Sonoluminescence: Bogolubov Coefficients for the QED Vacuum of a Time-Dependent Dielectric Bubble

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We extend Schwinger's ideas regarding sonoluminescence by explicitly calculating the Bogolubov coefficients relating the QED vacuum states associated with changes in a dielectric bubble. Sudden (nonadiabatic) changes in the refractive index lead to an efficient production of real photons with a broadband spectrum, and a high-frequency cutoff that arises from the asymptotic behavior of the dielectric constant.

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Sonoluminescence occurs when acoustic energy induces the collapse of small bubbles, and the collapse of these bubbles results in a brief intense flash of visible light [1]. There are several competing mechanisms proposed to explain this phenomenal concentration of kilohertz acoustic energy into petahertz electromagnetic energy. An interesting mechanism, originally proposed by Schwinger [2], is based on changes in the zero point fluctuations of the QED vacuum. In his model, Schwinger estimates the static Casimir energy of an expanded dielectric bubble, compares it with the Casimir energy of a collapsed dielectric bubble, and argues that this Casimir energy difference (difference in zero point energies) would be converted into real photons during collapse of the bubble. This model is often described in terms of the dynamic Casimir effect (even if the calculation is quasistatic). Several authors have argued that this model is not relevant to SL (cf. [3] and references therein). Nevertheless, we feel that the underlying idea of SL as a QED vacuum effect has been prematurely discarded without sufficient analysis.

In this Letter, we consider a variant of Schwinger's proposal which is obtained by focusing our attention on changes in the refractive index rather than on the bubble motion. We explicitly compute the Bogolubov coefficients relating two vacuum states characterized by two different values of the refractive index. Nontrivial Bogolubov coefficients imply the production of real photons. The spectrum is qualitatively compatible with those experimentally observed. Calculations are most easily carried out for extremely large bubbles (large compared to the cutoff wavelength), where the Bogolubov coefficients take on particularly simple forms in terms of delta functions. The spectrum and total energy emission are analytically calculable. For finite bubbles, the delta functions are smeared by finite-volume effects, and the spectrum can be written down as an integral over a suitable sum of spherical Bessel functions. This integral must be evaluated numerically and can then be compared to both the large-volume estimate, and to the experimental situation. Even with a rather crude (step function) model for the refractive index as a function of frequency the resemblance between observed and predicted spectra is quite reasonable. For technical details of the computation, and additional discussion of the history of this proposal, see [3].

Basic features.—One of the key aspects of photon production by a space-dependent and time-dependent refractive index is that for a change occurring on a time scale τ, and a photon of frequency ω, then in the high frequency limit the amount of photon production is exponentially suppressed by an amount $\exp(-\omega\tau)$. The adiabatic approximation always implies such a suppression [3]. The importance for SL is that the experimental spectrum is *not* exponentially suppressed at least out to the far ultraviolet. Therefore any mechanism of Casimir-induced photon production based on an adiabatic approximation is destined to failure: Since the exponential suppression is not visible out to $\omega \approx 10^{15}$ Hz, it follows that *if* SL is to be attributed to photon production from a time-dependent dielectric bubble (i.e., the dynamical Casimir effect), *then* the time scale for change in the dielectric bubble must be of order a *femtosecond.* Thus any Casimir–based model has to take into account that *it is no longer the collapse from $R_{\max}$ to $R_{\min}$ that is important*.

The SL flash is known to occur at or shortly after the point of maximum compression. The light flash is emitted when the bubble is at or near minimum radius $R_{\min} \approx 0.5\ \mu\text{m} = 500$ nm. Note that to get an order femtosecond change in refractive index over a distance of about 500 nm, the change in refractive index has to propagate at relativistic speeds. To achieve this, we must adjust basic aspects of the model: We will move away from the original Schwinger suggestion and *we will postulate a rapid (order femtosecond) change in refractive index of the gas bubble when it hits the van der Waals hard core.* The underlying idea is that there is some physical process that gives rise to a sudden change

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of the refractive index inside the bubble when it reaches maximum compression. Given the fact that the time scale of such a change is much shorter than that typical of the bubble collapse, we shall consider the bubble radius as fixed and equal to the minimal one $R_{\min}$. For the sake of simplicity we take, as Schwinger did, only the electric part of QED, reducing the problem to a scalar electrodynamics. The equations of motion are

$$\epsilon \frac{\partial^2}{\partial t^2} E - \nabla^2 E = 0 . \tag{1}$$

We shall consider two different asymptotic configurations for the gas inside the bubble. An "in" configuration with refractive index n_{in}, and an "out" configuration with a refractive index n_{out}. These two configurations will correspond to two different bases for the quantization of the field. The two bases will be related by Bogolubov coefficients in the usual way. Once we determine these coefficients we easily get the number of created particles per mode and from this the spectrum. Using the inner product

$$(\phi_1, \phi_2) = i \int_{\Sigma_t} \epsilon(r,t)\ \phi_1^* \overleftrightarrow{\partial}_0 \phi_2 d^3x , \tag{2}$$

the Bogolubov coefficients are defined a

$$\alpha_{ij} = (E_i^{\text{out}}, E_j^{\text{in}}), \qquad \beta_{ij} = (E_i^{\text{out}*}, E_j^{\text{in}}) . \tag{3}$$

We focus on the coefficient β_{ij} because it is related to the spectrum, number, and energy:

$$\frac{dN}{d^3\vec{k}_{\text{out}}} = \int |\beta(\vec{k}_{\text{in}}, \vec{k}_{\text{out}})|^2\, d^3\vec{k}_{\text{in}} , \tag{4}$$

$$N = \int \frac{dN}{d\omega_{\text{out}}}\, d\omega_{\text{out}} , \tag{5}$$

$$E = \hbar \int \frac{dN(\omega_{\text{out}})}{d\omega_{\text{out}}}\, \omega_{\text{out}}\, d\omega_{\text{out}} . \tag{6}$$

Homogeneous dielectric.—In the infinite volume limit the eigenmodes are plane waves:

$$E(\vec{x}, t) = \frac{1}{(2\pi)^{3/2}} \frac{\exp(i[\vec{k} \cdot \vec{x} - \omega t])}{\sqrt{2\ \omega}\ n} . \tag{7}$$

We now introduce a "pseudotime" parameter by defining $\partial/\partial\tau = \epsilon(t)\partial/\partial t$, that is, $\tau(t) = \int dt/\epsilon(t)$. Then

$$\frac{\partial^2}{\partial \tau^2} E = c^2 \epsilon(\tau) \nabla^2 E . \tag{8}$$

Now pick a convenient profile for the refractive index

$$\epsilon(\tau) = \frac{1}{2}(n_{\text{in}}^2 + n_{\text{out}}^2) + \frac{1}{2}(n_{\text{out}}^2 - n_{\text{in}}^2)\tanh(\tau/\tau_0) . \tag{9}$$

Here τ_0 represents the typical (pseudotime) time scale for the change of the refractive index. After computation of the Bogolubov coefficient we have to convert back to physical time. Defining $\langle n^2 \rangle = (n_{\text{in}}^2 + n_{\text{out}}^2)/2$ we get

$$|\beta(\vec{k}^{\ \text{in}}, \vec{k}^{\ \text{out}})|^2 = \frac{V}{(2\pi)^3}\, \delta^3(\vec{k}^{\ \text{in}} + \vec{k}^{\ \text{out}})\, \frac{\sinh^2[\pi |n_{\text{in}}^2 \omega_{\text{in}} - n_{\text{out}}^2 \omega_{\text{out}}| t_0 / (2\langle n^2 \rangle)]}{\sinh(\pi n_{\text{in}}^2\ \omega_{\text{in}} t_0 / \langle n^2 \rangle) \sinh(\pi n_{\text{out}}^2 \omega_{\text{out}} t_0 / \langle n^2 \rangle)} , \tag{10}$$

where t_0 is now the physical time scale of the change in the refractive index. For the particular temporal profile we have chosen for analytic tractability this evaluates to $t_0 = \frac{1}{2}\tau_0(n_{\text{in}}^2 + n_{\text{out}}^2)$. The sudden approximation is valid provided

$$\omega \ll \Omega_{\text{sudden}} = \frac{1}{2\pi t_0} \frac{n_{\text{in}}^2 + n_{\text{out}}^2}{n_{\text{out}} \max\{n_{\text{in}}, n_{\text{out}}\}} . \tag{11}$$

Thus the frequency up to which the sudden approximation holds is not just the reciprocal of the time scale of the change in the refractive index: there is also a strong dependence on the initial and final values of the refractive indices. This implies that we can relax, for some ranges of values of n_{in} and n_{out}, our figure of $t_0 \sim O(\text{fs})$ by up to a few orders of magnitude. Unfortunately the precise shape of the spectrum is heavily dependent on all the experimental parameters $(K, n_{\text{in}}, n_{\text{out}}, R)$. This discourages us from making any sharp statement regarding the exact value of the physical time scale required in order to fit the data. In the region where the sudden approximation holds the various $\sinh(x)$ functions in Eq. (10) can be replaced by their arguments x. Then

$$|\beta(\vec{k}^{\ \text{in}}, \vec{k}^{\ \text{out}})|^2 \approx \frac{1}{4} \frac{(n_{\text{in}} - n_{\text{out}})^2}{n_{\text{in}} n_{\text{out}}} \frac{V}{(2\pi)^3} \times \delta^3(\vec{k}^{\ \text{in}} + \vec{k}^{\ \text{out}}) . \tag{12}$$

In the real physical situation n_{in} is a function of ω_{in} and n_{out} is a function of ω_{out}. Schwinger's sharp momentum-space cutoff for the refractive index is equivalent to the choice

$$n(k) = n\Theta(K - k) + 1\Theta(k - K) . \tag{13}$$

More complicated models for the cutoff are, of course, possible at the cost of obscuring the analytic properties of the model. Taking into account the two photon polarizations

$$|\beta(\vec{k}^{\text{in}}, \vec{k}^{\text{out}})|^2 \approx \frac{1}{2} \frac{(n_{\text{out}} - n_{\text{in}})^2}{n_{\text{in}} n_{\text{out}}} \frac{V}{(2\pi)^3} \Theta(K - k^{\text{in}}) \times \Theta(K - k^{\text{out}}) \delta^3(\vec{k}^{\text{in}} + \vec{k}^{\text{out}}) . \tag{14}$$

The spectrum is

$$\frac{dN(\omega_{\rm out})}{d\omega_{\rm out}} \approx \frac{n_{\rm out}}{2\ c} \frac{(n_{\rm out}-n_{\rm in})^2}{n_{\rm out}n_{\rm in}} \frac{V}{(2\pi)^3} 4\pi (k^{\rm out})^2 \times \Theta(K-k^{\rm out}). \tag{15}$$

Thus at low frequencies, where the sudden approximation holds strictly, the spectrum should show a polynomial behavior (instead of the linear one expected for a thermal distribution). (This statement is independent of the explicit form of the profile for the change of the refractive index. Only very special profiles (exponential and never-ending in time) provide an exactly thermal spectrum [4].) The number of emitted photons is

$$N \approx \frac{1}{9\pi} \frac{(n_{\rm out}-n_{\rm in})^2}{n_{\rm out}n_{\rm in}} (RK)^3. \tag{16}$$

The total emitted energy is approximately

$$E \approx \frac{1}{16\pi^2} \frac{(n_{\rm out}-n_{\rm in})^2}{n_{\rm in}n_{\rm out}^2} \hbar c K V K^3 = \frac{3}{4} N\hbar\omega_{\rm max}. \tag{17}$$

So the average energy per emitted photon is approximately $\langle E\rangle = \frac{3}{4}\hbar\omega_{\rm max} \sim 3$ eV. To extract some numerical estimates, recall that in our new variant of Schwinger's model we have $R_{\text{light-emitting-region}} \approx R_{\rm min} \approx 500$ nm. We take $K_{\rm observed} = K n_{\rm liquid} \approx 2\pi/(200\ \text{nm})$ so that $K_{\rm observed}R \approx 5\pi \approx 15$. To get about one million photons we now need, for instance, $n_{\rm in} \approx 1$ and $n_{\rm out} \approx 12$, or $n_{\rm in} \approx 2\times 10^4$ and $n_{\rm out} \approx 1$, or even $n_{\rm out} \approx 25$ and $n_{\rm in} \approx 71$, though many other possibilities could be envisaged. Note that the estimated values of $n_{\rm gas}^{\rm out}$ and $n_{\rm gas}^{\rm in}$ are extremely sensitive to the precise choice of cutoff, and the size of the light emitting region.

More systematically, using $K_{\rm observed}R \approx 15$ we get

$$N = \frac{119}{n_{\rm liquid}^3} (n_{\rm out}-n_{\rm in})^2 \frac{n_{\rm out}^2}{n_{\rm in}}. \tag{18}$$

Solving for $n_{\rm in}$ as a function of $n_{\rm out}$ and N, and taking $N = 10^6$, the result is plotted in Fig. 1. For any specified value of $n_{\rm out}$ there are exactly two values of $n_{\rm in}$ that lead to one million emitted photons.

Finite volume effects.—In finite volume the eigenmodes are combinations of Bessel functions and spherical harmonics, subject to

$$\epsilon = \begin{cases} \epsilon_1 = \epsilon_{\text{bubble-contents}} & \text{if } r < R, \\ \epsilon_2 = \epsilon_{\rm medium} & \text{if } r > R, \end{cases} \tag{19}$$

and satisfying appropriate junction conditions at $r = R$. We limit ourself to just quoting the key result [3]. Introducing $\Delta n \equiv n_{\rm gas}^{\rm in} - n_{\rm gas}^{\rm out}$,

$$\frac{dN}{d\omega_{\rm out}} = \frac{1}{4} R^2(\Delta n)^2 \sum_{l=1}^{\infty} (2l+1) \int d\omega_{\rm in} \left\{\frac{n_{\rm gas}^{\rm out}\omega_{\rm out}^2 + n_{\rm gas}^{\rm in}\omega_{\rm in}^2}{\omega_{\rm out}+\omega_{\rm in}}\right\}^2 |A_\nu^{\rm in}|^2 |A_\nu^{\rm out}|^2 \times \left[\frac{W[J_\nu(n_{\rm gas}^{\rm out}\omega_{\rm out}r/c), J_\nu(n_{\rm gas}^{\rm in}\omega_{\rm in}r/c)]_R}{(n_{\rm gas}^{\rm out}\omega_{\rm out})^2 - (n_{\rm gas}^{\rm in}\omega_{\rm in})^2}\right]^2, \tag{20}$$

where $W(f,g)$ is the Wronskian and the $A_\nu^{\rm in}$ are calculable coefficients depending on the frequency, refractive index of the bubble, and that of the ambient medium. The above is a general result applicable to any dielectric sphere that undergoes a sudden change in refractive index. This expression is far too complex to allow a practical analytical resolution of the general case. For the specific case of sonoluminescence, we have developed suitable numerical approximations. In the infinite volume limit there were two continuous branches of values for $n_{\rm gas}^{\rm in}$ and $n_{\rm gas}^{\rm out}$ that led to approximately one million emitted photons. If we now place the same values of refractive index into the spectrum obtainable from the Bogolubov coefficients derived above, numerical integration again yields approximately one million photons. The total number of photons is changed by at worst a few percent, while the average photon energy (3/4 times the cutoff energy) is almost unaffected. (Some specific sample values are reported in Table I.) The basic result is this: as expected, finite volume effects do not greatly modify the results estimated by using the infinite volume limit. Note that $\hbar\omega_{\rm max}$ is approximately 4 eV, so that average photon energy in this crude model is about 3 eV. In addition, for the specific case $n_{\rm gas}^{\rm in} = 2\times 10^4$, $n_{\rm gas}^{\rm out} = 1$, we have calculated and plotted the form of the spectrum. We find that the major result of including finite volume effects

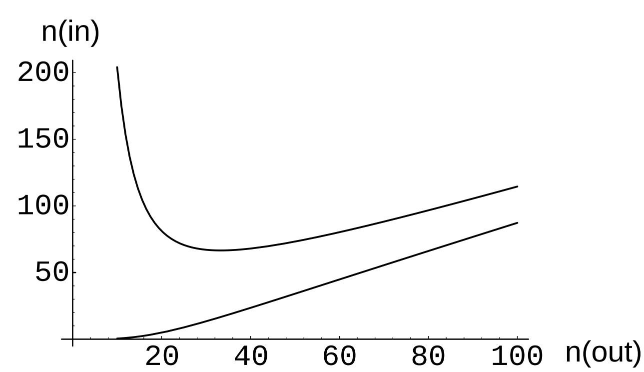

FIG. 1. The initial refractive index $n_{\rm in}$ plotted as a function of $n_{\rm out}$ when one million photons are emitted in the sudden approximation.

TABLE I. Some typical cases.

n_{gas}^{in}	n_{gas}^{out}	Number of photons	$\langle E\rangle/\hbar\omega_{max}$
2×10^4	1	1.06×10^6	0.803
71	25	1.00×10^6	0.750
68	34	1.06×10^6	0.751
9	25	0.955×10^6	0.750
1	12	0.98×10^6	0.765

is to smear out the otherwise sharp cutoff coming from Schwinger's step-function model for the refractive index. Other choices of refractive index lead to qualitatively similar spectra. These results are in reasonable agreement with experimental data.

In conclusion.—We suggest that the key to the SL phenomenon is not the details of the bubble collapse, but rather the way in which the refractive index changes as a function of space and time. Sudden changes in the refractive index will lead to efficient conversion of zero point fluctuations into real photons. Fitting the details of the observed SL spectrum then becomes an issue of building a robust model for such sudden changes of the refractive index of the entrained gases as functions of frequency, density, and composition.

A viable conjecture is that the refractive index of the trapped gas undergoes major changes near the moment of maximum compression, when the molecules in the gas bounce off the van der Waals' hard core. Femtosecond time scales for changes in the refractive index have already been envisaged in the literature [5], but we feel that the present model should encourage a more detailed experimental investigation regarding the dynamics of the refractive index, both that of the entrained gas as well of the water surrounding the bubble.

We stress that *any mechanism* that provides subpicosecond time scales for the change of the refractive index would imply an important contribution of Casimir photons in sonoluminescence. It is not inconceivable that for some such mechanisms (e.g., sudden ionization) and for some regions of the parameter space the dynamical Casimir effect could play an important role in addition to or in opposition to other proposed explanations of photon emission (e.g., bremsstrahlung or shock waves).

Generic features for testing our proposal are [3], nonthermal behavior of the spectrum at low frequencies, absence of hard UV photons (no dissociation of the water molecules), emission of photons bounded in angular momentum ($l \leq KR$). Detection of correlations [6] in the emitted photons has also been identified as a possibly efficient tool for discriminating between vacuum-effect-based models of SL and thermal light emission models.

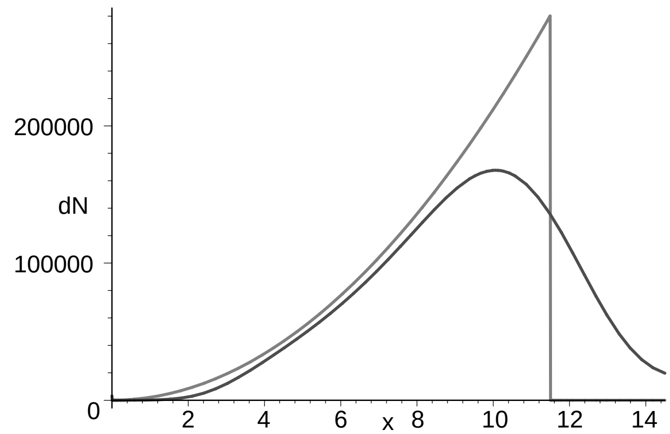

FIG. 2. Spectrum dN/dx obtained by integrating the approximated Bogolubov coefficient. For $n_{out} = 1$ and $R = 500$ nm the relation between the nondimensional quantity x and the frequency ν is $x \sim \nu(10.5 \times 10^{-15}$ s). So $x \approx 11.5$ corresponds to $\nu \approx 1.1$ PHz. The curve with the sharp cutoff is the infinite volume approximation. Finite volume effects tend to smear out the sharp discontinuity, but do not greatly affect the total number of photons emitted.

We argue, both here and elsewhere [3,6], that Casimir-like mechanisms for SL are viable, that they make both qualitative and quantitative predictions, and that they are now sufficiently well defined to be experimentally falsifiable.

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[1] B. P. Barber, R. A. Hiller, R. Löfstedt, and S. J. Putterman, Phys. Rep. **281**, 65–143 (1997).
[2] J. Schwinger, Proc. Natl. Acad. Sci. **89**, 4091–4093 (1992); **89**, 11 118–11 120 (1992); **90**, 958–959 (1993); **90**, 2105–2106 (1993); **90**, 4505–4507 (1993); **90**, 7285–7287 (1993); **91**, 6473–6475 (1994).
[3] S. Liberati, M. Visser, F. Belgiorno, and D.W. Sciama, quant-ph/9805031; quant-ph/9904008; quant-ph/9904013; quant-ph/9905034.
[4] B.L. Hu, in *Thermal Field Theories and Their Applications*, edited by Y.X. Gui, F.C. Khanna, and Z.B. Su (Word Scientific, Singapore, 1996).
[5] E. Yablonovitch, Phys. Rev. Lett. **62**, 1742 (1989).
[6] F. Belgiorno, S. Liberati, M. Visser, and D.W. Sciama, quant-ph/9904018.