On the Origin of Shrimpoluminescence

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shrimpity shrimp shrimp shrimp. abstract

I. INTRODUCTION

Snapping shrimp, like our cute friend in Fig. 1 (left), have long been known to produce cavitating bubbles by *snapping* their claws [1; 2; 3]. They have a strong appendage (the *dactyl*) in thier claw [Fig. 1 (center)] that is used to create a very high-velocity jet of water. The low pressure region in the jet's wake forms a bubble [Fig. 1 (t=0 ms)] that, among other things, produces an exceptionally loud noise when it collapses. The sound (i.e. the pressure wave) produced by even a *single* shrimp's snap is detectable over a mile away [4]. The noise produced by groups of shrimp is so intense that the U.S. Navy used them as "sonar-camoflauge" in the Pacific ocean during World War II [1].

The shrimp were not patriots helping the war-effort, however; they snapped for food. The shock-wave produced by the cavitating bubble is used to stun and even kill prey [1]. If the shrimp's prey had very sensitive eyes (and also were not dead) they might notice a flash of light is also produced through an effect referred to as "shrimpoluminescence" in the case of the pistol shrimp [2], but more generally known as *sonoluminescence*.

Sonoluminescence (SL) is more precisely defined as the process by which a "driven gas bubble collapses so strongly that the energy focusing at collapse leads to light emission" [5]. Sonoluminescence comes in two forms: (i) single-bubble sonoluminescence and (ii) multi-bubble sonolumiscence. The distinction is self-explanatory: multi-bubble sonolumiscence (MBSL) consists of "the simultaneous creation and destruction of many seperate, individual cavitation bubbles" [5; 6], whereas in single-bubble sonoluminescence (SBSL), rather obviously, only a single bubble is present [7]. The discovery of MBSL predates SBSL by ∼60 years but due to the more-or-less random and fleeting nature

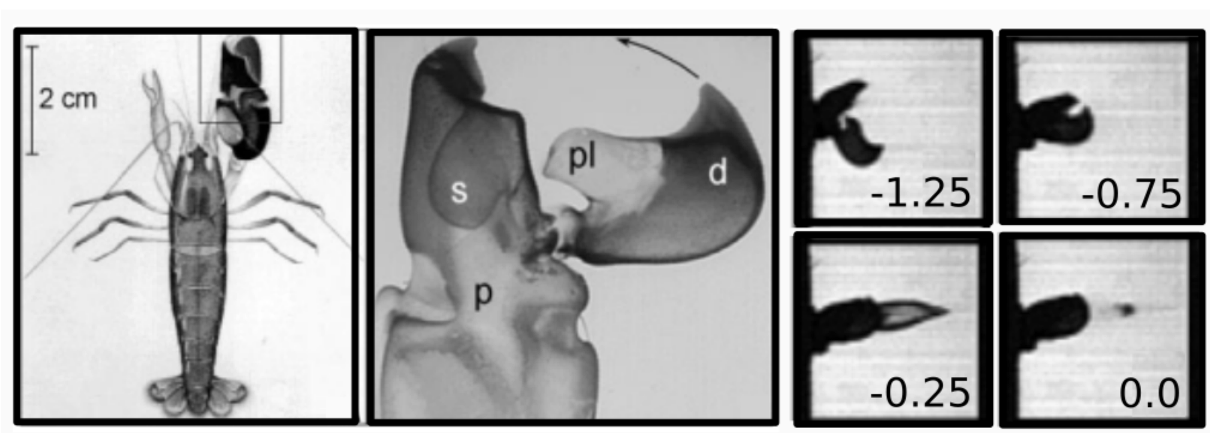

FIG. 1 (left) Snapping shrimp (*Alpheus heterochaelis*). (center) Blown-up view of the shrimp's claw. The *plunger* (pl) on the *dactyl* (d) rapidly enters the *socket* (s), ejecting a high-velocity jet of water. The water ejection and subsequent bubble formation and, finally, bubble collape (at t=0) are shown on the right. The time-offset are shown in the panels in ms. Adapted from ref. [1]

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of the bubbles, SL in general wasn't well studied until the early 1990's when it was discovered that single bubbles could be created and periodically driven to produce light with very high precision [5; 6; 7; 8]. We will discuss MBSL breifly throughout this paper, but for now it suffices to say that nearly all theoretical and experimental progress to explain sonoluminescence has been made using SBSL, with some authors even calling it "the hydrogen atom of sonoluminescence" [6; 9].

In SBSL, emission from a single bubble is not complicated from multiple scattering (off other bubbles), volumetric-light emission from the bubble cloud, etc. Similiarly, the bubble is not perturbed by interaction with other bubbles and, due to its tiny size, by interaction with the container walls and the theory is greatly simplified compared to MBSL. SBSL, while offering advantages, also comes with complications. Due to the tiny volume of the bubble, directly measuring the temperature or pressure *inside* the bubble is practically impossible [10]. Still, the discovery of SBSL created a rush of effort to explain the phenomena with arguments ranging from the simplest models [1] to more exotic explanations based of quantum field theory [11; 12; 13].

But besides the obvious case of shrimpoluminescence, why is SL interesting? Well, at a glance, it is not obvious why SL occurs. The acoustic wave in the liquid displaces molecules on the order of nanometers, costing an elastic energy of $\sim 1 \times 10^{-12}$ eV/molecule while emitting visible light (through e.g. electronic transitions) costs an energy ~ 1 eV/molecule [6; 9]... there is a 12-orders-of-magnitude concentration of energy. That is huge! Estimates of the temperature at the center bubble are ...

After 3 decades of studying SBSL, the physical mechanism of light production in SL is still not understood. This can be seen by glancing at the literature and noticing that there are many recent papers with different arguments for *where* [2] the light comes from [14; 15; 16; 17]. Numerous aspects of the process are conveniently accessible to the experimentalist, while at the same time the theory of the bubble's interior is quite mature. Many theories require experimental inputs as parameters, while in other cases experimental results are indirectly *inferred* by fitting to a theory. Put simply, the history of studying SBSL is a rather beautiful example of the scientific process of explaining nature. Accorind to Brenner "SBSL has become a rather sophisticated testing ground for the ability of mathematical models and numerical simulations to explain detailed experimental data from a complicated physical process" [5].

The goal of this paper is to catch the reader up on recent efforts to explain SBSL. While the author of this paper is particularly excited about shrimpoluminescence, it is important to stress that this work will focus on SBSL in general. It is apparently simpler to create and characterize SBSL in the laboratory without involving shrimp (for example, convincing the shrimp to snap requires tickling them [1; 2; 9]), so practically all work to study SL has not involved shrimp [3]. Also notably absent will be any detailed discussion of MBSL since this will take us too far afield. Instead, we will summarize the history of SBSL research up to now starting at the discovery of MBSL leading to the subsequent discovery of SBSL and a rush of effort to explain it. We will explore both the experimental and theoretical discoveries along the way. Finally, we will review the current state of agreement between what is known experimentally and theoretically about SBSL.

II. HISTORICAL OVERVIEW

SL was discovered by accident in 1933 (in the form of MBSL) by Marinesco and Trillat [18] and was subsequently characterized by Frenzel and Schultes [19] [4] Marinesco et al. were trying to accelerate photo development by *insonating* developing fluid. They discovered that a photosensitive plate immersed in the insonated fluid became "foggy" which they attributed to exposure to light. Shortly after, Frenzel et al. repeated the experiment and confirmed that the insonated fluid emits light in the form of a faintly glowing cloud of bubbles.

This result was not particularly surprising to the community since it had been known for a while that cavitating bubbles could do tremendous damage to e.g. ship's propellers [5]. The discovery's impact is consicesly summarized by Brenner: "if the cloud [of cavitating bubbles] collapses violently enough to break molecular bonds in a solid, why should it *not* emit photons" [5]. In fact, cavitating bubbles had been of interest to engineers working on fluid mechanics for a while. The discovery of cavitation is credited to Euler (as early as 1754) who hypothesized that if the velocity in a fluid was large enough, negative pressures could become so large as to "break the fluid" [7; 20]. Cavitation was confirmed to exist (and named "cavitation") in 1895 by engineers studying the failure of a British

[1] There are far too many to try list here and we will not be interested in the vast majority of early models which are now irrelevant. References to these can be found in Gaitan [8] and Brenner et al. [5].

[2] We emphasive "where" since it's not even certain if the light is *surface* or *volumetric* emission, i.e. it is literally not known *where* the light comes from [].

[3] With notable exceptions [3]

[4] Neither the paper due to Marinesco et al. nor the one due to Frenzel et al. can be found by the author; in any case he can't read French or German so having the papers wouldn't be much use. As such, the story of how the effect was discovered is taken from more recent sources whose authors hopefully could read French and German [5; 6; 8]

[5] A more modern example could be my van's oil pump, which is in need of replacing.

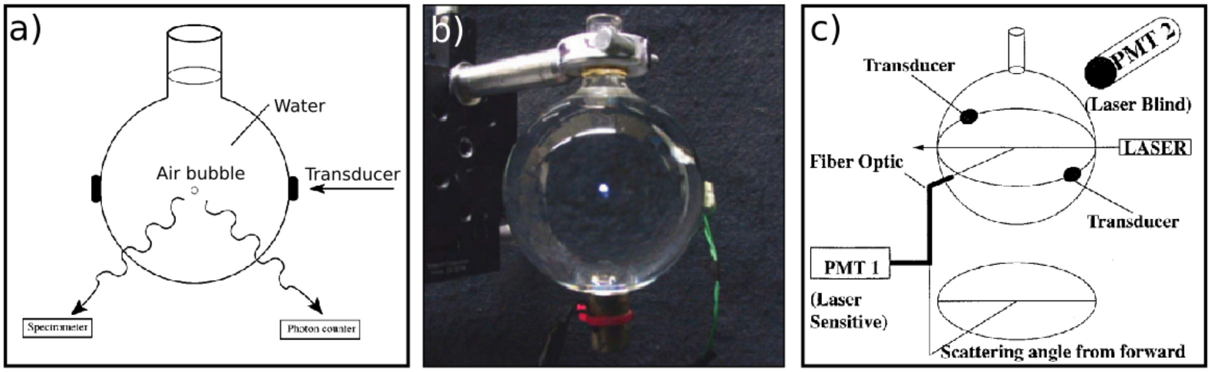

FIG. 2 (a) Schematic of a spherical acoustic levitation cell. The piezoelectirc transducers that drive the bubble trapped in the flask are labeled in the diagram. (b) Photograph of the same. Here, the fluid is H_2SO_4 instead of water. A sonoluminescing bubble is clearly visible in the center of the flask. The photo was taken in a fully lit room with exposure time 2 s. (c) Schematic of a Mie scattering experimental setup. A similar acoustic levitation cell as (a) is shown with an added laser source and photomultiplier tubes (PMT) to analyze the scattered and emitted light. (a), (b), and (c) are from refs. [5], [10], and [28] respectively.

Navy ship's propeller [20]. Shortly after, Lord Rayliegh wrote down and solved the differential equation for a vapor filled cavity collapsing in water (the so-called Rayleigh equation), giving the first rigoruous theoretical treatment of cavitation [21; 22]. Rayleigh found that, for a bubble at a lower pressure than the surrounding fluid (and both pressures held constant), the bubble wall diverges during collapse. Still, very little information was accessible about the light emission until the 1990's when stable [6], single bubbles could be created and driven to emit light [6; 7; 8]. With this discovery, serious interest took root.

Details of a typical SBSL experiment will be given later, but for now it suffices to note that SBSL, unlike previous studies on MBSL, allowed very precise control and measurment of the SL process. With unprecidented experimental control, many discrepencies in previous assumptions about SL were discovered. Experiments measuring the duration of the light pulse found that it was orders or magnitude smaller than the time in which the bubble was compressed to its smallest radius [23; 24]. This discovery implied that SL was nearly decoupled from the bubble's dynamics, contradicting models based on Rayleigh's equation. New models were proposed based on *converging shock waves* at the bubble's center with estimates of the temperature at the center of bubbles $\sim 10^8$ K [25; 26]. At the same time, experimentalists fit the bubble's light emission spectra as a *black body emitter* and concluded that the temperature in the bubble was atleast 25,000 K [27].

These very high estimates for the temperature in the bubbles lead to a spark of interest more broadly: if the temerature in the bubble were really that large, then it should be possible for nuclear *fusion* to occur []. An action movie featuring Keanu Reeves was even made with this at the center of the plot [7]. In the early 2000's, several papers were even published claiming that stable nuclear fusion was possible in the lab using a method based on SBSL []. Unfortunately these results turned out to be fraudulent []. Shortly after this fiasco, evidenced emerged that there are shape instabilities in the bubbles dynamics indicating that the converging shock-wave hypothesis is probably not right. This severely reduced the estimates for the temperature to below $10,000$ K, ruling out the possibility of fusion occuring at all []. Moreover, it was shown that the light is most likely emitted from the *bulk* of the bubble as opposed to its surface, making fits to a black body specturm (i.e. surface emission) incorrect []. Current estimates for the temperature at the center of a sonoluminescing bubble are ... []

Still, the very high pressures and temperatures in the bubble during sonoluminescence see technological applications in chemistry and biosciences. ...

III. WHAT DO WE KNOW?

The conditions in the bubble's *interior* are still not entirely understood and will be the subject of an entire section of this paper. Before we look at what we don't know, it is useful to summarize what we *do* know about SBSL. This

[6] *Stable* is compared with *transient* when discussing caviation. A stable bubble persists through multiple cycles of cavitation, oscillating nonlinearly around an equilibrium size. Transient bubbles appear and then collapse to dissapear

[7] It is called *Chain Reaction* and I am unwilling to watch it.

has two parts: on the one hand, the dynamics of the bubble are experimentally accessible and have been well studied by *Mie scattering* experiments. As we shall see, a (relatively) simple theory describes the dynamics of the bubble quite well. On the other hand, a great deal of effort has been invested in measuring what is going on in the bubble's interior. We will look at what we have been able to figure out.

A. The Bubble Wall

As it turns out, the dynamics of the bubble in SBSL (regardless of the light emission) are quite well described both qualitatively and *quantitatively* by the classical theory of bubble dynamics [5; 10; 22; 29; 30; 31; 32]. This isn't particularly surprising since the light emission and bubble collapse occur at different timescales [5; 22; 29; 30]. The starting point for any theory of the bubble dynamics is Rayleigh's original work expounded upon by many others [22; 30; 33]: the so called "the Rayleigh-Plesset" (RP) equation [8].

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho}\left[\Delta p(t) - \frac{1}{R}\left(2\sigma + 4\mu\dot{R}\right) - P(t)\right]. \tag{1}$$

This is the equation of motion for a driven spherical bubble in an infinite, incompressible fluid. On the left hand side, R is the radius of the bubble, $\dot{R}$ is the velocity of the bubble's radius i.e. the velocity of the *bubble wall*, and $\ddot{R}$ is the bubble wall's acceleration. On the right hand side, ρ is the (constant) density of the fluid. $\Delta p(t) = p_g(t) - p_\infty$ is the deviation of the gas pressure in the bubble, $p_g(t)$, from the static fluid pressure at infinite distance, p_∞. $p_g(t)$ is evaluated at the bubble wall. In the second term, the $\sim \sigma$ part gives the restoring force from surface tension stresses while the $\sim \mu$ term is due to the fluid viscos stresses: σ and μ are the surface tension and viscosity coefficients respectively. The last term on the right, $P(t)$, is the *external pressure*. In the context of SBSL, this is a sinusoidal function driven by the transducers connected to the flask containing the bubble (see the next section).

The Rayleigh equation, as opposed to the RP equation, is equivalent to eq. 1 with the pressure term at a constant value. Write down Rayleighs equation, show that it is recovered from the RP equation, discuss its solution, show that its wall velocity diverges. [5] discusses these and how it is necessary for something to diverege with the same power to oppose the collapse. They call this the Rayleigh collapse.

While the RP equation is pretty, it is non-linear in R, making its analytical solution intractable. In some cases, linearizating in R is reasonable and makes an analytical solution possible but in general, this equation is solved numerically. We will explore this below. Go thru [5] and maybe [8]. Different models include, to different extents, mass and heat transfer across the interface, with good results. These are included by integegrating the Navier-Stokes equations assuming energy is not conserved, mass is not conserved, etc. Just state the different equations and heurestically describe how they come about. The point of all of them is the dynamics are modified somewhat. In [29], these different formulations are all reduced to a particular form of *one parameter* equations with different numerical results. Discuss these and show which one is the most accurate so far.

B. Experimental Results

Creating stable, single bubbles in a laboratory experiment is not particulary challenging and can be done with standard and low-cost materials [9]. A typical experimental setup is shown in Fig. 2; it can be summarized as follows [5; 7; 8; 28; 31; 32; 34]. A sample of degassed liquid is placed into a flask. Coupled to the outside of the flask are piezoelectric transducers of some sort.

what conditions are necessary for light emission (i.e. what is the phase space [9]) (Ar content, driving frequency, P_{max})

[7] has good discussion of early experimental errors. Measuring phase of SL relative to the acoustic field was an early attempt to explain SL. Phase here means how far in time is the SL light emission offset from the driven pressure. In the case of periodic oscillations, this is a phase. A handful of early models assumed that the light emission occured during expansion of the bubble. Others assumed that it was at some point during. Most of these were ruled out when it was observed that the light emission is for a very short duration near maximum compression, though the phase

[8] The RP equation is derived in the appendix

[9] Actually, the phase-space for SBSL is "small". The phase-space being over the variables driving pressure, driving frequency, fluid composition, temperature, ambient pressure, gas content, gas composition, etc... What we are claiming is simple is the experimental setup once the correct parameters are identified is relatively simple.

depends quite a bit on the initial bubble radius, the driving frequency, the resononances of the flask, the physical properties of the liquid, etc [5; 7]. These complicated relationships were unkown to the early investigators of SL and were quite difficult to measure in MBSL. For this reason, it wasn't until SBSL was discovered that many of these models were dismissed [7]. In this paper, they present the discovery of SBSL...

IV. WHAT DON'T WE KNOW

A. The Bubble Wall (Again)

B. Experimental Anomalies

V. LET THERE BE LIGHT!

A. Thermal Arguments

B. Electric Arguments

C. The Casimir Effect

D. The Casimir Argument

VI. SUMMARY

VII. ACKNOWLEDGMENTS

Appendix A: The Rayleigh-Plesset Equation

The Rayleigh-Plesset (RP) equation can be deduced from the Navier-Stokes equations [5; 10; 22; 29; 30; 31]. To that end, we first remind ourselves of the Navier-Stokes equations.

The *total* change (in time) of mass in an arbitrary element of a fluid (whose coordinates and boundary may depend on time) is given by $\frac{d}{dt}\int_{\Omega_t}\rho dV$ where Ω_t is the domain of the fluid element under consdiration and $\rho(\boldsymbol{x},t)$ is the local, time-dependent density of the fluid. Using the transport theorem of fluid dynamics [35], Gauss's theorem, and (with only some loss of generality) taking Ω_t to be a *fluid parcel* (so that the boundary of the fluid parcel, $\partial\Omega_t$, moves with the fluid's velocity, $\boldsymbol{u}(\boldsymbol{x},t)$), this becomes

$$\frac{d}{dt}\int_{\Omega_t}\rho dV=\int_{\Omega_t}\partial_t\rho dV+\int_{\partial\Omega_t}\rho\boldsymbol{u}\cdot\boldsymbol{n}dA=\int_{\Omega_t}\partial_t\rho+\nabla\cdot(\rho\boldsymbol{u})dV. \tag{A1}$$

Now to have *mass-continuity*, which we will require on physical grounds, we set eq. A1 to 0. Basically, we are saying no additional material is being added or removed from the fluid. Since the fluid parcel's boundary Ω_t is arbitrary, we find that $\partial_t\rho+\nabla\cdot(\rho\boldsymbol{u})\equiv 0$ which is true for every point in the fluid volume.

Newton's second law for fluids (*momentum-continuity*) is defined similarly [35; 36]. We write the momentum-density as $\rho\boldsymbol{u}$ with momentum given by $\int_{\Omega_t}\rho\boldsymbol{u}dV$. To simiplify what follows, we focus on only the i^{th} component of velocity at a time. Then Newton's second law can be written as

$$\frac{d}{dt}\int_{\Omega_t}\rho u_i dV=\int_{\Omega_t}\partial_t(\rho u_i)dV+\int_{\partial\Omega_t}\rho u_i(\boldsymbol{u}\cdot\boldsymbol{n})dA=F_i. \tag{A2}$$

Now using Gauss's theorem, the product rule, and mass-continuity we find $\frac{d}{dt}\int_{\Omega_t}\rho u_i dV=\int_{\Omega_t}\rho\frac{du_i}{dt}dV=F_i$. In vector notation this reads

$$\int_{\Omega_t}\rho\frac{d\boldsymbol{u}}{dt}=\boldsymbol{F} \tag{A3}$$

We now turn to the forces. Let us seperate the forces into *body-* and *surface-forces*: $\boldsymbol{F}=\int_{\Omega_t}\boldsymbol{f}_B dV+\int_{\partial\Omega_t}\boldsymbol{f}_S dA$. The body-force density, $\boldsymbol{f}_B$, is due to forces acting in the bulk of the liquid e.g. gravity, electrostatic forces, etc. and is typically from sources external to the liquid. We won't be concerned with these forces here. Instead, let us concentrate on the suface part. If we only consider an ideal fluid, the surface part can be written as $\int_{\partial\Omega_t}\boldsymbol{f}_S dA=$

$-\int_{\partial\Omega_t} p\boldsymbol{n}dA = -\int_{\Omega_t} \nabla p dV$ with p the pressure and the minus sign being due to the sign convention: *out* of the surface $\partial\Omega_t$ is positive.

If instead the fluid is real, we can't neglect viscous forces (i.e. dissipation). We suppose that the viscous surface term can be represented by a tensor, $\hat{\boldsymbol{\tau}}$ such that $\boldsymbol{F}_v = \int_{\partial\Omega_t} \hat{\boldsymbol{\tau}} \cdot \boldsymbol{n}dA = \int_{\Omega_t} \nabla \cdot \hat{\boldsymbol{\tau}}dV$. (Note, here the dot product is to be understood as matrix multiplication). To leading order in gradients of the velocity, (and under the physical constraints that the tensor be symmetric), the viscous-stress tensor is $\hat{\boldsymbol{\tau}} = \hat{\boldsymbol{\mu}} \cdot \hat{\boldsymbol{\epsilon}}$ where $\hat{\boldsymbol{\mu}}$ is the viscosity-tensor and $\hat{\boldsymbol{\epsilon}}$ is the strain-rate tensor: $\epsilon_{ij} = \frac{1}{2}\left[\partial_i u_j + \partial_j u_i\right]$.

In general, $\hat{\boldsymbol{\mu}}$ is a symmetric rank-4 tensor. However, if we assume an isotropic fluid [10], we find that the the viscous-stress tensor can be seperated into two irreducible parts: a scalar, $\hat{\boldsymbol{\epsilon}}^{(v)}$, and traceless symmetric part, $\hat{\boldsymbol{\epsilon}}^{(s)}$ [37; 38]:

$$\begin{aligned} \hat{\boldsymbol{\tau}} &= \xi\hat{\boldsymbol{\epsilon}}^{(v)} + \mu\hat{\boldsymbol{\epsilon}}^{(s)} \\ \hat{\boldsymbol{\epsilon}}^{(v)} &= \delta_{ij}\epsilon_{kk} \\ \hat{\boldsymbol{\epsilon}}^{(s)} &= \hat{\boldsymbol{\epsilon}} - \frac{1}{3}\delta_{ij}\epsilon_{kk}. \end{aligned} \tag{A4}$$

Thus, we see that the viscosity-tensor only has two free components: ξ, the normal viscosity and μ the shear viscosity. It is usual to combine the pressure (i.e. elastic stresses) and the viscous-stresses into the stress-tensor $\hat{\boldsymbol{\sigma}} = -p\hat{\boldsymbol{I}} + \xi\hat{\boldsymbol{\epsilon}}^{(v)} + \mu\hat{\boldsymbol{\epsilon}}^{(s)}$. Combined with this, Newton's second law becomes

$$\int_{\Omega_t} \rho\frac{d\boldsymbol{u}}{dt} - \boldsymbol{F} = \int_{\Omega_t} \rho\frac{d\boldsymbol{u}}{dt} - \nabla\cdot\hat{\boldsymbol{\sigma}} - \boldsymbol{f}_B dV = 0 \tag{A5}$$

Again, since we allow the volume of integration to be arbitrary, the integrand must vanish everywhere: $\rho\frac{d\boldsymbol{u}}{dt} - \nabla\cdot\hat{\boldsymbol{\sigma}} - \boldsymbol{f}_B = 0$. This is the momentum-continuity equation for a fluid. Now we would rather have this explicitly in terms of the velocity $\boldsymbol{u}$. To that end, we may subsitute eq. A4 into the momentum-continuity equation. For e.g. the i^{th} component we may expand out the divergence part as

$$(\nabla\cdot\hat{\boldsymbol{\sigma}})_i = -\partial_i p + \left[\xi + \frac{2}{3}\mu\right]\partial_i(\nabla\cdot\boldsymbol{u}) + \mu\nabla^2 u_i \equiv -\partial_i p + \eta\partial_i(\nabla\cdot\boldsymbol{u}) + \mu\nabla^2 u_i \tag{A6}$$

where we have introduced the *bulk-viscosity* $\eta = \xi + \frac{2}{3}\mu$. Collecting all of the components into vector form and, combined with the mass-continuity equation A1, we arrive at the *Navier-Stokes* (NS) equations for a compressible fluid:

$$\begin{aligned} \rho\frac{d\boldsymbol{u}}{dt} = \rho\left[\partial_t\boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\,\boldsymbol{u}\right] &= -\nabla p + \eta\nabla\left(\nabla\cdot\boldsymbol{u}\right) + \mu\nabla^2\boldsymbol{u} + \boldsymbol{f}_B \\ \partial_t\rho + \nabla\cdot(\rho\boldsymbol{u}) &= 0. \end{aligned} \tag{A7}$$

Some comments are in order. The total-derivative of a moving fluid $\frac{d}{dt} \equiv D_t$ is sometimes called the *material derivative*. In the case of velocity $D_t\boldsymbol{u}$ is the *acceleration* of the fluid. It records not only explicit time dependence of the fluid's velocity $\partial_t\boldsymbol{u}$ but also how the fluid's velocity varies *in space* as it moves past us: $(\boldsymbol{u}\cdot\nabla)\,\boldsymbol{u}$. The pressure term on the right hand side is to be understood as forces from the *elastic* energy. On the other hand, the terms $\sim\boldsymbol{u}$ are viscous (damping) terms that tend to make the velocity field spatially uniform: they vanish when the (spatial) derivatives of the velocity vanish. The bulk-viscosity term $\sim\eta$ damps radial changes in the fluids velocity (e.g. dilation/contraction) while the shear-viscosity term $\sim\mu$ resists shearing. The body term $\boldsymbol{f}_B$ is from external sources: from here on, we will assume it is zero.

Even with these simplifying assumptions, solving eqs. A7 is a formidable task [11] To that end, several more-drastic approximations, valid for our problem, have to be made that allow us to make progess.

1. We will assume irrotational flow (i.e. $\nabla\times\boldsymbol{u}\equiv 0$) and only radial motion in the liquid (i.e. $\boldsymbol{u} = u\boldsymbol{r}$). Note that, since the flow is assumed irrotational, we can represent the velocity as the gradient of a scalar function: $u\boldsymbol{r} = \partial_r\phi\boldsymbol{r}$ [40]. This amounts to assuming that the bubble is always spherical. This seems like a rather drastic approximation but has been validated experimentally in many cases []. The fact that the bubble tends to remain spherical can be understood by accounting for surface-tension at the liquid-bubble interface [].

[10] Definitely true for water which is usually the fluid used in sonoluminescence experiments.

[11] In fact, it's not even clear that the NS equations *can* be solved even for incompressible fluids. In 2000, it was announced that proving smooth, sensible solutions exist earns $1,000,000 [39]. Better hurry though... inflation was 6.8% in Nov. 2021.

2. Next, we assume that the viscous terms are negligible in the bulk dyanimcs of the liquid. For (relatively) low viscosity fluids such as water which is also nearly incompressible, this is an accurate approximation and is widely used [5; 22; 29; 30]. We will acount, to some extent, for viscosity later when looking at the bubble-liquid interface. Moreover, we are assuming that the flow is *isentropic*, i.e. that it is reversible (no damping) and that no heat is exchanged between fluid parcels. We will further assume that the liquid is isothermal, i.e. its temperature is constant (in space). Then pressure p is determined from an instantaneous equation-of-state $p = p(\rho, T)$. In the case of SBSL, this is valid since the bubble makes up a *tiny* fraction of the total volume and, as we will later see, heat-transport across the liquid-bubble interfacing is usually neglected anyway (i.e. the bubble is compressed adiabatically in the thermodynamic sense [12])

3. Related to the fact that the bubble is tiny, we assume that the extent of the liquid is so large compared to the bubble that we may consider the dynamics of the liquid as if there were no bubble present; similarly, we consider the dynamics of a bubble in an infinite, isotropic medium. Of course the bubble-liquid interface enters both systems as a boundary condition [30].

With these simplifying assumptions (and a vector identity [13]) we rewrite eqs. A7 as a system of $1 + 1$ dimensional partial differential equations:

$$\begin{aligned} \partial_r \left[\partial_t \phi + \frac{1}{2} \left(\partial_r \phi \right)^2 \right] &= -\frac{1}{\rho} \partial_r p \\ \partial_t \rho + \partial_r \phi \partial_r \rho + \rho \partial_r^2 \phi &= 0. \end{aligned} \tag{A8}$$

We can integrate the first equation above using the fact that the compressibility is negligible to move the density through the integration [41]

$$\int_{r_\infty}^{r} \frac{\partial}{\partial r'} \left[\frac{\partial \phi}{\partial t} + \frac{1}{2} \left(\frac{\partial \phi}{\partial r'} \right)^2 \right] dr' = -\frac{1}{\rho} \int_{r_\infty}^{r} \frac{\partial p}{\partial r'} dr'. \tag{A9}$$

On the left side, we take the velocity to vanish at infinity. On the right side, we assume that only the static and unperturbed pressures, p_∞ and $P(t)$ respectively, are relevant so that [29; 30; 41]

$$\partial_t \phi + \frac{1}{2} \left(\partial_r \phi \right)^2 = \frac{(p_\infty + P(t)) - p(t, r)}{\rho}. \tag{A10}$$

Eq. A10 gives the pressure at cordinate (r, t) in terms of the velocity of the fluid, the density, and the appied pressure.

Now, let us assume that the velocity potential satisfies a wave equation $\nabla^2 \phi - \frac{1}{c^2} \partial_t^2 \phi = 0$ [14] with c the speed of sound in the fluid. We can again include incompressibilty by recalling that for an incompressible fluid $c \to \infty$ which implies $\nabla^2 \phi = 0$. [15] In this case, we can ignore retardation effects and, remembering that we have spherical symmetry, write the solution of $\nabla^2 \phi$ as

$$\phi = \frac{\psi(t)}{r} \tag{A11}$$

with ψ the time-dependent coefficient. We now use the boundary condition for the velocity at the bubble wall, $u(R) \equiv \dot{R}$ with R the bubble radius, to determine ψ:

$$\dot{R} = -\frac{\psi}{R^2} \Rightarrow \psi = -R^2 \dot{R}. \tag{A12}$$

Thus, we find $\phi(r, t) = -\frac{R^2 \dot{R}}{r}$. Plugging this into the left hand side of eq. A10 and evaluating at the bubble wall, $r \equiv R$

$$\begin{aligned} \partial_t \phi|_{r=R} &= -2\dot{R}^2 - R\ddot{R} \\ \partial_r \phi|_{r=R} &= \dot{R} \equiv u(R) \end{aligned} \tag{A13}$$

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[12] According to Wikipedia, *adiabatic* means *fast* in thermodynamics lingo. This is relevant to us since we are claiming the bubble wall moves so quickly that it is compressed to its maximum pressure before any heat can flow out. On the otherhand, the mechanics lingo implies *adiabatic* to mean *slow*. This is, e.g. the adiabatic theorem in quantum mechanics: a perturbation acts so slowly that the system is in its groundstate at all times.

[13] $(\nabla \cdot \boldsymbol{u})\, \boldsymbol{u} = \frac{1}{2} \nabla(u^2) - \boldsymbol{u} \times (\nabla \times \boldsymbol{u})$

[14] This doesn't come from thin air. We could write the r.h.s. of eq. A10 as the enthalpy, $h \equiv \int \frac{dp}{\rho}$, and similarly introduce $dh = \frac{dp}{\rho}$ in the first line of eq. A8. Eliminating h between the two and dropping small terms $\sim \frac{u}{c}$, we arrive at the homogeneous wave equation [5; 29; 41].

[15] For our problem, it is true that $\left(\frac{1}{c^2} \partial_t \phi \right) / \nabla^2 \phi \sim \left(\frac{R}{\lambda} \right)^2 \ll 1$, i.e. the long wave-length approximation is accurate [29; 30; 41].

Finally, sticking all this together, we arrive at the Rayleigh-Plesset equation [22; 29; 30; 33; 41]

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{p_B(t) - (p_\infty + P(t))}{\rho} \tag{A14}$$

with $p_B \equiv p(r = R, t)$ the pressure in the fluid at the bubble wall. Evaluated beyond the bubble wall, $r > R$, eq. A14 gives the pressure radiated by the fluctuating bubble. Some comments are in order. The combination $p_\infty + P(t)$ is the *ambient* pressure [29; 30] which gives the pressure in the fluid in the absence of the bubble. The left hand side of eq. A14 may be regarded as the kinetic energy (density). The right hand side is the change in enthalpy (density) due to the fluctuating bubble, i.e. the dyanmics of the bubble given on the left hand side are determined by the enthalpy in the fluid due to the bubbles motion. In deriving this equation, we have assumed that the speed of propogation in the fluid is infinite. In reality, this is not true but this approximation will be very good close to the bubble (in the *near field*) where retardation effects are negligible anyway. In a similar way, we have assumed that the energy in the fluid due to distorting its volume is negligible. This approximation will be accurate near the bubble as well since the kinetic energy in this region will dominate [29].

We can refine eq. A14 a little further. Since the motion in the fluid is purely radial, we expect the only relevant stresses to be normal to the bubble wall. Let's denote the radial stress in the liquid at some point r by $s_r(r) = -p(r,t) + 2\mu\partial_r u$. Recall, μ is the fluid viscosity from earlier. Using conservation of mass and momentum, and our expression above for the velocity, we can equate the normal stresses in the fluid to those in the bubble:

$$s_r(R) = -p_B - 4\frac{\mu\dot{R}}{R} = -p_g + 2\frac{\sigma}{R} \tag{A15}$$

where p_g is the pressure in the gas and σ is the surface tension. Solving for $p_B(t)$, we have the relation [5; 29; 30]

$$p_B(t) = p_g(t) - \frac{1}{R}\left(2\sigma + 4\mu\dot{R}\right). \tag{A16}$$

Note that in deriving the equations A8 we neglected the viscosity terms. This was valid since only combinations of viscosity and compressibility $\nabla \cdot \boldsymbol{u}$ appeared, both of which are assumed small. Here, however, we opt to keep the viscosity terms coupling the bubble wall to the fluid. With this, we can rewrite the RP equation as

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho}\left[\Delta p(t) - \frac{1}{R}\left(2\sigma + 4\mu\dot{R}\right) - P(t)\right] \tag{A17}$$

with $\Delta p = p_g(t) - p_\infty$ the deviation of the pressure inside the bubble from the static pressure. With the above assumption that the compression is adiabatic, the conditions inside the bubble are decoupled from the liquid (besides the coupling through the RP equation) and we may regard $p_g(t)$ in the bubble as a given quantity to be determined from an equation of state. Recall $P(t)$ is the time-dependent part of the pressure in the absence of the bubble: we may regard this as an external pressure from e.g. a driving stress (this will be important in the main text).

To summarize, the Rayleigh-Plesset equation eq. A17 gives the equation of motion for a spherical bubble in an incompressible liquid driven by a time dependent pressure $P(t)$. This is one of the more important results for the subject of this paper.

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