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Priority Percent Calculation Notes [WIP]

The math note is an expansion on the combinatorics calculation implemented in the MLEC sim's flat DP mode. The concept is simple - when we have a disk failure, we are interested in how many data on the drive (priority percent) belongs to the disk's priority and then we can calculate the time needed for repair in relation to normal RAID. Knowing the repair time in relation to normal RAID, we can then calculate the MTTDL of network SLEC DP in relation to normal RAID and mathematically calculate the number of nines for network SLEC DP durability.

The notations used in the note is as follows

- r: number of racks
- B: number of disks per rack
- k: number of data chunks
- m: number of parity chunks
- n: stripe width $(k + m)$
- f: number of failures in the stripe (number of disks that we need to repair)
- $\mathcal{CAP}$: capacity of the disk
- $S_{net_W} = S_{net_R} = S_{net}$: speed of network IO

I. WHEN THERE IS A SINGLE FAILURE [DISTINCT RACK], THIS MEANS THAT

- All the affected stripes must not have another drive on the same rack
- The remaining good drives to select affected stripes out of are $(r-1)B$
- The number of failed drive is 1 and has priority of 1
- The priority percent equation then becomes

$$\frac{ncr((r-1)B, n-1) * ncr(1-1, 1-1)}{ncr((r-1)B + 1 - 1, n-1)} = \frac{ncr((r-1)B, n-1)}{ncr((r-1)B, n-1)} = 1$$

We can see that in terms of parallel repair of DP, we can only read/write from and to the disks that do not reside on the same rack. Therefore we will have the parallelism as $(r-1)B$.

The amplification is reading from k chunks and writing to f chunk.

Therefore the time needed for repair calculation is as follows

$$\frac{\mathcal{CAP}(k+f)}{S_{net}(r-1)B/f}$$

When there are two failures [same rack], this means that the _priority 1 stripe_ works as follows

- All the affected stripes must not have another drive on the same rack
- The remaining good drives to select affected stripes out of are **still** $(r-1)B$ because damaged stripes will have other chunks sitting on other racks, and the damaged chunk be sitting on either one of the failed drive
- The number of failed drive is 2 and both still has priority of 1
- The priority percent equation then becomes

$$\frac{ncr((r-1)B, n-1) * ncr(2-1, 1-1)}{ncr((r-1)B + 2 - 1, n-1)} = \frac{(r-1)B - n + 2}{(r-1)B + 1}$$

Making an example with 10 racks, 10 drives per rack, and (8+2) config, and there are two failures on the same rack, the priority percent would be

$$\frac{(10-1) * 10 - 10 + 2}{(10-1)10 + 1} = \frac{82}{91} \approx 0.901$$

When there are two failures [same rack], this means that _priority 2 stripe_ works as follows

- Basically same as two failures same rack, priority 1 stripe, except priority
- The remaining good drive is $(r-1)B$
- The number of failed drive is 2
- The priority percent equation then becomes

$$\frac{ncr((r-1)B, n-2) * ncr(2-1, 2-1)}{ncr((r-1)B + 2 - 1, n-1)} = \frac{n-1}{(r-1)B + 1}$$

Using the same $r = 10, B = 10, n = 10$ example, we have priority percent equals $\frac{9}{91} \approx 0.0989$

When there are two failures [distinct rack], this means that _priority 1 stripe_ works a follows

- The remaining good drive is still $(r-1)B - 1$ because the priority 1 stripe will have all surviving chunks in all racks except the one that contains the failed chunk. The minus 1 is because in one of the rack containing one of the surviving chunk, there is one failed disk that happens to not impact this stripe.
- The number of failed drive is 2
- The priority percent equation then becomes

$$\frac{ncr((r-1)B - 1, n-1) * ncr(2-1, 1-1)}{ncr((r-1)B - 1 + 2 - 1, n - 1)} = \frac{(r-1)B - n + 1}{(r-1)B}$$

Using the same $r = 10, B = 10, n = 10$ example, we have priority percent equals $\frac{81}{90} = 0.9$

When there are two failures [distinct rack], this means that *priority 2 stripe* works a follows

- All the stripes with priority 2 have both of the chunks sitting on each of the failed drive residing in two racks. This means that the remaining good drives to select from is $(r-2)B$.
- The number of failed disk is 2
- The priority of the stripes is 2
- The priority percent equation then becomes

$$\frac{ncr((r-2)B, n-2) * ncr(2-1, 2-1)}{ncr((r-2)B + 2 - 1, n - 1)} = \frac{n-1}{(r-2)B + 1}$$

When there are three failures [distinct rack], this means that *pirority 3 stripe* works as follows

- The remaining good drive is $(r-3)B$
- The number of failed disk is 3
- The priority of the stripe is 3
- The priority perent equation then becomes

$$\frac{ncr((r-3)B, n-3) * ncr(3-1, 3-1)}{ncr((r-3)B + 3 - 1, n - 1)} = \frac{(n-1)(n-2)}{[(r-3)B + 1][(r-3)B + 2]}$$

When there are four failures [distinct rack], this means that *priority 4 stripe* works as follows

- The remaining good drive is $(r-4)B$
- The number of failed disk is 4
- The priority of the stripe is 4
- The priority percent equation then becomes

$$\frac{ncr((r-4)B, n-4) * ncr(4-1, 4-1)}{ncr((r-4)B + 4 - 1, n - 1)} = \frac{(n-1)(n-2)(n-3)}{[(r-4)B + 1][(r-4)B + 2][(r-4)B + 3]}$$

When there are n failures across n distinct racks, the stripes with priority n

The priority percent calculation should be the following. First we let the number of failures be f

$$\prod_{i=1}^{f-1} \frac{(n-i)}{[(r-f)B + i]}$$