GEOS F431 / F631
Foundations of Geophysics

– Strain –

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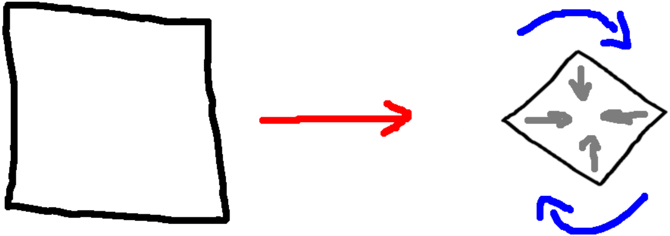
September 08, 2020

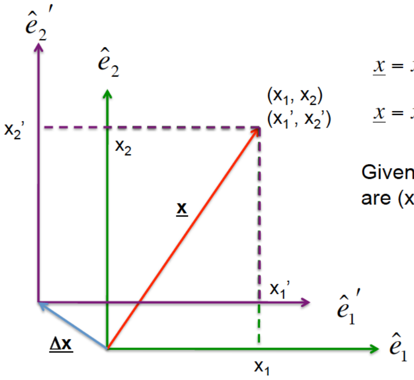
Deformation



deformation = translation + rotation + dilatation

- translation, rotation: rigid body deformation (angles, volume preserved)
- dilatation: volume changes, angles change

Transformations: Translation



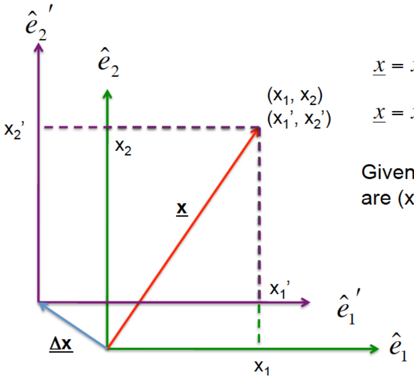
$$\underline{x} = x_1\hat{e}_1 + x_2\hat{e}_2$$

$$\underline{x} = x_1'\hat{e}_1' + x_2'\hat{e}_2'$$

Given (x_1, x_2), what are (x_1', x_2')?

J. Freymueller

Transformations: Translation



$\underline{x} = x_1\hat{e}_1 + x_2\hat{e}_2$

$\underline{x} = x_1'\hat{e}_1{}' + x_2'\hat{e}_2{}'$

Given (x_1, x_2), what are (x_1', x_2')?

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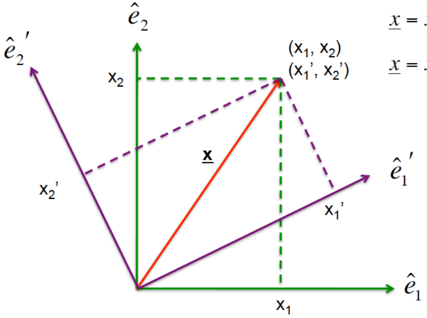
$$\begin{aligned} x_1' &= x_1 + \Delta x_1 \\ x_2' &= x_2 + \Delta x_2 \end{aligned}$$

(indices indicate vector components!)

Transformations: Rotation



$$\underline{x} = x_1\hat{e}_1 + x_2\hat{e}_2$$

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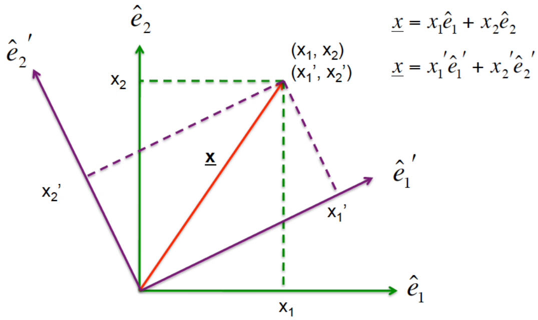
Transformations: Rotation



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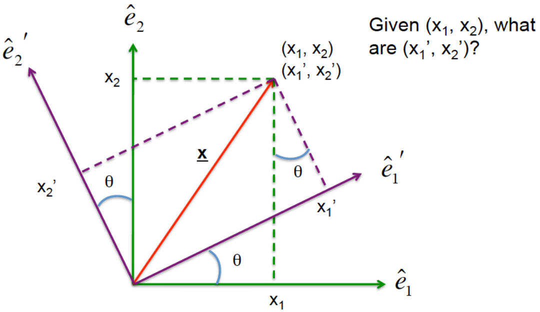
Given 2 systems, how are vector components related?

Transformations: Rotation



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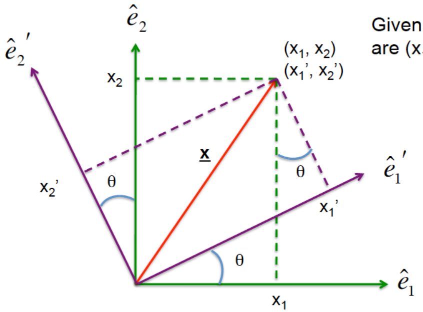
Transformations: Rotation



Given (x_1, x_2), what are (x_1', x_2')?

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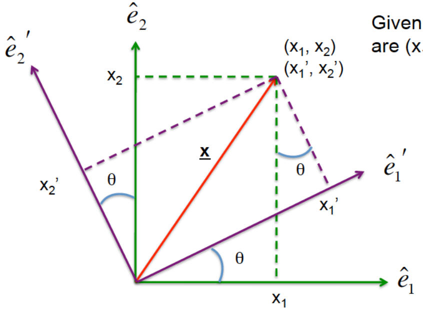
Given 2 systems, how are vector components related?

$$x_1 = x_1' cos(\theta) - x_2' sin(\theta)$$
$$x_2 = x_1' sin(\theta) + x_2' cos(\theta)$$

$$x_1' = x_1 cos(\theta) + x_2 sin(\theta)$$
$$x_2' = -x_1 sin(\theta) + x_2 cos(\theta)$$

Transformations: Rotation



Given (x_1, x_2), what are (x_1', x_2')?

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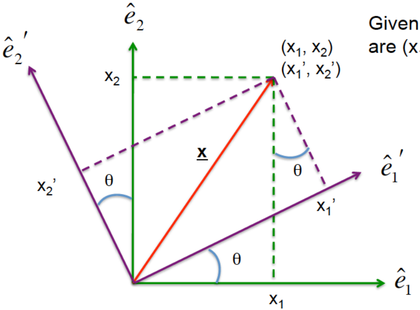
Given 2 systems, how are vector components related?

$$\mathbf{x} = \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix} \begin{bmatrix} x_1' \\ x_2' \end{bmatrix} \quad \mathbf{x}' = \begin{bmatrix} cos(\theta) & sin(\theta) \\ -sin(\theta) & cos(\theta) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Transformations: Rotation



Given (x_1, x_2), what are (x_1', x_2')?

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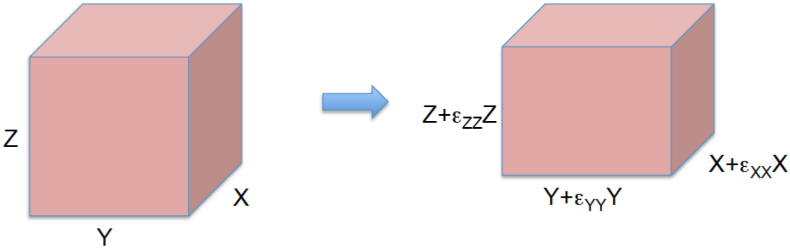
Rotating a vector is the same as rotating the coordinate system in the opposite direction

Transformations: Dilatation



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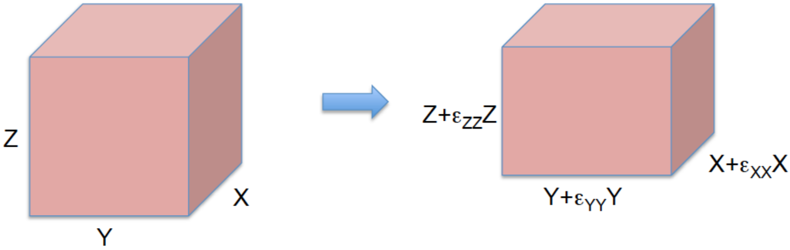
Transformations: Dilatation



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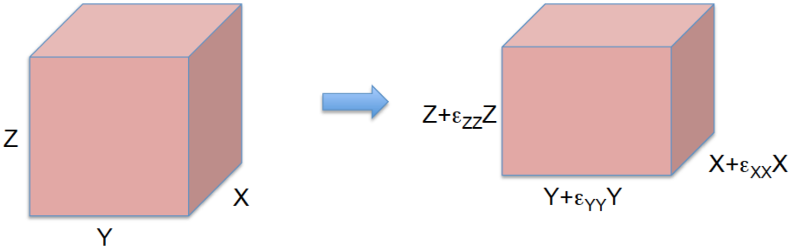
fractional length changes are **normal strains**:

$$\begin{aligned}\frac{change\_in\_length}{original\_length} &= strain \\ du_1/X &= \varepsilon_{xx} \\ du_2/Y &= \varepsilon_{yy} \\ du_3/Z &= \varepsilon_{zz}\end{aligned}$$

convention important: geologists often use positive = contraction, can be extension, too. Check!

Transformations: Dilatation



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think in finite differences (infinitesimal lengths):

$$\lim_{length \to 0} \frac{length - new\_length}{length} = derivative$$

$$\partial u_1 / \partial x = \varepsilon_{xx}$$

$$\partial u_2 / \partial y = \varepsilon_{yy}$$

$$\partial u_3 / \partial z = \varepsilon_{zz}$$

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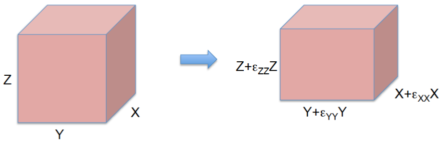
Transformations: Dilatation



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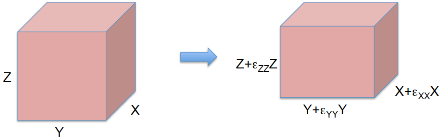
Dilatation (Δ) defined as fractional volume change:

$$
\begin{aligned}
\Delta &= \frac{X(1+\varepsilon_{xx}) * Y(1+\varepsilon_{yy}) * Z(1+\varepsilon_{zz}) - X * Y * Z}{X * Y * Z} \\
&= \frac{X * Y * Z((1+\varepsilon_{xx}) * (1+\varepsilon_{yy}) * (1+\varepsilon_{zz}) - 1)}{X * Y * Z} \\
&= (1+\varepsilon_{xx}) * (1+\varepsilon_{yy}) * (1+\varepsilon_{zz}) - 1
\end{aligned}
$$

Transformations: Dilatation



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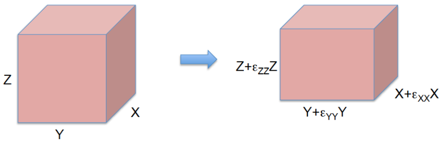
We use infinitesimal strain, products of strain can be dropped:

$$\begin{aligned}\Delta &= 1 + \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} - 1 \\ &= \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}\end{aligned}$$

Transformations: Dilatation



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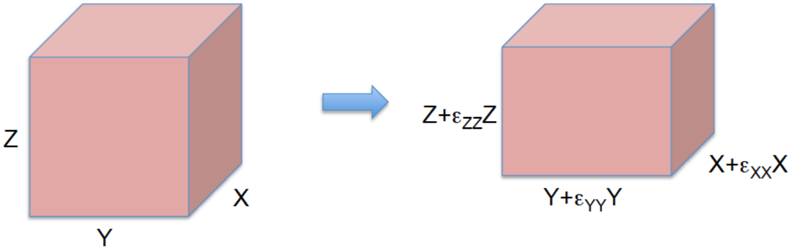
$$\begin{aligned}\Delta &= 1 + \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} - 1 \\ &= \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}\end{aligned}$$

seismic P waves are travelling oscillations of Δ

Strain: Normal Strain



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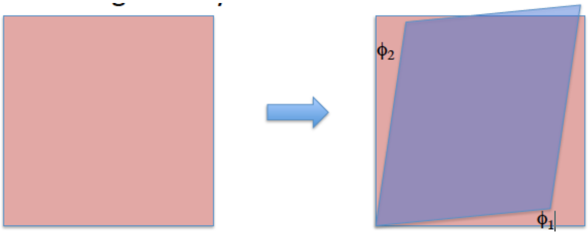
fractional length changes are **normal strains**:

$$\begin{aligned}\partial u_1/\partial x &= \varepsilon_{xx}\\ \partial u_2/\partial y &= \varepsilon_{yy}\\ \partial u_3/\partial z &= \varepsilon_{zz}\end{aligned}$$

components of strain proportional to derivatives of displacements in respective directions

Strain: Shear Strain



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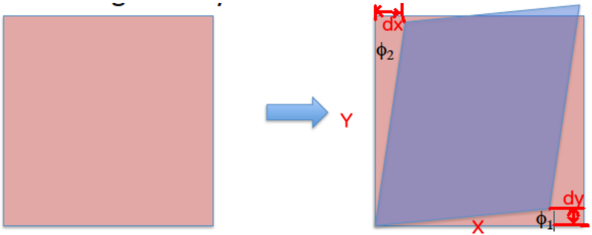
shear components of strain measure change in shape / angles

$$\varepsilon_{xy} \quad = \quad \varepsilon_{yx} = -\frac{1}{2}(\phi_1 + \phi_2)$$

Strain: Shear Strain



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shear components of strain measure change in shape / angles

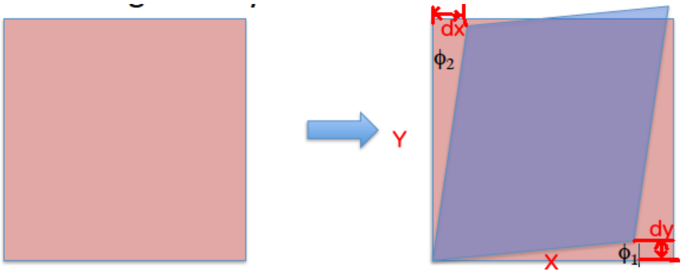
$$\varepsilon_{xy} = \varepsilon_{yx} = -\frac{1}{2}(\phi_1 + \phi_2)$$

angles related to displacements (small angle approx: $tan(\phi) \approx \phi$):

$$\phi_1 = tan(\phi_1) = \frac{opposite}{adjacent} = -\frac{dy}{X}$$

$$\phi_2 = tan(\phi_2) = \frac{opposite}{adjacent} = -\frac{dx}{Y}$$

Strain: Shear Strain



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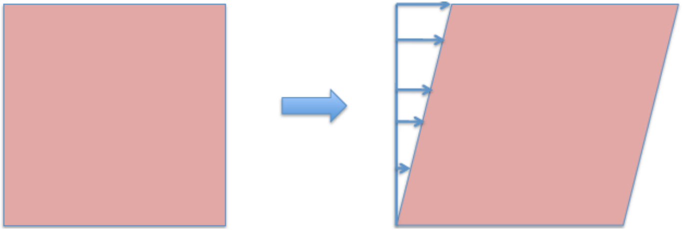
shear components of strain measure change in shape / angles

$$\varepsilon_{xy} \quad = \quad \varepsilon_{yx} = \frac{1}{2}\left(\frac{\partial u_2}{\partial x} + \frac{\partial u_1}{\partial y}\right)$$

subscripts: 1st – direction normal to element, 2nd – direction of shear

Strain: Shear Strain



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shear strain results in solid body rotation if $\phi_1 \neq \phi_2$:

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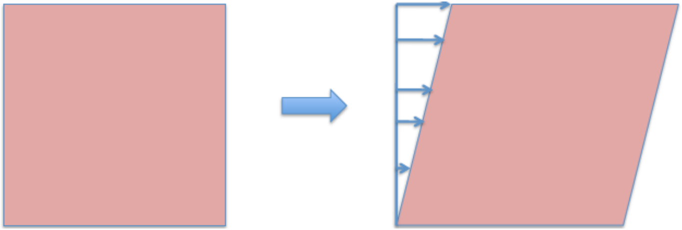

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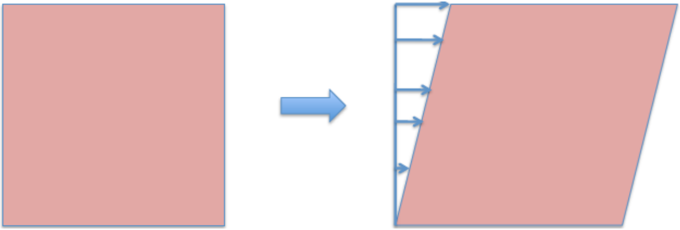
shear strain results in solid body rotation if $\phi_1 \neq \phi_2$:

$$\omega_z = -\frac{1}{2}(\phi_1 - \phi_2) = \frac{1}{2}\left(\frac{\partial u_2}{\partial x} - \frac{\partial u_1}{\partial y}\right)$$

Strain: Shear Strain



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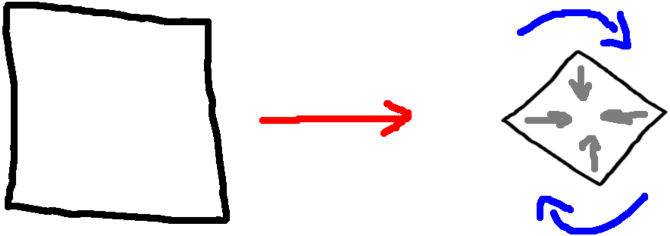
- if $\phi_1 = \phi_2$: no solid body rotation – **pure shear**
- if $\phi_1 = 0$: solid body rotation + shear – **simple shear** (strike slip faulting)

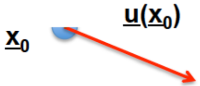
Putting it all together



$$
\begin{aligned}
\textit{deformation} &= \textit{translation} + \textit{dilatation} + \textit{rotation} \\
u &\approx x + dx + \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)
\end{aligned}
$$

correct formal description follows ...

Deformation



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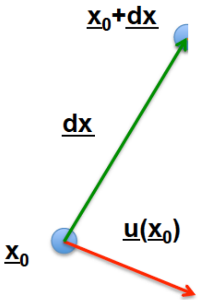
Deformation



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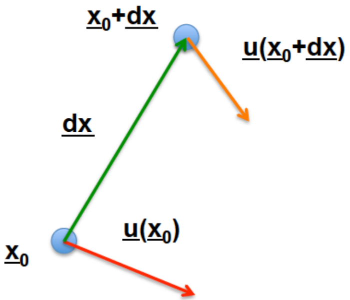
Deformation



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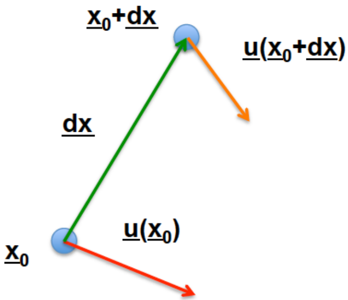
Deformation



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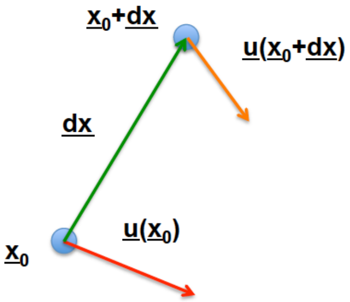
use Taylor Series expansion to relate the two vectors:

$$u_i(\mathbf{x_0} + \mathbf{dx}) \quad = \quad u_i(\mathbf{x_0}) + \left(\frac{\partial u_i}{\partial x_1}\right) dx_1 + \left(\frac{\partial u_i}{\partial x_2}\right) dx_2 + \left(\frac{\partial u_i}{\partial x_3}\right) dx_3$$

Deformation



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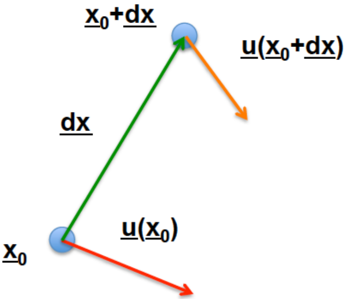
3 equations: i=1,2,3

first term: translation, remainder: rotation + dilatation

9 values $\partial u_i / \partial x_j$ for $i, j = 1 \ldots 3$

Deformation Tensor



$$u(\mathbf{x_0} + \mathbf{dx}) \quad = \quad u(\mathbf{x_0}) + \begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3} \\ \frac{\partial u_2}{\partial x_1} & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3} \\ \frac{\partial u_3}{\partial x_1} & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3} \end{bmatrix}$$

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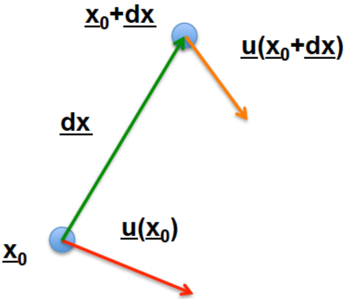
Deformation Tensor



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- matrix describes dilatation and rotation
- is a 2-direction (rank 2) tensor: contains normal strain, and strain perpendicular to face on which it acts
- think of tensors as extension of vectors (magnitude and direction), which are an extension of scalars (magnitude)

Separate Rotation and Strain

We can separate gradient tensor into the sum of two tensors: strain tensor and rotation tensor from:

$$\textit{deformation} = \textit{translation} + \textit{dilatation/strain} + \textit{rotation}$$

rotation is anti-symmetric (see rotation matrix), strain part is symmetric.

Separate Rotation and Strain

We can separate gradient tensor into the sum of two tensors: strain tensor and rotation tensor from:

$$\begin{aligned} \textit{deformation} &= \textit{translation} + \textit{dilatation/strain} + \textit{rotation} \\ u_i(\mathbf{x_0} + \mathbf{dx}) &= u_i(\mathbf{x_0}) + \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) dx_j + \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right) dx_j \end{aligned}$$

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$$\begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3} \\ \frac{\partial u_2}{\partial x_1} & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3} \\ \frac{\partial u_3}{\partial x_1} & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3} \end{bmatrix} = \begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right) \\ \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right) & \frac{\partial u_2}{\partial x_2} & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\right) \\ \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\right) & \frac{\partial u_3}{\partial x_3} \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} - \frac{\partial u_2}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) \\ \frac{1}{2}\left(\frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2}\right) & 0 & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} - \frac{\partial u_3}{\partial x_2}\right) \\ \frac{1}{2}\left(\frac{\partial u_3}{\partial x_1} - \frac{\partial u_1}{\partial x_3}\right) & \frac{1}{2}\left(\frac{\partial u_3}{\partial x_2} - \frac{\partial u_2}{\partial x_3}\right) & 0 \end{bmatrix}$$

rotation is anti-symmetric (see rotation matrix), strain part is symmetric.

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$$\begin{aligned} \textit{deformation} &= \textit{translation} + \textit{dilatation/strain} + \textit{rotation} \\ u_i(\mathbf{x_0} + \mathbf{dx}) &= u_i(\mathbf{x_0}) + \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) dx_j + \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right) dx_j \end{aligned}$$

$$\begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3} \\ \frac{\partial u_2}{\partial x_1} & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3} \\ \frac{\partial u_3}{\partial x_1} & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3} \end{bmatrix} = \begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right) \\ \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right) & \frac{\partial u_2}{\partial x_2} & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\right) \\ \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\right) & \frac{\partial u_3}{\partial x_3} \end{bmatrix} +$$

$$\begin{bmatrix} 0 & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} - \frac{\partial u_2}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) \\ \frac{1}{2}\left(\frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2}\right) & 0 & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} - \frac{\partial u_3}{\partial x_2}\right) \\ \frac{1}{2}\left(\frac{\partial u_3}{\partial x_1} - \frac{\partial u_1}{\partial x_3}\right) & \frac{1}{2}\left(\frac{\partial u_3}{\partial x_2} - \frac{\partial u_2}{\partial x_3}\right) & 0 \end{bmatrix}$$

rotation is anti-symmetric (see rotation matrix), strain part is symmetric. We generally define the deformation gradient tensor as:

$$\varepsilon_{ij} = \epsilon_{ij} + \omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) dx_j + \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right) dx_j$$

Strain and Rotation Tensors

Strain tensor can be written:

$$\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{bmatrix}$$

symmetric, with 6 independent components since

$\epsilon_{21} = \epsilon_{12}, \epsilon_{31} = \epsilon_{13}, \epsilon_{32} = \epsilon_{23}$

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Rotation tensor can be written:

$$\omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right) = \begin{bmatrix} 0 & \omega_{12} & \omega_{13} \\ -\omega_{12} & 0 & \omega_{23} \\ -\omega_{13} & -\omega_{23} & 0 \end{bmatrix}$$

antisymmetric, with 3 independent components

Strain and Rotation from GPS Data

- Can estimate all components of strain and rotation tensors directly from GPS data
- Equations in terms of 6 independent strain tensor components and 3 independent rotation tensor components
- …or in terms of the 9 components of the displacement gradient tensor ε_{ij}

Strain and Rotation from GPS Data

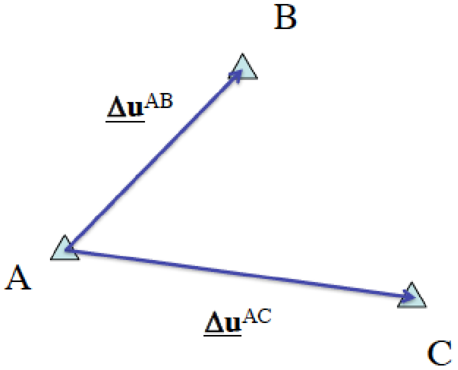
- Can estimate all components of strain and rotation tensors directly from GPS data
- Equations in terms of 6 independent strain tensor components and 3 independent rotation tensor components
- ... or in terms of the 9 components of the displacement gradient tensor ε_{ij}
- Write motions relative to reference site or reference point in terms of distance from reference ("remove translation"):

$$u_i(\mathbf{x_0} + \mathbf{dx_0}) - u_i(\mathbf{x_0}) \;=\; \epsilon_{ij} dx_j + \omega_{ij} dx_j$$

- $\mathbf{x_0}$ is reference location, $\mathbf{dx}$ is vector from reference to data location

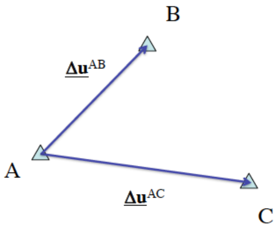
Example: Strain from 3 GPS sites

- simple, general way to calculate average strain and rotation from 3 GPS sites
- (average strain for the area enclosed by the 3 sites)
- with more than 3 sites: divide network into triangles
- Delaunay triangulation implemented in GMT is a quick way to do so



J. Freymueller

Example: Strain from 3 GPS sites



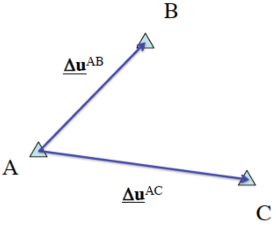
Let's look at this for a single baseline (horizontal deformation): *J. Freymueller*

$$\begin{bmatrix} \Delta u_1^{AB} \\ \Delta u_2^{AB} \end{bmatrix} = \begin{bmatrix} \epsilon_{11}\Delta x_1^{AB} + \epsilon_{12}\Delta x_2^{AB} + \omega_{12}\Delta x_2^{AB} \\ \epsilon_{12}\Delta x_1^{AB} + \epsilon_{22}\Delta x_2^{AB} - \omega_{12}\Delta x_1^{AB} \end{bmatrix}$$

$$=$$

$$= G \cdot \mathbf{m}$$

Example: Strain from 3 GPS sites



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$$= \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \cdot \begin{bmatrix} \quad \\ \quad \\ \quad \\ \quad \end{bmatrix}$$

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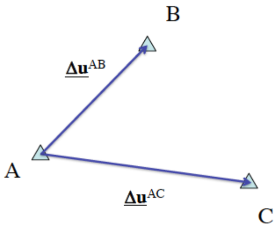

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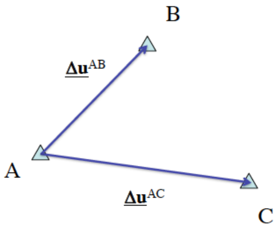
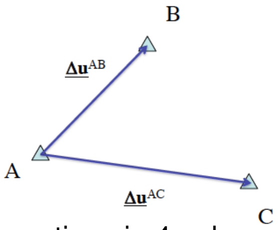


Let's look at this for a single baseline (horizontal deformation): *J. Freymueller*

$$
\begin{aligned}
\begin{bmatrix} \Delta u_1^{AB} \\ \Delta u_2^{AB} \end{bmatrix} &= \begin{bmatrix} \epsilon_{11}\Delta x_1^{AB} + \epsilon_{12}\Delta x_2^{AB} + \omega_{12}\Delta x_2^{AB} \\ \epsilon_{12}\Delta x_1^{AB} + \epsilon_{22}\Delta x_2^{AB} - \omega_{12}\Delta x_1^{AB} \end{bmatrix} \\
&= \begin{bmatrix} \Delta x_1^{AB} & \Delta x_2^{AB} & 0 & \Delta x_2^{AB} \\ 0 & \Delta x_1^{AB} & \Delta x_2^{AB} & -\Delta x_1^{AB} \end{bmatrix} \cdot \begin{bmatrix} \epsilon_{11} \\ \epsilon_{12} \\ \epsilon_{22} \\ \omega_{12} \end{bmatrix} \\
&= G \cdot \mathbf{m}
\end{aligned}
$$

Example: Strain from 3 GPS sites

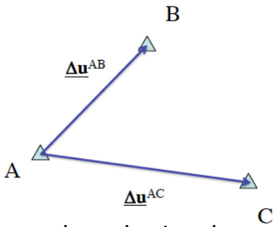


Using all sites we get 4 equations in 4 unknowns:

J. Freymueller

$$\begin{bmatrix} \Delta u_1^{AB} \\ \Delta u_2^{AB} \\ \\ \\ \end{bmatrix} = \begin{bmatrix} \Delta x_1^{AB} & \Delta x_2^{AB} & 0 & \Delta x_2^{AB} \\ 0 & \Delta x_1^{AB} & \Delta x_2^{AB} & -\Delta x_1^{AB} \\ & & & \\ & & & \end{bmatrix} \cdot \begin{bmatrix} \epsilon_{11} \\ \epsilon_{12} \\ \epsilon_{22} \\ \omega_{12} \end{bmatrix}$$

Example: Strain from 3 GPS sites



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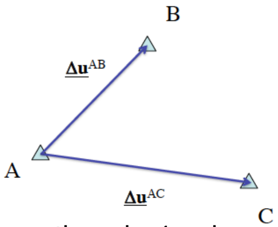
$$\begin{bmatrix} \Delta u_1^{AB} \\ \Delta u_2^{AB} \\ \Delta u_1^{AC} \\ \Delta u_2^{AC} \end{bmatrix} = \begin{bmatrix} \Delta x_1^{AB} & \Delta x_2^{AB} & 0 & \Delta x_2^{AB} \\ 0 & \Delta x_1^{AB} & \Delta x_2^{AB} & -\Delta x_1^{AB} \\ \Delta x_1^{AC} & \Delta x_2^{AC} & 0 & \Delta x_2^{AC} \\ 0 & \Delta x_1^{AC} & \Delta x_2^{BC} & -\Delta x_1^{AC} \end{bmatrix} \cdot \begin{bmatrix} \epsilon_{11} \\ \epsilon_{12} \\ \epsilon_{22} \\ \omega_{12} \end{bmatrix}$$

Example: Strain from 3 GPS sites



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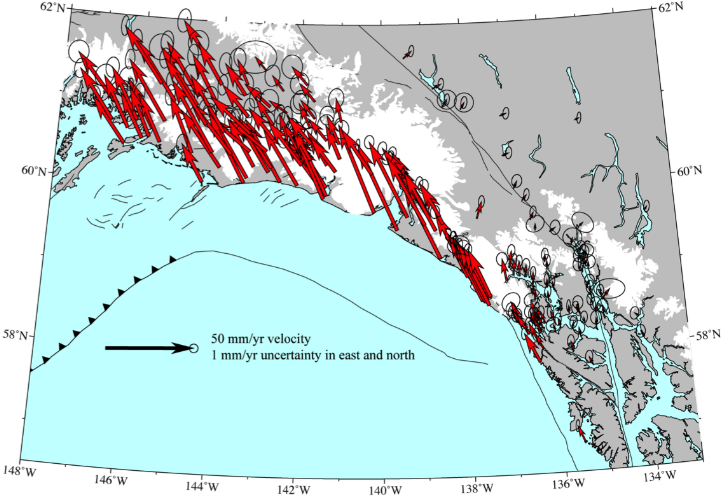
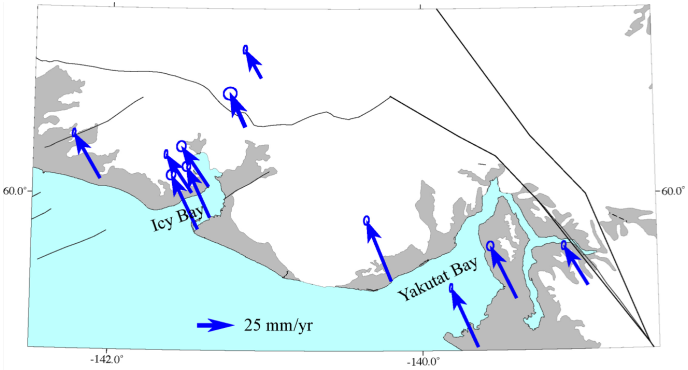
$$\begin{bmatrix} \epsilon_{11} \\ \epsilon_{12} \\ \epsilon_{22} \\ \omega_{12} \end{bmatrix} = \begin{bmatrix} \Delta x_1^{AB} & \Delta x_2^{AB} & 0 & \Delta x_2^{AB} \\ 0 & \Delta x_1^{AB} & \Delta x_2^{AB} & -\Delta x_1^{AB} \\ \Delta x_1^{AC} & \Delta x_2^{AC} & 0 & \Delta x_2^{AC} \\ 0 & \Delta x_1^{AC} & \Delta x_2^{AC} & -\Delta x_1^{AC} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \Delta u_1^{AB} \\ \Delta u_2^{AB} \\ \Delta u_1^{AC} \\ \Delta u_2^{AC} \end{bmatrix}$$

Example: SE Alaska



Julie Elliott

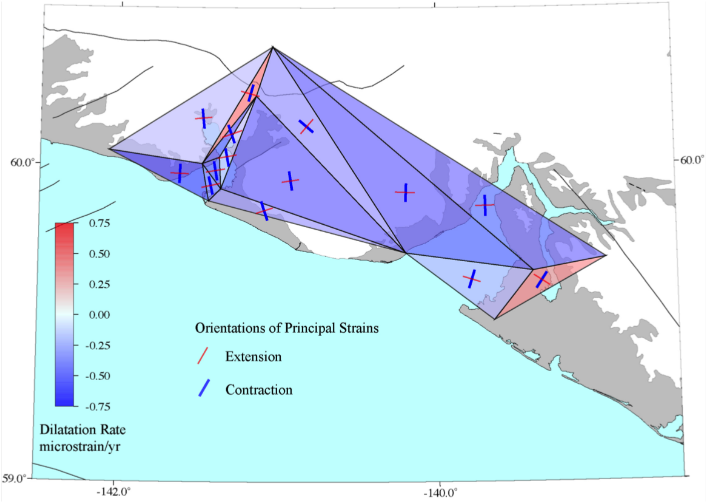
Example: SE Alaska, Icy Bay



Julie Elliott

- velocities relative to stable North America *(Sella et al, 2007)*
- velocities corrected for GIA using model of *Larson et al (2005)*

Example: SE Alaska, Icy Bay



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