$ energy required to play note b using position p

$$\forall p \in P, \forall b_i \in B, C(p, b_i) = \begin{cases} E(p, b_i) & \text{, if } i = |B| \\ E(p, b_i) + \min\limits_{p' \in P}(T(p, p') + C(p', b_{i+1})) & \text{, otherwise} \end{cases}$$

$C(p, b_i)$ is (should be) the minimum energy required to play the song starting at beat b_i in position p

Proof:

Base case: $\forall \boldsymbol{p} \in \boldsymbol{P},\ \boldsymbol{C}(\boldsymbol{p}, \boldsymbol{b}_{|\boldsymbol{B}|})$ is optimal

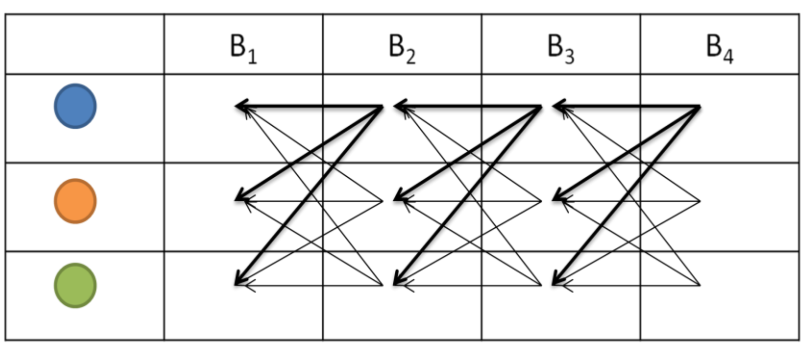
This is obviously true.

Induction: $\forall \boldsymbol{p}' \in \boldsymbol{P}\ \boldsymbol{C}(\boldsymbol{p}', \boldsymbol{b}_{i+1})$ is optimal $\Rightarrow \forall \boldsymbol{p} \in \boldsymbol{P}\ \boldsymbol{C}(\boldsymbol{p}, \boldsymbol{b}_i)$ is optimal

1. Assume $\forall p' \in P\ C(p', b_{i+1})$ is optimal
2. Pick an arbitrary $p \in P$ and prove $C(p, b_i) = E(p, b_i) + \min_{p' \in P}(T(p, p') + C(p', b_{i+1}))$ is optimal
3. From first assumption, $\min_{p' \in P}(T(p, p') + C(p', b_{i+1}))$ is the true minimum energy required to play the rest of the song
4. Therefore, $E(p, b_i) + \min_{p' \in P}(T(p, p') + C(p', b_{i+1}))$ must be the minimum energy required to play the current note and then play the rest of the song

Data Representation:

Can be represented as a |P| x |B| array. The following figure shows the dependency graph:



To calculate all values, we just need to sweep from the last beat to the first beat. For the sake of caching performance, the beats and the positions would be switched in implementation.

The data complexity for the algorithm is O(|B||P|), the time complexity is also O(|B||P|)