The Optimal Strength-Retaining Hole Pattern for Sheet Material

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Words:

Abstract

One of the greatest challenges of structural engineering is to reduce the weight of a system without compromising its strength. Hole patterns are a go-to solution to make parts lighter and maintain the majority of their rigidity. The problem is, what hole pattern is best? With many 2D tessellation patterns to choose from, it can be difficult to determine the optimal pattern to use. This investigation will simulate stresses on test sheet parts with a variety of polygon hole patterns to determine which shape maintains the highest strength in an array of scenarios.

Words:

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1 Background

Sheet metal is one of the most commonly used materials in engineering, but especially finds its place in the field of robotics. Sheet metal provides a stiff, structural component while taking little space and weight. Robotics frequently requires complicated geometry in small volumes. Sometimes sheet metal can be used as a light structural material instead of solid or tube extrusion. In the FIRST Robotics Competition (FRC), teams use sheet metal to make custom gearboxes, gussets, base-plates, even entire drive-trains. For all of these applications, applying a lightening hole pattern is a great way to reduce weight while maintaining an acceptable durability and stiffness. As seen in Figure **??**, not every team uses the same pattern.

Figure 1: FRC Examples of Hole Patterns
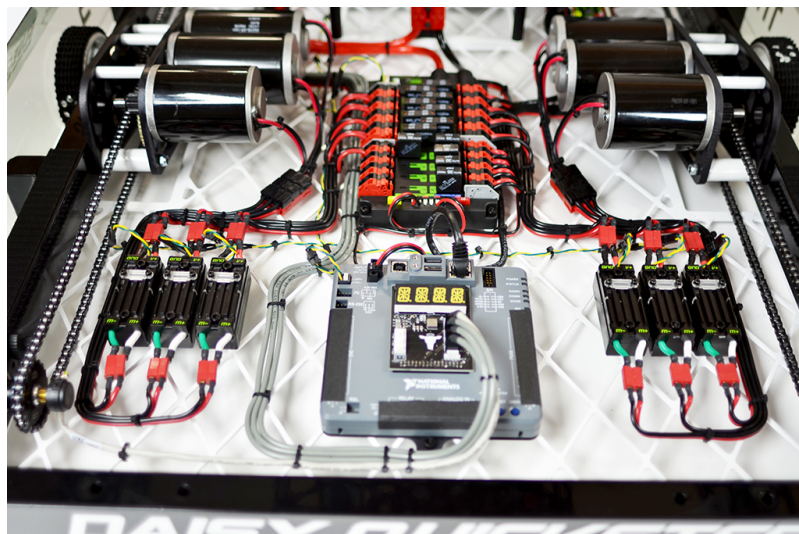
(a) FRC robot (team 1538) with a rectangular diamond hole pattern

(b) FRC robot made almost entirely out of sheet metal (team 118) with a triangular hole pattern

1.1 Statics

Statics is an area of physics thoroughly used in engineering, and particularly in the area of solid mechanics and structural engineering. For this investigation of a solid object under various loads and forces, solid mechanics provides a paradigm of study. Statics analyzes torque and force moments applied to a system, which is exactly what these simulations will do.

$$|\vec{M}_O| = \vec{F} \cdot \vec{r} = \vec{\tau} \tag{1}$$

$|\vec{M}_O|$ = vector force moment

$\vec{F}$ = vector force applied

$\vec{r}$ = perpendicular distance to axis of rotation

For each simulation, displacement between t_i and t_f is recorded. Displacement is the vector change in position between initial position and final position, generically described as $\vec{r}_i - \vec{r}_f = \vec{d}$. In the Solidworks Simulation, the models for force moments and displacement are far more accurate, complicated, and do not need to be understood for analysis.

1.2 Solid Mechanics

The field of solid mechanics analyzes the deformations of a solid under system changes. Deformation is the change (Δ) between the rest shape and the final shape. These changes can include changes in forces, temperature, phase changes and other internal or external changes[chou'elasticity:'1992]. A vari-

ety of models have been developed to describe different types of deformations of solids:

- Stress: The restoring force over a specific cross-sectional area

- Strain: The response of a system to an applied stress

- Plasticity: The degree to which an object can deform

- Elasticity: The degree to which an object can return to rest shape after deformation

Stress is the force exerted by a material, against the external force, to return to rest shape, divided by the cross sectional area. For this investigation, the stress and strain model will be used to describe the characterize the simulated deformations in tension, compression and torsion. The model is an extension of Hooke's law, which describes the behavior of a spring. This similarity makes sense because all materials have some degree of elasticity, and therefore, can behave as a spring.

$$\sigma_x \equiv \lim_{\Delta A \to 0} \frac{\Delta N}{\Delta A} \tag{2}$$

σ_x = stress

ΔN = fraction of normal force N

ΔA = cross-sectional area element

$$\varepsilon_x \equiv \frac{\sigma_x}{E} + \alpha \Delta T \tag{3}$$

ε_x = strain

σ_x = stress

E = modulus of elasticity

α = temperature coefficient

ΔT = change of temperature

Equations **??** and **??** are basic models of stress and strain. The simulations that will be run using the SolidWorks Simulation add-in use the Von Mises model for stress and strain by default (Equations **??** and **??**). The Von Mises model is well beyond the scope of understanding for this investigation, however it is an industry standard and is suitable for use in this case.

$$\sigma_{ij} = \frac{1}{3}\delta_{ij}\sigma_{kk} + \sigma'_{ij} \tag{4}$$

$$\varepsilon_{ij} = \frac{1}{3}\delta_{ij}\varepsilon_{kk} + \varepsilon'_{ij} \tag{5}$$

2 Simulation

2.1 Variables

To keep the stress analysis of each part fair, certain properties were controlled for every part.

- All parts have the dimensions of 1000mm x 1000mm x 5mm

- All parts are set to the material 6061 Aluminum Alloy

- All parts have a mass of 2.525kg, within ±2.5% error (except the solid test part)

- All parts have a 10mm perimeter with no holes

- All polygon holes have a 10mm wide edge

- An equal force will be applied to each part for each test

Each part is not exactly 2.5kg because the skill in SolidWorks and time required to reach that target are beyond the scope of this investigation and the researcher. 2.5kg was chosen as the target mass because it was the approximate mass of the 'square-pattern' part. The 10mm perimeter was included to ensure equal mass where forces will be applied in the various simulations, and that the parts would have a closed perimeter.

Variables that will change based on the shape used, and be recorded, are as follows:

- Hole Area

- Part Surface Area

- Number of holes

A total of 6 different patterns will be tested in simulated stress tests.

1. Filled (control)
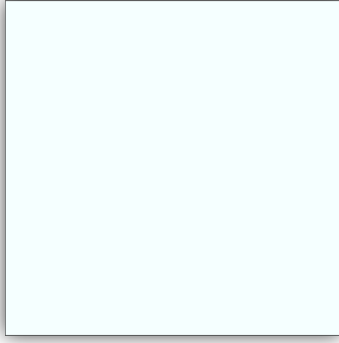
2. Square

3. Square Diamond

4. Hex

5. Hex Diamond

6. Triangle

The 'filled' pattern is a solid sheet. It is used as a reference test as well as a data point to show stress properties when no holes are used. It should be noted that not all holes have the same area since hole patterns that do not match the shape of the part will not fit an integer number of shapes, and some shapes will be 'cut off'. The vertices of incomplete shapes are not identical to the vertices of whole holes. The inconsistent vertices may have an effect on the results of the investigation. The surface area of each part is the 2D surface area of the flat sheet.
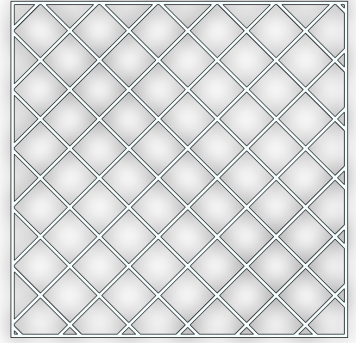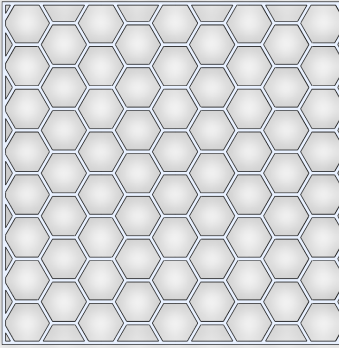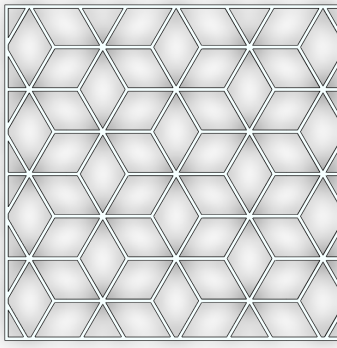
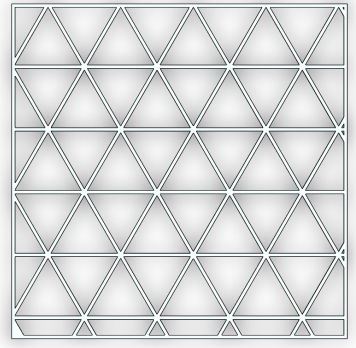Figure 2: Test Parts

(a) Filled
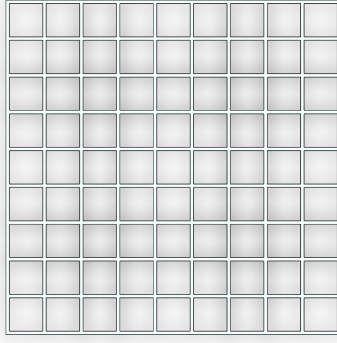
(b) Square

(c) Square Diamond

(d) Hex

(e) Hex Diamond

(f) Triangle

Pattern	Mass (kg)	Surface Area (m^2)	Max Hole Area (cm^2)	n Holes
Filled	13.50	1.000	0	0
Square	2.565	0.190	100.00	81
Square Diamond	2.545	0.188	129.37	62.77
Hex	2.506	0.186	114.55	71.06
Hex Diamond	2.497	0.185	155.38	52.45
Triangle	2.500	0.185	170.65	47.76

(g) Part Properties

2.2 Procedure

A series of stress simulations will be performed on each of the 6 test parts.

1. Linear Tension

2. Linear Compression

3. Torsion

The same input parameters will be used in each simulation, with different directions of force.

1. Fixed geometry includes on side face of the sheet

2. Force of 100N is applied to the face or edge opposite to the fixed face

3. Mesh node size is 20mm^2

For linear tension, the force is applied on the face opposite to the fixed face and directed normal to the face and outward. For linear compression, the force is applied on the face opposite to the fixed face and directed normal to the face and inward. For torsion, the force is applied on the top corner edge of the face opposite to the fixed face.
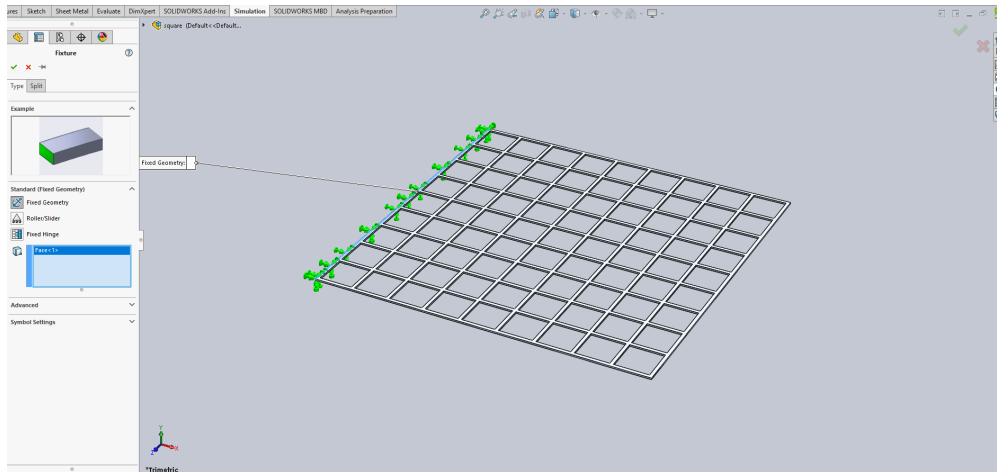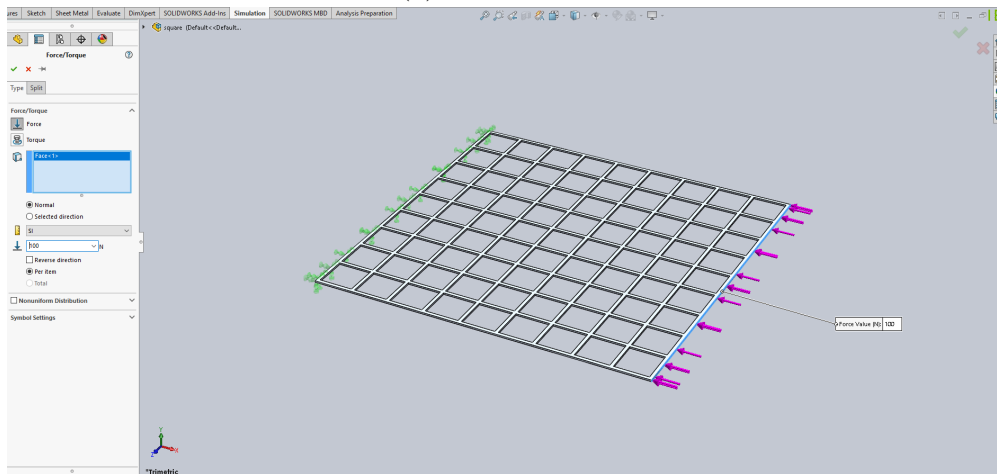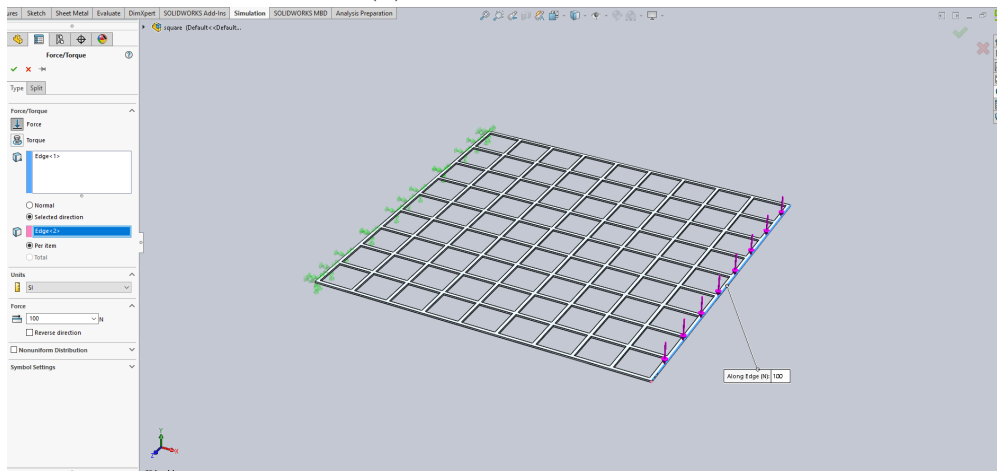
(a) Fixed geometry
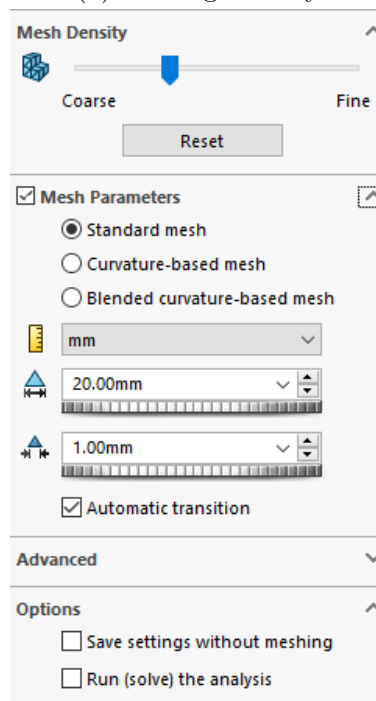
(b) Mesh size
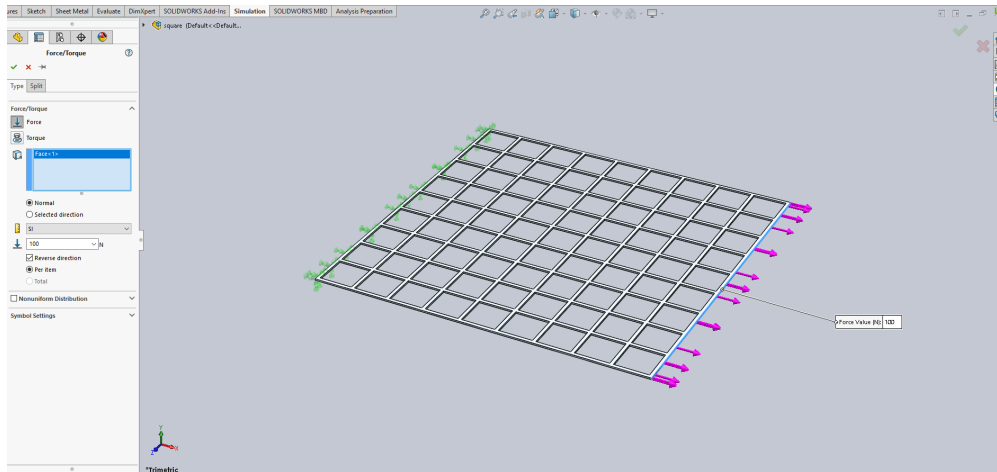
(a) Tension force

(b) Compression force

(c) Torsion force

2.3 Results

Figure 5: Maximum values reached for displacement, stress, and strain, per test and part

Part	Displacement	Strain	Stress
Filled	2.91E+04	5.94E-07	5.92E+04
Square	4.43E-03	1.02E-05	9.44E+05
Square Diamond	2.72E-02	3.28E-05	3.26E+06
Hex	3.15E-02	2.32E-05	2.39E+06
Hex Diamond	5.73E-02	3.52E-05	3.20E+06
Triangle	1.46E-02	2.78E-05	2.93E+06

(a) Maximums in tension

Part	Displacement	Strain	Stress
Filled	2.91E-04	5.94E-07	5.92E+04
Square	4.43E-03	1.02E-05	9.44E+05
Square Diamond	2.72E-02	3.28E-05	3.26E+06
Hex	3.15E-02	2.32E-05	2.39E+06
Hex Diamond	5.73E-02	3.52E-05	3.20E+06
Triangle	1.46E-02	2.78E-05	2.93E+06

(b) Maximums in compression

Part	Displacement	Strain	Stress
Filled	4.23E+01	2.15E-04	2.22E+07
Square	3.90E+02	2.17E-03	2.57E+08
Square Diamond	3.78E+02	2.47E-03	3.65E+08
Hex	3.93E+02	2.30E-03	2.92E+08
Hex Diamond	4.08E+02	2.82E-03	3.49E+08
Triangle	4.02E+02	2.52E-03	2.87E+08

(c) Maximums in torsion

3 Analysis

4 Conclusion

5 Appendix

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