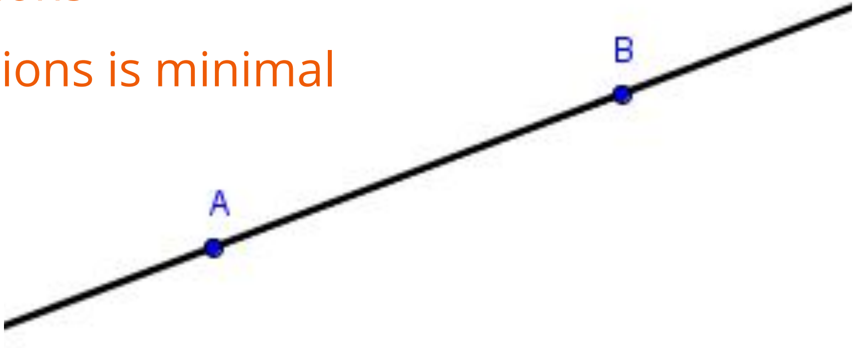
Robust Geometry Estimation

(RANSAC)

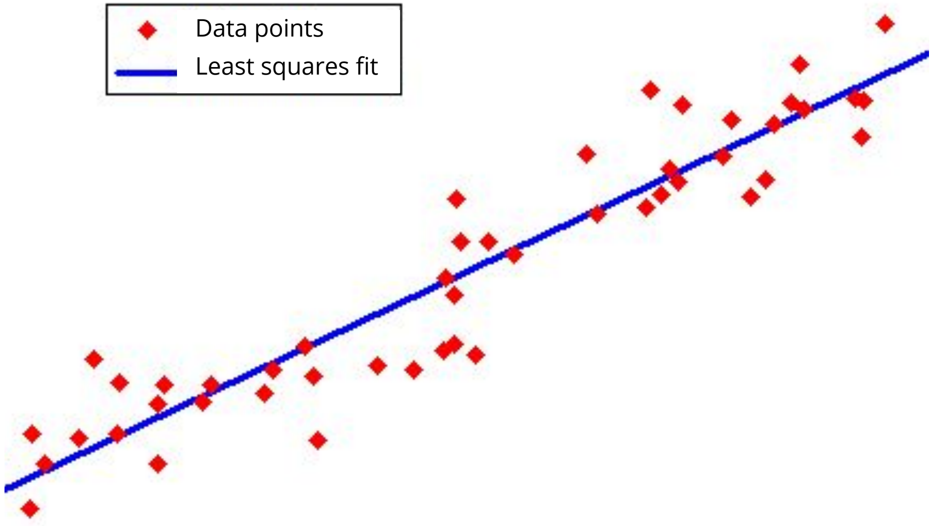
Line Fitting Example

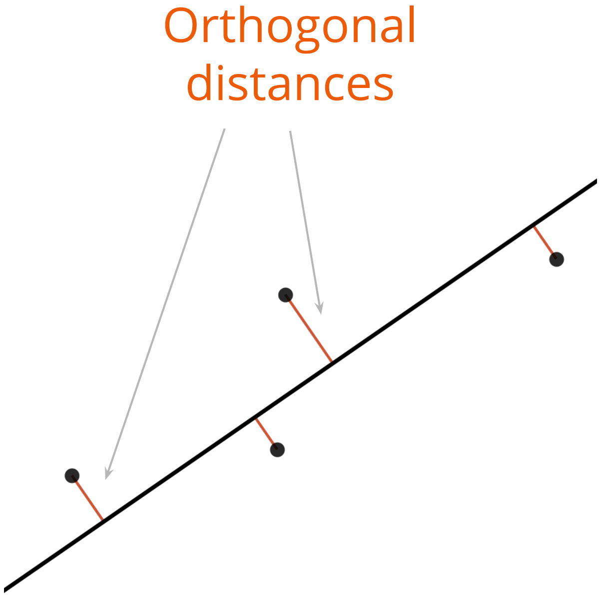
Exact solution to $ax + by + c = 0$

- Line has **2** degrees of freedom (DoF)
- **1** point introduces **1** independent constraint equation
- Thus we need at least **2** points
- **2** is a **minimal** number of observations
- The solution to the system of equations is minimal
- Also called a **minimal solver**



Lots of Noisy Points -> MLE -> Least Squares

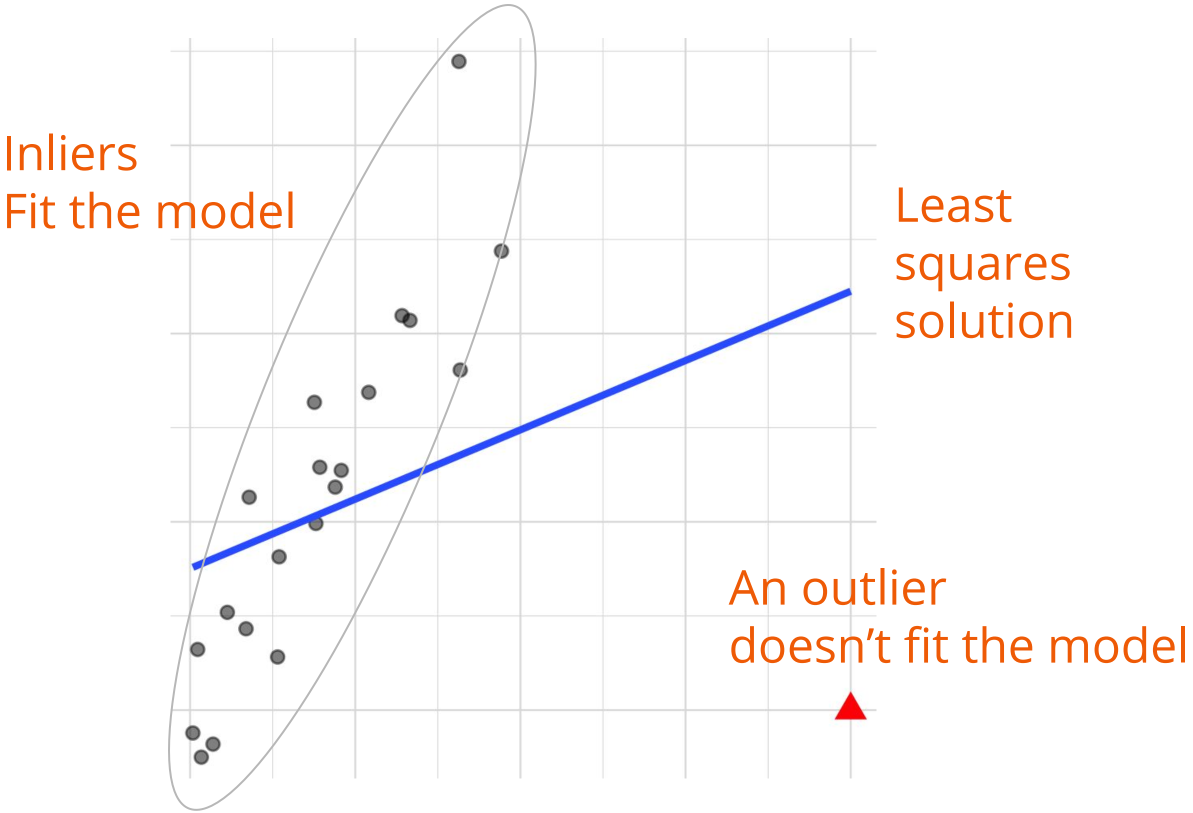




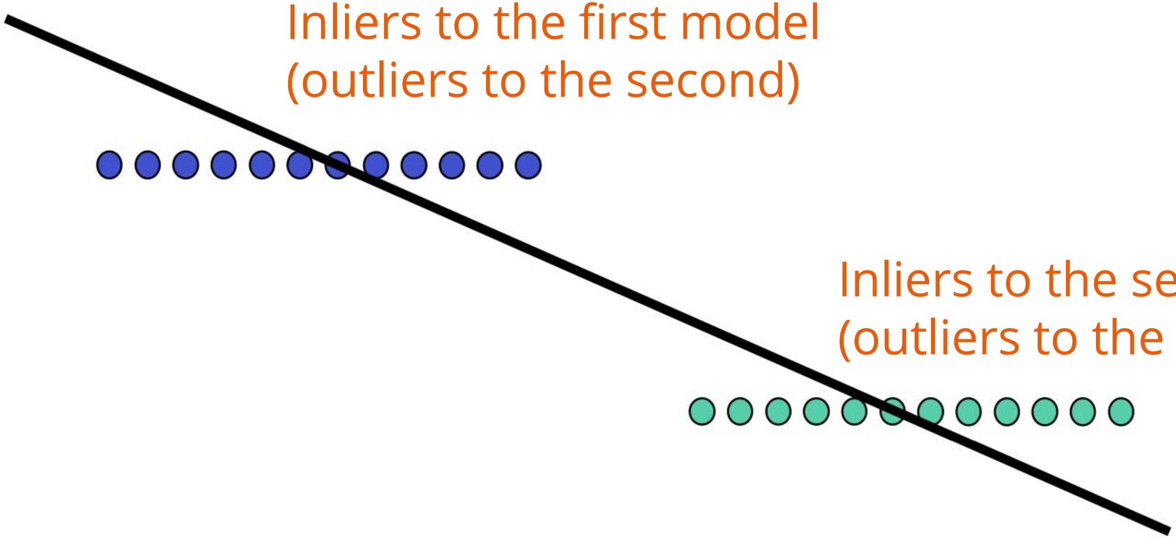
Total Least Squares (TLS)

Both x- and y- coordinates are observed subject to error. Solution — eigenvector corresponding to the smallest eigenvalue of A^TA, where A is $Kx2$ data matrix.

Outliers?


Inliers
Fit the model
Least
squares
solution
An outlier
doesn't fit the model

Multiple Models (e.g. Lines) ?



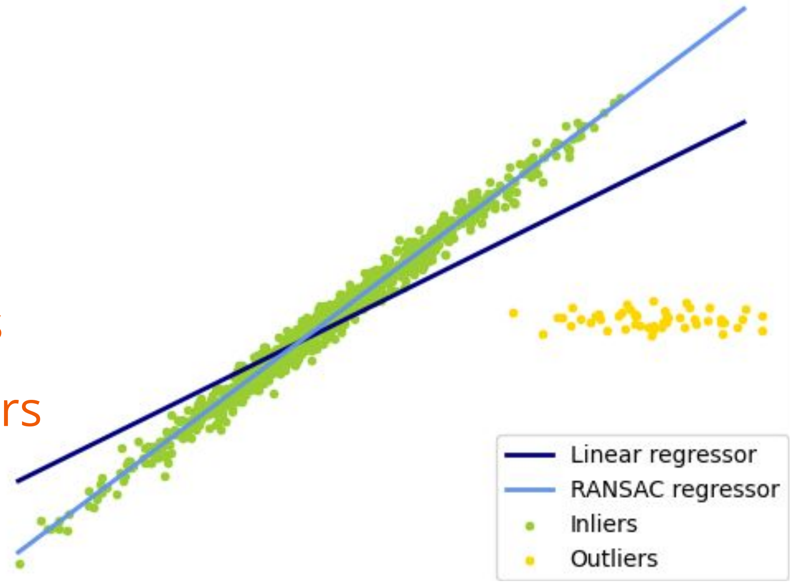
RANSAC: Robust to Outliers

*(**RAN**dom **SA**mple **C**onsensus)*

Two "stages":

(1) Classify data points as outliers or inliers

(2) Fit model to inliers while ignoring outliers

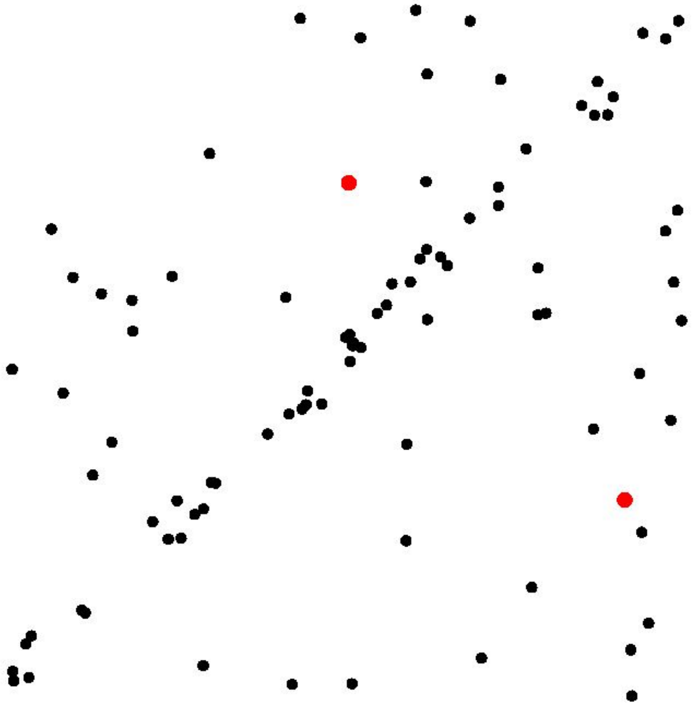


[1] M. A. Fischler and R. C. Bolles. Random Sample Consensus: A Paradigm for Model Fitting with Applications to Image Analysis and Automated Cartography (1981)

RANSAC Procedure

RANSAC Procedure

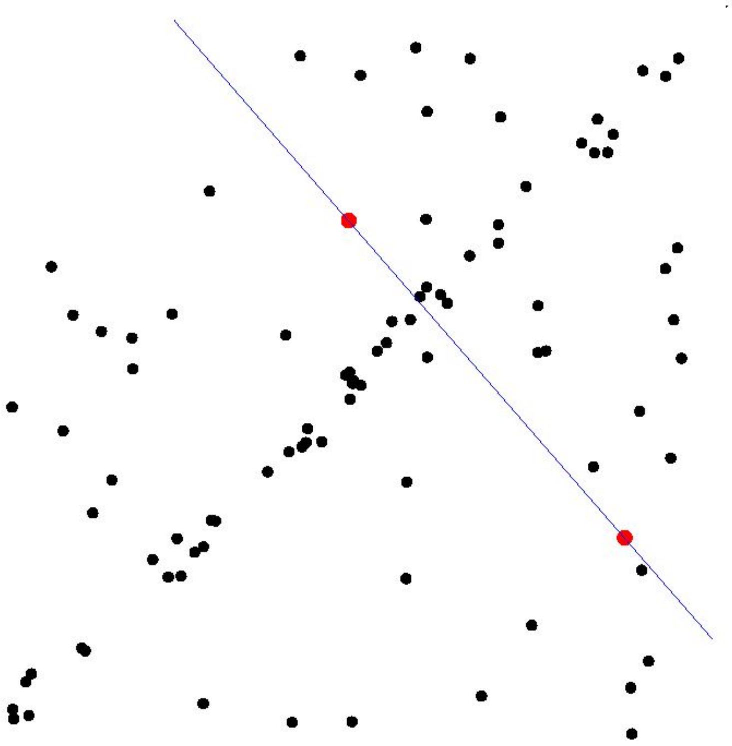
Randomly pick the minimal number of observations — the ***minimal sample***



Hypothesize

RANSAC Procedure

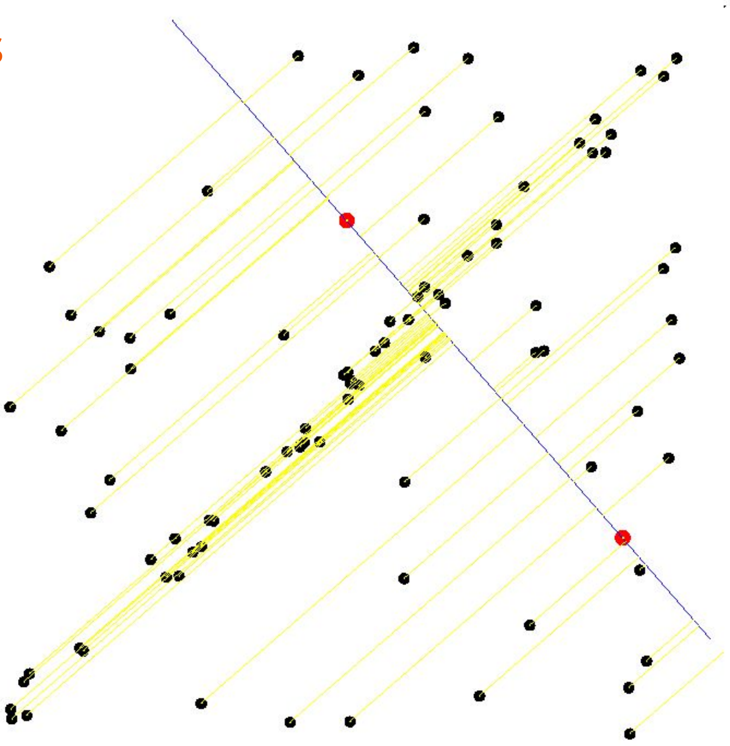
Fit the model exactly



Hypothesize

RANSAC Procedure

Calculate the residuals



Verify

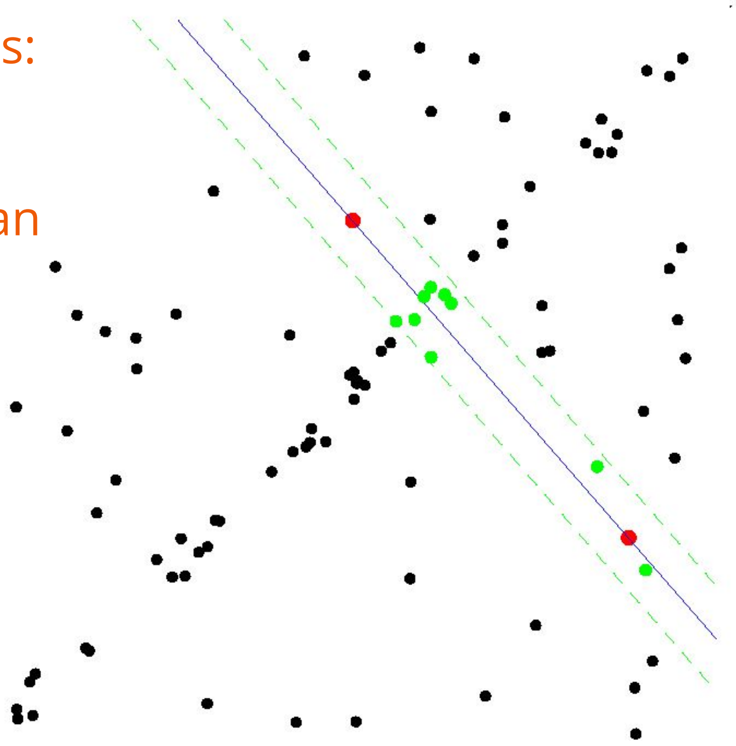
RANSAC Procedure

Classify the data points:

- **inlier** if the residual is less than threshold
- **outlier** otherwise

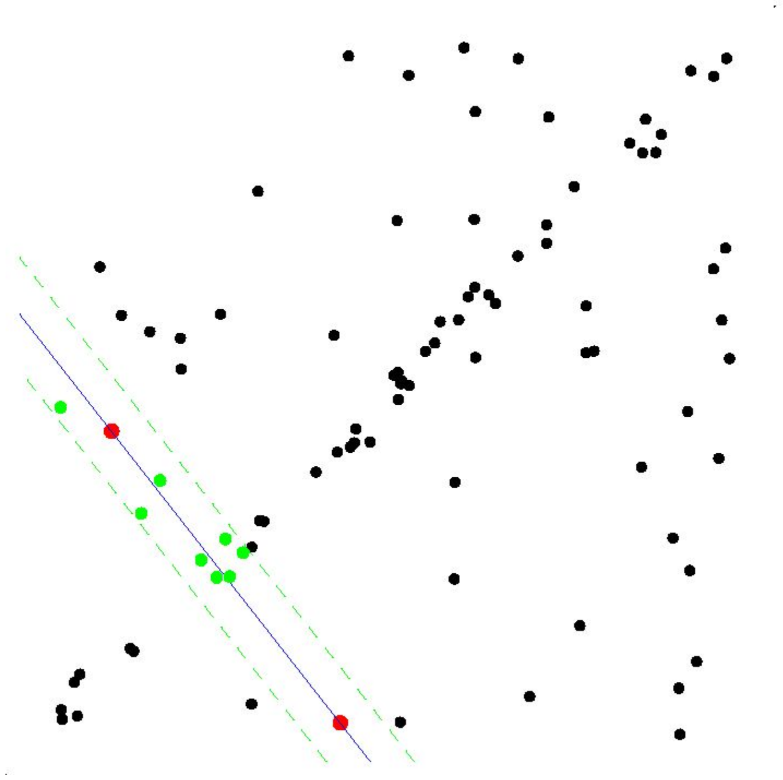
Compute the number of inliers — the **consensus set**

Verify



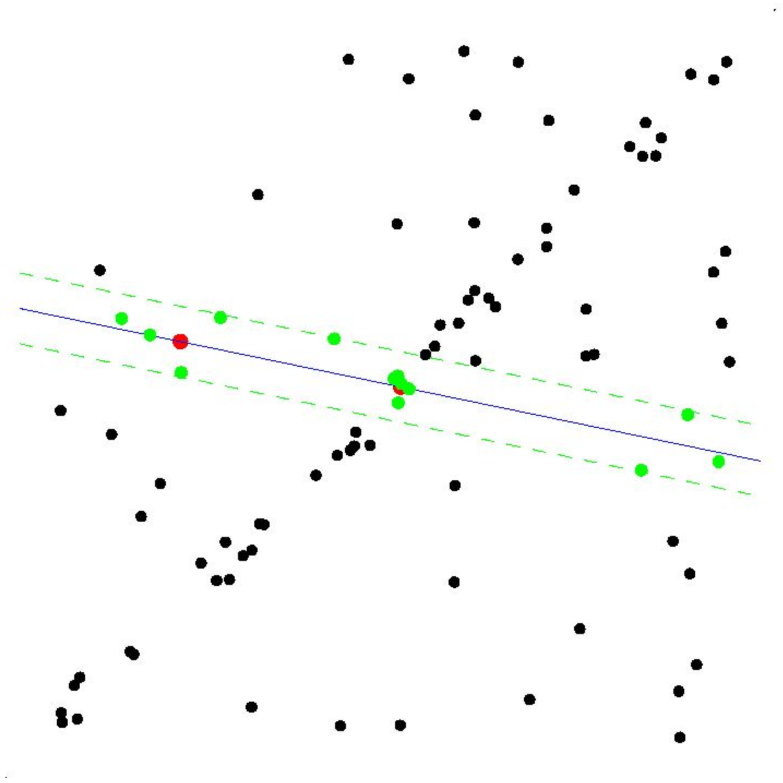
RANSAC Procedure

Repeat



RANSAC Procedure

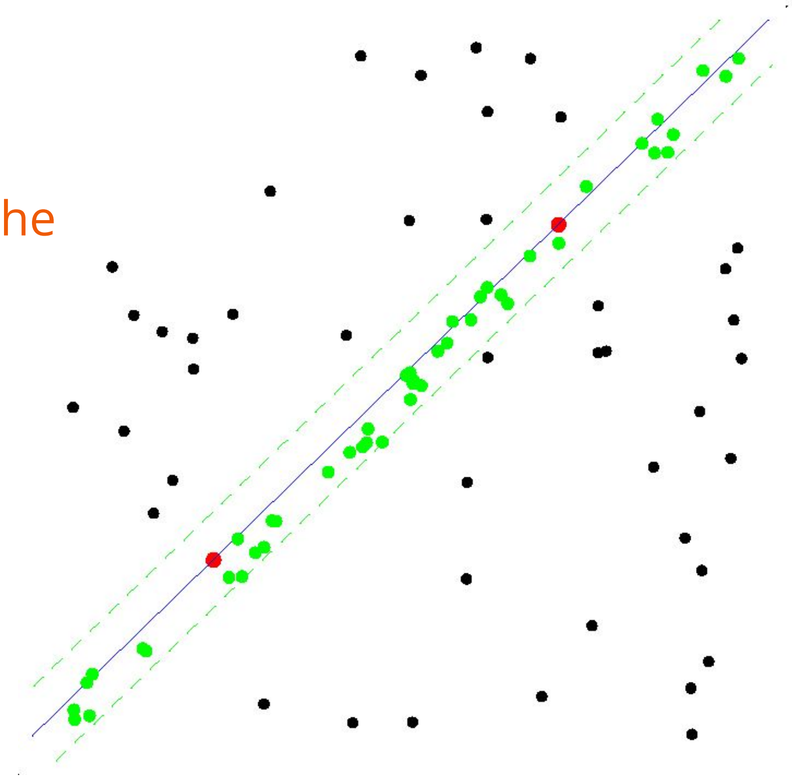
Repeat



RANSAC Procedure

Repeat

Goal: to maximize the consensus set



Number of Iterations (Samples)

Sample all models? That's a lot..

$$k = \binom{K}{2} = \frac{K!}{2(K-2)!} = \frac{K(K-1)}{2} = \Theta(K^2)$$

K — number of all data points

k — number of samples

Number of Iterations

$$1 - p = (1 - w^n)^k \longrightarrow k = \frac{\log(1-p)}{\log(1-w^n)}$$

p — desired probability of success

w — inlier ratio — number of inliers in data / number of points in data

n — minimal sample size

k — number of samples

Number of Iterations

Percentage of outliers

Number of samples

Minimal sample size

s	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177