Gaussian Elimination

$$x_1 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + x_2 \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \overset{?}{=} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{aligned} x_1 + 3x_2 &= 1 \\ 2x_1 + 2x_2 &= 0 \\ 3x_1 + x_2 &= 0 \end{aligned}$$

$$x_1 \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \overset{?}{=} \begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix} \qquad \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \overset{?}{=} \begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix}$$

Solve $\quad A\underline{x} = \underline{b}$

* Algorithm: Gaussian elimination (M).

$$M = \begin{pmatrix} \textcircled{1} & -1 & 3 & 2 \\ \textcircled{0} & 0 & 1 & 1 \\ \textcircled{2} & -2 & 2 & 0 \end{pmatrix}$$

$$\boxed{\begin{aligned} \text{col 2} &= -\text{col 1.} \\ \text{col 4} &= \text{col 3} - \text{col 1.} \end{aligned}}$$

first pivot $\quad = \quad 1$

$\quad$ row 3 $\leftarrow$ row 3 $-$ (2) row 1.

$$\begin{pmatrix} \textcircled{1} & -1 & 3 & 2 \\ 0 & 0 & 1 & 1 \\ 2 & -2 & 2 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 3 & 2 \\ 0 & \textcircled{0} & 1 & 1 \\ 0 & 0 & \textcircled{-4} & -4 \end{pmatrix} \rightarrow \begin{pmatrix} \textcircled{1} & -1 & 3 & 2 \\ 0 & \textcircled{0} & \textcircled{1} & \textcircled{1} \\ 0 & 0 & 0 & \textcircled{0} \end{pmatrix}$$

free (above col 2 & col 4 in last two matrices)

$$\underbrace{\begin{pmatrix} 1 & -1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}}_{\text{rref (M)}} \xleftarrow{\text{row 1} \leftarrow \text{row 1} - 3\text{row 2}} \begin{pmatrix} \textcircled{1} & -1 & 3 & 2 \\ 0 & 0 & \textcircled{1} & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

row 3 $\leftarrow$ row 3 $- (-4)$ row 2

col 2 $= \; -$col 1.

col 4 $= \; -$col 1 $+$ col 3.