New Approaches in Mortality Modelling and Forecasting

Assessment of PhD dissertation

Ugofilippo Basellini

Interdisciplinary Centre on Population Dynamics | SDU

ined INSTITUT NATIONAL D'ÉTUDES DÉMOGRAPHIQUES

Assessment Committee:
Assoc. Prof. Fanny Janssen
Dr. Iain D. Currie
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University of Southern Denmark
Department of Public Health
Odense, 24$^{\text{th}}$ February 2020

Thesis overview

Five chapters:

- Basellini, Canudas-Romo and Lenart (2019). Location–Scale Models in Demography: A Useful Re-parameterization of Mortality Models. *European Journal of Population*, **35**, 645 – 673.
- Basellini and Camarda (2019). Modelling and forecasting adult age-at-death distributions. *Population Studies*, **73**(1), 119 – 138.
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Main goal of PhD thesis:

- introduce innovative methods to provide novel insights into the analysis and forecasting of human mortality

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- few efforts to reunite most models within a single framework
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Objective: reconcile the most common parametric models of mortality within a single overarching family

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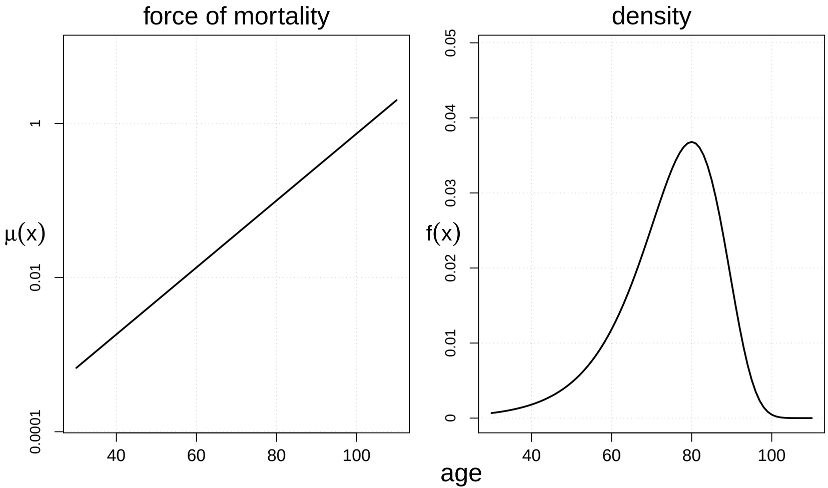
Objective: reconcile the most common parametric models of mortality within a single overarching family

Contribution: propose the location–scale (LS) family as unifying framework in mortality modelling

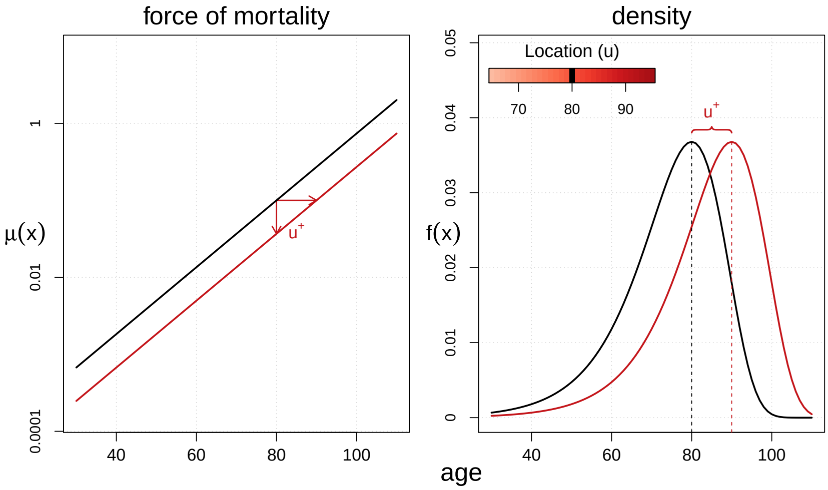
- several models belong to the family
- LS parameters simple and clear demographic interpretation: mortality shifting and compression

LS family


force of mortality
density
μ(x)
f(x)
1
0.01
0.0001
0.05
0.04
0.03
0.02
0.01
0
40
60
80
100
age

LS family

location (u): **shifting** mortality dynamic



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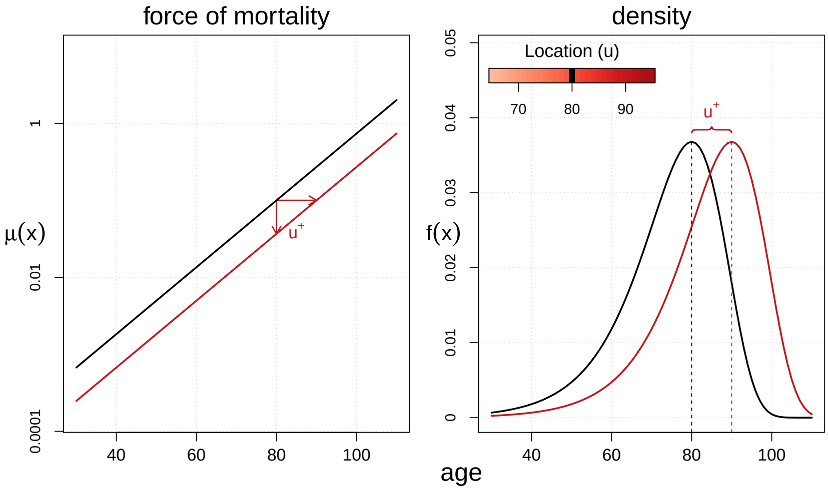
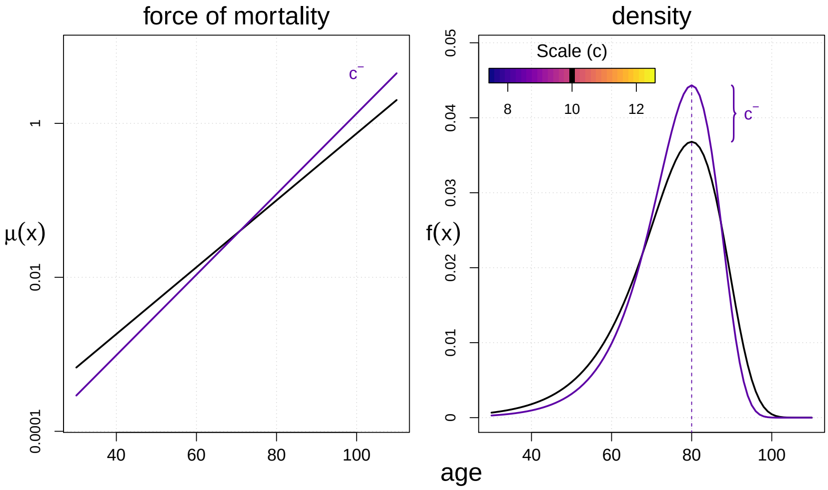


LS family

scale (c): **compression** mortality dynamic



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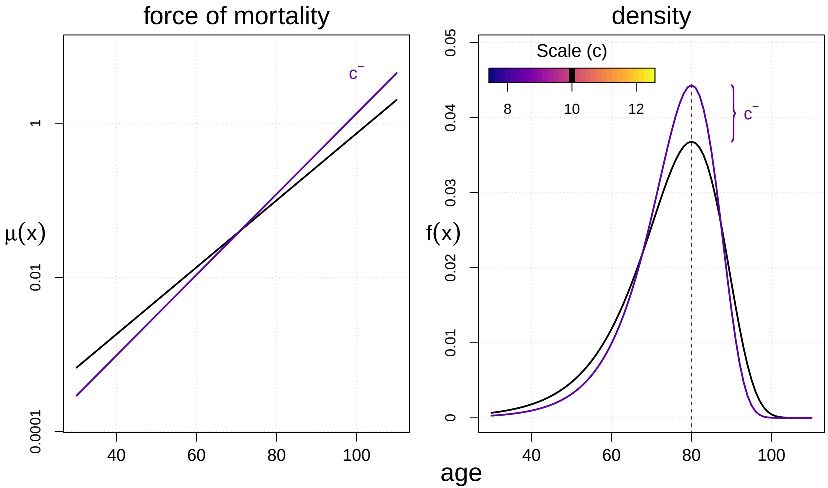
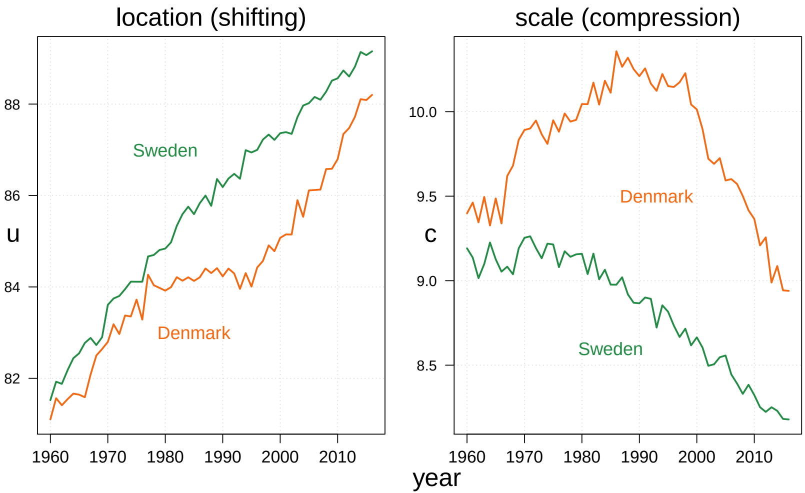


Application: mortality shifting and compression



Estimated LS parameters of the Minimal Generalized Extreme–Value model (lowest BIC).
Swedish and Danish females, ages 30–110+, years 1960–2016.
Source: HMD (2020)

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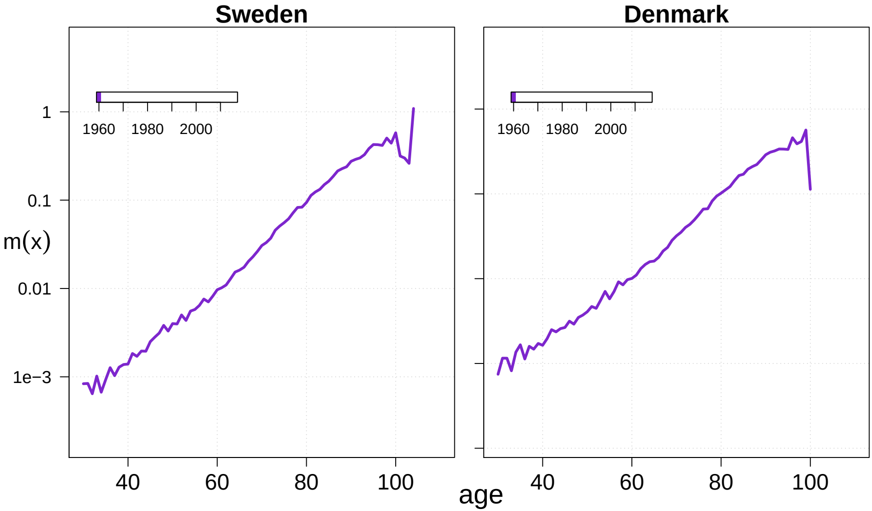
Objective: model and forecast adult mortality by studying changes in age-at-death distributions

Contribution: propose the Segmented Transformation Age-at-Death Distributions (STAD) model to analyze and forecast adult mortality

- parsimonious and efficient model
- mortality forecasts more accurate and optimistic than the Lee-Carter model and its variants

Motivation



Mortality rates (log scale).
Swedish and Danish females, ages 30–110+, years 1960–2016.
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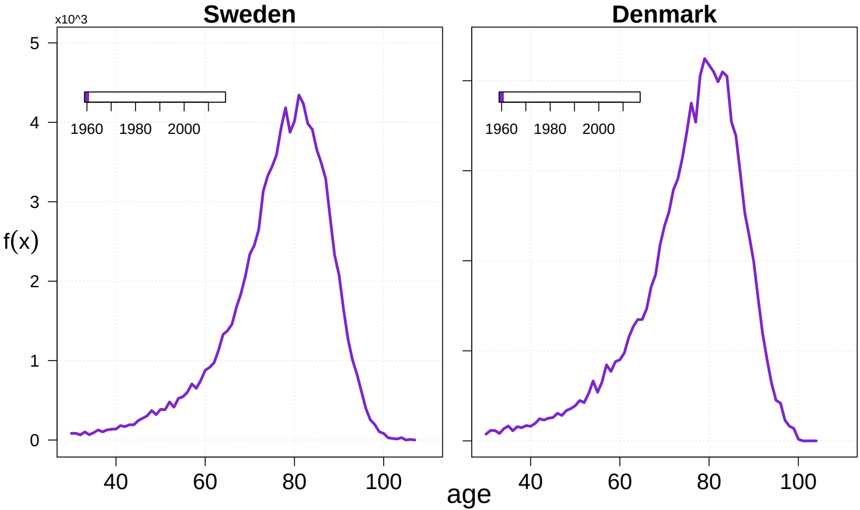

Age-at-death distributions.
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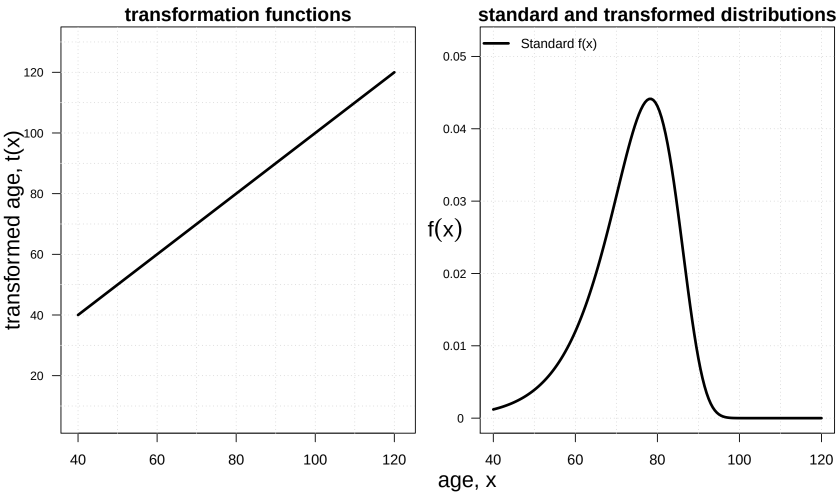
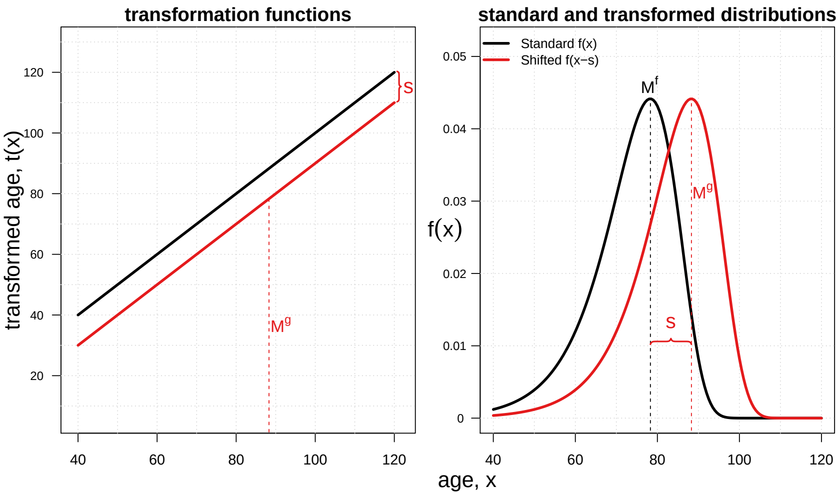
The STAD model



The STAD model

$$\color{red}{s} = M^g - M^f$$

Shifting mortality dynamic



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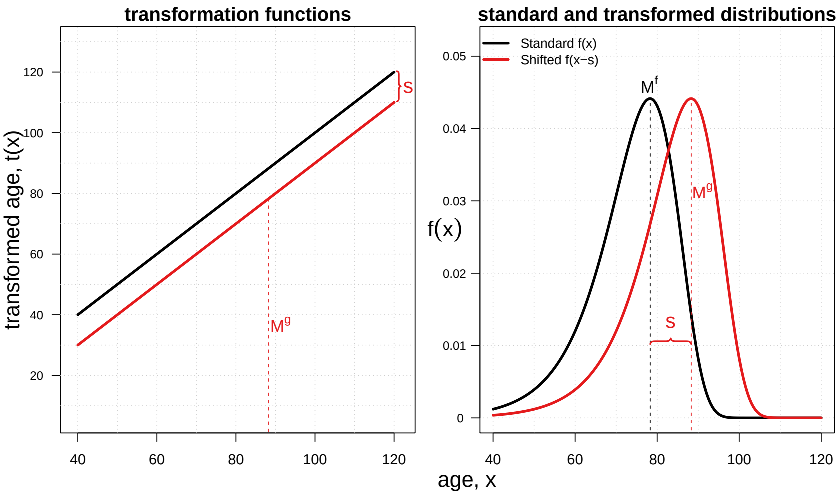
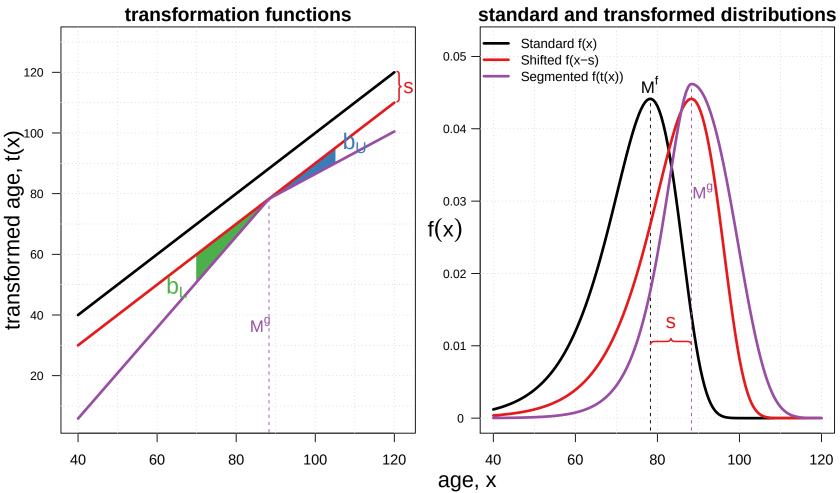


The STAD model

b_L, b_U: change in lifespan variability before and after M^g

Compression mortality dynamic



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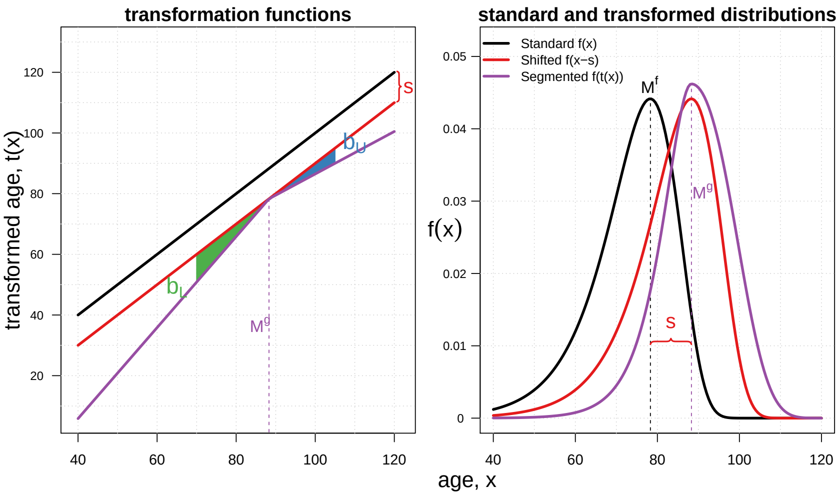
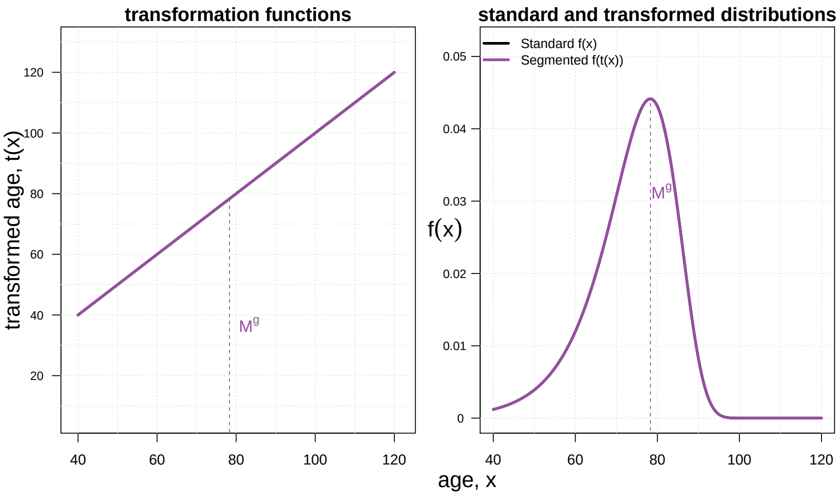
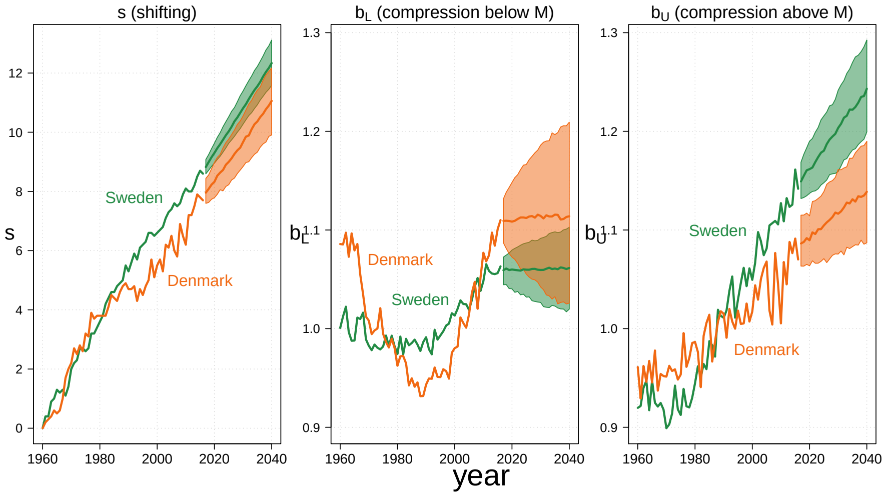


The STAD model

Flexible and parsimonious model to capture **mortality developments** over age and time



STAD parameters



Swedish and Danish females.
Ages 30–110+, fitted years 1960–2016, forecast years 2017–2040.

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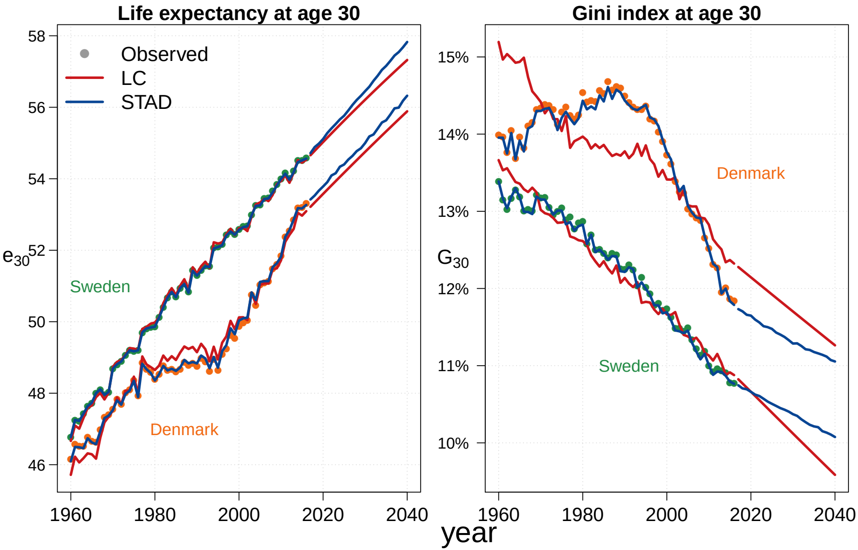

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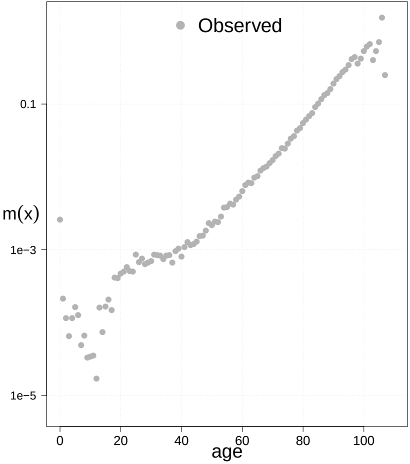
Contribution: propose the Three-Component STAD (3C-STAD) model to analyse and forecast mortality

- forecasts more accurate and optimistic than rate-based models
- forecasts have smooth age-profile, generally characterized by greater shifting and less compression

Motivation



Mortality rates (log scale).
Sweden, males, 2016, ages 0-110+
Source: HMD (2020)

Mortality decomposition (Thiele 1871):

$$\mu =$$

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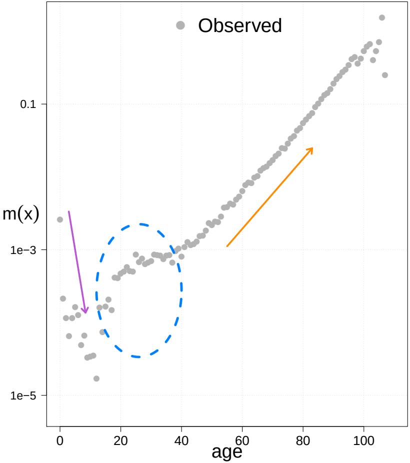

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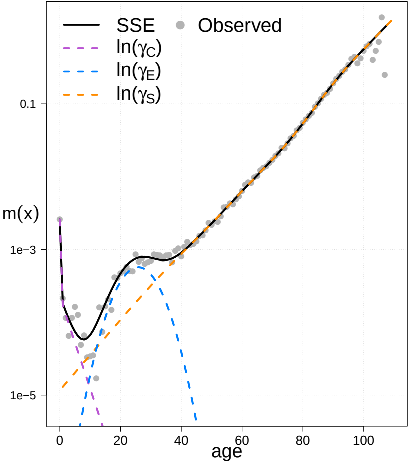

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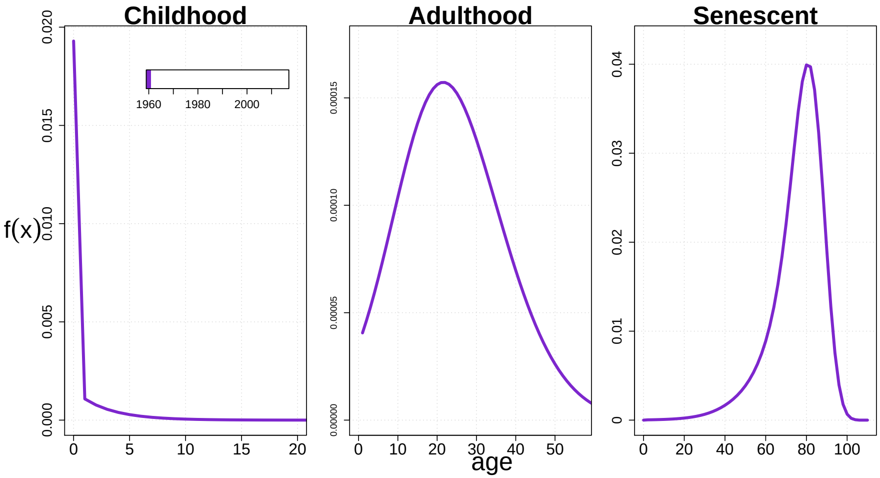
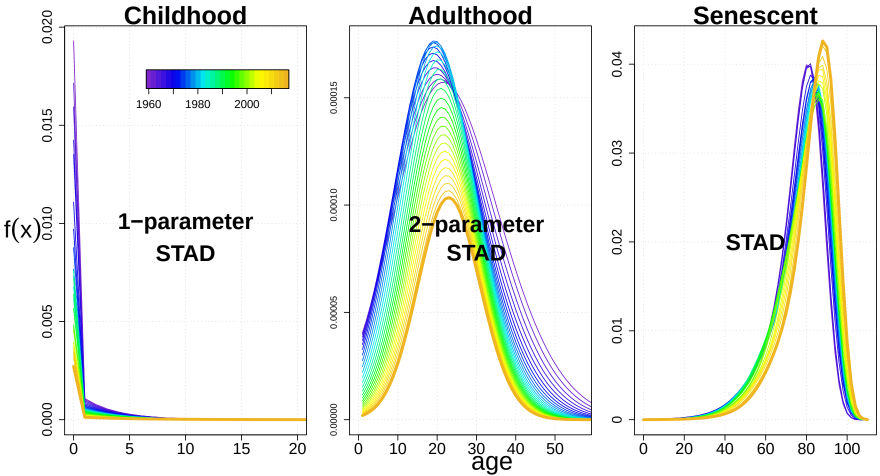
$\Rightarrow$ SSE model (Camarda et al. 2016)

Component-specific distributions



Danish females, ages 0–110+, years 1960–2016.
Source: estimates from the SSE model (Camarda et al. 2016)

The 3C-STAD model



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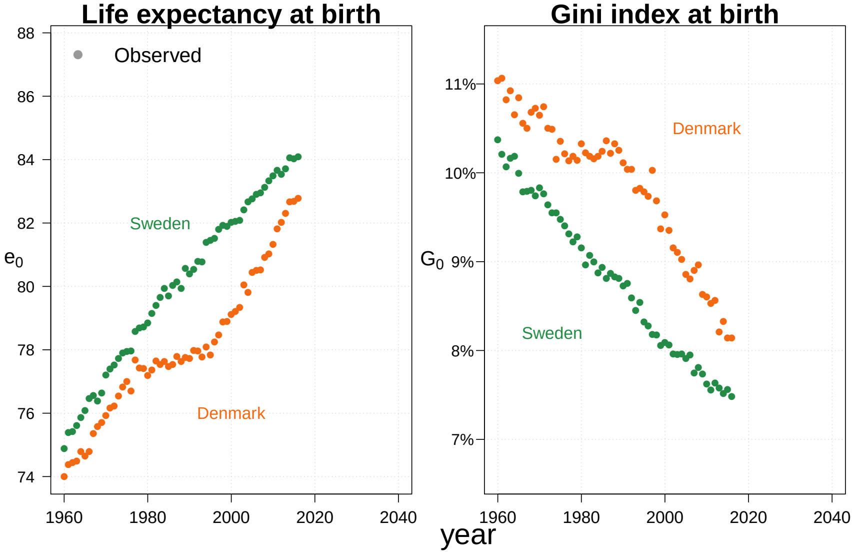
3C-STAD: summary measures



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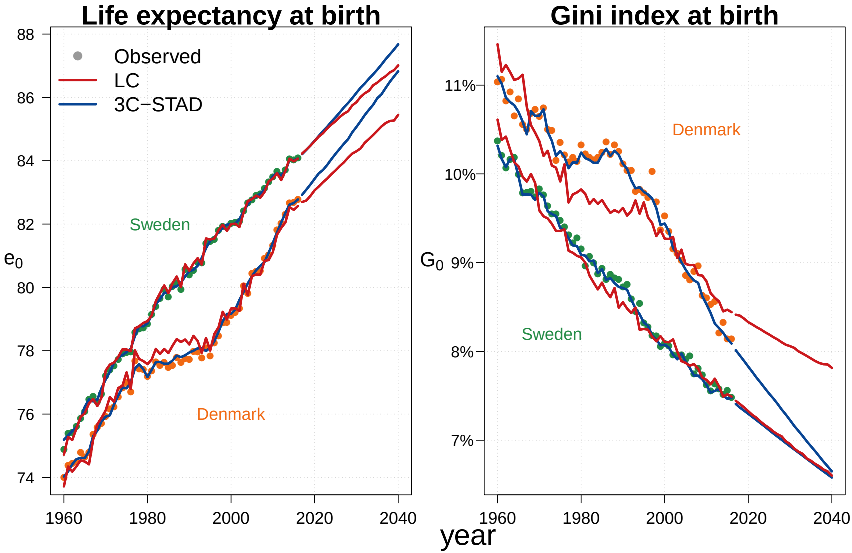

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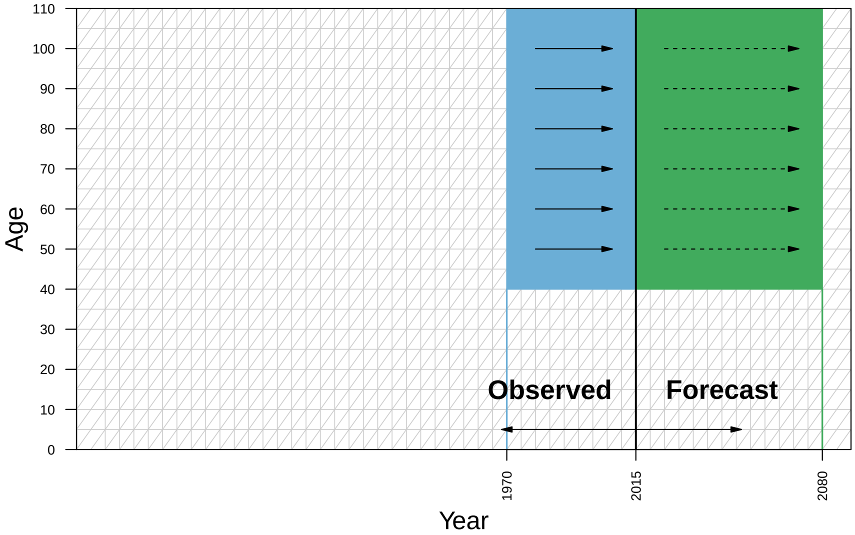
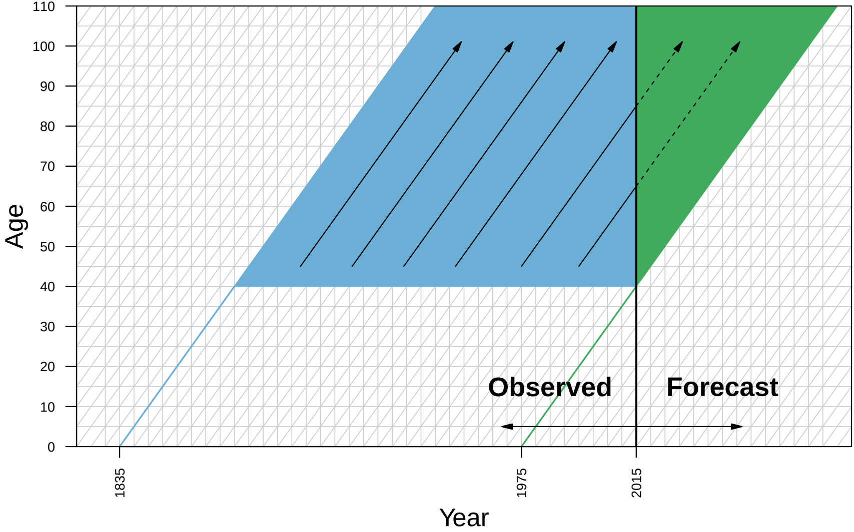
Contribution: propose the Cohort-STAD (C-STAD) model to analyse and forecast cohort adult mortality

- one of the few approaches to forecast cohort mortality
- more accurate forecasts than other cohort or period approaches

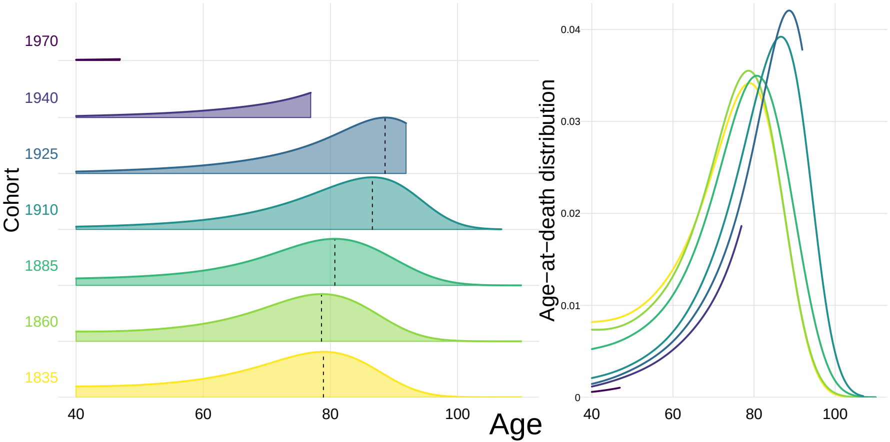
From period forecasting ...



... to cohort forecasting



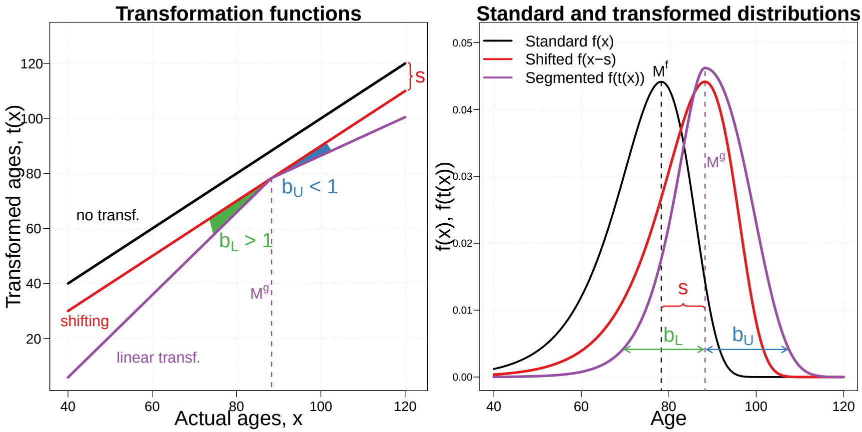
Cohort mortality data



Swedish female age-at-death distributions, ages 40–110+, selected cohorts between 1835 and 1970.
Source: HMD (2020)

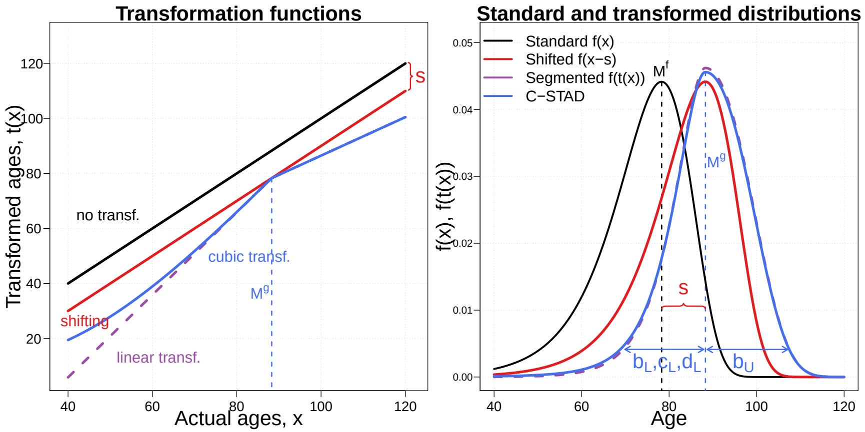
The C-STAD model

Extending the STAD segmented linear transformation

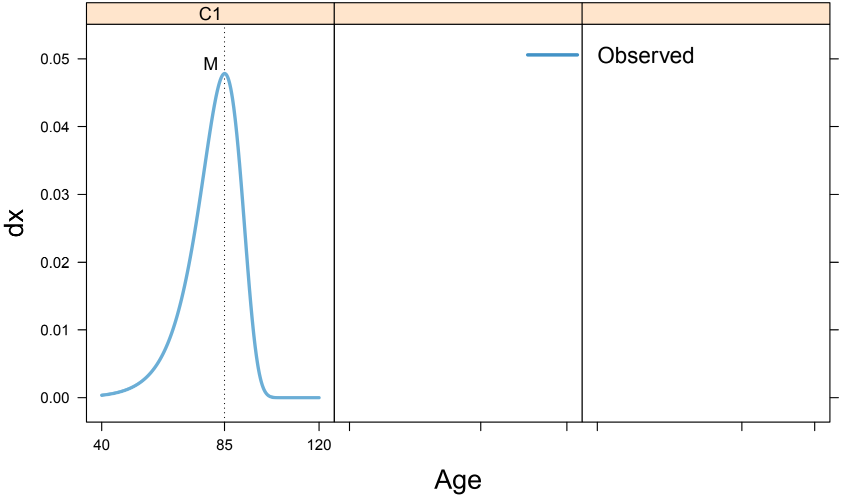


The C-STAD model

Extending the STAD segmented linear transformation
to **cubic** model below the mode (i.e. adding c_L and d_L)



C-STAD: fully observed d_x (c_1)



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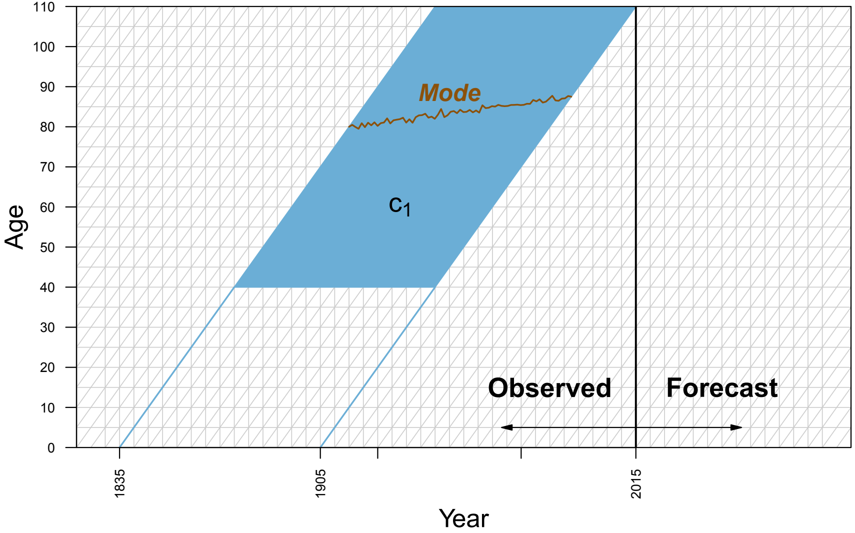
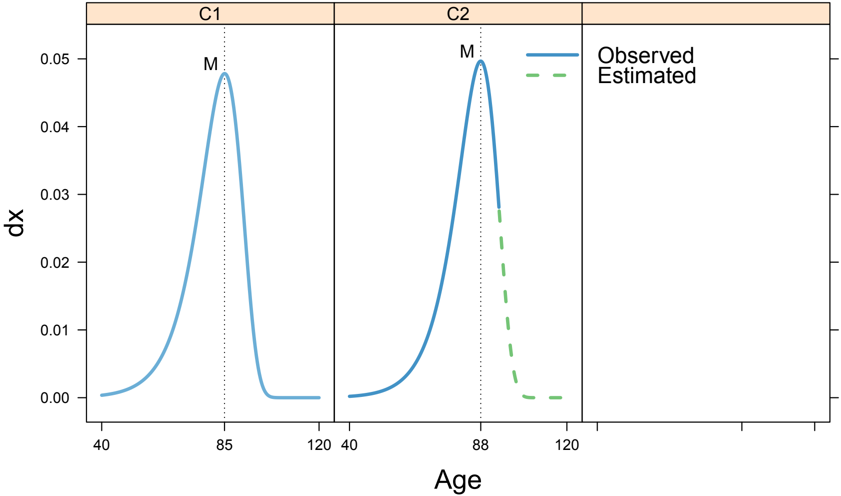


C-STAD: partially observed d_x (c_2)



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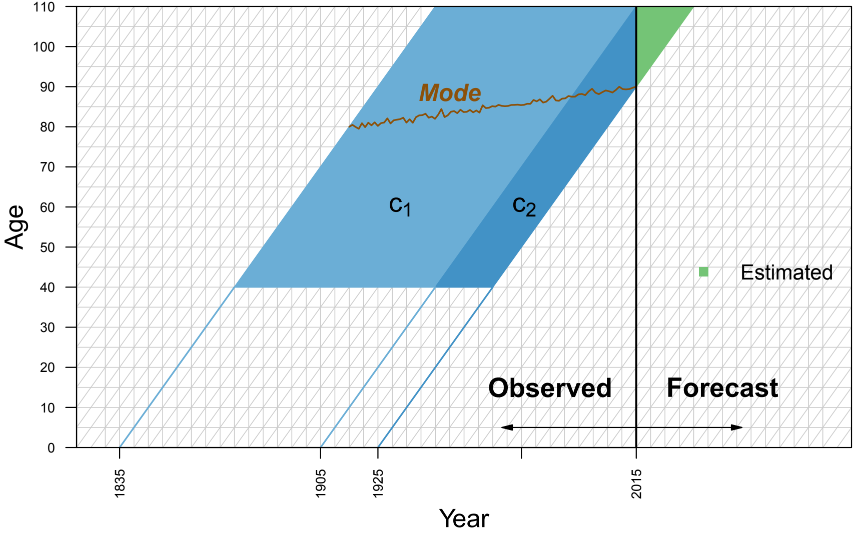
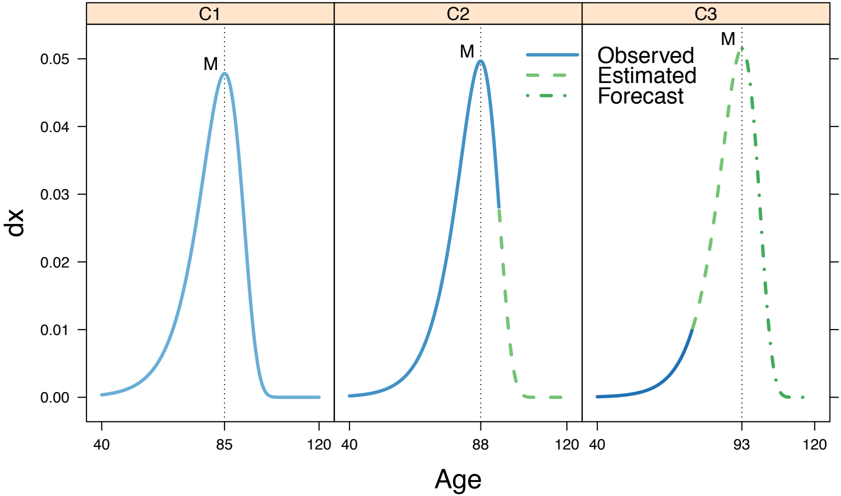


C-STAD: partially observed d_x (c_3)



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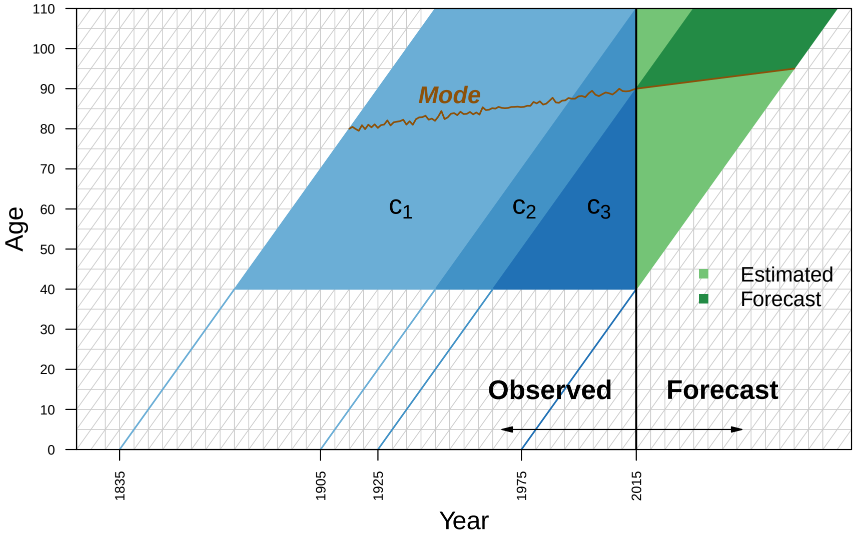
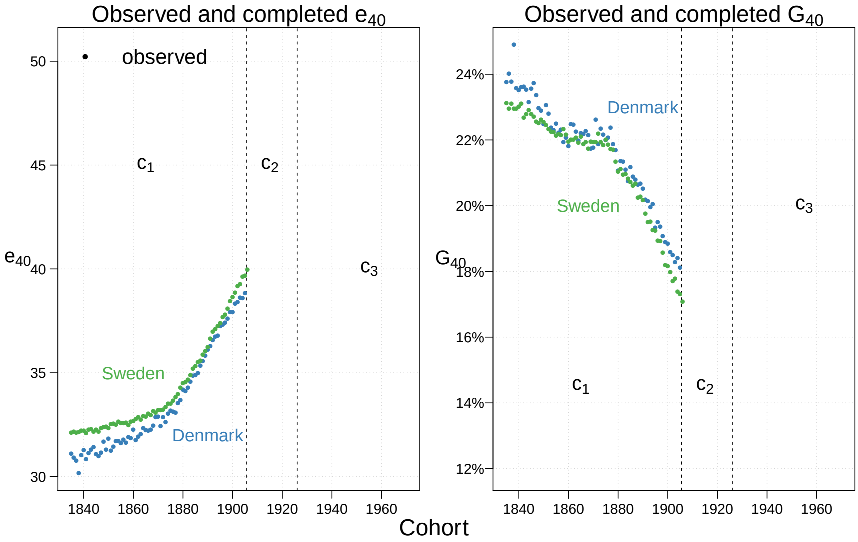


C-STAD: summary measures



Swedish and Danish females, ages 40–110+, cohorts 1835–1970.

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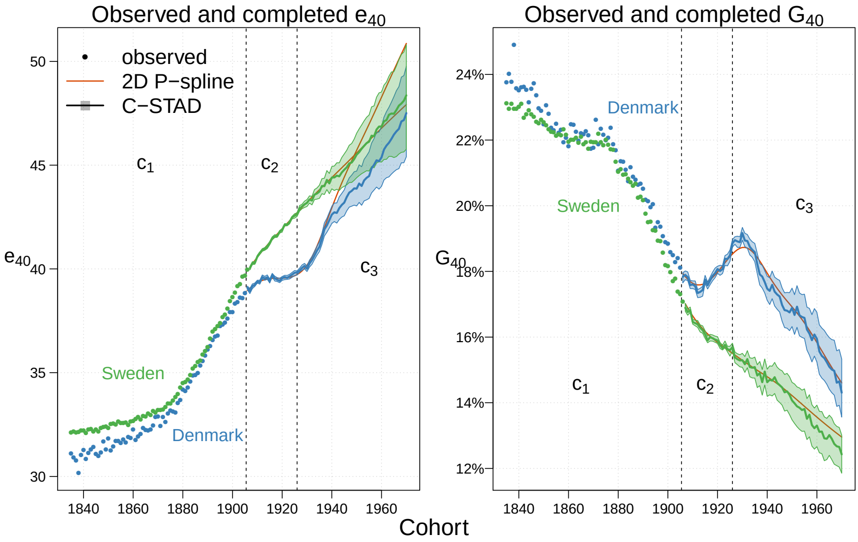

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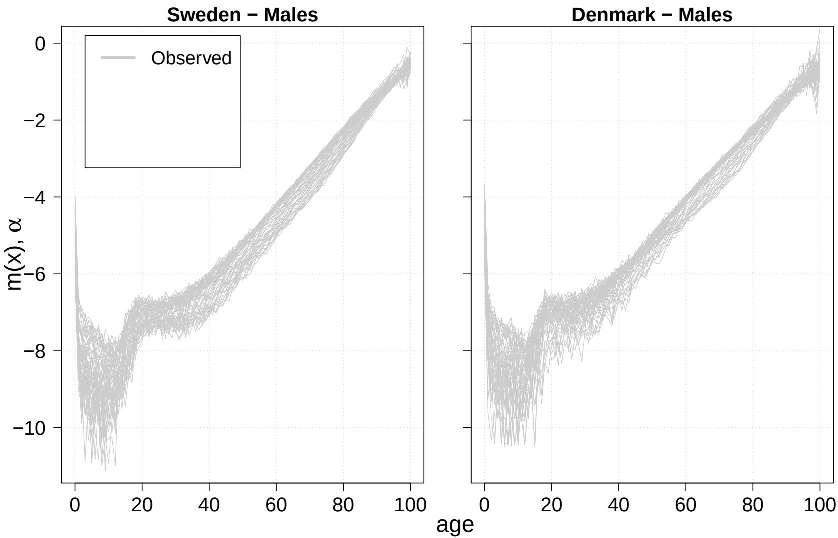
Contribution: propose the Three-Component smooth Lee-Carter (3C-sLC) model to analyze and forecast mortality

- more realistic outcomes, i.e. smooth age-profiles and wider prediction intervals
- enhanced flexibility and goodness-of-fit thanks to mortality decomposition

Shape of mortality pattern



Log-mortality rates and $\hat{\alpha}$ estimates from the LC and 3C-sLC model.
Swedish and Danish males, ages 0–100, years 1960–2016.

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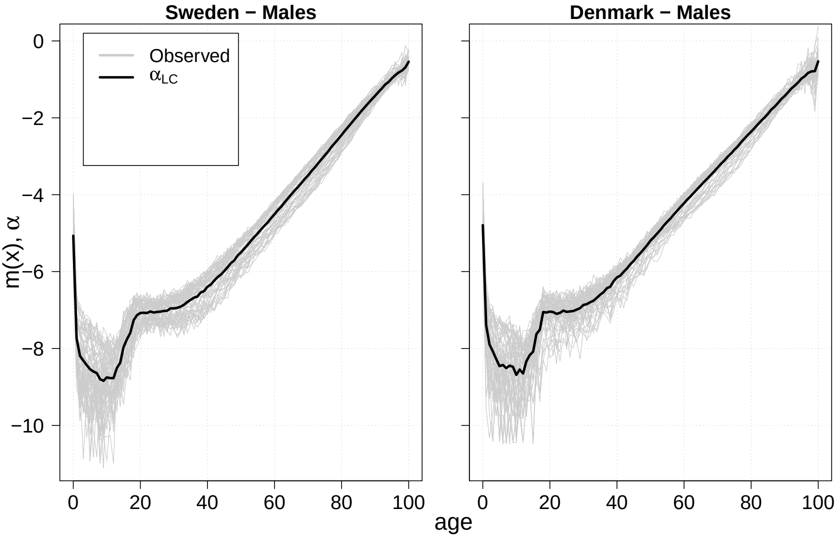

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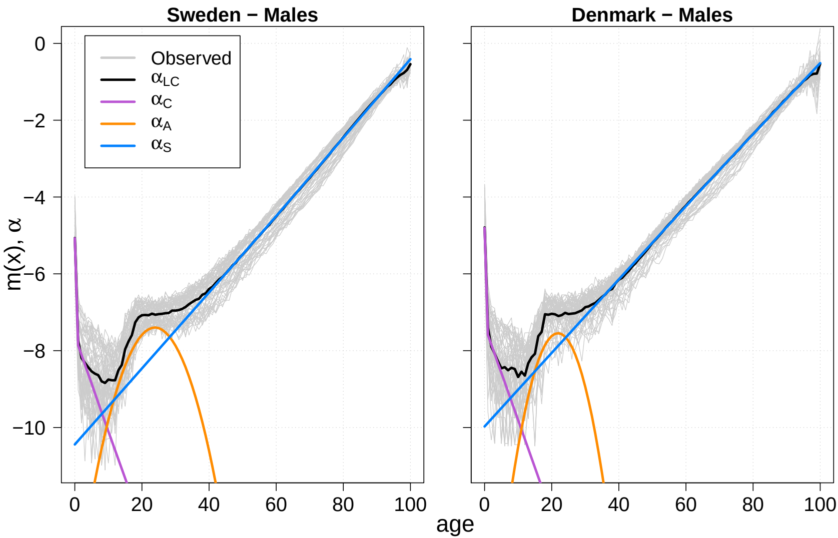
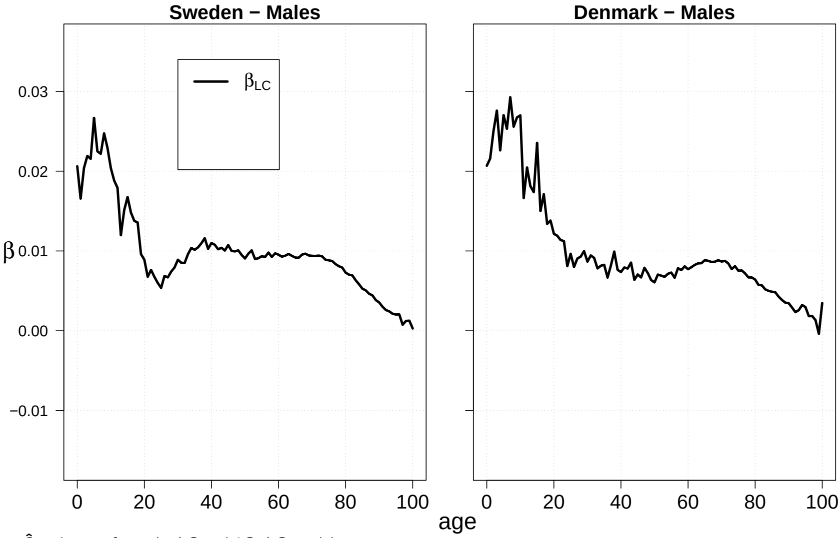


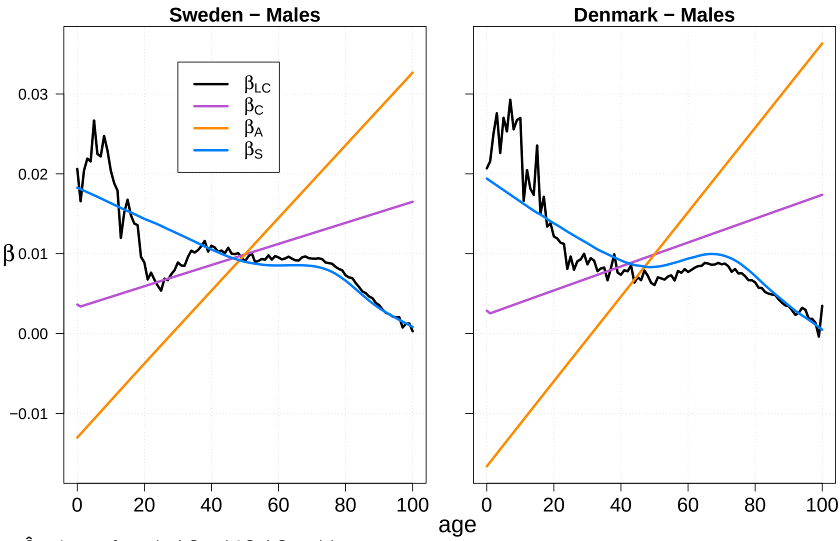
Log-mortality rates and $\hat{\alpha}$ estimates from the LC and 3C-sLC model.
Swedish and Danish males, ages 0–100, years 1960–2016.

Rates of mortality improvement



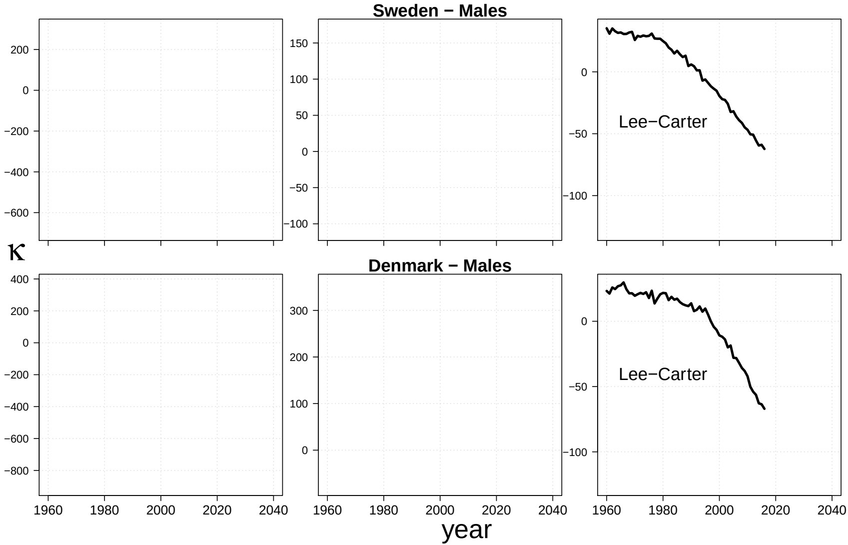
$\hat{\beta}$ estimates from the LC and 3C-sLC model.
Swedish and Danish males, ages 0–100, years 1960–2016.

Rates of mortality improvement



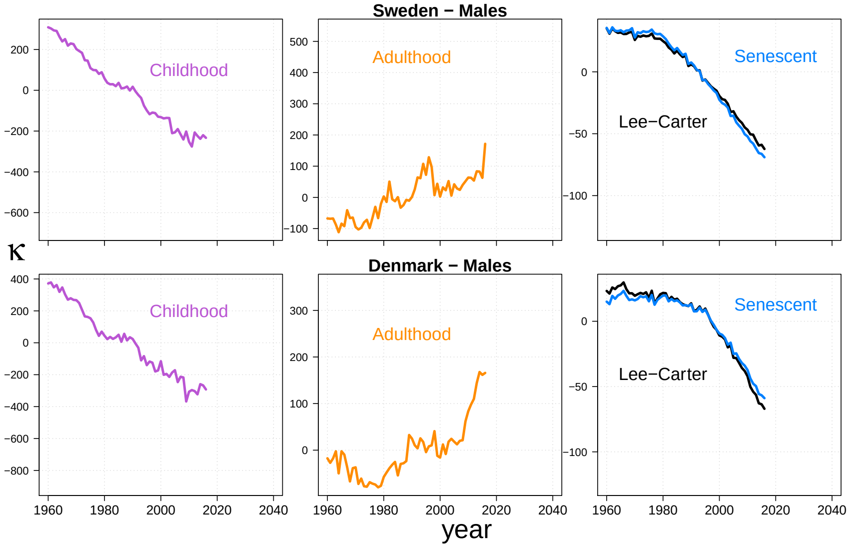
$\hat{\beta}$ estimates from the LC and 3C-sLC model.
Swedish and Danish males, ages 0–100, years 1960–2016.

Level of mortality



$\hat{\kappa}$ estimates and forecasts from the LC and 3C-sLC model.
Swedish and Danish males, ages 0–100, fitted years 1960–2016, forecast years 2017–2040.

Level of mortality



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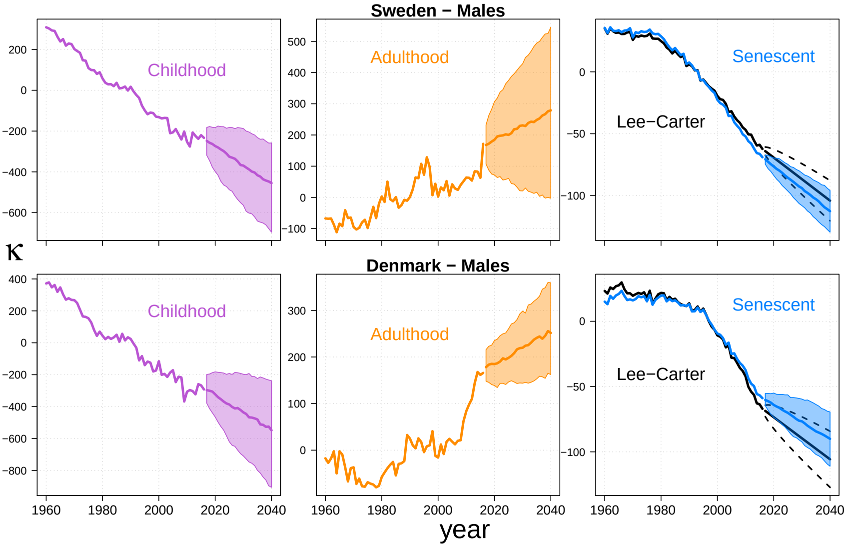
Level of mortality



$\hat{\kappa}$ estimates and forecasts from the LC and 3C-sLC model.
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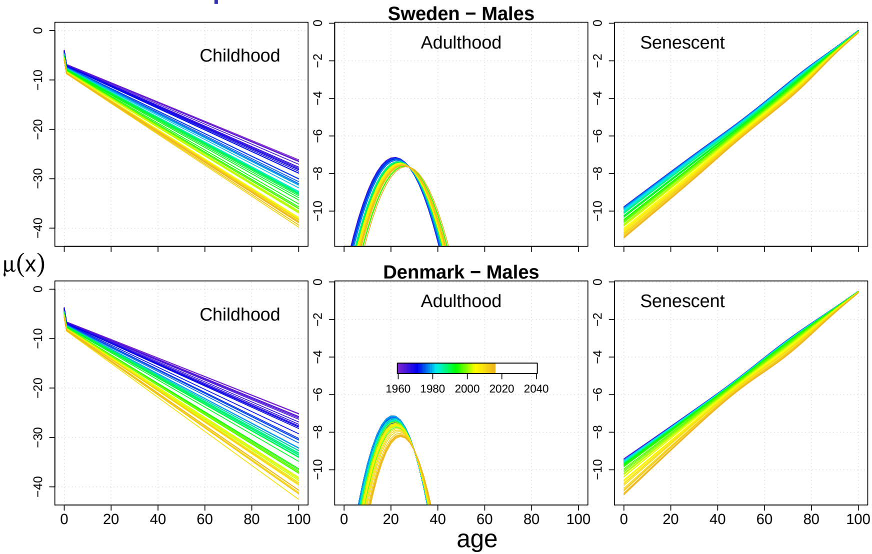
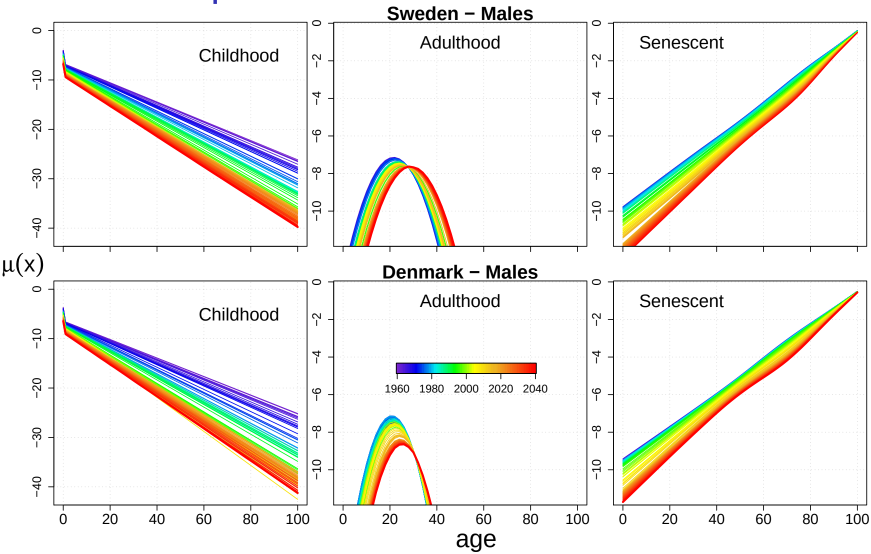
3C-sLC: components



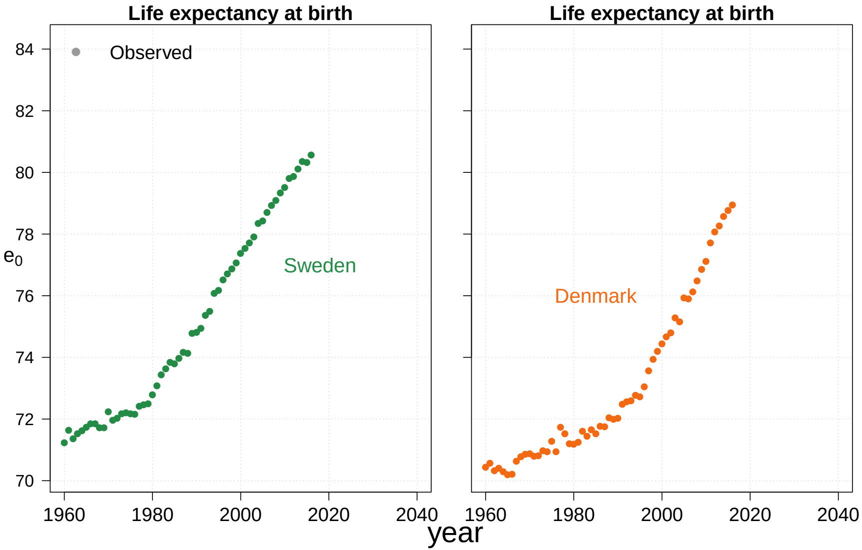
Fitted and forecast components (log-scale) of the 3C-sLC model.
Swedish and Danish males, ages 0–100, fitted years 1960–2016, forecast years 2017–2040.

3C-sLC: components



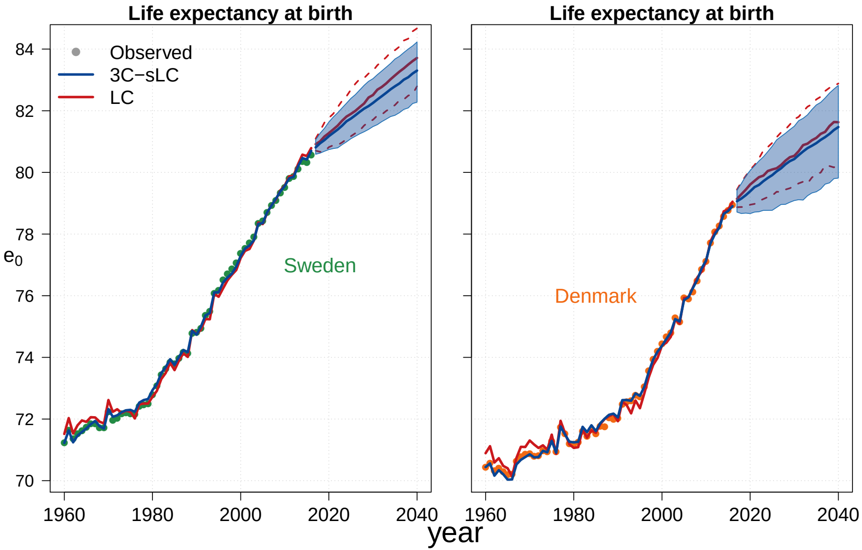
Fitted and forecast components (log-scale) of the 3C-sLC model.
Swedish and Danish males, ages 0–100, fitted years 1960–2016, forecast years 2017–2040.

3C-sLC: summary measures



Swedish and Danish males.
Ages 0–110+, fitted years 1960–2016, forecast years 2017–2040.

3C-sLC: summary measures



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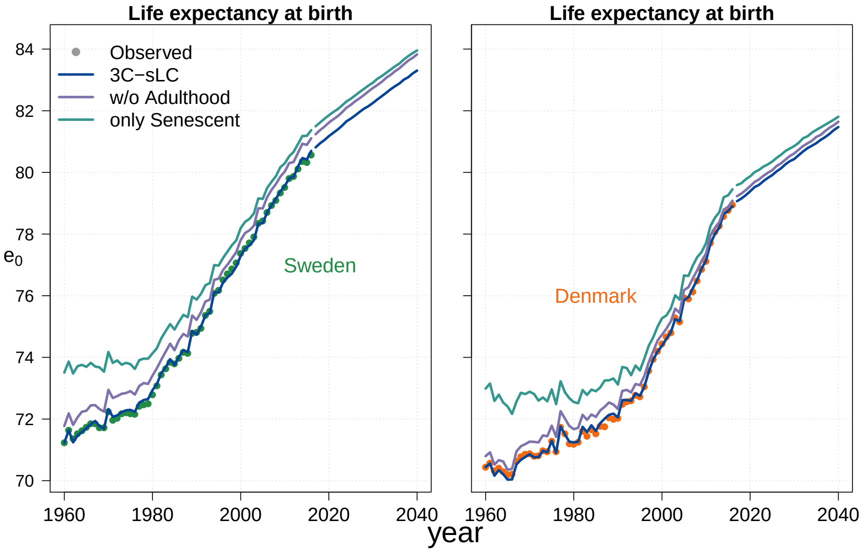

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Thesis summary

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Currie, Durban and Eilers (2004). Smoothing and forecasting mortality rates. *Statistical Modelling*, **4**(4), 279 – 298.

Currie (2016). On fitting generalized linear and non-linear models of mortality. *Scandinavian Actuarial Journal*, **4**, 356 – 383.

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Thiele (1871). On a Mathematical Formula to express the Rate of Mortality throughout the whole of Life. *Journal of the Institute of Actuaries*, **16**(5), 313 – 329.

Acknowledgments

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Alessandro, Matteo, Giulia, Russel, Davide, Sebastian, Namrata, Matthew, Jenny, Enrica, Alessandra, Linh, Enrique, Marcus and Angelo

Marília, my parents Elena and Aldo, my brother Carlo and Olivia, my grandparents Michela and Gustavo

New Approaches in Mortality Modelling and Forecasting

Assessment of PhD dissertation

Ugofilippo Basellini

Thank you for your attention!

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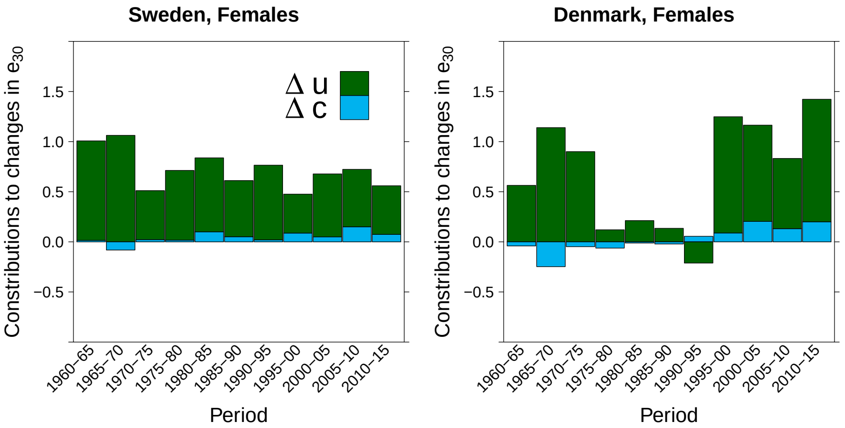
Future research

- Applicability: broader application of methodologies to other populations, e.g.:
  - other low-mortality populations
  - populations with lower data quality
  - populations with atypical mortality developments
- Generalization: extension of methodologies to other datasets, e.g. causes-of-death, fertility, coherent mortality projections
- Distribution: development of an R package for fast(er) implementation and use of the methodologies

LS mortality models

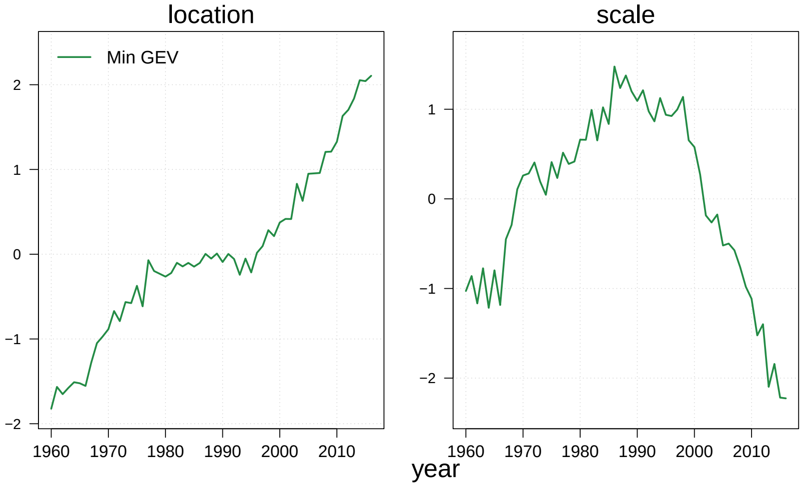
Models belonging to LS family	Models belonging to LLS family	Models related to LS family
Logistic	Log–Logistic	Gompertz
Normal	Log–Normal	Kannisto
Smallest Extreme–Value	Weibull	Gamma–Gompertz
Largest Extreme–Value		Minimal Generalized Extreme–Value
		Maximal Generalized Extreme–Value

LS contributions to changes in e_{30}



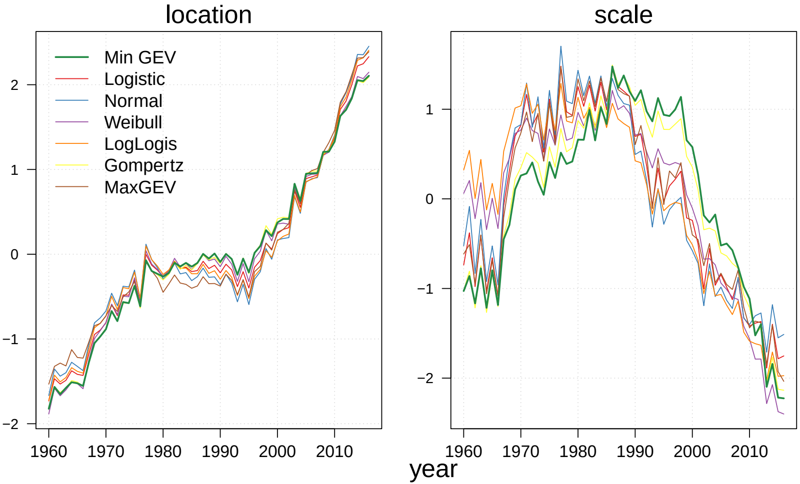
Contributions of the location u (green) and scale c (light blue) Gompertz parameters to changes in e_{30}.
Swedish and Danish females, ages 30–110+, years 1960–2015.
Source: Basellini et al. (2019)

LS estimates



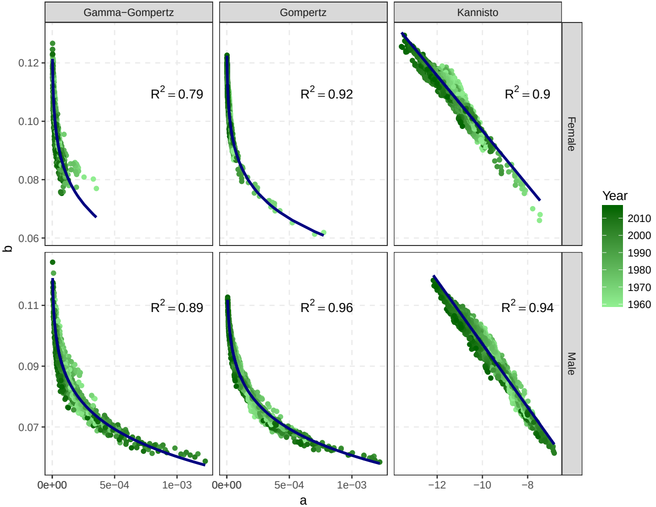
Rescaled LS estimated parameters of the Minimal Generalized Extreme–Value model.
Danish females, ages 30–110+, years 1960–2016.
Source: HMD (2020)

LS estimates



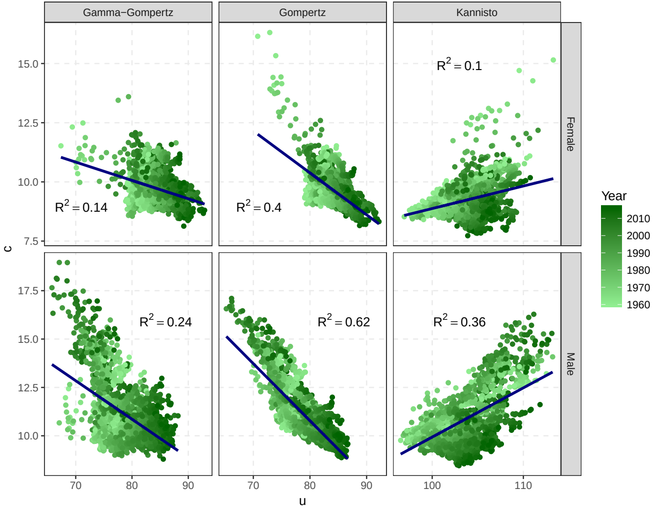
Rescaled LS estimated parameters of six LS models.
Danish females, ages 30–110+, years 1960–2016.
Source: HMD (2020)

LS correlations



MLEs and R^2 for the classic parameterization of three mortality models.
33 countries by gender, ages 30–110+, years 1960–2016.
Source: HMD (2018), Basellini et al. (2019)

LS correlations



MLEs and R^2 for the LS parameterization of three mortality models.
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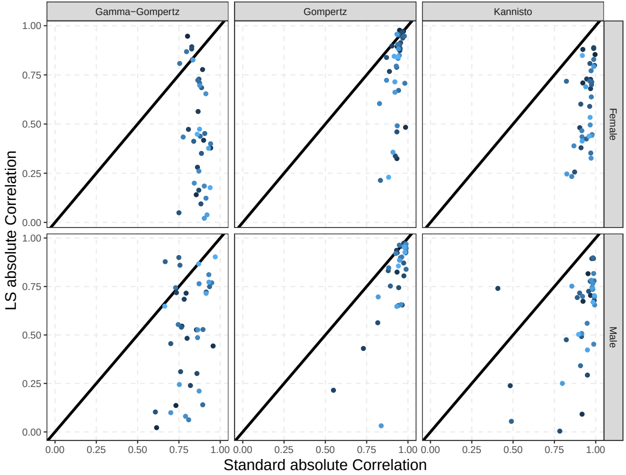
Source: HMD (2018), Basellini et al. (2019)

LS correlations



Within-country absolute correlation of the classic and LS parameterization of three mortality models.
33 countries by gender, ages 30–110+, years 1960–2016.
Source: HMD (2018), Basellini et al. (2019)

STAD: the model

Notation:

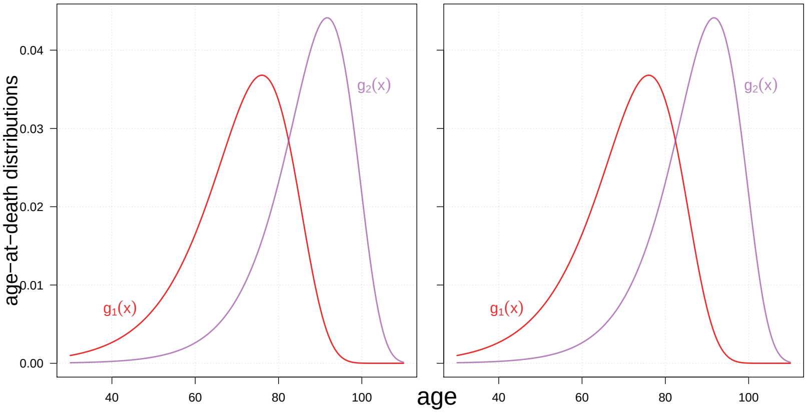
- x: age
- $f(x)$: standard distribution
- $g(x)$: observed distribution
- $t(x)$: transformation function

Aim: Look for a $t(x)$ such that:

- $g(x)$ conforms to $f(x)$ on the warped axis, i.e. $g(x) = f(t(x))$
- $t(\cdot)$ is a **segmented function** of the difference in modal ages and the change in the variability before and after M:

$$t(x; s, b_L, b_U) = \begin{cases} M^f + b_L\,(x - s - M^f) & \text{if } \; x \leq M^g \\ M^f + b_U\,(x - s - M^f) & \text{if } \; x > M^g \end{cases}$$

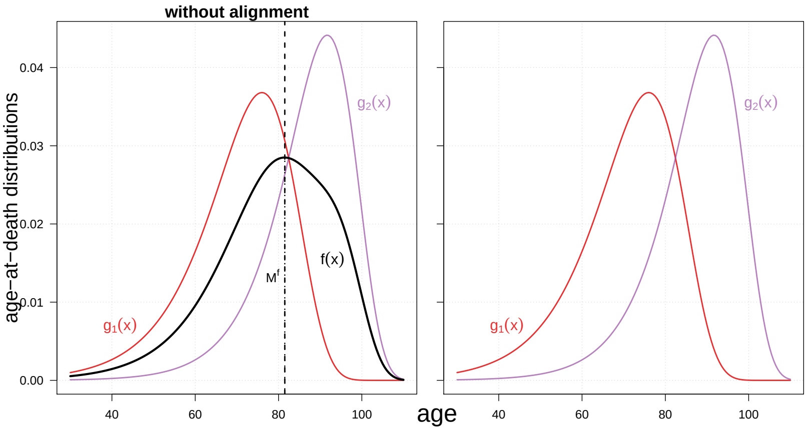
STAD: standard distribution



Source: Basellini et al. (2019)

STAD: standard distribution

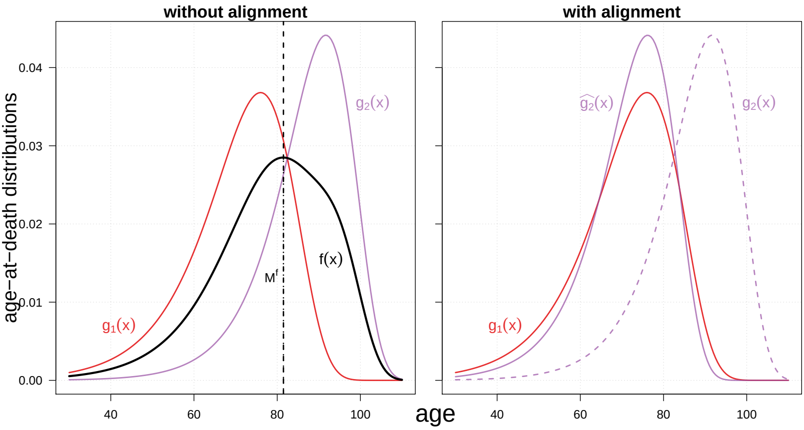
Simple mean over ages: can lead to biased / unreasonable standard distribution



Source: Basellini et al. (2019)

STAD: standard distribution

Landmark registration (Ramsay & Silverman 2005): alignment of observed densities to a common mode



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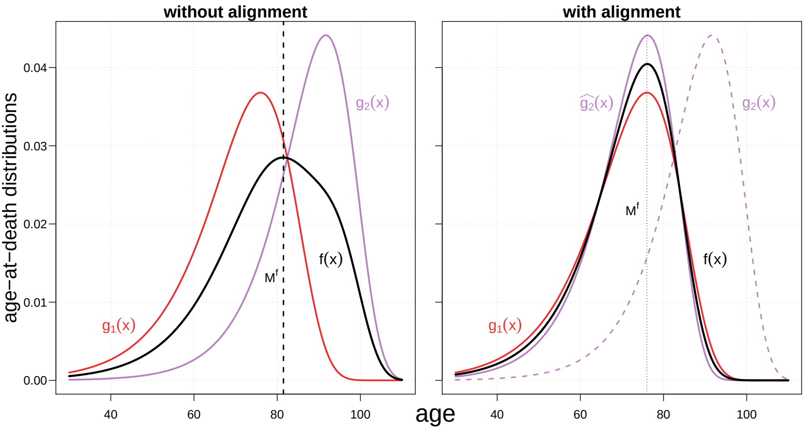


Source: Basellini et al. (2019)

STAD: application to observed data

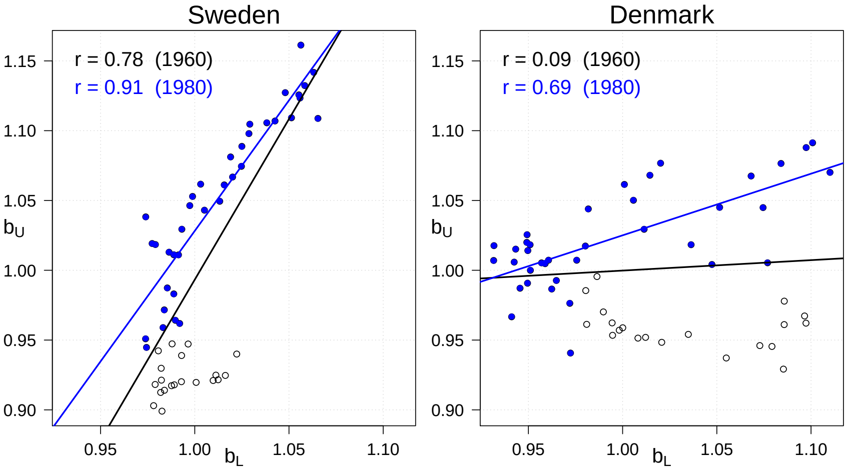
- **Smoothing** (Eilers & Marx 1996):
    - apply continuous model to discrete data
    - avoid rigid parametric mortality structure
    - derive M and parameter s (Ouellette & Bourbeau 2011)
    - derivation of standard distribution

- **Estimation:** for each year, b_L and b_U estimated by maximum likelihood from the assumption:

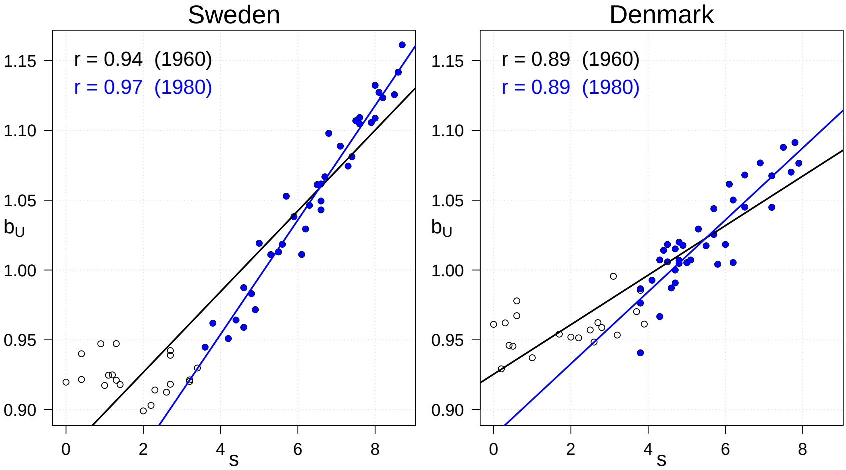
$$D_x \sim \mathcal{P}\left(E_x \, \mu_x\right)$$

where μ_x is evaluated numerically from the STAD segmented distribution $f(t(x))$

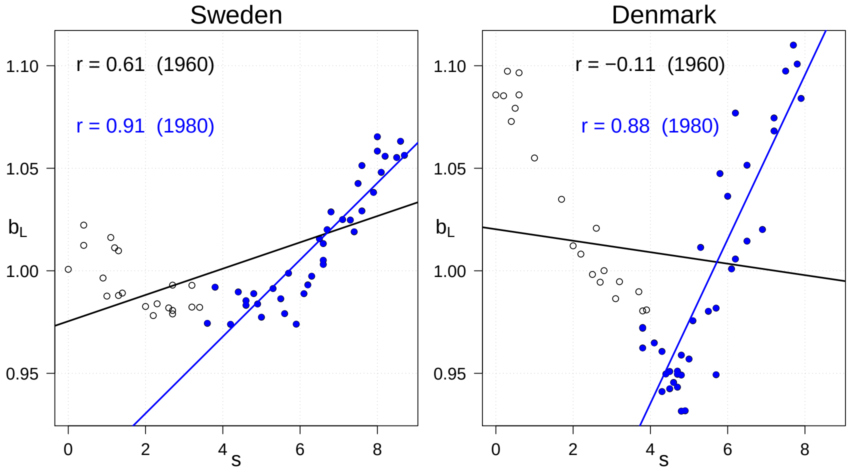
STAD: correlations



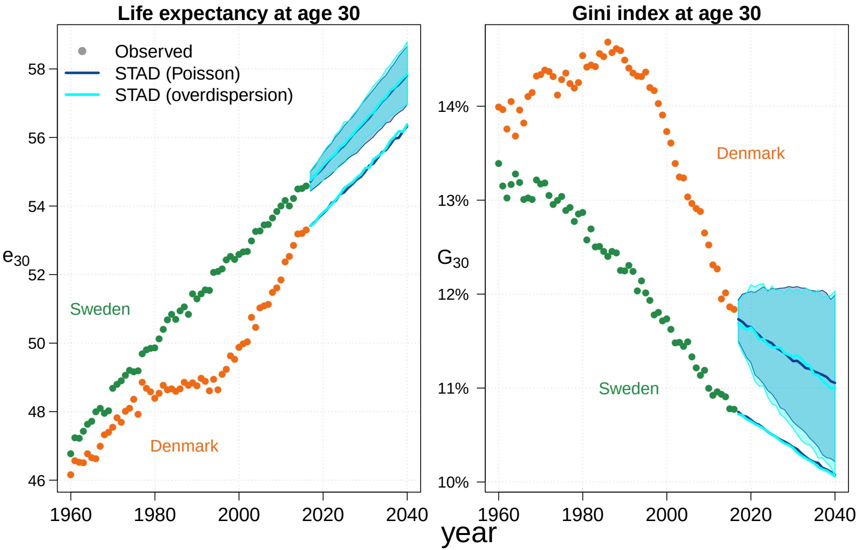
STAD: correlations



STAD: correlations

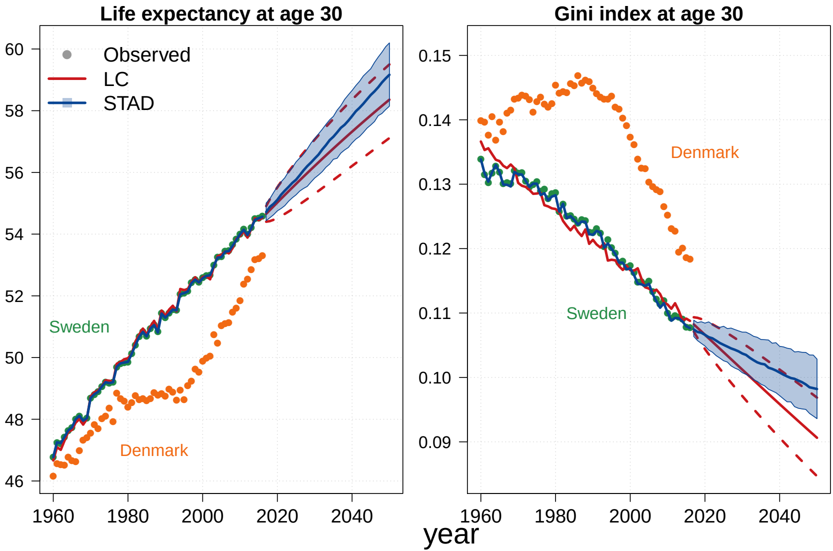


STAD: overdispersion

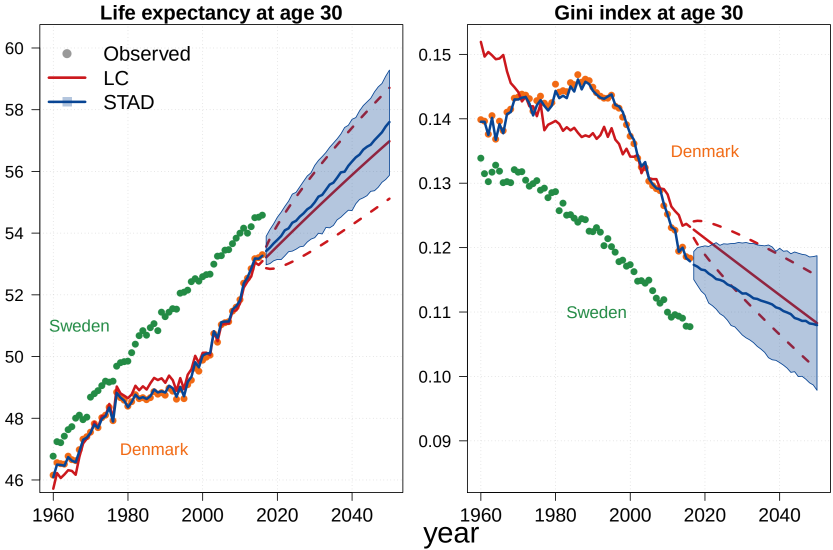


Swedish and Danish females.
Ages 30–110+, fitted years 1960–2016, forecast years 2017–2040.

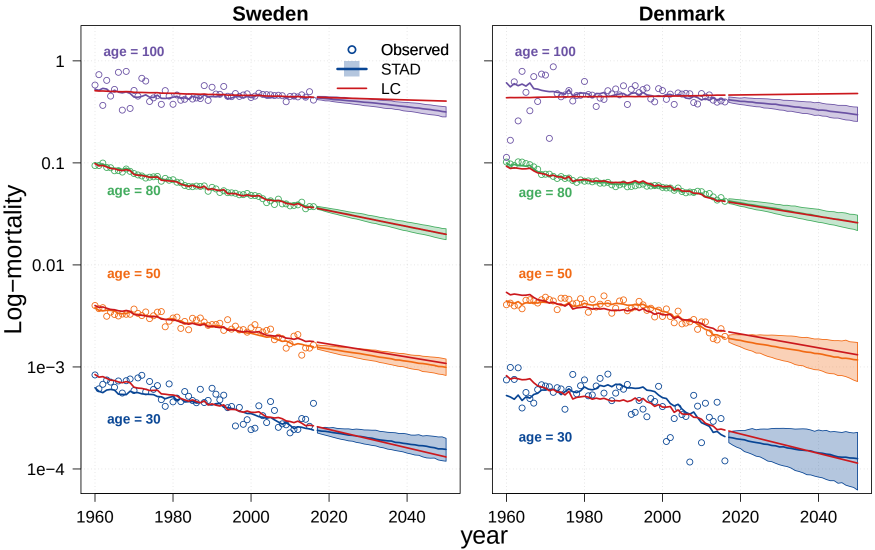
STAD: CI


Life expectancy at age 30
Gini index at age 30
Observed
LC
STAD
Sweden
Denmark
60
58
56
54
52
50
48
46
0.15
0.14
0.13
0.12
0.11
0.10
0.09
1960
1980
2000
2020
2040
year

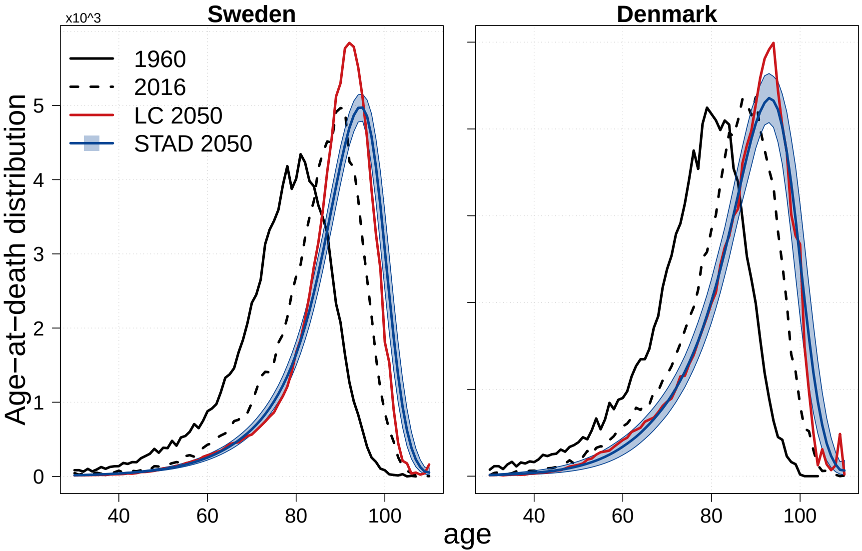
STAD: CI


Life expectancy at age 30
Gini index at age 30
Observed
LC
STAD
Sweden
Denmark
60
58
56
54
52
50
48
46
0.15
0.14
0.13
0.12
0.11
0.10
0.09
1960
1980
2000
2020
2040
year

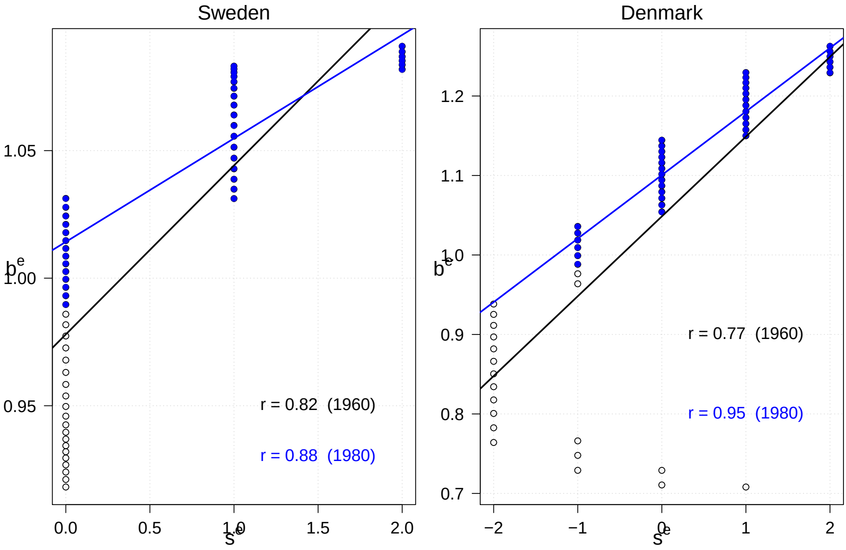
STAD: rates


Sweden
Denmark
Observed
STAD
LC
age = 100
age = 80
age = 50
age = 30
Log−mortality
1
0.1
0.01
1e−3
1e−4
1960
1980
2000
2020
2040
year

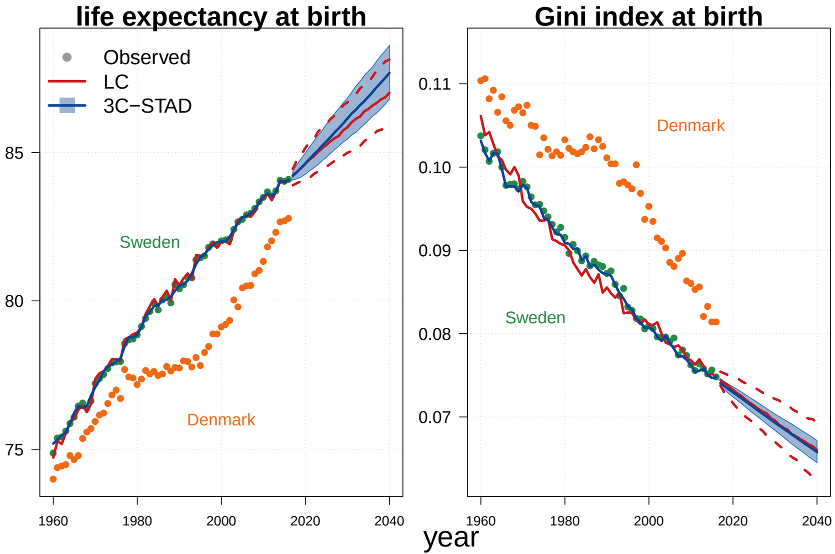
STAD: distributions



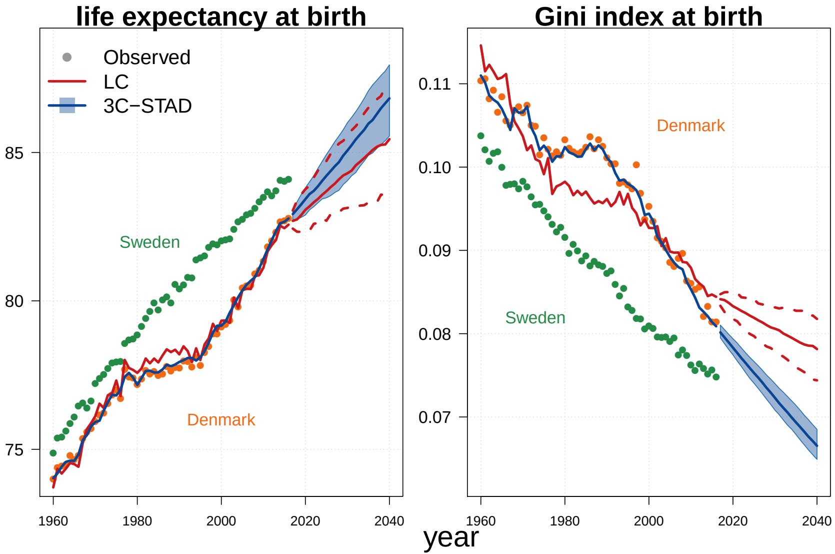
3C-STAD: correlations



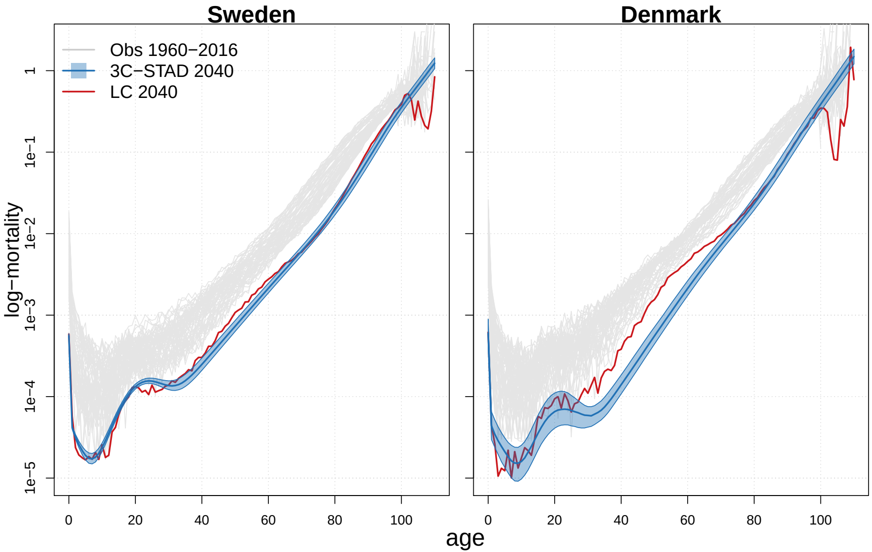
3C-STAD: CI


life expectancy at birth
Observed
LC
3C−STAD
Sweden
Denmark
85
80
75
1960
1980
2000
2020
2040
Gini index at birth
0.11
0.10
0.09
0.08
0.07
Denmark
Sweden
1960
1980
2000
2020
2040
year

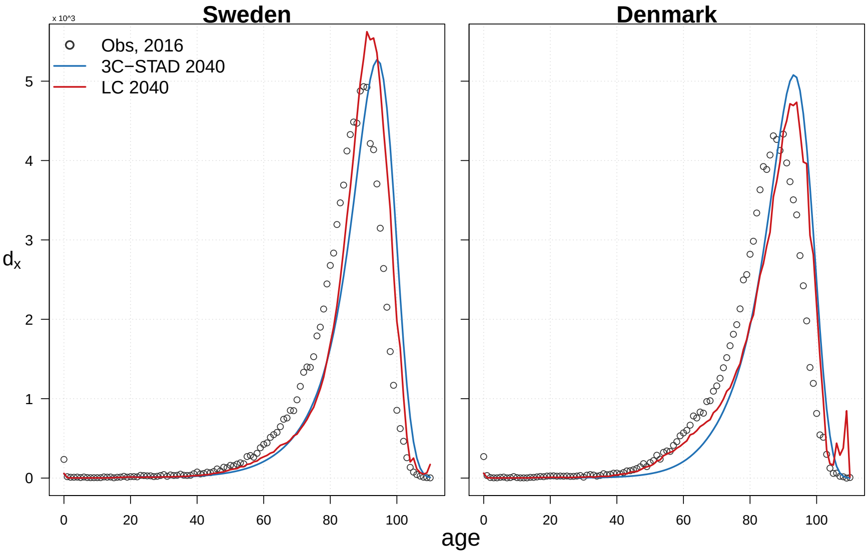
3C-STAD: CI


life expectancy at birth
Gini index at birth
Observed
LC
3C−STAD
Sweden
Denmark
85
80
75
0.11
0.10
0.09
0.08
0.07
1960
1980
2000
2020
2040
year

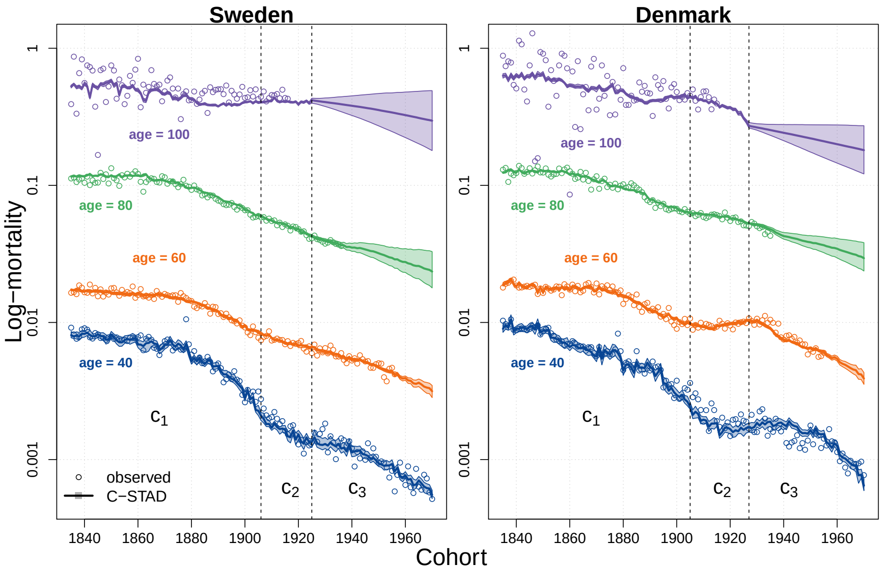
3C-STAD: rates



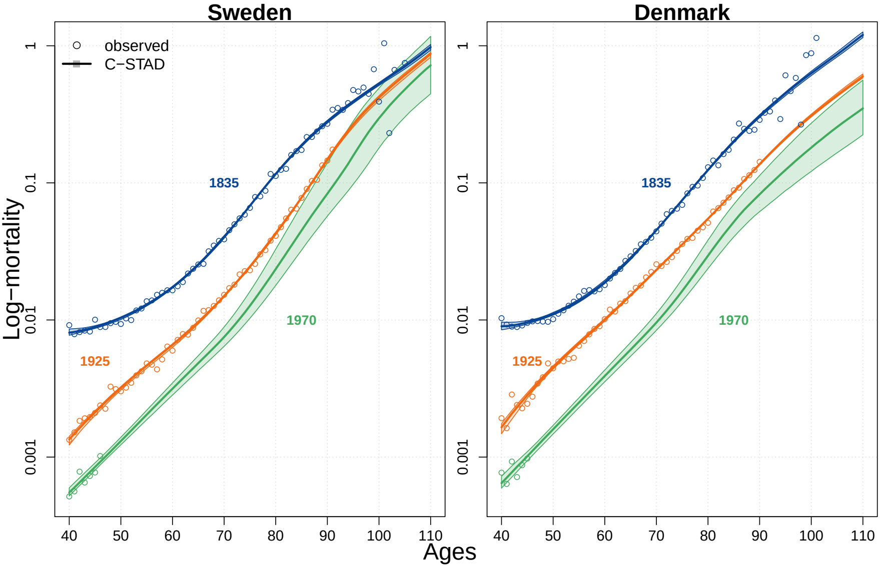
3C-STAD: distributions



C-STAD: rates I



C-STAD: rates II



C-STAD: distributions