Reply to Lambrecht, Jaekel, and Reynaud and to Garcia and Levanyuk:

The two preceding Comments heavily criticize my recent suggestion that the light emission in sonoluminescence might be explained in terms of quantum vacuum radiation [1,2]. I find that the criticism of both of the Comments is unfounded. Lambrecht *et al* fallaciously base general assertions on one particular model which is physically ill-chosen, and Garcia and Levanyuk mistake two illustrational models of Ref. [2] for realistic and make estimates for them under inappropriate approximations. As neither of the two Comments is based on an independent calculation but both rely entirely on formulae taken from my papers [1,2], it appears to me that they have arisen from an incomplete understanding of my theory. I will try and elucidate a number of points.

Lambrecht *et al* estimate, in the short-wavelength limit, the number of photons radiated by a spherical cavity whose time-dependent radius $R(t)$ follows a Lorentzian model profile. I find no fault with their back-of-the-envelope calculation, but I do not agree with their rather general conclusion about fundamental limits of the amount of radiation from a moving spherical cavity. The radiated energy and the number of radiated photons are functionals of the radius $R(t)$ as a function of time, and the properties of a functional can in general not be derived by choosing just one particular kind of function as the argument of the functional. In fact, all that Lambrecht *et al* show is that a Lorentzian profile for $R(t)$ is not a physically realistic choice, and this point has already been made in Ref. [2] on p. 2780. Lambrecht *et al*'s conclusion that the total number of radiated photons is fundamentally limited by the speed of the bubble surface is proven erroneous by a simple counter-example from Ref. [2], pp. 2780–2781. The function $R(t) = R_{\min} + \beta_0 t \tanh(t/\gamma)$ leads to a radiated energy whose leading term behaves like $\mathcal{W} \propto \beta_0^2/\gamma^3$. For this choice of $R(t)$ the velocity $\dot{R}(t) = \beta(t)$ is bounded by $1.2\beta_0$ but $\lim_{\gamma\to 0}\mathcal{W}$ diverges for all non-zero β_0. Hence the radiated energy is not bounded from above, QED. Ditto for the number of radiated photons. As discussed in Refs. [1,2] (cf. e.g. the paragraph below eq. (10) of [1]) the physical reason for this is that the amount of quantum vacuum radiation is to leading order governed not by the velocity $\beta(t)$ nor by the acceleration $\dot{\beta}(t)$ but by the fourth time-derivative $\beta^{(4)}(t)$ of the velocity. Lambrecht *et al*'s problems arise because Lorentzians form a one-parameter family of functions, and hence the velocity β and its fourth derivative $\beta^{(4)}$ are necessarily governed by the same one parameter.

Turning to the Comment by Garcia and Levanyuk, I should like to point out that although the above argument illuminates some principal properties of quantum vacuum radiation it can hardly be used for reliable estimates of the intensity. Apart from the fact that, as illustrated by the above discussion, the result of any such estimate does indeed depend very strongly on the particular choice of a model function for $R(t)$, both Lambrecht *et al* and Garcia and Levanyuk seem to have overlooked the fact that all the equations for the photon number and the radiated energy they quote contain substantial approximations, most notably a short-wavelength expansion (cf. fn. [18] of Ref. [1] and the extensive discussion on pp. 2779–2782 of Ref. [2]), which is to say that these expressions are valid only if the wavelength of the emitted light is very much shorter than the size of the cavity during emission — a condition that is not satisfied for sonoluminescent bubbles. When the wavelength and the cavity radius are of the same order of magnitude, resonance effects occur, as is well-known from scattering theory (cf. e.g. [3]). For a dielectric sphere in an optically thinner medium such resonances lie at real arguments kR of the spherical Bessel functions and lead to the widely known whispering-gallery modes. In the converse case of a spherical cavity in an optically denser medium these resonances occur for complex arguments of the Bessel functions and hence are more difficult to keep track of. As an additional complication the case of quantum vacuum radiation from a sphere brings about products of four spherical Bessel functions (cf. eq. (4.3) of [2]) and not just two as in the standard Mie theory of light scattered from spheres, so that no known analytical techniques can be resorted to. That is why the spectrum of quantum vacuum radiation was calculated numerically in Ref. [2]. For comparison of the numerical results with those obtained analytically in the short-wavelength limit, a Lorentzian model function for $R(t)$ was introduced. The reason for choosing a Lorentzian was that it is governed by just one parameter and its Fourier transform is a pure exponential which forestalls the need for any asymptotic approximations in the analytical calculations. The comparison showed that resonance effects lead to a substantial enhancement over the estimates made in the short-wavelengths approximation.

The purpose of the second model function introduced in Ref. [2] and picked up by Garcia and Levanyuk in their eq. (4) was then just to show explicitly, in the short-wavelength limit, that the amount of quantum vacuum radiation is not principally limited by any limit to the velocity, as discussed above, and that the superluminal velocities arising for Lorentzian model functions are an artefact of a particular choice of model function but not of any physical meaning. The attempts by Garcia and Levanyuk of fitting this model function to experimental data may be correct but bear no significance to any general estimate of the amount of quantum vacuum radiation from a collapsing gas bubble in a fluid.

The difficulty of finding a realistic model for the dynamics of the cavity radius $R(t)$ is twofold. First, there is no good way of estimating the magnitude of the fourth time-derivative of the velocity from general physical arguments; and second, the system of a moving spherical cavity is subject to a back-reaction from the emitted light, i.e. one is dealing with a coupled and highly nonlinear system, as has been emphasized before (cf. Sec. V.C of

Ref. [2]).

Neither Lambrecht *et al* nor Garcia and Levanyuk seem to be sufficiently familiar with the phenomenon of sonoluminescence to have realized that the crucial part of the dynamics of a collapsing sonoluminescent bubble, namely the dynamics in the vicinity of the collapse, is not described by the Rayleigh-Plesset equation. Indeed much current research effort about sonoluminescence is directed towards the accurate description of the bubble dynamics. The works cited by Lambrecht *et al* and by Garcia and Levanyuk emphasize that the Rayleigh-Plesset equation is valid only for low Mach numbers and in the absence of any shock waves and that it therefore can serve at best as a crude approximation to the rather complicated problem of a collapsing sonoluminescent bubble. Extensive numerical simulations of the dynamics inclusive of shock waves have been performed by several authors [4–7]. In particular, Vuong's and Szeri's recent calculations have shown very clearly that the dynamics of the bubble radius $R(t)$ close to its minimum must be expected to involve at least picosecond timescales [7]. In view of the discussion on pp. 2781 and 2782 of Ref. [2] this raises hopes on quantum vacuum radiation as a possible candidate for the explanation of the light emitting mechanism.

As regards the experimental technique of measuring the time-dependence of the bubble radius by Mie scattering from the bubble, the authors of the preceding Comments are apparently unaware that these experiments are fundamentally limited to at least several nanoseconds in their time resolution, although this is pointed out in the papers cited by Lambrecht *et al* and by Garcia and Levanyuk. One reason for this limited resolution is that the photomultiplier tubes have a finite rise time; the other is that the data collection takes place over at least several thousands of acoustic cycles so that the jitter in the bubble dynamics washes out any fast timescales in the recorded data. Hence one cannot expect to see picosecond or even femtosecond timescales in these experiments which therefore cannot be cited as proof of the absence of such timescales.

Finally, when it comes to estimating the fastest timescale in the system, I do not think I agree with Garcia and Levanyuk. At least, I do not know what sound should be at interatomic distances. If 10^6 photons are radiated by a bubble of 0.5μm radius then this means that only 1% of the water molecules in the top layer of the boundary send off a photon each on average. This being the case, the shortest timescale in the system with regard to quantum vacuum radiation is that of the dynamics of this innermost layer of water molecules and this is given by the interparticle collision time of the water molecules with the gas molecules inside the bubble. The interparticle collision time depends very strongly on density and local pressure and under the given circumstances it can certainly lie in the subpicosecond range. Refs. [1,2] have argued that quantum vacuum radiation could presumably still be a viable explanation for the light emission in sonoluminescence if the shortest timescale in the dynamics of the bubble surface is as long as 100fs, because the crude estimate in eq. (9) of [1] does not account for the resonant enhancement away from the short-wavelength limit.

In summary, I find that although both of the preceding Comments are expressed with much emphasis they are based on inconclusive arguments. On the basis of the present experimental evidence there is no reason to eliminate quantum vacuum radiation as a possible candidate for the explanation of the light observed in sonoluminescence.

I would like to thank Andrew J. Szeri for sending me an advance copy of his paper [7]. Financial support through the Ruth Holt Research Fellowship at Newnham College Cambridge is gratefully acknowledged.

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