Welcome to EE 6451

Linear Regression

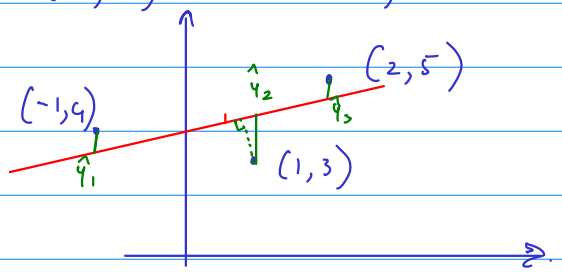
X : $n \times p$ (Data).

$\underline{y}$: $n \times 1$ (target).

$$X = \begin{pmatrix} 2 & 1 \\ -1 & 0 \\ 1 & 1 \end{pmatrix} \qquad \underline{y} = \begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix}$$

$$X' = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \qquad \underline{y} = \begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix}$$

$(2, 5)$ $(-1, 4)$ $(1, 3)$



$(\hat{y}_1 - 4)^2 + (\hat{y}_2 - 3)^2 + (\hat{y}_3 - 5)^2$

$$X = \begin{pmatrix} 2 & 1 \\ -1 & 0 \\ 1 & 1 \end{pmatrix} \qquad \underline{y} = \begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix}$$

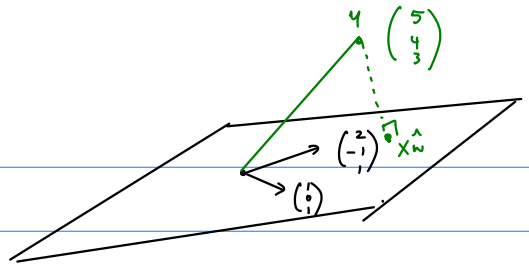
$\alpha 2 + \beta 1$
$\alpha(-1) + \beta 0$.
$\alpha(1) + \beta 1$

$$\left.\begin{array}{l} \alpha 2 + \beta 1 \\ \alpha(-1) + \beta 0. \\ \alpha(1) + \beta 1 \end{array}\right\} \quad \alpha \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 1 \\ -1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

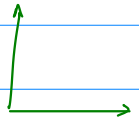
If $\alpha = \beta = 1$ Predictions $\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$

$\alpha = 0$ $\beta = 1$ Predictions $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$



$y - X\hat{w}$: error vector perpendicular to every col of X.

$$\begin{pmatrix}5\\4\\3\end{pmatrix} \cdot \begin{pmatrix}1\\0\\-5/3\end{pmatrix} = 5\cdot 1 + 4\cdot 0 + 3\left(\frac{-5}{3}\right) = 0.$$



$$X^T(y - X\hat{w}) = 0$$

$$X^T y = X^T X \hat{w} \qquad \hat{w} = (X^T X)^{-1} X^T y.$$