Casimir light: A glimpse

(radiation/spectrum)

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ABSTRACT   Light emission produced by the reversible collapse of a cavity in a dielectric medium is given an initial, simplified treatment. The agreement between planar and spherical shapes indicates the volume nature of the effect.

The interpretation of coherent sonoluminescence (1) as Casimir light (2, 3) requires a very rapid collapse of dielectric material into a vacuum. In this initial study, I adopt the simplification of instantaneous collapse, while restricting the ensuing infinite spectrum to an upper limit at $\bar{\omega}$. And, to avoid an immediate face-off with the spherical geometry appropriate to a bubble, I first consider parallel planes as contact surfaces between different dielectric regions. Further, as this is an unexplored subject that involves large orders of magnitude, concern with some details can be inappropriate. Such simplifications are signaled by $\sim$. As in ref. 3, only the electric field is considered.

The starting point is the action (2)

$$W = \int (dx)\left[\frac{1}{2}\epsilon(x)(\partial_0 A)^2 - \frac{1}{2}(\vec{\nabla}A)^2 + AJ\right].$$

The implied equation for G, Green's function, relating A and J, is

$$[\partial_0\epsilon(x)\partial_0 - \nabla^2]G(x, x') = \delta(x - x').$$

For a completely uniform dielectric $[\epsilon(x) = \epsilon]$, with a 2+1 $(\vec{r}_\perp + z)$ spatial analysis,

$$G(x, x') = \int \frac{(d\vec{k}_\perp)}{(2\pi)^2} \{\exp[i\vec{k}_\perp\cdot(\vec{r} - \vec{r}')_\perp]\}$$

$$\times \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \{\exp[-i\omega(t - t')]\}g(z, z'; \omega, \vec{k}_\perp),$$

where g, which obeys the differential equation $(k = |\vec{k}_\perp|)$

$$[-\partial_z^2 - \kappa'^2]g(z, z'; \omega, k) = \delta(z - z'),$$

$$\kappa'^2 = \epsilon\omega^2 - k^2,$$

is given explicitly by

$$g = \frac{i}{2\kappa'} \exp[i\kappa'|z - z'|].$$

In this circumstance of dielectric uniformity, and with a causal arrangement of sources, $J = J_1 + J_2$, the factor in the vacuum persistence probability amplitude $(\exp iW)$ that

represents the multiphoton exchange between the emission source J_2 and the detection source J_1 is

$$\exp\left[i \int J_1 G J_2\right] = \exp\left[\sum_\lambda (iJ^*_{1\lambda})(iJ_{2\lambda})\right].$$

Here, λ represents a discrete labeling of ω, $\vec{k}_\perp$, and, for example,

$$J_\lambda =$$

$$\left[\frac{(d\vec{k}_\perp)}{(2\pi)^2}\frac{d\omega}{2\pi}\frac{1}{2\kappa'}\right]^{1/2} \int (dx) \exp\{-i[\vec{k}_\perp\cdot\vec{r}_\perp + \kappa'z - \omega t]\}J(x).$$

In ref. 2 it is recognized that an infinitesimal change in the general form of G produces an additional two-photon emission from the infinitesimal source

$$\delta(JJ) = i\delta G^{-1}$$

$$= i\partial_0\delta\epsilon(x)\partial_0,$$

where the second version is appropriate to an infinitesimal alteration of $\epsilon(x) = \epsilon(z, t)$, and the latter form refers to the geometry under consideration.

To be more specific about that geometry, I recall that, in ref. 3, a limiting connection is made between a sphere and a plane surface, such that the field vanishes at $z = 0$. I accept that boundary condition here for the domain $z > 0$, and consider an initial static regime in which $\epsilon(z) = \epsilon$, $z > l$, and $\epsilon(z) = 1$, $0 < z < l$. The time dependence for instantaneous, near total, collapse and subsequent recovery is contained in the interval $-\frac{1}{2}\tau < t < \frac{1}{2}\tau$, when

$$\epsilon(z, t) = \epsilon, \ z > 0$$

or, $|t| < \frac{1}{2}\tau$,

$$\epsilon(z, t) - \epsilon(z) = \epsilon - 1, \ z < l,$$

and is zero otherwise.

One might presume that the important values of $\epsilon - 1$ are of order unity. Nevertheless, because of its relative simplicity, one should not pass over the situation $|\epsilon - 1| \ll 1$. Then, the infinitesimal expression for $\delta(JJ)$ becomes a first-order statement in which [the function of $\eta(x)$ is the rejection of negative x values]

$$\epsilon(z, t) - \epsilon(z) = \delta\epsilon(z, t)$$

$$= (\epsilon - 1) \eta\left(\frac{1}{2}\tau - |t|\right) \eta(l - z).$$

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In addition, the boundary condition at $z = 0$ implies suitable image sources, or, equivalently, the replacement, in J_λ, of $\exp(-i\kappa z)$ by $\sim\sin(\kappa z)$.

The resulting source, $J_\lambda J_\lambda'$, for the emission of two photons is

$$\sim \left[(d\vec{k}_\perp)(d\vec{k}_\perp')\, d\omega\, d\omega'\, \frac{1}{\kappa\kappa'}\right]^{1/2} \omega\omega' \int_0^{1/2\tau} dt\, \cos(\omega + \omega')t$$

$$\times \int (d\vec{r}_\perp)\, \exp\{-i[\vec{k} + \vec{k}']_\perp\cdot\vec{r}_\perp\}$$

$$\times \left[\int_0^l dz\, \sin\kappa z\, \sin\kappa' z\right](\epsilon - 1).$$

The two-dimensional spatial integral is a quasi-delta function, the square of which is $\sim A\delta(\vec{k}_\perp + \vec{k}'_\perp)$, where A is the finite area of the dielectric body in the x-y plane.

The squared integral

$$\left[\int_0^l dz\, \sin\kappa z\, \sin\kappa' z\right]^2 \sim \left(\frac{\sin(\kappa - \kappa')l}{\kappa - \kappa'}\right)^2 \to \sim l\delta(\kappa - \kappa')$$

exploits the relative largeness of l on the atomic scale. Then one has, effectively, with $lA = V$, a volume,

$$|J_\lambda J_\lambda'|^2 \sim Vdk^2 d\omega\, \frac{\omega}{\kappa}\, \sin^2(\omega\tau)(\epsilon - 1)^2.$$

One focuses on the spectrum by using the integral

$$\int dk^2\, \frac{1}{\kappa} = \int_0^{\omega^2} \frac{dk^2}{(\omega^2 - k^2)^{1/2}} \sim \omega,$$

which yields the differential probability

$$dp \sim Vd\omega\omega^2\sin^2(\omega\tau)(\epsilon - 1)^2.$$

Then, if $\bar{\omega}$ is large compared with $1/\tau$, and dispersion is ignored, the probability of emitting a pair of photons is

$$p \sim V\bar{\omega}^3\, (\epsilon - 1)^2,$$

provided p is small.

Having reached this minor goal, for parallel planes, I now turn to the collapsing spherical cavity, with $|\epsilon - 1| \ll 1$. The replacement of 2+1-dimensional geometry by 3-dimensional geometry gives

$$J_\lambda J_\lambda' \sim \left[(d\vec{k})(d\vec{k}')\, \frac{1}{\omega\omega'}\right]^{1/2} \omega\omega' \int_0^{1/2\tau} dt\, \cos(\omega + \omega')t$$

$$\times \left[\int (d\vec{r})\, \exp\{-i[\vec{k} + \vec{k}']\cdot\vec{r}\}\, \eta(R - r)\right](\epsilon - 1),$$

where R is the initial radius of the cavity.

The square of the quasi-delta function provided by the spatial integral is

$$\sim V\delta(\vec{k} + \vec{k}'), \qquad V \sim R^3.$$

That yields

$$dp \sim Vd\omega\omega^2\sin^2(\omega\tau)(\epsilon - 1)^2,$$

quite the same as before. Indeed, from the dependence on total volume, one might have anticipated that shape is of secondary importance.

1. Barber, B. & Putterman, S. (1991) *Nature (London)* **352**, 318–320.
2. Schwinger, J. (1992) *Proc. Natl. Acad. Sci. USA* **89**, 4091–4093.
3. Schwinger, J. (1992) *Proc. Natl. Acad. Sci. USA* **89**, 11118–11120.