Priority Percent Calculation Notes

I. When there is a single failure [distinct rack], this means that

- All the affected stripes must not have another drive on the same rack
- The remaining good drives to select affected stripes out of are $(r-1)B$
- The number of failed drive is 1 and has priority of 1
- The priority percent equation then becomes

$$\frac{ncr((r-1)B, n-1) * ncr(1-1, 1-1)}{ncr((r-1)B+1-1, n-1)} = \frac{ncr((r-1)B, n-1)}{ncr((r-1)B, n-1)} = 1$$

We can see that in terms of parallel repair of DP, we can only read/write from and to the disks that do not reside on the same rack. Therefore we will have the parallelism as $(r-1)B$.

The amplification is reading from k chunks and writing to f chunk.

Therefore the time needed for repair calculation is as follows

$$\frac{\mathcal{CAP}(k+f)}{S_{net}(r-1)B/f}$$

When there are two failures [same rack], this means that the *priority 1 stripe* works as follows

- All the affected stripes must not have another drive on the same rack
- The remaining good drives to select affected stripes out of are **still** $(r-1)B$ because damaged stripes will have other chunks sitting on other racks, and the damaged chunk be sitting on either one of the failed drive
- The number of failed drive is 2 and both still has priority of 1
- The priority percent equation then becomes

$$\frac{ncr((r-1)B, n-1) * ncr(2-1, 1-1)}{ncr((r-1)B+2-1, n-1)} = \frac{(r-1)B-n+2}{(r-1)B+1}$$

Making an example with 10 racks, 10 drives per rack, and (8+2) config, and there are two failures on the same rack, the priority percent would be

$$\frac{(10-1)*10-10+2}{(10-1)10+1} = \frac{82}{91} \approx 0.901$$

When there are two failures [same rack], this means that *priority 2 stripe* works as follows

- Basically same as two failures same rack, priority 1 stripe, except priority
- The remaining good drive is $(r-1)B$
- The number of failed drive is 2
- The priority percent equation then becomes

$$\frac{ncr((r-1)B, n-2) * ncr(2-1, 2-1)}{ncr((r-1)B+2-1, n-1)} = \frac{n-1}{(r-1)B+1}$$

Using the same $r = 10, B = 10, n = 10$ example, we have priority percent equals $\frac{9}{91} \approx 0.0989$

When there are two failures [distinct rack], this means that *priority 1 stripe* works a follows

- The remaining good drive is still $(r-1)B-1$ because the priority 1 stripe will have all surviving chunks in all racks except the one that contains the failed chunk. The minus 1 is because in one of the rack containing one of the surviving chunk, there is one failed disk that happens to not impact this stripe.
- The number of failed drive is 2
- The priority percent equation then becomes

$$\frac{ncr((r-1)B-1, n-1) * ncr(2-1, 1-1)}{ncr((r-1)B-1+2-1, n-1)} = \frac{(r-1)B-n+1}{(r-1)B}$$

Using the same $r = 10, B = 10, n = 10$ example, we have priority percent equals $\frac{81}{90} = 0.9$

When there are two failures [distinct rack], this means that *priority 2 stripe* works a follows

- All the stripes with priority 2 have both of the chunks sitting on each of the failed drive residing in two racks. This means that the remaining good drives to select from is $(r-2)B$.
- The number of failed disk is 2
- The priority of the stripes is 2

- The priority percent equation then becomes

$$\frac{ncr((r-2)B, n-2) * ncr(2-1, 2-1)}{ncr((r-2)B+2-1, n-1)} = \frac{n-1}{(r-2)B+1}$$

When there are three failures [distinct rack], this means that *pirority 3 stripe* works as follows

- The remaining good drive is $(r-3)B$
- The number of failed disk is 3
- The priority of the stripe is 3
- The priority perent equation then becomes

$$\frac{ncr((r-3)B, n-3) * ncr(3-1, 3-1)}{ncr((r-3)B+3-1, n-1)} = \frac{(n-1)(n-2)}{[(r-3)B+1][(r-3)B+2]}$$

When there are four failures [distinct rack], this means that *priority 4 stripe* works as follows

- The remaining good drive is $(r-4)B$
- The number of failed disk is 4
- The priority of the stripe is 4
- The priority percent equation then becomes

$$\frac{ncr((r-4)B, n-4) * ncr(4-1, 4-1)}{ncr((r-4)B+4-1, n-1)} = \frac{(n-1)(n-2)(n-3)}{[(r-4)B+1][(r-4)B+2][(r-4)B+3]}$$

When there are n failures across n distinct racks, the stripes with priority n

The priority percent calculation should be the following. First we let the number of failures be f

$$\prod_{i=1}^{f-1} \frac{(n-i)}{[(r-f)B+i]}$$