Cross-asset Strategy with Dynamic Asset Allocation

Mathematical Models of Investment Course Project

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Abstract

This report analyzed the factor exposure for 15 different assets using Lasso regression with six different factors. The resulting exposure was then used to perform dynamic asset allocation based on factors, under different settings including weight and target exposure constraints. The performance of this approach was compared to an asset-based model, and conclusions were drawn based on the analysis. Our findings suggest that dynamic asset allocation based on factors can outperform asset-based models in certain scenarios, and that the lasso regression approach can be an effective tool for analyzing factor exposure in a portfolio.

1 Datasets

1.1 Asset Data

The table below showcases the assets we have selected along with their corresponding Bloomberg indices. There are a total of five categories and 15 assets, with data spanning from 2017 to 2022, comprising of 72 monthly data points.

Asset	Index	Bloomberg Code
Equity	S&P 500 Index	SPX Index
	NASDAQ 100 Stock Index	NDX Index
	Russell 2000 Index	RTY Index
	MSCI Emerging Markets Index	MXEF Index
	MSCI World Excluding United States Index	MXWOU Index
Bond	J.P. Morgan EMBI Global Diversified Composite	JPEIDIVR Index
	U.S. TIPS Return Un	I01551US Index
	Bloomberg Global High Yield Total Return Index Value	LG30TRUU Index
	Bloomberg Global Agg Treasuries Total Return Index	LGTRTRUU Index

Commodity	Refinity/CoreCommodity CRB(R) Index	CRY Index
Alternative	S&P Listed Private Equity Index	SPLPEQTY Index
	HFRX Global Hedge Fund Index	HFRXGL Index
	MSCI World Real Estate Index	MXWO0RE Index
	S&P Global Infrastructure Index	SPGTIND Index
Cash	ICE LIBOR USD 1 Month	US0001M Index

1.2 Factor Data

Below is a table showing the 6 types of risk factors we focused on, along with their meanings and calculation formulas (referenced from examples in the course materials).

Factors	Description	Proxy
Equity	Risk associated with global equity markets	MSCI AC World Total Return Index
Inflation	Risk of bearning exposure to changes in nominal prices	Barclays Global Government Bond Index - Barclays Global Inflation Linked Bond Index
Real Rates	Risk of bearning exposure to real interest rate changes	Barclays Global Inflation Linked Bond Index
Commodity	Risk associated with commodity markets	Bloomberg Commodity Index
Credit	Risk of default or spread widening	Barclays Global Credit Index - Barclays Global Government Bond Index
Emerging Market	Risk that emerging soverign governments will change capital market rules	MSCI EM Index - MSCI World Index

2 Factor Exposure Calculation Using Lasso

2.1 Lasso Regression

Instead of using Stepwise regression, we chose to use Lasso regression to calculate factor exposure for each asset. Lasso regression has a penalty coefficient λ to control the number of regression coefficients, achieving the desired effect of a sparse solution. Additionally, Lasso regression can handle variable collinearity.

The algorithm of Lasso is as follows:

$$(\hat{\alpha},\widehat{\boldsymbol{\beta}}) = \underset{\alpha,\boldsymbol{\beta}}{\operatorname{argmin}}\{\frac{1}{N}\|\boldsymbol{Y}-\alpha\mathbf{1}-X\boldsymbol{\beta}\|_2^2+\lambda * Penalty(\boldsymbol{\beta})\}$$

Where

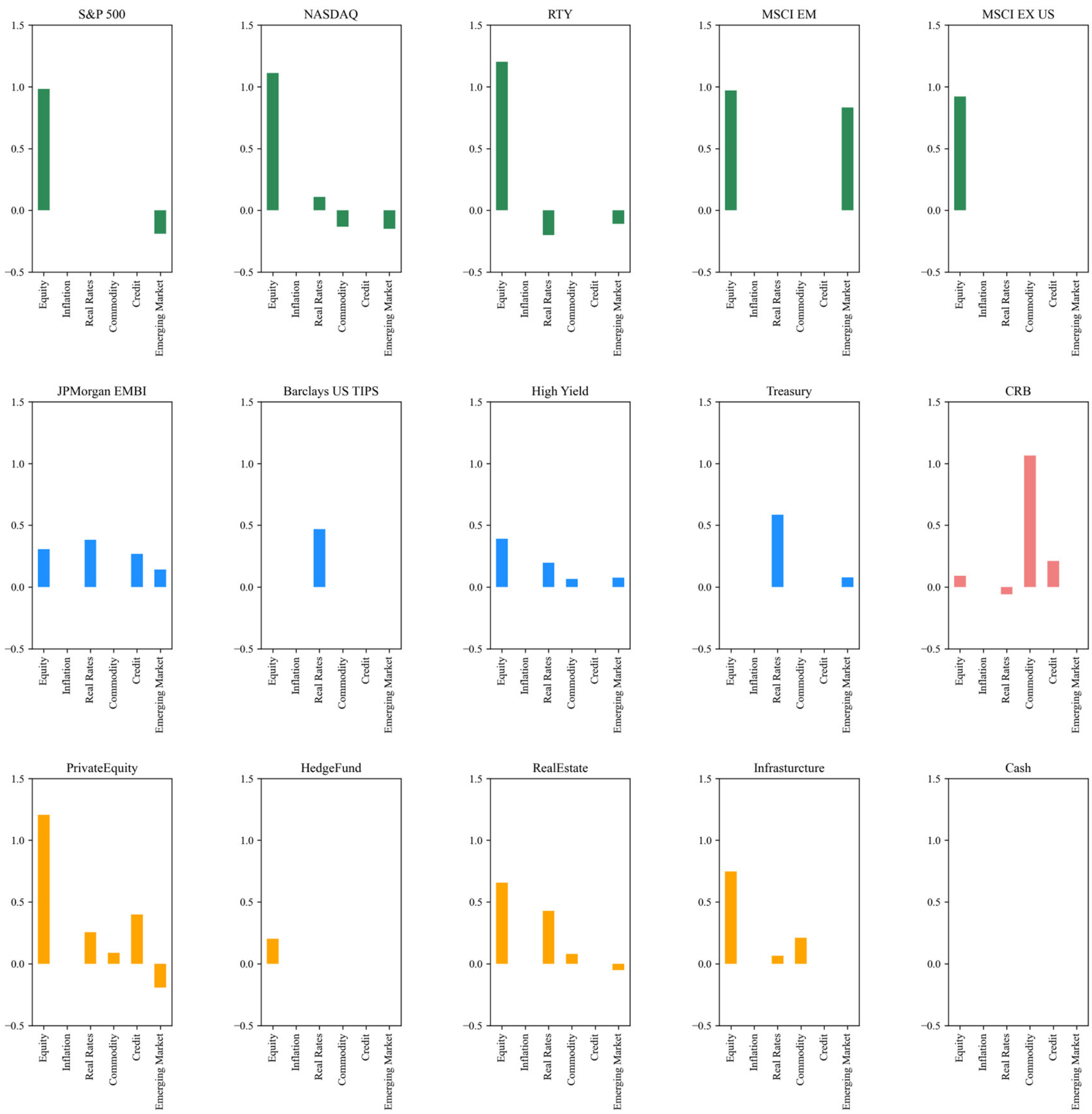
$$Penalty(\boldsymbol{\beta}) = \|\boldsymbol{\beta}\|_1$$

In this report we set $\lambda = 0.0001$ in the regression.

2.2 Factor Exposure for Each Asset

The figure below displays the results of the Lasso regression, with each color representing a different asset category. In fact, we had previously tried Stepwise regression and found that Lasso was indeed capable of solving the collinearity problem compared to Stepwise regression. For example, under the commodity risk factor, inflation would have a negative correlation coefficient under Stepwise regression, which contradicts our understanding.

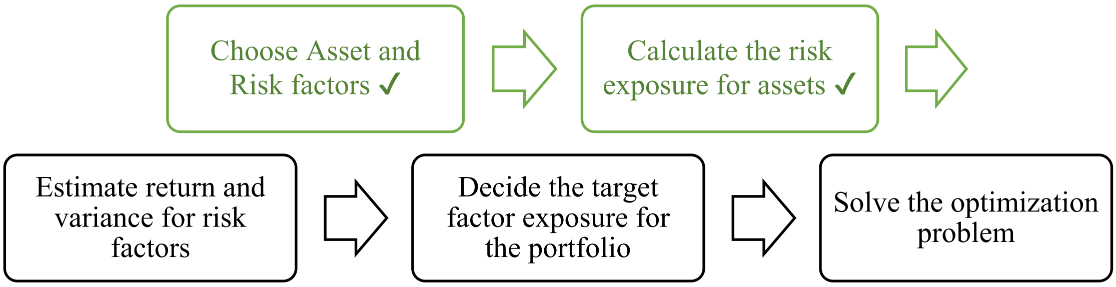
We obtained a sparse solution, and the results are consistent with our expectations. For example, assets under the equity category (S&P, NASDAQ, etc.) have significant positive exposure to equity risk, with coefficients around 1, and negative exposure to the emerging market risk. MSCI EM has positive exposure to the emerging market risk, and the commodity index shows a clear correlation with its corresponding factor. Besides, cash (LIBOR) has no correlation with any of factors.



3 Factor-based Asset Allocation

3.1 Framework and Settings

The following image is a flowchart for asset allocation. We have already completed the first two steps.

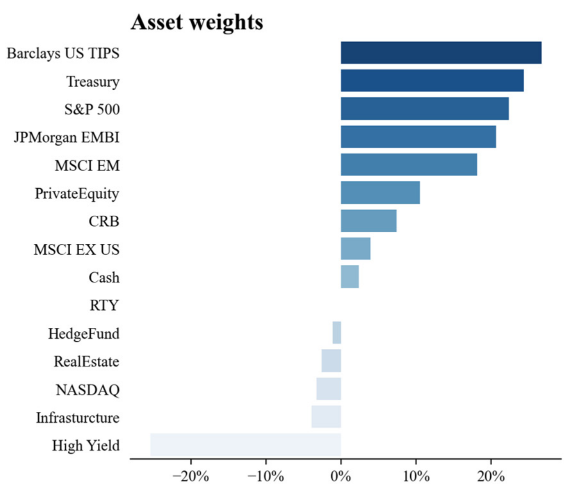


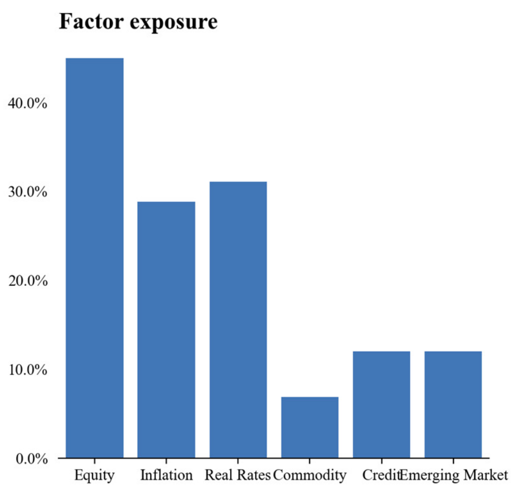
For the third step, we use mean and variance of the historical factor returns as the estimates.
For step 4, we will set four different constraints to solve the problem:

Case	Target Factor Exposure						Asset Weight Range	Other Constrains
	Equity	Inflation	Real Rates	Commodity	Credit	Emerging Market		
1	0.45	0.29	0.31	0.07	0.12	0.12	[-100%, 100%]	-
2	0.45	0.29	0.31	0.07	0.12	0.12	[0%, 100%]	-
3	0.45	0.29	0.31	0.07	0.12	0.12	[0%, 100%]	Exclude illiquid assets
4	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	[-50%, 50%]	-

3.2 Result and Visualization

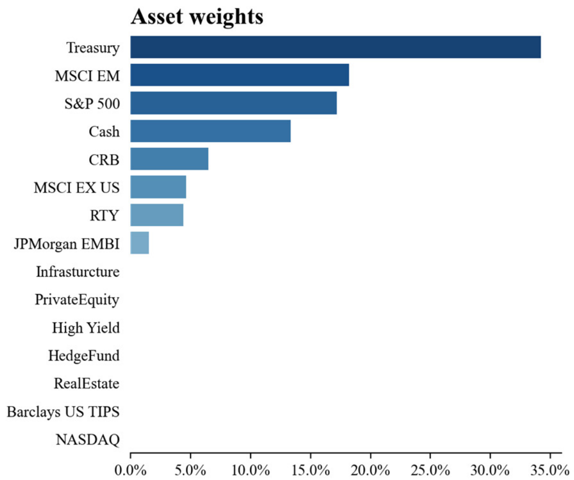
3.2.1 Case 1

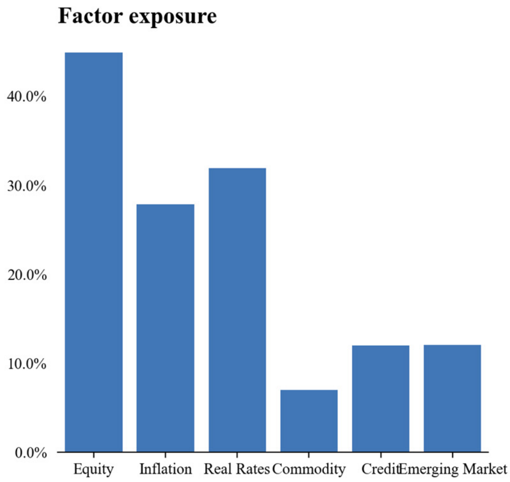




Case 1 set a target exposure, referring to the weight provided in class, which will be used in Case 2 and Case 3. There were not many restrictions on weight setting, which ranged from -100% to 100%. From the results, the weight of short positions in high yield bonds was high, and most of the short positions were in the alternative asset class. Factor exposure met our target.

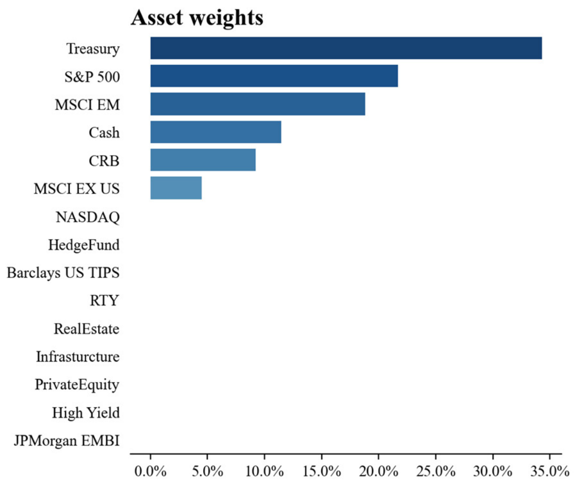
3.2.2 Case 2

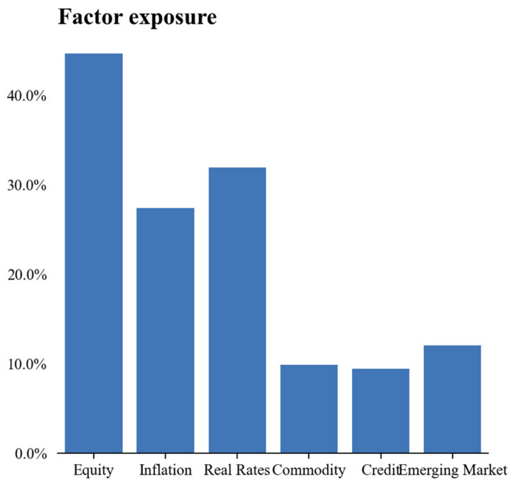




In case 2, this is a long only and fully investment strategy, which means all weights are greater than 0. We set the minimum asset weight to 0 and found that there were no small weights that made the strategy impractical. If we set a lower limit greater than 0, such as 2.5%, we would not reach the target exposure. Therefore, we chose [0%, 100%] as the interval for weights. The results show that the strategy's final factor exposure also reached our target, with weights concentrated in equity and bond assets.

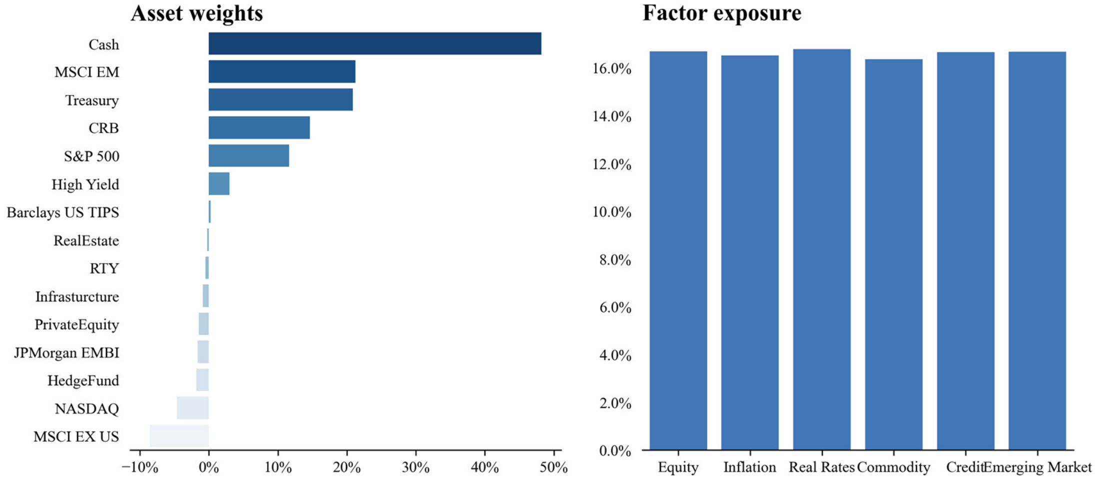
3.2.3 Case 3





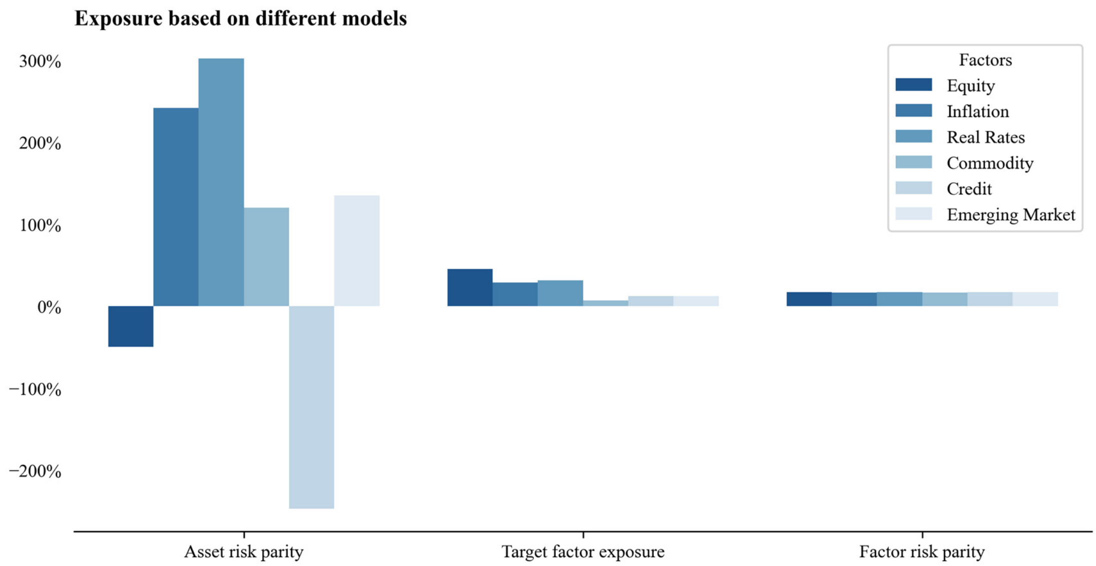
In Case 3, we removed assets with weak liquidity. Similar to Case 2, we set the minimum weight to 0 because we did not observe any small weights in the results. We found that besides some assets with zero weight, such as those that are forced to be zero, the weight of assets such as NASDAQ and US TIPS also decreased to 0. As for factor exposure, except for credit risk exposure being slightly lower than the target, all other exposures were consistent with the settings.

3.2.4 Case 4



Different from the target exposure of the first three strategies, Case 4 adopts Factor Risk Parity, i.e., equal exposure to each risk factor. The results show that factor exposure has met our expectations. In terms of asset weight, the weight of cash is high. If we limit the weight range between -30% to 30%, this imbalance will significantly decrease. However, if we further shorten the range, there will be a significant change in the distribution of strategy weights.

3.3 Comparison



In the figure, we compare the factor exposure based on the Asset risk-parity model (target risk = 0.02) with the results from section 3.2. Despite the apparent reasonableness of the Asset risk-parity model, which controls for the same marginal contribution to risk, it is evident that its exposure is highly imbalanced. This is not conducive to risk management.

4 Conclusion

Compared to an asset-based model, the factor-based model has a significant advantage in that it clearly explains the main driver of portfolio return and risk. The factor-based model also allows for the control of factor exposure within an ideal range, while asset-based models often result in unbalanced outcomes. However, there are various methods for setting targets in the factor-based model, and this report used the weight provided by the instructor. This setting is highly subjective, leading to differences in results. Additionally, we did not make other adjustments to correlations between risk factors, but the Lasso regression used to calculate exposure reduced the impact of correlation to some extent.