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Physics

Casimir light: Field pressure

(pulse/conservation)

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ABSTRACT **The electromagnetic field is assigned a self-consistent role in which abrupt slowing of the collapse produces radiation and the pressure of the radiation produces abrupt slowing. A simple expression is introduced for the photon spectrum. Conditions for light emission are proposed that imply a high degree of spatial localization. Some numerical checks are satisfied. A study of the mechanical equations of motion suggests an explanation of the very short time scale in terms of oppositely directed field pressures and the speed of light.**

The remarkable phenomenon of coherent sonoluminescence (1) has posed the following question: How does a macroscopic, classical, hydromechanical system, driven by a macroscopic acoustical force, generate an astonishingly short time scale and an accompanying high electromagnetic frequency, one that is at the atomic level? Bubble collapse in itself [Lord Rayleigh had already described it in 1917 (2)] gives no hint of light-creation-capability.

To the latter point, I offer the hypothesis that light plays a fundamental role in the mechanism. Provocatively put:

The collapse of the cavity is slowed abruptly by the pressure of the light that is created by the abrupt slowing of the collapse.

A more manageable approach would separate this into two parts: finding the radiation pulse emitted by an abrupt slowing, and finding the abrupt slowing produced by the emission of a radiation pulse; to be followed by a self-consistent choice of parameters.

I begin with the weak, single pair emission probability of ref. 3, but with instantaneous collapse replaced by a highly localized, but finite loss of dielectric kinetic energy. The initial steps are those of ref. 3 that lead to the pair emission probability amplitude

$$\langle|\vec{k}|\vec{k}'|0\rangle = -i\left[\frac{(d\vec{k})}{(2\pi)^3}\frac{(d\vec{k}')}{(2\pi)^3}\frac{\omega\omega'}{4}\right]^{1/2} \times \int (d\vec{r})dt\,\delta\epsilon(x)\exp[it(\omega+\omega') - i\vec{r}\cdot(\vec{k}+\vec{k}')].$$

What is different is the structure of $\delta\epsilon(x)$, now

$$\delta\epsilon(x) = -(\epsilon - 1)\,\eta(r(t) - r).$$

The situation thus described, for $|\epsilon - 1| \ll 1$, has the uniform dielectric constant ϵ for all radii $r > r(t)$ but is the vacuum of unit dielectric constant for $r < r(t)$.

The differential probability for pair emission, the absolute square of the probability amplitude, is given by the product of the factor

$$\frac{(d\vec{k})}{(2\pi)^3}\frac{(d\vec{k}')}{(2\pi)^3}\frac{\omega\omega'}{4}(\epsilon-1)^2,$$

with the multiple integrals

$$\int\int d^*t\,dt\,\exp[-i(^*t - t)(\omega+\omega')] \times (d^*\vec{r})(d\vec{r})\exp[i(^*\vec{r} - \vec{r})\cdot(\vec{k}+\vec{k}')],$$

where the latter coordinate integrals are restricted by $^*r < {}^*r(^*t)$, $r < r(t)$.

It helps to introduce the new variables

$$\vec{\rho} = \frac{1}{2}(^*\vec{r} + \vec{r}), \qquad \vec{\sigma} = {}^*\vec{r} - \vec{r},$$

$$(d^*\vec{r})(d\vec{r}) = (d\vec{\rho})(d\vec{\sigma}),$$

so that the coordinate integral reads

$$\int (d\vec{\rho})\int(d\vec{\sigma})\exp[i\vec{\sigma}\cdot(\vec{k}+\vec{k}')].$$

Then, in the spirit of ref. 3, for the circumstance $rk \gg 1$, the $\vec{\sigma}$ integral is approximated by $(2\pi)^3\delta(\vec{k}+\vec{k}')$, and the $\vec{\rho}$ integral becomes the volume of a sphere with radius $\frac{1}{2}(r(^*t) + r(t))$.

On eliminating explicit reference to $\vec{k}' = -\vec{k}$, the differential probability for the emission of a photon pair with equal and opposite momenta emerges as

$$\frac{(d\vec{k})}{(2\pi)^3}(\epsilon-1)^2\frac{\omega^2}{4}\frac{4\pi}{3}\int d^*t\,dt\,e^{-2i\omega(^*t-t)}\left[\frac{1}{2}(r(^*t)+r(t))\right]^3.$$

There should be no significant contributions from end points in carrying out a partial integration on $^*t$ and on t. That transforms the integrand of the double time integral into

$$e^{-2i\omega(^*t-t)}\left(\frac{1}{2\omega}\right)^2\frac{3}{2}v(^*t)v(t)\left[\frac{1}{2}(r(^*t)+r(t))\right],$$

in which the respective vs are the time derivatives of the corresponding rs.

The phrase "abrupt slowing of the collapse" is more usefully presented as follows: during the short time interval in which the $|v|$s decrease significantly, the rs undergo a small relative change. Thus the latter bracket is effectively the constant r. Then the double time integral emerges as the absolute square of a single integral, which, through an additional partial integration, introduces the acceleration: $d/dt\ v(t)$. The outcome for the differential probability is

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$$\frac{(d\vec{k})}{(2\pi)^3}(\epsilon-1)^2\frac{\pi}{8}r\left|\int dt\,\frac{1}{2\omega}e^{2i\omega t}\frac{d}{dt}v(t)\right|^2.$$

Perhaps the simplest model for the acceleration is the Gaussian

$$\frac{d}{dt}v(t)=|v|\gamma\pi^{-1/2}\exp[-\gamma^2(t-t_0)^2],$$

where v is (approximately) the velocity at the onset of slowing, just prior to time t_0. The implied Fourier transform is

$$\int dte^{2i\omega(t-t_0)}\frac{d}{dt}v(t)=|v|\exp\left[-\frac{\omega^2}{\gamma^2}\right].$$

The somewhat remote aspect of these considerations ($|\epsilon-1|\ll 1$) makes them—at best—qualitative. Accordingly, I omit all fixed factors and focus on the shape of the energy spectrum (energy per wavelength):

$$\sim\frac{1}{\lambda^3}\exp\left[-\frac{3}{2}\left(\frac{\lambda_o}{\lambda}\right)^2\right],$$

where λ_o locates the peak intensity. For long wavelengths ($\lambda\gg\lambda_o$), the intensity varies as $1/\lambda^3$, whereas the Gaussian structure dominates for $\lambda\ll\lambda_o$. For $\lambda=\frac{1}{2}\lambda_o$, the intensity drops, relative to the peak value, by the factor 1/11.3; the analogous factor for $\lambda=2\lambda_o$ is 1/2.6.

What is so remarkable about coherent sonoluminescence is not that macroscopic bodies can produce lights—our early ancestors could manage that—but that the light emission is confined to a very short time interval. In order to be quantitative, I submit this more specific hypothesis.

The conditions for light emission are at hand when the fluid kinetic energy becomes independent of t, for a short time interval, and similar remarks apply immediately after the emission act. In effect, one is picking out the circumstances for spontaneous radiation, from a coherent state of definite energy, to another such state of definite, lower energy.

Outside regions of electromagnetic activity, the conserved mechanical energy is (4)

$$E=2\pi\rho_0\left[r^3v^2+\Omega^2r^3+\frac{p_0}{\rho_0^3}\frac{2}{3}r_0^3\left(\frac{r_0}{r}\right)^{3(\gamma-1)}\right]$$
$$\equiv T+V_C+V_p.$$

For a conserved E, and a locally constant in time T, so must V_C+V_p be locally constant in time, which means that V_C+V_p have a vanishing r derivative. This singles out an equilibrium point,† where the attractive force of V_C balances the repulsive force of V_p. Notice that the statements for the two circumstances are the same; there is a unique radial distance, r_ℓ, where the light is emitted. It is given by

$$1=\frac{p_0}{\rho_0\Omega^2}\frac{2}{3}(\gamma-1)\left(\frac{r_0}{r_\ell}\right)^{3\gamma},$$

or

$$\frac{r_0}{r_\ell}=\left[\frac{3}{2}\frac{\rho_0\Omega^2}{p_0}\frac{1}{\gamma-1}\right]^{1/3\gamma}.$$

†In ref. 4, the term "equilibrium radius" was misleadingly applied to r_0. It is more appropriately assigned to r_ℓ.

For $\gamma=1.4$, and $p_0=\rho_0\Omega^2$, $r_0/r_\ell=1.37$. Thus, in the example of ref. 5, where $r_0\simeq 4\times10^{-4}$ cm, one has $r_\ell\simeq 3\times10^{-4}$ cm.

I enquire again about the magnitude of $v_{a\ell}$, the radial velocity immediately *a*fter the blast of *l*ight. This time there is no appeal to information about the first bounce, and, of course, r_0 and r_ℓ are no longer equated. The assumption of energy conservation over the path between r_ℓ and $\mathbf{r}$, the minimum radius, as expressed by

$$r_\ell^3\left[\frac{v_{a\ell}^2}{\Omega^2}+1+\frac{2}{3}\left(\frac{r_0}{r_\ell}\right)^{3\gamma}\right]=\mathbf{r}^3\left[1+\frac{2}{3}\left(\frac{r_0}{\mathbf{r}}\right)^{3\gamma}\right]$$

with the experimental inputs $r_0=4\times10^{-4}$ cm, $\mathbf{r}=0.7\times10^{-4}$ cm, yields

$$\frac{v_{a\ell}^2}{\Omega^2}=10.4,\qquad |v_{a\ell}|\simeq 3\times10^3\ \text{cm/s}.$$

One sees the possibility of calculating $|v_{a\ell}|$ either by introducing the first bounce parameter r_1, but not the minimum radius $\mathbf{r}$ (ref. 4, without the distinction between $r_{a\ell}$ and r_0), or, by using $\mathbf{r}$, without reference to r_1. The two procedures are connected by the statement of energy conservation for the first bounce:

$$\left(\frac{r_1}{r_0}\right)^3+\frac{2}{3}\left(\frac{r_0}{r_1}\right)^{3(\gamma-1)}=\left(\frac{\mathbf{r}}{r_0}\right)^3+\frac{2}{3}\left(\frac{r_0}{\mathbf{r}}\right)^{3(\gamma-1)}.$$

The introduction of $\mathbf{r}=0.7\times10^{-4}$ cm yields $r_1=7\times10^{-4}$ cm, and conversely.

Finally, I look at the Lagrangian that includes both mechanical and electromagnetic effects:

$$L=T-V_C-V_p+L_e,$$

where the $\epsilon(\vec{r}t)$-dependent part of L_e is

$$L_\epsilon=\int(d\vec{r})\,\frac{1}{2}\epsilon(\vec{r}t)\left[\frac{1}{c}\frac{\partial}{\partial t}A(\vec{r}t)\right]^2.$$

The contributions of the two regions, on opposite sides of the moving surface $r(t)$, are conveyed by

$$\epsilon(\vec{r}t)=\epsilon\times\eta[r-r(t)-0]+1\times\eta[r(t)-r-0],$$

where 0 is approached through positive values.

The stationary property of the actions, for the infinitesimal variations $\delta r(t)$, yields

$$[4\pi\rho_0r^3]\frac{d}{dt}v(t)+\cdots$$
$$=\int(d\vec{r})\,\frac{1}{2}\left[\frac{1}{c}\frac{\partial}{\partial t}A(\vec{r}t)\right]^2\{\delta[r(t)-r-0]-\epsilon\delta[r-r(t)-0]\},$$

where $\delta[\ ]$ is a delta function, and $\cdots$ refers to terms of lesser degree in the time derivative, which will be of secondary importance for abrupt slowing.

Just before that begins, there is no significant field, and $v(t)$ is essentially constant. Then the field strength rises rapidly in the vacuum region, giving a positive value to the right side of the above equation. Accordingly, $v(t)$, which is negative, must move toward smaller magnitudes—the slowing has begun. That process will cease when the field, flowing at the speed of *light* toward the outer dielectric region, has produced the countering pressure.

For some specific results, it is useful to separate radial and transverse aspects,

$$(d\vec{r}) = dr\, dS,$$

and then perform the r integration in the respective volumes. On defining F_+, the outward surface force, and F_-, the inward force, which include the unwritten contributions of the other field type, one gets

$$[4\pi\rho_0 r^3]\frac{d}{dt}v(t) = F_+(t) - F_-(t).$$

First, multiply by the positive quantity $-v(t)$, and then integrate over the time span of the emitted pulse:

$$T_{b\ell} - T_{af} = \int dt(-v)(F_+ - F_-)$$

$$= \int |dr|(F_+ - F_-).$$

One sees that the kinetic energy released equals the work done on—the energy transferred to—the electromagnetic field.

Momentum, defined mechanically by

$$p = \frac{\partial T}{\partial V} = [4\pi\rho_0 r^3]v,$$

now enters as

$$\frac{d}{dt}p(t) = F_+(t) - F_-(t).$$

The integrated consequence here is

$$|p|_{b\ell} - |p|_{a\ell} = \int dt(F_+ - F_-):$$

the momentum removed from the mechanical system is transferred to the electromagnetic field.

The somewhat mysterious initial hypothesis has emerged clarified, as an unusual example of a familiar fact—spontaneous emission of radiation by an electrical system is a single, indivisible act that obeys the laws of energy and momentum conservation.

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