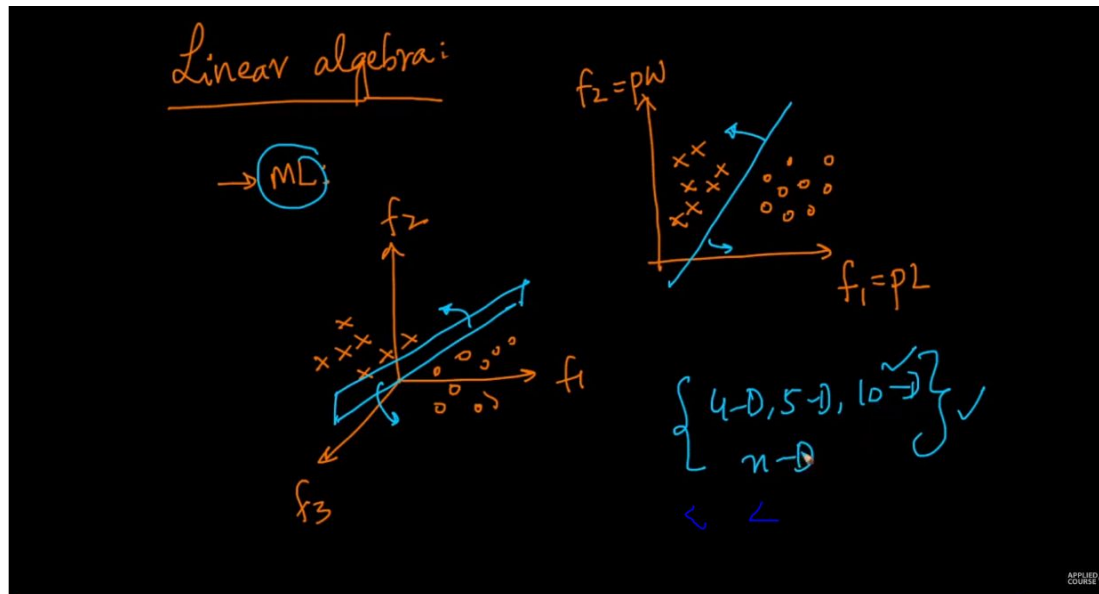


# LINEAR ALGEBRA

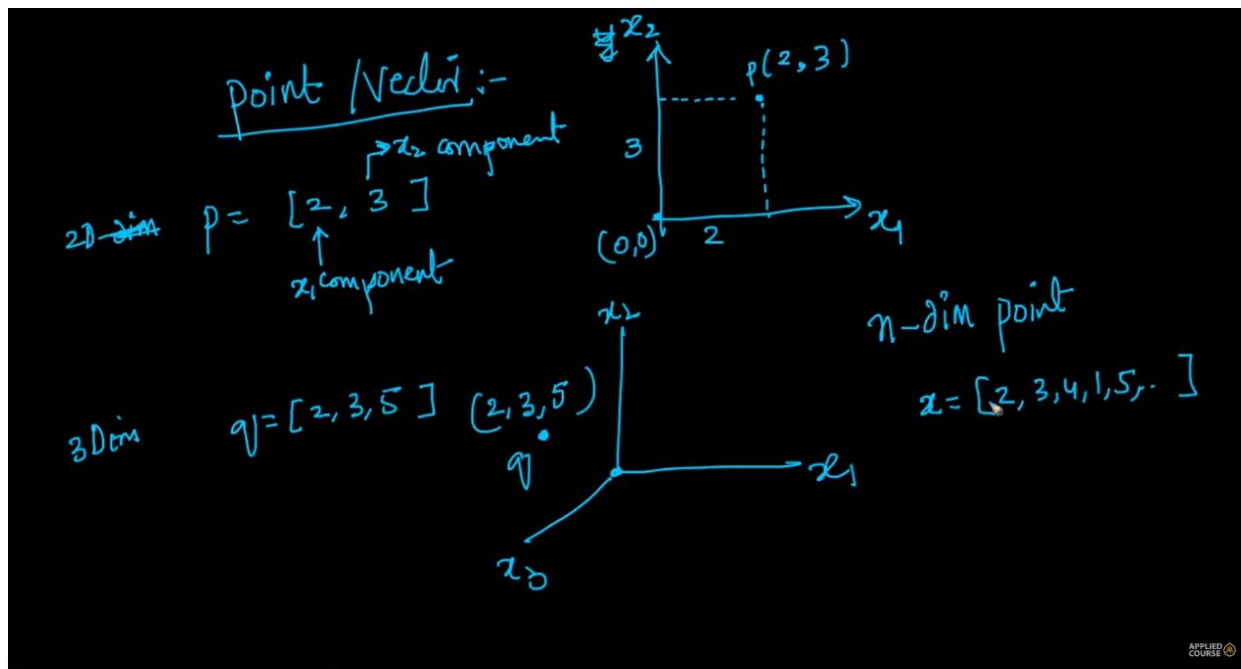
## Why Linear Algebra is used in Machine Learning?



As we can see the line or the plane separates the data points perfectly w.r.t 2-D and 3-D but what if we have 10 dimensions or 1000 dimensions.

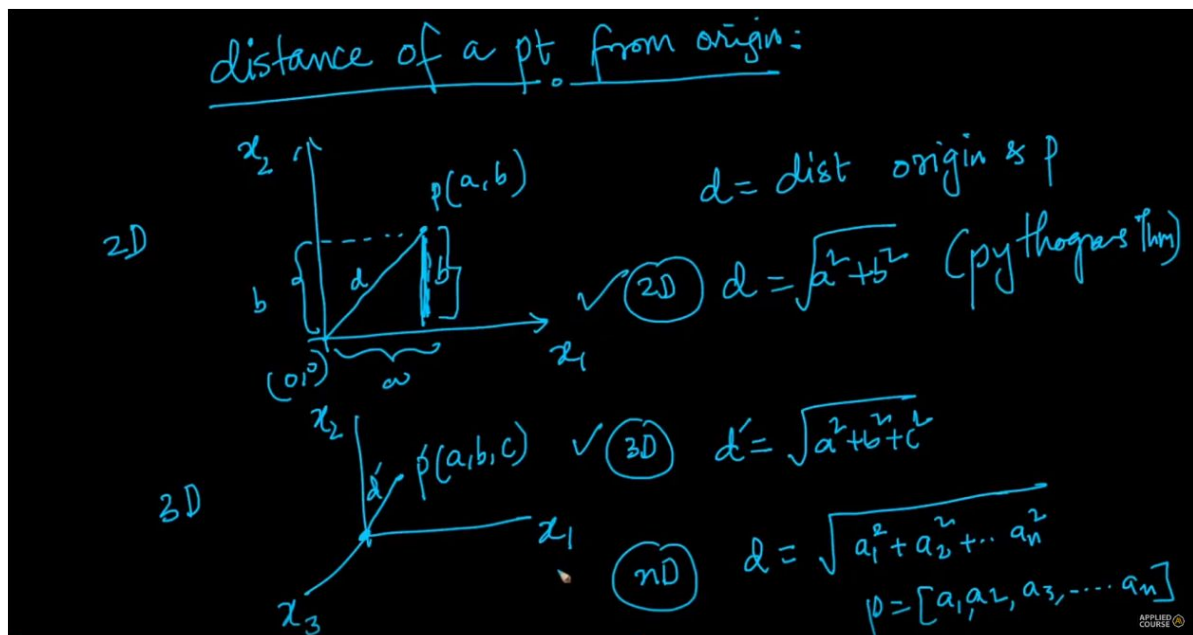
Linear Algebra provides us the necessary tools to operate in higher dimensions

## Point/ Vector



Self explanatory but suppose if we have  $n$ - points then we can represent it as  $x = [2, 3, 4, 1, 6, \dots, n]$

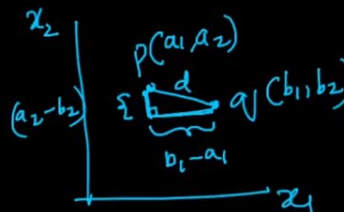
## Distance of a point from origin



## Distance between two points

dist b/w 2 pts

(2D)



$$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$$

(3D)

$$p(a_1, a_2, a_3)$$

$$q(b_1, b_2, b_3)$$

$$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$$

(nD)

$$p(a_1, a_2, \dots, a_n)$$

$$q(b_1, b_2, \dots, b_n)$$

$$d_{pq} = \sqrt{\sum_{i=1}^n (a_i - b_i)^2}$$

## Dot product (Linear algebra point of view)

Multiplication: dot product ; cross product

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 + \dots + a_n b_n$$

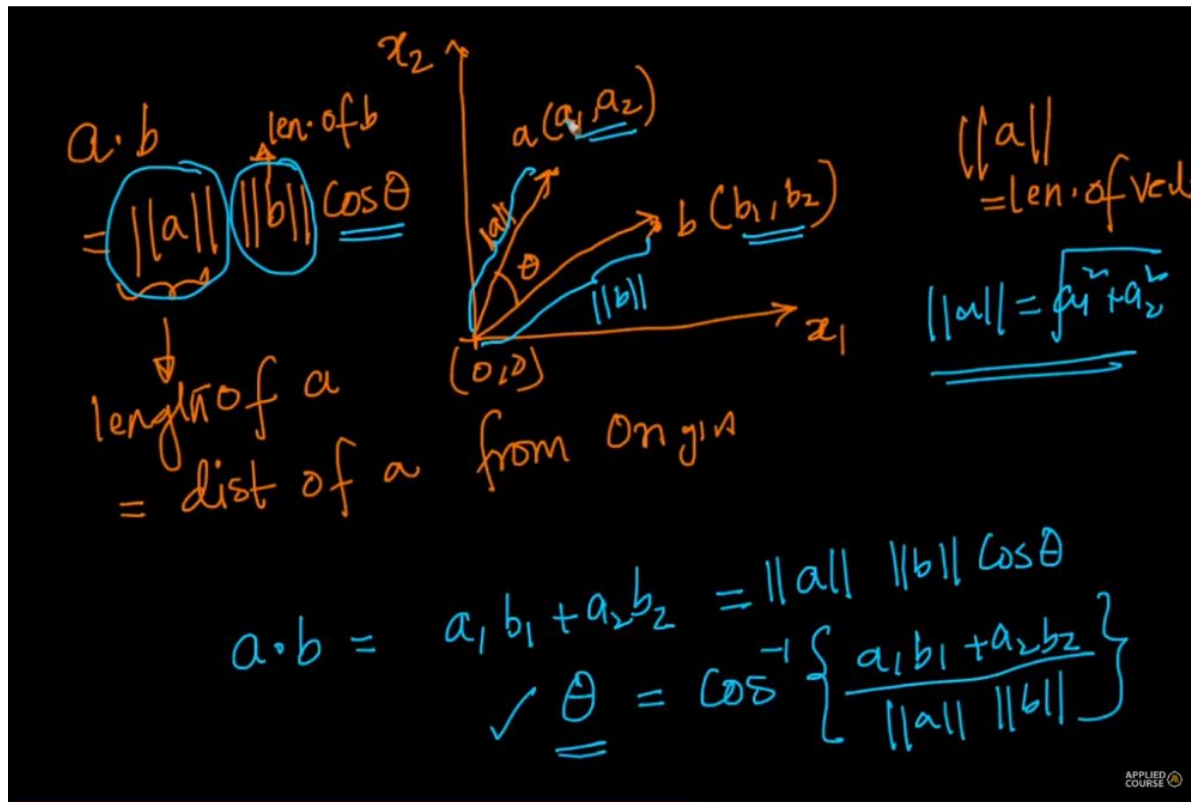
$$a \cdot b = a^T b = \sum_{i=1}^n a_i b_i$$

$$= \underbrace{[a_1, a_2, \dots, a_n]}_{1 \times n} \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}}_{n \times 1}$$

$a^T b$

One question : Why are we summing up the products ? Will be answered in the geometrical point of view of dot product.

## Dot Product (Geometric understanding)



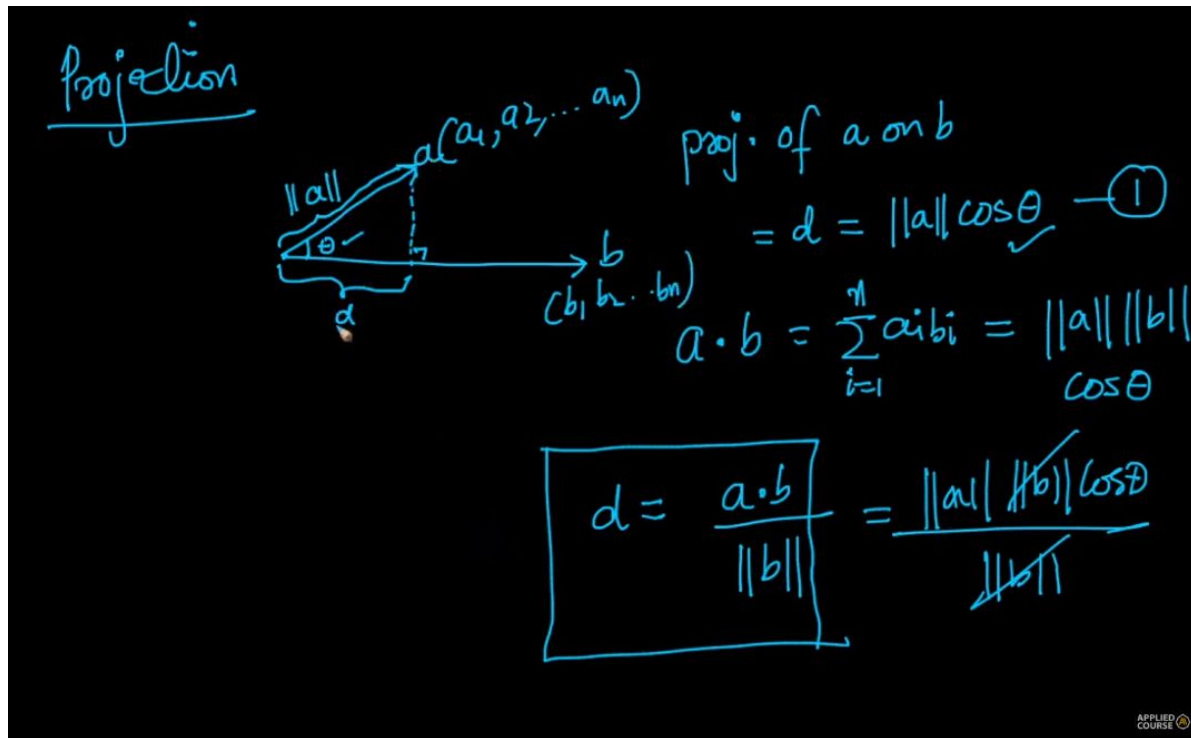
This theory can be applied to any dimensions of vectors.

Note : If the angle between two vectors is 90 then dot is 0 ( $\cos 90 = 0$ )

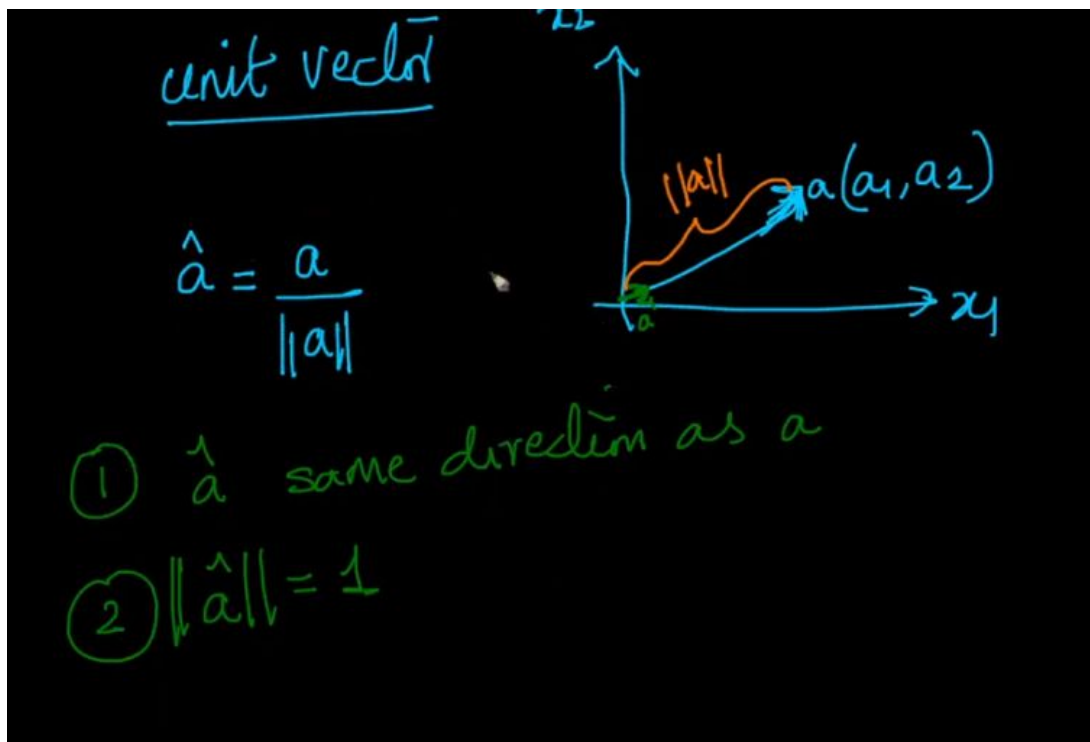
For dot product intuition and the answer to above question

<https://math.stackexchange.com/questions/348717/dot-product-intuition>

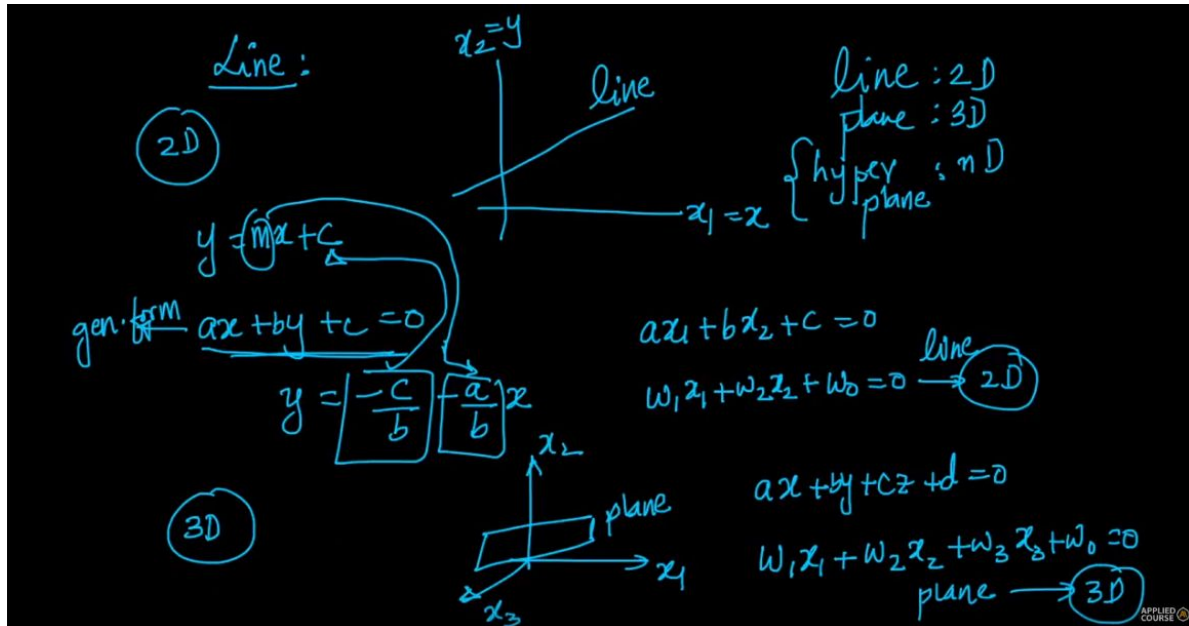
## Projection



## Unit Vector



## Line and Plane (or for n-d array)



A line divides the space into two parts. Above it and below it. Same exists for plane in 3d. For n= dimensional vector it is hyperplane.

Gen eqn of line :  $ax + by + c = 0$

$w_1x_1 + w_2x_2 + w_0 = 0$  (Commonly used for representation)

General Equation of plane :  $ax + by + cz + d = 0$

For N- d vectors

$n$ -D (hyperplane)

$$\hookrightarrow w_0 + w_1 x_1 + w_2 x_2 + \dots + w_n x_n = 0$$

Summation  $\rightarrow \checkmark w_0 + \sum_{i=1}^n w_i x_i = 0$

Vector notation  $\rightarrow w_0 + \underbrace{[w_1, w_2, \dots, w_n]}_{w_{1 \times n}} \cdot \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}}_{x_{n \times 1}} = 0$

$$w_0 + w_1 x_1 + w_2 x_2 + \dots + w_n x_n = 0$$

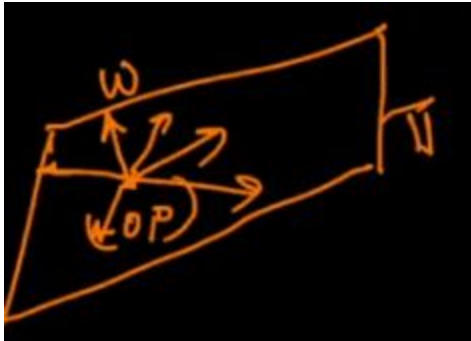
$W^T x = 0$  — eqn. of a plane passing through origin

$W^T x + w_0 = 0$

If both  $\mathbf{W}$  and  $\mathbf{x}$  are **both**  $(1 \times n)$  vectors then this equation .  $w_0$  is the intercept same as  $c$  in Equation of line :  $y = mx + c$  (where  $c$  is the  $y$  intercept)

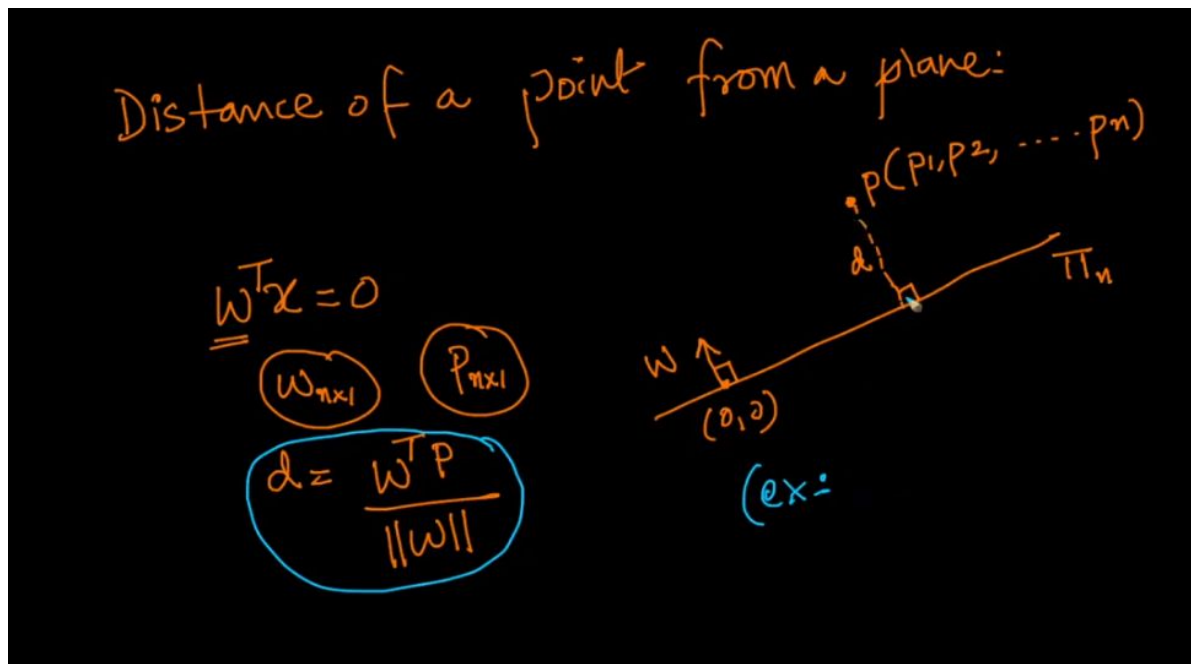
**What is the significance of  $W$  in  $W^T x = 0$ ?**

We know that  $W^T x = 0 \Rightarrow \|W\| \cdot \|x\| \cos \Theta$  therefore  $\Theta$  is 90 which leads us to know that  $W$  is vector perpendicular to the plane passing through **origin**



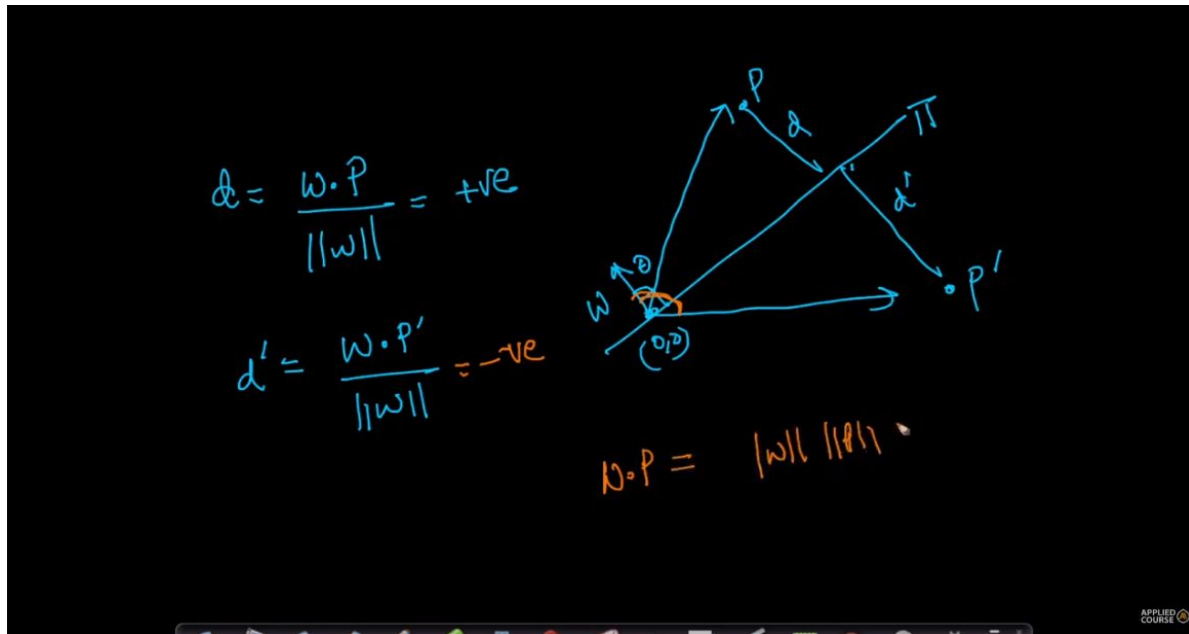
It also means that  $W$  is perpendicular to any vector in the plane which makes their dot product = 0 as well

**Distance of a point from a plane**



Let us cover what it can tell below





The angle between W and P vector is less than  $90^\circ$  it is +ve because cos is +ve between 0 and  $90^\circ$ . The angle between W and P' vector is more than  $90^\circ$  it is -ve because cos is -ve between  $90^\circ$  and  $180^\circ$ . It can also be inferred that the distance points lying in half space of P will be positive and negative for P'. It can be used in **classification**.

For circle

