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Casimir light: The source

(energy/spectrum)

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ABSTRACT The release of Casimir energy in filling a dielectric hole is identified as the source of coherent sonoluminescence. Qualitative agreement with recently acquired data is found for the magnitude and shape of the spectrum.

The recent attempt to explain coherent sonoluminescence (1) as a dynamical Casimir effect was limited to the circumstance of very small change in dielectric constant (2). But it already reveals two essential characteristics of this phenomenon: the dominance of high frequencies and the volume nature of the effect.

The conditions under which volume effects dominate surface effects merit study. I begin with a static medium of uniform dielectric constant ϵ, which is observed for the time interval T. The effect, on the energy, of an infinitesimal change in ϵ is given by (ref. 3, for the electric field example)

$$\delta W_0 = -T\delta E = \frac{i}{2}\,\mathrm{Tr}[\partial_0 \delta\epsilon \partial_0 G],$$

$$G = [\partial_0 \epsilon \partial_0 - \nabla^2 - i0_+]^{-1},$$

in which 0_+ signifies the approach to zero through positive values.

The Tr(ace) can be evaluated by phase space integrals:

$$\mathrm{Tr}\ldots = \int \frac{(d\vec{r})\,(d\vec{k})}{(2\pi)^3} \int \frac{dt d\omega}{2\pi}\cdots.$$

For a finite spatial volume V and time interval T, one gets

$$\delta E = \frac{i}{2} V\delta\epsilon \int \frac{(d\vec{k})}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{k^2/\epsilon}{k^2 - \epsilon\omega^2 - i0_+}.$$

Then the ω-integration can be carried out in any of several ways: the introduction of Euclidean time ($\omega \to i\zeta$), the deformation of the ω-path around the pole at $k/\epsilon^{1/2}$, or the use of the P(rinciple value)

$$\frac{1}{x - i0_+} = \mathrm{p}\frac{1}{x} + \pi i \delta(x),$$

all of which yield

$$\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{k^2 - \epsilon\omega^2 - i0_+} = \frac{i}{2}\frac{1}{k\epsilon^{1/2}}.$$

Accordingly, one has

$$\delta E = -\frac{1}{4}\frac{\delta\epsilon}{\epsilon^{3/2}} V \int \frac{(d\vec{k})}{(2\pi)^3} k$$
$$= \delta\left[V \int \frac{(d\vec{k})}{(2\pi)^3} \frac{1}{2} \frac{k}{\epsilon^{1/2}}\right],$$

which is then presented as dielectric energy, relative to the zero energy of the vacuum, by

$$E = -V \int \frac{(d\vec{k})}{(2\pi)^3} \frac{1}{2} k \left(1 - \frac{1}{\epsilon^{1/2}}\right).$$

So the Casimir energy of a uniform dielectric medium is negative. It is not hard to believe that, with an $\epsilon(\vec{r})$ that is slowly varying, on an appropriate physical scale,

$$E = -\int \frac{(d\vec{r})\,(d\vec{k})}{(2\pi)^3} \frac{1}{2} k \left(1 - \frac{1}{(\epsilon(\vec{r}))^{1/2}}\right).$$

A small test of this assertion, even under the sharper circumstance of an abrupt dielectric discontinuity, can be found in ref. 4. There, dielectrics of area A and widths a and $2L$ can be examined in the limit where these widths are large on the scale provided by inverse wavenumbers (wavelengths). Then the two dielectric regions have energies that are proportional to the respective volumes, with the factor $1/\epsilon^{1/2}$ distinguishing the $\epsilon > 1$ region from the vacuum.

A dielectric medium, with a spherical vacuum cavity of radius R, has a higher Casimir energy than that of the uniform medium produced by filling the cavity. That excess energy is

$$E_c = \frac{4\pi}{3} R^3 \int \frac{(d\vec{k})}{(2\pi)^3} \frac{1}{2} k \left(1 - \frac{1}{\epsilon^{1/2}}\right)$$
$$= \frac{1}{12\pi} R^3 K^4 \left(1 - \frac{1}{\epsilon^{1/2}}\right),$$

where K is a cut-off wavenumber.

Just such a release of Casimir energy accompanies the collapse of a water bubble. Is one not led to relate the release of electric Casimir energy to the electromagnetic emission of photons? If that relation is taken to be an equality, one identifies the average number of photons as

$$N = \frac{4\pi}{3} R^3 \int \frac{(d\vec{k})}{(2\pi)^3} \frac{1}{2} (\epsilon^{1/2} - 1)$$
$$= \frac{1}{9\pi} (RK)^3 (\epsilon^{1/2} - 1),$$

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which is certainly reminiscent of the result in ref. 2, with its dependence on volume and the cube of the cut-off frequency or wavenumber.

It so happened that the theoretical development had reached this point before I became aware (thanks to Seth Putterman) of experimental data more recent than that of ref. 1. Perhaps the most striking result in ref. 5 is a strong dependence of photon production on the ambient temperature: increasing, by a factor $\gtrsim 10$, when the temperature is lowered from 22°C to 3°C.

My simplified theory makes no reference to temperature. I adopt the photon output at the lowest temperature, divided by 2, appropriate to a single polarization:

$$N = 3 \times 10^6.$$

In ref. 6 one finds a number for the maximum bubble radius

$$R \simeq 4 \times 10^{-3} \text{ cm}.$$

Combined with the index of refraction of water for visible light,

$$\epsilon^{1/2} \simeq \frac{4}{3},$$

the simplified theory then produces the cut-off wavenumber

$$K = 1.7 \times 10^5 \text{ cm}^{-1}$$

or the cut-off wavelength

$$\Lambda = 3.6 \times 10^{-5} \text{ cm}.$$

This is within the ultraviolet region. As such, it is a qualitative success—although coherent sonoluminescence appears to the eye as blue light, the spectrum does extend into the ultraviolet region.

The spectra of ref. 5 use the wavelength scale. For comparison, the Casimir energy is presented as

$$E_c = \frac{2}{3}(2\pi R)^3\left(1 - \frac{1}{\epsilon^{1/2}}\right)\int_\Lambda^\infty d\lambda\lambda^{-5}.$$

Thus, the simplified spectral density is independent of Λ and proportional to λ^{-5}, for $\lambda > \Lambda$, and vanishes for $\lambda < \Lambda$.

The experimental data for temperatures 10°C and 22°C (figure 4 of ref. 5) are roughly reproduced in the visible range between 7×10^{-5} cm and 4×10^{-5} cm. The growing separation between the two curves, in progressing to shorter wavelength, could be an artifact of the presentation, which gives them equal area, despite the unequal total energies.

Finally, I return to the remarkable fact that the number of photons created in a cavity collapse increases with decreasing temperature. This is restated as follows: (*i*) the cut-off frequency increases with decreasing temperature, or (*ii*) the time scale characterizing the final stage of the collapse decreases with decreasing temperature. Perhaps this is nature's reminder that an air bubble and an evacuated cavity are significantly different in some respects.

1. Barber, B. & Putterman, S. (1991) *Nature (London)* **352,** 318–320.
2. Schwinger, J. (1993) *Proc. Natl. Acad. Sci. USA* **90,** 958–959.
3. Schwinger, J. (1992) *Proc. Natl. Acad. Sci. USA* **89,** 4091–4093.
4. Schwinger, J. (1992) *Lett. Math. Phys.* **24,** 227–230.
5. Hiller, R., Putterman, S. & Barber, B. (1992) *Phys. Rev. Lett.* **69,** 1182–1184.
6. Barber, B. & Putterman, S. (1992) *Phys. Rev. Lett.* **69,** 3839–3842.