

THESIS TITLE - CHANGE POINT DETECTION

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Abstract

Thesis Title - Change Point Detection

Tyler Manning-Dahan

Text of abstract.

Acknowledgments

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Contents

List of Figures	vi
List of Tables	vii
1 Introduction	1
1.1 Motivation	1
1.1.1 Health Care	1
1.1.2 Financial Applications	2
1.2 Characteristics of the change point problem	2
1.3 Problem Formulation	4
1.3.1 Performance Measures	4
1.4 Related Work	4
1.5 Our Contributions	5
2 Chapter 2	6
2.1 Chapter 2 Section	6
2.1.1 Chapter 2 Subsection	6

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Motivation

Almost 100 years ago, Walter Shewart recognized the need for monitoring manufacturing processes in real-time in a systematic way. The use of statistical control charts allowed for real-time detection of changes in variation that may have indicated a degradation in quality in the production process [15] . Shewart’s method was one of the first formal frameworks to solve the problem of detecting changes in a distribution of a sequence of random variables. This problem would come to be known more generally as the *change point detection problem* and would come to apply across various industries. The following are a few motivating examples.

1.1.1 Health Care

Health care is an important area for quickly detecting signal changes in heart rate monitoring [19] [16], epilepsy signal segmentation [11], and multi-modal MRI lesion detection [3] to name a few. Quickly detecting changes to a patient’s health is absolutely necessary for any system to be of practical use. However, this quick detection must be balanced with high accuracy as false positives or missed detections could have life-threatening consequences.

1.1.2 Financial Applications

The application of accurate and timely change point detection is also very appealing to the finance sector where shifts in asset prices can happen suddenly. Change point detection is further complicated in financial applications because of the non-stationarities typically observed in asset pricing along with long memory processes. Note, in the financial literature, change-points are also referred to as structural breaks, but for this thesis will use the broader term change-points.

An on-line, quick detection technique is proposed in [14], where a modified Shiryaev - Roberts procedure is used in a case study to detect a change-point on a single stock's daily returns. They compare their non-parametric method with other classic control chart methods using speed of detection and false alarm rate as measures of performance.

Detecting changes in variance is specifically explored in [8]. The authors propose an off-line change-point algorithm that minimizes a global cost function by using an adaptive regularization function. The algorithm is applied to the absolute returns of the FTSE 100 stock index and the US dollar-Japanese Yen foreign intra-day exchange rate to detect changes in asset price volatility. The change-points identified in the FTSE 100 coincided with key market events such as the stock market crash that occurred on October 14th, 1987 and breaking the 5000 price barrier in August 1997.

See section 1.3.6 of [17] for more applications to options markets and arbitrage opportunities.

1.2 Characteristics of the change point problem

A number of surveys of the literature already exist [1] [12], therefore we will not cover all existing methods but rather touch upon several, important factors to consider when tackling the change point detection problem. Across the body of literature, these factors determine what methods are available to practitioners.

The first factor is selecting between *parametric* and *non-parametric* techniques. Deciding between these two broad techniques is dependent on the prior knowledge one wants to encode into the problem. For example, if it is known that data is generated by a distribution from the exponential family of distributions, then we can subset the

problem from the space of all possible distributions to a smaller space of distributions. For example, Shewart control charts and CUSUM change-point techniques are all parametric techniques based on Gaussian-family of distributions [13] [5]. In certain settings, it is not possible to leverage information about the data and non-parametric techniques must be used instead [4].

The second factor is to deciding whether change-points should be detected *offline* or *online*. Some algorithms are off-line—also referred to as batch algorithms or retrospective or a posteriori change-point detection—and they are applied in an ex-post fashion after the dataset has been completely acquired [18]. The aforementioned Shewart control chart and CUSUM algorithm were both designed for data that is streamed in a real-time fashion. In the literature, on-line methods of change point detection are also referred to sequential change point detection [17]. For this thesis, we will use the terms interchangeably.

The third factor is determining if there are multiple change points or to assume there is only a single change point to detect. This is an important factor for off-line change point detection where the decision to detect 1 or more change-points is chosen at the outset. Detecting multiple change-points could also be relevant for the on-line case if a situation arises where the window of time series under consideration may contain more than one change point. However, most on-line change-point methods are designed to detect a single change-point at a time.

Finally, the last factor to address is determining exactly what kinds of statistical changes the algorithm should detect. Many methods focus solely on detecting changes in the mean of a distribution [9]. Some methods are more general and can detect changes in the variance or higher order moments and do not focus on any particular one. Methods like kernel change point detection can typically detect any distributional changes.

This thesis will concern itself with on-line change point detection, where data is received in a streaming nature. We assume no prior distributional characteristics on the data and operate in a completely non-parametric setting.

1.3 Problem Formulation

The basic change-point problem is set up as hypothesis test between two segments of a time series. Since we are concerned with the on-line setting, we will always consider a time series of fixed size, n . Let X_1, \dots, X_n be a series of independent random variables observed. Then, one of the following hypotheses holds:

$$\begin{cases} H_0 : X_1, X_2, \dots, X_n \sim F_0 & \text{(no change-point occurred)} \\ H_1 : X_i \sim F_1, X_j \sim F_2 & \text{(a change-point occurred).} \end{cases} \quad (1)$$

Where $i = 1, 2, \dots, t-1$ and $j = t, \dots, n$ are two distinct segments separated by change-point t that is within the time series window. Furthermore, F_0, F_1, F_2 are cumulative distribution functions (CDFs) with corresponding probability density functions (PDFs), f_0, f_1, f_2 . Because we are operating in a non-parametric setting, the CDFs are assumed to be completely unknown.

1.3.1 Performance Measures

Add information about what performance measures will be used to determine the utility of algorithm. E.g. time to detection, F1 measure, etc.

1.4 Related Work

In 2005, Desobry et al. [6] developed an on-line kernel change point detection model based on single class support vector machines (ν -SVMs). The authors train a single class support vector on a past set, $\mathbf{x}_{t,1} = x_{t-m_1}, \dots, x_{t-1}$ of size m_1 and train another single class support vector on a future set $\mathbf{x}_{t,2} = x_t, \dots, x_{t+m_2-1}$ of size m_2 . A ratio is then computed between the two sets that acts as the dissimilarity measure in Hilbert space. If the points are sufficiently dissimilar over some predetermined threshold, η , then a change point is assigned to the time splitting the two sets of data.

In 2007, Harchoui and Cappe [7] approached the off-line change point problem with a fixed number of change points by using kernel change point detection. This was further extended to an unknown number of change points in 2012 by Arlot et al. [2]. Finally, Garreau and Arlot extended this in line of research kernel change points in the off-line setting of detecting change points. Fundamentally, their method is the kernel version of the following least squares optimization problem:

The benefits of this kernel change point detection is that it operates on any kind of data for which a kernel that properly reproduces a Hilbert space can be applied. In an application setting, the user would use some training set to calibrate the kernel and the penalty parameters, to then be tested with appropriate accuracy measures on some out of sample data set.

Other related work include a 2015 study by Li et al. [10] that propose the use of M-statistics based on the kernel maximum mean discrepancy (MMD) B-test statistic for two-sample testing.

1.5 Our Contributions

Lay out the contributions of this work.

Chapter 2

Chapter 2

2.1 Chapter 2 Section

2.1.1 Chapter 2 Subsection

Subsection Test

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