LINEAR REGRESSION

$X : n \times p$ $\quad \underline{y} : n \times 1$

$$X = \begin{pmatrix} 2 & 1 \\ -1 & 0 \\ 1 & 1 \end{pmatrix} \qquad \underline{y} = \begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix}$$

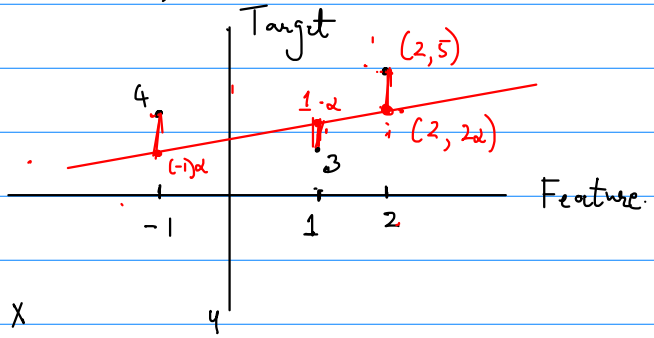
Weight of features : α, β (say).

Predictions	Target	Errors
$2\alpha + 1\cdot\beta$	5	$(5 - 2\alpha - 1\beta)^2$
$(-1)\alpha + 0\cdot\beta$	4	$(4 - \alpha(-1) - 0\cdot\beta)^2$
$1\cdot\alpha + 1\cdot\beta$	3	$(3 - \alpha(1) - 1\cdot\beta)^2$

How to visualize this?

Simpler (only one feature).

Predictions	Target	Error
2α	5	$(5 - 2\alpha)^2$
$\alpha(-1)$	4	$(4 - \alpha(-1))^2$
$\alpha(1)$	3	$(3 - \alpha)^2$



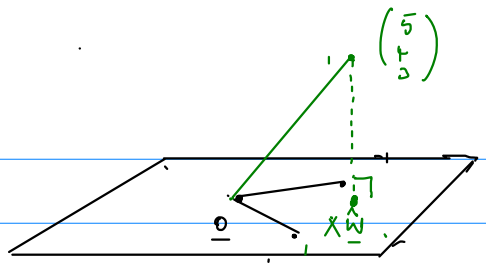
$$X = \begin{pmatrix} 2 & 1 \\ -1 & 0 \\ 1 & 1 \end{pmatrix} \qquad \underline{y} = \begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix}$$

Predictions $\quad 2\alpha + \beta$, $(-1)\alpha + 0\cdot\beta$, $1\alpha + 1\cdot\beta$

$$= \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}\alpha + \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\beta = \begin{pmatrix} 2 & 1 \\ -1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

For example $\quad \alpha = 1 \quad \beta = 0 \quad \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ $\qquad \alpha = 1 \quad \beta = 1 \quad \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$

$\alpha = 0 \quad \beta = 1 \quad \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

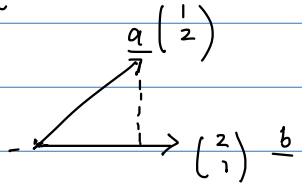


Error $y - X\hat{w}$

$y - X\hat{w}$ is perpendicular/orthogonal to each col of X.

Orthogonality: dot product

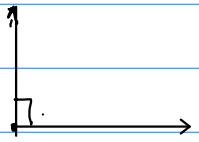
$a = \begin{pmatrix}1\\2\end{pmatrix}$ $b = \begin{pmatrix}2\\1\end{pmatrix}$



$1\cdot 2 + 2\cdot 1 = 4.$

$(1\ \ 2)\cdot\begin{pmatrix}2\\1\end{pmatrix} = 4$

$\begin{pmatrix}1\\2\end{pmatrix}$ $\begin{pmatrix}-2\\1\end{pmatrix}$



$(1\ \ 2)\begin{pmatrix}-2\\1\end{pmatrix} = 1(-2)+2(1)$

$= 0.$

$X^T(y - X\hat{w}) = 0$ $\qquad X^T y = (X^T X)\hat{w}$

$\hat{w} = (X^T X)^{-1} X^T y.$

vector of coeff.

$\begin{pmatrix}2 & 1 & 3\\ -1 & 0 & 1\\ 1 & 1 & 1\end{pmatrix}$ $\qquad \begin{pmatrix}5\\4\\3\end{pmatrix}$

X $\qquad y$

1: 2 x