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THE PHENOMENOLOGY OF SMALL-SCALE TURBULENCE

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> I have sometimes thought that what makes a man's work *classic* is often just this multiplicity [of interpretations], which invites and at the same time resists our craving for a clear understanding.
>
> *Wright (1982, p. 34), on Wittgenstein's philosophy*

ABSTRACT

Small-scale turbulence has been an area of especially active research in the recent past, and several useful research directions have been pursued. Here, we selectively review this work. The emphasis is on scaling phenomenology and kinematics of small-scale structure. After providing a brief introduction to the classical notions of universality due to Kolmogorov and others, we survey the existing work on intermittency, refined similarity hypotheses, anomalous scaling exponents, derivative statistics, intermittency models, and the structure and kinematics of small-scale structure—the latter aspect coming largely from the direct numerical simulation of homogeneous turbulence in a periodic box.

1. INTRODUCTION

1.1 *Preliminary Remarks*

Fully turbulent flows consist of a wide range of scales, which are classified somewhat loosely as either large or small scales. The large scales are of the order of the flow width; contain most of the energy; and dominate the transport

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of momentum, mass, and heat. The small scales include the dissipative range responsible for most of the energy dissipation and the inertial range; inertial-range scales are large compared to dissipative scales but small compared to the large scales. Although there is no difficulty in defining these scale-ranges formally at high Reynolds numbers (e.g. Batchelor 1953, Monin & Yaglom 1975), there is always the experimentalist's ambiguity that prevents precise definitions, especially at moderate Reynolds numbers.

It is useful to say a few words on why one ought to be interested in small-scale turbulence: Indeed, from the perspective of turbulent transport—which is clearly a problem of great practical importance—the emphasis placed on the small scale may seem unrewarding. One reason for the interest is that a proper theory of turbulence, if one were to emerge, may well relate to the small scale, which has the best prospect of being universal or quasi-universal. Any aspect of turbulence that can be understood precisely would be valuable in itself. For instance, inquiries into the nature of singularities of the Navier-Stokes equations—a problem of some theoretical significance—have a natural relation to the far-dissipation range. Small-scale turbulence is a fertile ground for studies bearing on vortex breakdown and reconnection, and it provides impetus for incorporating geometry into analytical studies of the Navier-Stokes equations. The research on small-scale turbulence has had a remarkable influence on neighboring fields (e.g. nonlinear waves), and as remarked by Wilson (1976), has been far ahead—at least it was when the statement was made—of other branches of physics in dealing with a wide range of different length scales.

There are also several practical reasons for engaging in this research: The influence of intermittent fluctuations of small-scale turbulence on the propagation of light and sound waves in the atmosphere is nontrivial; in combustion problems, different species are brought together at the molecular level as a result of small-scale turbulent mixing; a flame front passing through a region of large dissipation is likely to be torn asunder and perhaps quenched locally. If one understands the small scales well enough, one may be able to parametrize them realistically for use in the so-called large-eddy-simulation of high-Reynolds-number flows; this understanding is equally important for practical calculation methods such as the k-ε model. Adequate parametrization of the small scale is a proper ingredient of the so-called turbulence problem. An impressive list of applications of the knowledge of small-scale turbulence was given by Batchelor (1962) almost thirty-five years ago.

These are sufficient reasons for reviewing the progress made in the subject. Although several reviews on various aspects of turbulence have appeared in *Annual Reviews*, none of them especially focuses on the small scale. Although the intensity of research in this area has waxed and waned, it has been sustained remarkably over the past sixty or so years, with a strong influx in recent years from the physics and mathematics communities.

1.2 *The Scope of the Review*

Work on small-scale turbulence prior to the early 1970s has been reviewed admirably by Monin & Yaglom (1975); for the most part, we shall not consider it here. Many intertwining developments have occurred since then, and we estimate that between three and four hundred papers have appeared on the subject. Although it is not clear that lasting progress has been made, many interesting issues have surfaced and increasingly reliable information about small-scale turbulence is rapidly becoming available. We cannot do justice to all of these issues in the space available here, and the review is necessarily restrictive. After providing a short background in Section 2, we review a few topics currently under active research. We then discuss briefly in Section 4 some qualitative aspects of intermittency models and summarize in Section 5 results from direct numerical simulations (DNS) of Navier-Stokes equations. We conclude with a few broad remarks in Section 6. Nelkin's recent review (1994) and Frisch's book (1995) cover some of the same ground, but the focus here is different and complementary.

2. THE BACKGROUND

The notion of universality provided a great impetus for the study of small scales. This notion is generally associated with Kolmogorov (1941a), its principal originator, and is often designated as K41 for short. Two premises are basic to K41. The first is that when the fluid viscosity ν is small, the average energy dissipation rate $\langle\varepsilon\rangle$ is independent of ν. The second premise (known as local isotropy) is that small-scale turbulence at sufficiently high Reynolds numbers is statistically independent of the large scale, and is homogeneous, isotropic, and steady. Both these premises need detailed assessments of their own, but they cannot be attempted here. Taking them for granted, K41 states that (*a*) the statistical properties of the dissipation scales are determined universally by ν and $\langle\varepsilon\rangle$ and (*b*) those in the inertial range, if the Reynolds number is high enough for one to exist, are determined by $\langle\varepsilon\rangle$ only. An implication of (*b*) is that the probability density function (*pdf*) of normalized velocity increments $\Delta u_r/(r\langle\varepsilon\rangle)^{1/3}$—where $\Delta u_r \equiv u(x+r)-u(x)$, u is the velocity in the direction x, and r is any inertial range distance measured along x—should be universal; that is, independent of the flow, the separation distance r, and the Reynolds number. Equivalently, for a positive integer n, the moments of velocity increments, or the so-called longitudinal structure functions, obey the relation

$$\left\langle \Delta u_r^n \right\rangle = C_n(r\langle\varepsilon\rangle)^{n/3}, \tag{1}$$

where C_n are universal constants. A similar equation can be written for velocity increments with the separation distance transverse to the direction of the velocity component.

Of these structure function relations, an exact relation is known only for the third order (Kolmogorov 1941b). This so-called Kolmogorov 4/5$^{\text{ths}}$ law[1] is given for the inertial range by

$$\left\langle \Delta u_r^3 \right\rangle = -\frac{4}{5} r \langle \varepsilon \rangle. \tag{2}$$

The classical interpretation of this nonvanishing third movement (e.g. Monin & Yaglom 1975, p. 398) is that the energy flux from large to small scales is unidirectional on the average; other attempts have been made to extract more information from this equation (e.g. Kailasnath et al 1992a, Novikov 1992, Chorin 1994 (Chapter 3), Vainshtein & Sreenivasan 1994). The equation has played an important role in experiment, for example in fixing the extent of the inertial range and in estimating $\langle \varepsilon \rangle$ with arguably less ambiguity than by the local isotropy relation $\langle \varepsilon \rangle = 15\nu \langle (\partial u/\partial x)^2 \rangle$. Note, however, that Kolmogorov derived this result for globally homogeneous and isotropic turbulence. Since the equation involves only velocity differences, it is often thought to exact for all types of flows, provided local homogeneity and isotropy hold. The requirement for global isotropy can be relaxed, but not, apparently, the assumption of global homogeneity (Frisch 1995). Zubair (1993) shows that the large-scale contributions to the third-order structure function may be nontrivial even at atmospheric Reynolds numbers. This same remark might be made for the effect of large-scale forcing in numerical simulations. According to Barenblatt & Goldenfeld (1995), the notion of incomplete similarity (Barenblatt 1979) implies corrections of $O(\ln \mathrm{Re})^{-1}$ to the Kolmogorov law. More theoretical and experimental work is needed to clarify the role of this important relation.

Much effort has been devoted to verifying Equation 1, especially to the spectral equivalent of the case with $n = 2$, which can be written as

$$\phi(\kappa_1) = C \langle \varepsilon \rangle^{2/3} \kappa_1^{-5/3}, \tag{3}$$

where $\phi(\kappa_1)$ is the one-dimensional spectrum of energy in the wavenumber component κ_1 in direction x, and C is often called the Kolmogorov constant. In a landmark experiment in a tidal channel, Grant et al (1962) verified Equation 3.

[1]For the vector form of this law, see, for example, Novikov (1992). We shall not consider here the form of this equation appropriate to the dissipation region. The Kolmogorov law is then a relation between the second- and third-order structure functions. The leading terms of that equation for small r express a balance between the production and dissipation of mean square vorticity, or enstrophy. Alternatively, they allow us to express the skewness of the velocity derivative $\partial u/\partial x$ in terms of the energy spectral density.

Subsequent investigators (e.g. Gagne 1987, Zocchi et al 1994) have also found the spectral slope to be close to 5/3. A compilation of existing data (Sreenivasan 1995a) shows the Kolmogorov constant is *approximately* constant (0.5 ± 0.05) over a wide range of Reynolds numbers. (The most suitable Reynolds number to consider is the so-called microscale Reynolds number $R_\lambda \equiv \langle u^2 \rangle^{1/2} \lambda / \nu$ where λ is the Taylor microscale.)

In the dissipation range, K41 yields for the spectral density the form

$$\phi(\kappa_1) = \langle \varepsilon \rangle^{2/3} \kappa_1^{-5/3} f(K), \tag{4}$$

where $K = \kappa_1 \eta$ is the wavenumber normalized by the Kolmogorov length scale $\eta = (\nu^3 / \langle \varepsilon \rangle)^{1/4}$, and the universal function $f(K)$ is unknown (except that it approaches C for small K). Note that in conformity with the first hypothesis of K41, only $\langle \varepsilon \rangle$ and ν enter Equation 4. Foias et al (1990) argued that $f(K)$ for large K is of the form $\exp(-gK)$ (to within a wavenumber-dependent prefactor), where the constant $g \geq 1$ under certain assumptions of smoothness of the velocity field. The exponential supersedes the other forms summarized in Monin & Yaglom (1975, Section 22.3). From numerical simulations at low Reynolds numbers, Chen et al (1993a) showed the spectral density in the far-dissipation region to be of the form $K^a \exp(-gk)$, where $g \simeq 7.1$ and $a \simeq 3.3$. Kerr (1990) and Sanada (1992) obtained $g \simeq 5.2$ at higher Reynolds numbers. Sirovich et al (1994) offered an explanation for the Reynolds number dependence of g.

Experimental data collected by Monin & Yaglom (1975) (Figure 76c) and by Saddoughi & Veeravalli (1994)—among others—support Equation 4 roughly, but the data collapse is not fully satisfactory. She & Jackson (1993) and She et al (1993) normalized the wavenumber by κ_p corresponding to the peak of the dissipation spectrum, and the spectral density by that at κ_p. This rescaling appears to work better. A different type of spectral universality in the dissipation region has been proposed recently by Frisch & Vergassola (1991) and by Gagne &Castaing (1991). The basis for their proposal is the multifractality of the small scale (see Section 3.2). For a detailed discussion of this aspect, see Frisch (1995).

Regarding the crossover behavior of the spectral density between inertial and dissipative regions, it is tempting to presume that the function $f(K)$ decays monotonically with increasing K, but observations (e.g. Saddoughi & Veeravalli 1994) suggest that it increases through the inertial range and begins to decrease only around $K \simeq 1$. This so-called bottleneck phenomenon, studied recently by Falkovich (1994) and Lohse & Mueller-Groeling (1995), can be explained qualitatively by considering the triadic interaction among an inertial wavenumber, say p, with two others, say q and r, in the dissipation range. The conservation of energy flux in the inertial range implies that the viscous depletion of amplitudes of q and r will have to be compensated by an increase in the amplitude of p.

On the whole, it is not possible to assert that K41 works exactly, even for second order statistics, and as we shall see in Section 3.3, there appear to be unequivocal departures from Equation 1 for large enough n ($n \geq 4$). For example, in atmospheric boundary layers (Kailasnath et al 1992b, Praskovsky & Oncley 1994), high-Reynolds-number wind-tunnel flows (Castaing et al 1990) as well as the high-Reynolds-number helium flows (Zocchi et al 1994, Tabeling et al 1996), the *pdf*s of inertial-range velocity increments change continuously with r. Both Kailashnath et al and Tabeling et al fitted stretched exponentials of the tails of the *pdf*s, $p(x) \simeq \exp(-\beta|x|^c)$, and found that the stretching exponent c evolved smoothly with r (instead of being invariant according to K41), from about 0.5 for dissipative-range separations to about 2 for integral-scale separations. Similarly, the C_n for large n are flow dependent (although a careful assessment has not yet been made). Finally, it is implicit in K41 that the skewness and flatness factor of velocity derivatives should be constants independent of the Reynolds number, but experiments show otherwise (see Section 3.6). Thus, one is forced to give up K41 universality in its broadest generality.

One reason for this failure of K41 is the strong variability of the energy dissipation rate that itself was attributed by Obukhov (1962) to the strong "change of the large scale processes." Obukhov considered K41 to hold for the so-called "pure regions", defined by some fixed value of the energy dissipation rate ε. He suggested replacing $\langle\varepsilon\rangle$ in K41 by the local average of ε defined by

$$\varepsilon_r = \frac{1}{V}\int \varepsilon(\mathbf{x})d\mathbf{x}, \tag{5}$$

where $V = O(r^3)$ is a volume of linear dimension r. For $r \ll L$, where L is a characteristic large scale, the variable $\varepsilon_r/\langle\varepsilon\rangle$ is a fluctuating quantity and, according to Obukhov's suggestion, a function of the ratio r/L. Thus, whenever averages are taken over the so-called mixed regions containing varying levels of ε, the large scale enters inertial-range statistics explicitly. This is also the spirit of Barenblatt's (1979) incomplete similarity and a potential source of the infrared divergence in renormalization group theories. If Obukhov's picture is correct, the *pdf* of $\Delta u_r/(r\varepsilon_r)^{1/3}$ in the inertial range is not universal, but one may still expect power-laws of the form

$$\left\langle \Delta u_r^n \right\rangle / u_0^n = C_n'(r/L)^{\zeta_n}, \tag{6}$$

where the large-scale velocity u_0 and the prefactors C_n' are non-universal but the exponents ζ_n, although different from $n/3$, are presumed to be universal. Kolmogorov (1962) made Obukhov's suggestion more explicit by assuming that the dissipation rate is lognormally distributed (see Section 3.5 for some relevant comments). Kolmogorov also refined K41 in a vital way by taking note of Obukhov's suggestion. This refinement is discussed in Section 3.4. A study of the strong variability of the energy dissipation rate, now known as dissipation intermittency, is discussed in Section 3.2.

3. SOME PROBLEMS OF CONTINUING INTEREST

3.1 *The Meaning of Small-Scale Intermittency*

Batchelor & Townsend (1949) showed that the non-Gaussian behavior in the *pdf* of dissipation quantities increased with decreasing scale. In a complementary sense, dissipation quantities become increasingly non-Gaussian as the Reynolds number increases. These are the two hallmarks of dissipation-scale intermittency. For the inertial range, since the Reynolds number variation should be irrelevant, intermittency requires that the *pdf*s of wavenumber bands show increasingly flared-out tails for increasing midband wavenumber, or that the flatness of velocity increments increases with decreasing scale. To assess inertial-range intermittency, one has to determine the scale dependence of the *pdf* or the moments of the appropriate small-scale quantity.

Small-scale intermittency also implies a qualitative constraint on the behavior of scaling exponents. In the inertial range, if the ζ_n in Equation 6 follow the linear relation $\zeta_n = (n/2)\zeta_2$, one recovers a large measure of universality and the scaling is said to be 'trivial': For instance, any fractional Brownian process satisfies such a relation for even-order scaling exponents. If the *pdf*s of velocity increments depend on the separation distance in some complex way, the scaling exponents ζ_n will not satisfy the linear relation just mentioned. Intermittency is tantamount to a nonlinear or anomalous variation of the scaling exponents with the order of the moment. Because of the Hölder inequality, which guarantees the concavity of the ζ_n-n curve, the small-scale intermittency implies that the difference between the Kolmogorov value of $n/3$ and the actual exponent should increase with the order of the moment (alternatively, the difference $\zeta_{2n+2} - \zeta_{2n}$ is a decreasing function of n).

3.2 *Dissipation Intermittency*

3.2.1 SCALING EXPONENTS OF ENERGY DISSIPATION For convenience, experimentalists have nearly always used Obukhov averaging over a linear dimension. For example, they use for ε_r the dissipation ε averaged over an interval of size r residing in the inertial range. The scaling exponents for the energy dissipation are conventionally defined, for any real q, as

$$\langle \varepsilon_r^q \rangle \langle \varepsilon \rangle^q \propto (r/L)^{-\nu_q}. \tag{7}$$

The proportionality constants, omitted here, are not expected to be universal. The rationale for writing this power-law can be explained pragmatically in terms of the so-called breakdown coefficients or multipliers (Novikov

1971, Mandelbrot 1974, Van Atta & Yeh 1975, Chhabra & Sreenivasan 1992, Sreenivasan & Stolovitzky 1995, Pedrizzetti et al 1996). It is not clear that the multipliers, although quite useful, are fundamental to turbulence. Note that the exponents ν_q will all have to be zero in the absence of dissipation intermittency. Nontrivial scaling implies that ν_q is a nonlinear function of q.

Alternatively, one may write

$$\sum (r\varepsilon_r)^q \propto r^{(q-1)D_q}, \tag{8}$$

where the sum is taken over all intervals of size r, and Equation 8 defines the so-called generalized dimensions D_q (Mandelbrot 1974, Hentschel & Procaccia 1983). The relation between ν_q and D_q is readily shown (in one dimension) to be

$$\nu_q = (q-1)(1-D_q). \tag{9}$$

The generalized dimensions vary with q in some nontrivial fashion in the presence of intermittency. This nontrivial variation of D_q with q defines the so-called multifractal property of dissipation. For Kolmogorov's (1962) lognormal model, D_q varies linearly, and ν_q parabolically, with q.

Several efforts have been made to measure the ν_q in both high and low Reynolds number flows. The available data are compiled in Figure 1. Given the difficulties in obtaining them, the agreement among various data sets is surprisingly good. All the curves have been obtained by replacing the full dissipation by one of its components, namely $(\partial u/\partial x)^2$. The so-called surrogacy issue concerns the potential shortcomings of replacing ε by one of its components (e.g. Narasimha 1990). Chen et al's (1993b) work, which used DNS data for box-type homogeneous turbulence, suggests that surrogacy is a benign factor for some purposes, but Bershadskii et al (1993a) and Wang et al (1996) show that it could be problematic for others. There are indications that surrogacy is of dubious accuracy for the detailed structure of the *pdf*s, high-order moments, and exponents, and so our knowledge of these aspects is at best partial. This question will no doubt benefit from a more systematic analysis of the DNS data, even at the presently attainable Reynolds numbers.

Of particular interest is the so-called intermittency exponent, or the exponent ν_2 in Equation 7. Sreenivasan & Kailasnath (1993) have examined the various definitions used for the intermittency exponent and recommended, at high Reynolds numbers, a common value of 0.25 ± 0.05. This value is quite comparable to the 0.20 value found by Antonia et al (1981, 1982) and Chambers & Antonia (1984).

3.2.2 SCALING EXPONENTS FOR VORTICITY AND CIRCULATION For the streamwise vorticity component measured by Fan (1991) in several flows, including

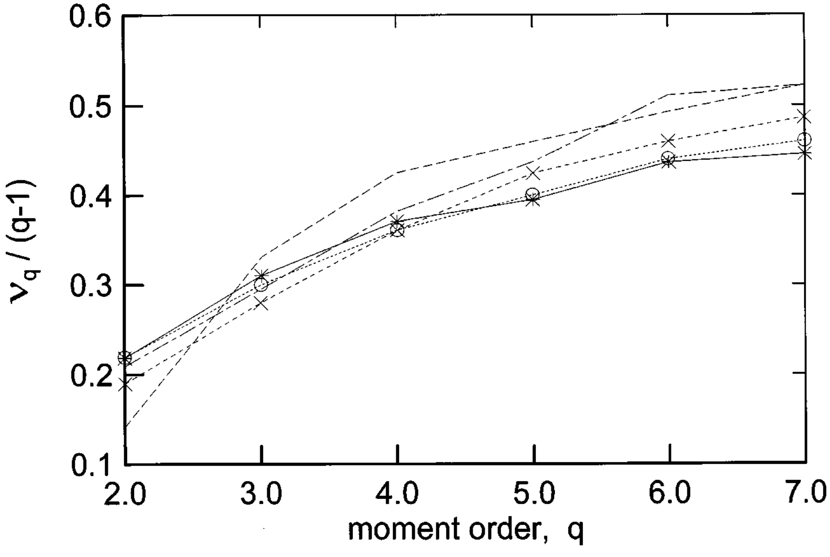

Figure 1 The scaling exponents for the energy dissipation, defined according to Equation 7. The continuous line with asterisks is from Meneveau & Sreenivasan (1987), the dot-dashed line from Meneveau & Sreenivasan (1991), the dashed line from Bershadskii et al (1993a), the dotted line with circles from Sreenivasan & Stolovitzky (1995), and the dashed line with crosses from Herweijer (1995). The differences among various data sources are comparable to the error bars on any one set, and so are not shown. For the quality of typical power-law fits from which these exponents have been obtained, see Meneveau & Sreenivasan (1991).

the atmospheric surface layer ($R_\lambda = 2000$), Meneveau et al (1990) obtained various scaling exponents for enstrophy using the so-called joint-multifractal formalism. They concluded that enstrophy is more intermittent than the energy dissipation rate. At low Reynolds numbers ($R \leq 200$), Sreenivasan et al (1995) and Cao et al (1996) obtained the scaling exponents λ_n defined by

$$\langle |\Gamma_r|^n \rangle \sim r^{\lambda_n}, \tag{10}$$

where Γ_r is the circulation around a contour of linear size r (in the inertial range). The proportionality sign here (and elsewhere) subsumes factors needed for proper non-dimensionalization. They found the exponents to be anomalous and confirmed that the enstrophy was more intermittent than the energy dissipation rate (see also Kerr 1985). However, the wake data of Shafi et al (1996) seem to suggest an opposite conclusion. This discrepancy is yet to be resolved.

3.2.3 SCALING EXPONENTS FOR SCALAR DISSIPATION These exponents are defined by

$$\langle \chi_r^q \rangle \propto r^{-\nu_q^{(\theta)}}, \tag{11}$$

where χ_r is the locally averaged scalar dissipation rate, analogous to the locally averaged energy dissipation rate ε_r. The symbol θ, here and elsewhere, stands for the passive scalar fluctuation. Unlike ε, all three components of χ can be measured directly using a suitable combination of cold wires (e.g. Sreenivasan et al 1977, Zhu & Antonia 1993); also, in both gas and liquid phases, it is possible to image the scalar field (e.g. Yip et al 1987, Dahm et al 1991) and obtain the dissipation without Taylor's hypothesis or the surrogacy assumption. For dye concentration fluctuations in water flows, there are two scaling regions (see e.g. Tennekes & Lumley 1972): the inertial-convective range (roughly between L and η) and the viscous-convective range (roughly between η and the Batchelor scale, $\eta_b = \eta Sc^{-1/2}$, where Sc is the Schmidt number). Prasad et al (1988) obtained, for $R_\lambda \simeq 200$, the exponents for the inertial-convective region (Figure 2); these exponents agree with those of Antonia & Sreenivasan (1977) for temperature fluctuations in air ($R_\lambda \simeq 150$) and show that the scalar

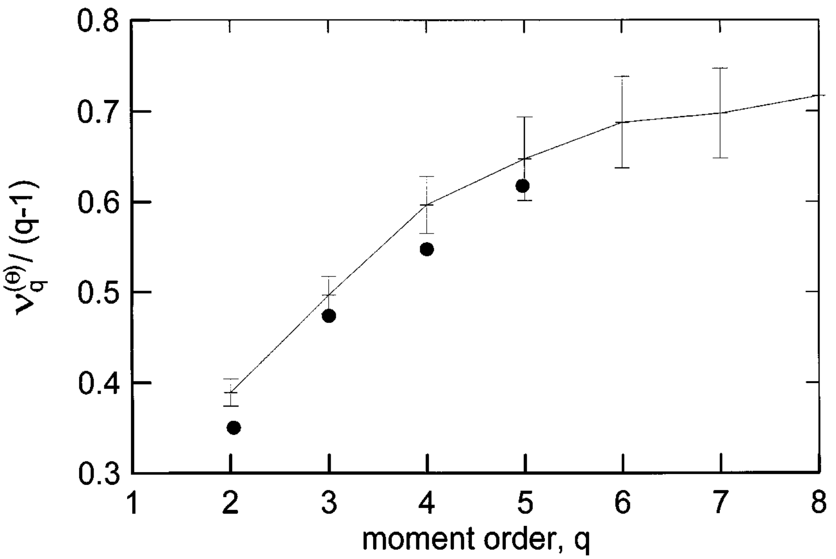

Figure 2 The scaling exponents for the scalar dissipation, defined according to Equation 11. The crosses with error bars are from the same data source as Prasad et al (1988), and the filled circles are from Antonia & Sreenivasan (1977).

dissipation is more intermittent than energy dissipation (i.e. $\nu_q^{(\theta)} > \nu_q$). In the viscous-convective range, Sreenivasan & Prasad (1989) found that the scaling exponents are trivial (that is, there is no intermittency and all intermittency exponents are essentially zero).

3.2.4 OTHER MEASURES OF DISSIPATION INTERMITTENCY Batchelor & Townsend (1949) associated γ_i with the flatness factors of differentiated or bandpass-filtered signals. Kuo & Corrsin's (1971) measurements in nearly isotropic turbulence and on the axis of a round jet indicated that γ_i decreased as R_λ increased, asymptoting to a constant at some large R_λ. Rao et al (1971) and Badri Narayanan et al (1977) obtained the intermittency factor γ_i for the active regions in narrow bandpass-filtered velocity signals. They also estimated the average width and frequency of the active regions. Sreenivasan (1985) provided an analytical framework for this methodology using the Hilbert transform, and clarified the role of the threshold setting and filter characteristics. The framework also showed that some of the properties attributed to small-scale turbulence are shared by bandpass-filtered random Gaussian white and non-white noise and underlined the stronger-than-algebraic roll-off of the spectrum in the far-dissipation range (Frisch & Morf 1981). Antonia et al (1987), Britz et al (1988), and Zubair (1995) have highlighted the important differences between intermittency characteristics of turbulent signals and those of a noise whose spectrum is chosen to coincide with the high wavenumber turbulent velocity spectrum. To emphasize the significance of the geometrical features associated with high amplitudes of small scales, She et al (1990) generated a Gaussian random-velocity field with an energy spectrum identical to that of isotropic turbulence: The intense vortical structures (see Section 5) that were prominent in turbulence were absent in the synthetic field.

3.3 *Inertial Range Intermittency*

3.3.1 SCALING EXPONENTS FOR VELOCITY STRUCTURE FUNCTIONS Grossmann & Lohse (1994) and Grossmann et al (1994) used the so-called reduced wave-vector representation of the velocity field and found essentially no intermittency in the inertial range. However, the technique admits only a geometrically scaling subset of wave vectors in the Fourier representation of the velocity field, and so the relevance of these interesting conclusions to real turbulence remains obscure.

Although anomalous scaling can be defined without referring to exponents, the latter have been the focus of many studies. Assuming that scaling occurs, Constantin & Fefferman (1994) provided analytical conditions for the existence of anomalous exponents. Kraichnan (1974) provided a thoughtful assessment of scaling itself. The experimental support for anomalous scaling appears overwhelming (e.g. Anselmet et al 1984, Maurer et al 1994), but several uncertainties remain: the finite Reynolds number effect, which is not understood and cannot be calculated *a priori*; the effects of finite shear on inertial-range quantities (Lumley 1965); as well as the purely practical question of how one defines the scaling range and obtains scaling exponents from power-laws of modest quality. (The quality of this scaling is poorer near the critical Reynolds number; Tong et al 1988.) The modest size of the scaling range has been extended in several ways. For example, Benzi et al (1993) proposed the Extended Self Similarity (ESS) method in which $\log\langle\Delta u_r^n\rangle$ is plotted against $\log\langle\Delta u_r^3\rangle$ instead of $\log r$. Although both methods are equivalent in principle because of Equation 2, ESS extends, for reasons that are not yet well-understood,[2] the scaling region significantly in unsheared flows; see Arneodo et al (1996) for a broad consensus among European researchers actively involved with ESS. One can also obtain ζ_n with less ambiguity by fitting the structure function data from inertial *and* dissipation ranges to a "scaling function" whose asymptotic form for the inertial range coincides with Equation 6 (Stolovitzky et al 1993). The difficulty, however, is the empirical content of the scaling functions. Perhaps more important, structure functions have been obtained without invoking Taylor's hypothesis (by Tong et al 1988 and Pak & Goldburg 1993 using photon correlation spectroscopy and by Noullez et al 1996 using the RELIEF method). These data also confirm—subject to continuing concerns about the modest quality of power-laws—the anomaly of the exponents. There are some differences among the exponents obtained by these methods, and it is likely that different classes of flows possess slightly different sets of exponents. Roughly speaking, the numbers obtained by Anselmet et al (1984) more than a decade ago seem to have held up rather well. Figure 3 summarizes typical data for ζ_n. Note that if one writes

$$K(r) \sim r^{-\alpha}, \quad H(r) \sim r^{-\beta}, \quad E(r) \sim r^{-\gamma}, \tag{12}$$

where $K(r)$, $H(r)$ and $E(r)$ are the normalized fourth, sixth, and eighth moments of Δu_r over an inertial-range separation distance r, experiments suggest that α, β, and γ are approximately 0.10, 0.29, and 0.47, respectively, essentially independent of the Reynolds number.

3.3.2 STRUCTURE FUNCTION EXPONENTS FOR PASSIVE SCALARS The scalar structure function exponents ξ_n may be defined by the relation $\langle\Delta\theta_r\rangle^n \propto r^{\xi_n}$.

[2]In practice, the adherents of ESS often use $\langle|\Delta u_r|^3\rangle$ instead of $\langle\Delta u_r^3\rangle$. There is no rigorous justification for this procedure, and it is unknown whether the improved scaling in the resulting plots improves our basic understanding. Experimental evidence, however, suggests that $\langle|\Delta u_r|^3\rangle$ scales as r rather closely (Vainshtein & Sreenivasan 1994, Benzi et al 1994). ESS does not seem to work when the shear is strong (Stolovitzky & Sreenivasan 1993, Benzi et al 1994).

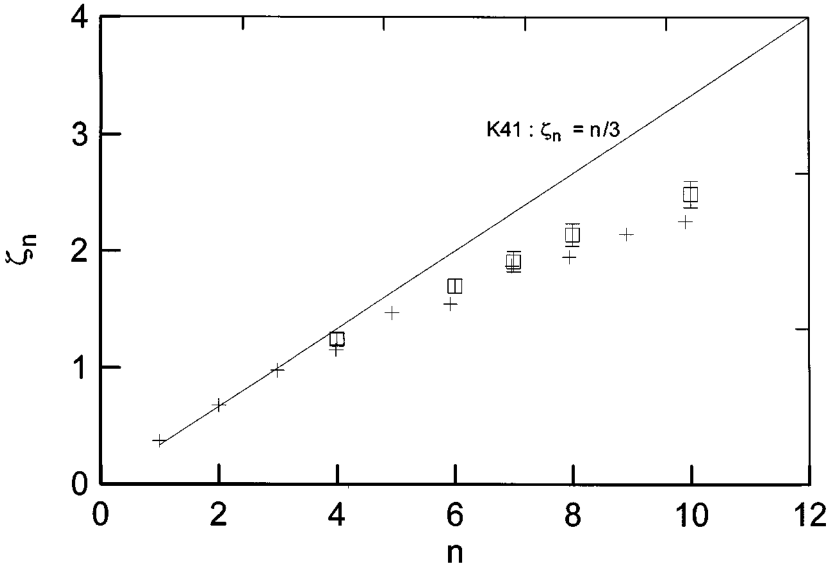

Figure 3 The scaling exponents for the velocity increments Δu_r with the separation distance in the inertial range. The unfilled squares are determined for shearless turbulence by the ESS method, using $|\Delta u_r|^3$ as the reference structure function (see text for more details). The crosses are for a boundary layer, obtained by the method of scaling function (see text for more details). The full line is $K41$.

Meneveau et al (1990) summarized the data on ξ_n (see Figure 4). This figure appears to support a stronger anomaly of the scalar than the velocity, but there are differences among the various sets of data. Antonia et al's (1984) data show an ever-increasing trend with the order of the moment; however, for high-order moments, these authors used an exponential extrapolation of the measured *pdfs* of temperature increments. On the other hand, Meneveau et al's (1990) data, obtained by a joint multifractal formalism, show a weaker tendency to increase with the moment order.

The notion of ESS has been applied to passive scalar data as well (Benzi et al 1994, Cioni et al 1995). The technique has poorer justification for scalars than for velocity. The ESS procedure yields scaling exponents that continue to increase with the order of the moment (as do exponents obtained from shell model simulations, Jensen et al 1992). It is unknown whether this is a real feature or an artifact of ESS. In our experience, the same scalar data at moderate Reynolds numbers, processed in different ways, could lead to different conclusions.

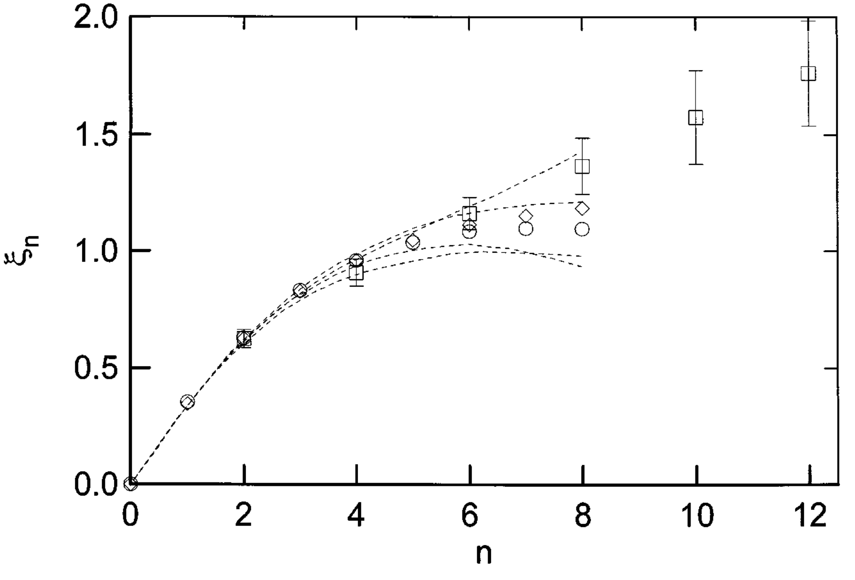

Figure 4 The scaling exponents for the scalar increments $\Delta\theta_r$ with the separation distance in the inertial range. The figure is taken from Meneveau et al (1990). Squares are experimental results from Antonia et al (1984), with vertical bars showing the uncertainty in the data. Circles and diamonds are from Meneveau et al, obtained by two different averaging procedures, both using a joint-multifractal formalism. The dashed lines are estimates of the uncertainty.

Rapid progress is being made in the understanding of anomalous scaling for a model problem that assumes that the velocity field advecting the scalar possesses very short correlation time (Kraichnan 1968, Majda 1993, Kraichnan 1994, Chertkov et al 1995, Gawedzki & Kupiainen 1995, Shraiman & Siggia 1995, Fairhall et al 1996). On the basis of a certain ansatz for the scalar dissipative term, Kraichnan (1994) derived an explicit expression for the scaling exponents, which for high-order structure functions vary as the square root of the order; some substantiation of the ansatz has been obtained from two-dimensional numerical simulations using 8192^2 gridpoints (Kraichnan et al 1995). Fairhall et al used renormalized perturbation theory and recovered Kraichnan's result. Gawedzki & Kupiainen noted that the exponent for the second-order structure function is exactly 2/3 (as in a straightforward extension of K41) and obtained an explicit correction for the fourth-order. This expression is consistent with the expression Chertkov et al obtained from an expansion in the parameter $1/d$, where d is the space dimension. In a somewhat different development, Yakhot (1996) obtained a nonlinear equation for the *pdf* of scalar increments. The equation's solution shows normal scaling for low-order moments and constant exponents for high-order exponents—the latter apparently resulting from the effective cut-off of the tails of the *pdf* by the scalar variance. Using another model for the scalar dissipation, Shraiman & Siggia (1995) focused on the third-order structure function and showed that the anisotropy at small scales persists, leading to a strongly non-Kolmogorov behavior. Although the details of this problem are still in flux and the approaches used by various authors differ, all of these results support anomalous scaling of structure functions of passive scalars.

3.4 *The Relation Between Inertial-Range Intermittency and Dissipation-Scale Intermittency*

The second Refined Similarity Hypothesis (RSH) of Kolmogorov (1962) can be stated (albeit in a somewhat milder form than was proposed originally) that the velocity increment over an inertial range separation r is related to ε_r through

$$\Delta u_r = V(r\varepsilon_r)^{\frac{1}{3}} \tag{13}$$

where V is a stochastic variable that is dimensionless, independent of r and $r\varepsilon_r$ and thus universal—provided the local Reynolds number $r(\varepsilon_r r)^{1/3}$ is large. The RSH says that the universal variable is not $\Delta u_r/(r\langle\varepsilon\rangle)^{1/3}$ but rather the variable $V = \Delta u_r/(r\varepsilon_r)^{1/3}$. This is the spirit of Obukhov's (1962) suggestion. [Kraichnan (1974) suggested that the local flux in wavenumber space would follow the RSH.] The RSH was not tested until recently, but several recent papers have purported to test various facets of the RSH (e.g. Hosokawa & Yamamoto 1992; Praskovsky 1992; Stolovitzky et al 1992, 1995; Thoroddsen & Van Atta 1992; Chen et al 1993b, 1995; Stolovitzky & Sreenivasan 1994; Thoroddsen 1995; Zhu et al 1995; Borue & Orszag 1996; Wang et al 1996). The support for the RSH is strong but not unequivocal. Some unpublished work at Yale suggests that the equivocation arises, at least in part, because the requirement of large local Reynolds number is often forgotten.

Another aspect of the RSH is its kinematic content. Stolovitzky & Sreenivasan's (1994) demonstration that the hypothesis holds for a variety of stochastic processes implies that there is a kinematic aspect to the RSH. This issue has been examined by Chen et al (1995), who concluded, on the basis of their DNS data at R_λ of 200, that the hypothesis has both kinematic and dynamical ingredients. These authors noted that the surrogacy issue, while not altering the basic conclusions, might exaggerate the degree to which Equation 13 is inferred as valid; the nature of averaging (along a line as opposed to that over a volume) also results in some differences. In particular, the hyperviscous simulations of Borue & Orszag (1996) suggested that the RSH may exhibit some limitations for high-order moments. Further experiments at high local

Reynolds numbers, and a companion analysis of numerical data, will go a long way in consolidating our fast-accumulating knowledge.

If we assume that the RSH holds, by raising both sides of Equation 13 to some integer power q and averaging, and by noting that the average of the right-hand side can be factored because V is independent of r and $r\varepsilon_r$, we obtain (for one-dimensional cuts)

$$\zeta_q = \left(\frac{q}{3} - 1\right) D_{q/3} + 1. \tag{14}$$

Note that if $D_q = 1$ for all q in Equation 14, that is, if the dissipation field is space-filling, we recover the (nonintermittent) K41 result that $\zeta_q = q/3$. Intermittency vanishes even if D_q is a constant independent of q. *Only if the generalized dimensions vary with q does one have intermittency.*

Thus, if the RSH is valid, inertial-range intermittency is inseparably tied to dissipative-scale intermittency, and Equation 14 holds. This relation was shown to work well by Meneveau & Sreenivasan (1987). There seems little doubt about its applicability as a working approximation, especially if one avoids high-order moments: In principle, the relation between the two intermittencies is almost definitely more complex than is implied by the RSH.

Although there is at present no complete theory validating RSH, the so-called fusion rules, which have their origin in the application of modern field theory to turbulence, validate Equation 14 (L'vov & Procaccia 1996). Fusion rules specify the analytic structure of many-point correlation functions of a turbulent field when some of the coordinates fuse or get very close to each other (see also Eyink 1993, 1994), and provide useful analytic constraints on the allowable theory. In this connection, the multiplicity of cut-off scales between inertial and dissipative ranges (Paladin & Vulpiani 1987, Sreenivasan & Meneveau 1988, Frisch & Vergassola 1991) is of some interest.

The extension of the RSH for passive scalars (RSHP) is straightforward (Hosokawa 1994, Stolovitzky et al 1995, Zhu et al 1995). The latter two references also roughly verified the RSHP experimentally. The following theoretical result concerns the RSHP: Starting from conservation equations for the scalar, Stolovitzky et al (1995) generalized the so-called Yaglom equation (Yaglom 1949) for Obukhov's (1962) pure ensembles and showed that several of the conclusions from the generalized Yaglom equation are consistent with the dictates of the RSHP.

3.5 *Probability Density Functions*

Despite their dubious utility in practice, the scaling exponents are considered theoretically important. Given the extensive preoccupation with them, it is somewhat disappointing that the final word may not have been said yet.

Researchers have done nearly as well as possible so far, but for future work to take us qualitatively further, measurements in controlled flows at much higher R_λ are needed. Such measurements should make little use of dubious conviences such as surrogacy and Taylor's hypothesis (see Section 6 for some additional comments).

On balance, the *pdf*s of small-scale quantities themselves might be worthy of study, even though some of their features are flow specific. A cursory study of the existing data suggests that the *pdf*s of the RSH variable V collapse in various flows, provided the large scale L is left as a free parameter. That is, the *pdf*s of velocity increments in the inertial range appear to be of the functional form $p(\Delta u_r/(r\varepsilon_r)^{1/3}; L)$. Other interesting proposals for the *pdf*s have been made by Gagne et al (1994); Polyakov (1995) implied certain qualitative forms for the case of turbulence without pressure.

Experiments have often focused on the tails of the *pdf*s of velocity increments and derivatives with the belief that their core is subject to central limit statistics (although a spiky shape near the origin is also indicative of intermittency). As discussed in Section 2, the tails of the *pdf*s of velocity increments can be fitted by stretched exponentials; other empirical fits also exist (Castaing et al 1990). The so-called third hypothesis of Kolmogorov (1962) implies that the *pdf* of $(\partial u/\partial x)^2$ for instance, is lognormal. In practice, the lognormal fit to the data is modest (e.g. Stewart et al 1970, Wang et al 1996). In principle, lognormality is ruled out for a variety of reasons emphasized by Orszag (1970), Novikov (1971), Mandelbrot (1974) and Narasimha (1990). Substantial parts of the dissipation *pdf* are fitted well by a square-root exponential form (Gagne 1987, Hosokawa 1991, Meneveau & Sreenivasan 1991, Bershadskii et al 1993b[3]), but there is no theoretical justification for this fit. Dowling (1991) briefly discussed the limitations of the square-root exponential form of the *pdf* of the scalar dissipation.

The *pdf*s of vorticity appear to be simpler. Unpublished work at Yale shows that, in boundary-layer flows, the tails can be fitted very well by an exponential and the core by a Gaussian. As the Reynolds number increases, the exponential tails creep into the core until, at high enough Reynolds numbers characteristic of the atmospheric surface layer, say, the Gaussian core vanishes nearly entirely. There is no theoretical understanding of this observation either, although some plausible explanations can be given.

Migdal (1994) has proposed a theory for the *pdf* of circulation. One of his predictions is that the *pdf* of circulation around a contour lying entirely within the inertial range is independent of the shape of the contour as long as the area

[3]These authors also examined the surrogacy aspect of the spectral densities of velocity and vorticity.

it circumscribes is the same. Cao et al (1996) verified this prediction. However, Migdal's prediction that the *pdf* itself is Gaussian was not substantiated. Apart from noting the presence of non-Gaussian tails, no deeper analysis of the shape of the *pdf*s has been made. Furthermore, Novikov's (1995) exact relations on conditional moments of circulation have not been followed up by experiment.

The *pdf*s of the temperature increments $\Delta\theta_r = \theta(x+r) - \theta(x)$ have stretched exponential tails as well (Antonia et al 1984, Ould-Rouis et al 1995). The most detailed analysis is that of Ching (1991) for Wu's (1991) convection data: The stretching parameter changed smoothly from about 0.5 for small separation distances to about 1.7 for large separation distances. (For scalars, the exponent does *not* always approach 2, even for large separation distances.) Starting with Sinai & Yakhot's (1989) method, Vaienti et al (1994) determined the *pdf* of $\Delta\theta_r$ in fully developed isotropic turbulence; the resulting distribution has a nearly Gaussian core with stretched exponential tails. The experimental *pdf*s of $\Delta\theta_r$ (e.g. Vaienti et al 1994, Hosokawa 1994) emphasize the *increasing* departure from isotropy for *decreasing* separation distances, a feature reflected in Shraiman & Siggia's (1995) analysis. The squares of spatial derivatives of temperature are closely but not exactly lognormal (Antonia & Sreenivasan 1977, Dowling 1991).

3.6 *Derivative Skewness and Flatness Factors*

3.6.1 VELOCITY FIELD The statistics that are measured most often are the flatness factor and skewness of velocity derivatives, which we might recall are constants in K41. Data obtained by a number of investigators in the atmosphere (e.g. Gibson et al 1970, Wyngaard & Tennekes 1970, Wyngaard & Pao 1972, Champagne et al 1977, Williams & Paulson 1977, Champagne 1978, Antonia et al 1981, Bradley et al 1985), in laboratory flows (e.g. Antonia et al 1982), and from numerical simulations (e.g. Jimenez et al 1993) show that the derivative skewness and flatness increase monotonically with Reynolds number. Van Atta & Antonia's (1980) compilation is reproduced in Figures 5 and 6, but with two differences, resulting mainly from Sreenivasan's (1995b) recent scrutiny: The Reynolds numbers of Gibson et al (1970) have been corrected and new data have been added.

Although the trends shown in Figures 5 and 6 are supported by other unpublished data (Y. Gagne & A. Praskovsky, private communication), they are contrary to Tabeling et al's data (1996). These latter authors made measurements in a closed flow created by two counter-rotating discs of 20 cm diameter enclosed in a cylindrical envelope, with the discs 13 cm apart vertically. The fluid was helium gas at 5° K. The microscale Reynolds number, R_λ, varied from about 180 to about 5000. The derivative skewness in these measurements followed the trend of the compiled data up to an R_λ of about 800 and decreased

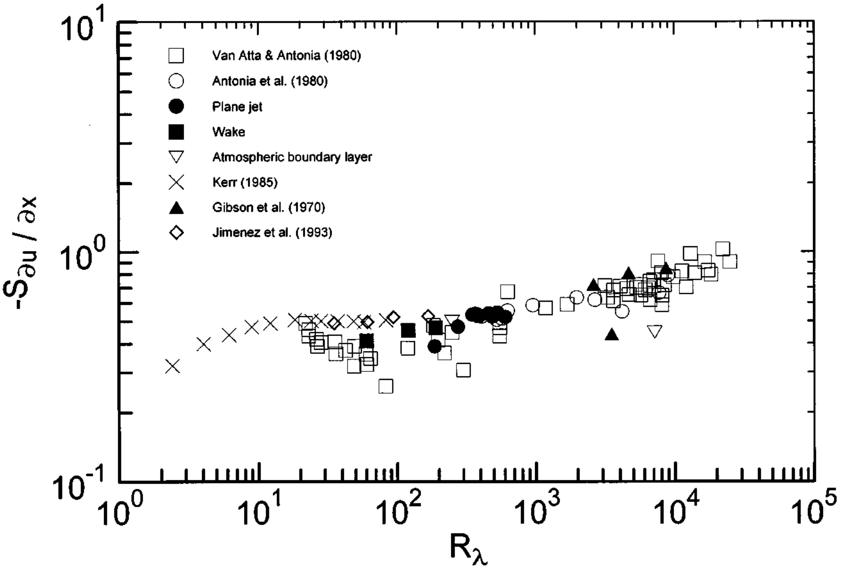

Figure 5 The Reynolds number variation of the skewness of the streamwise (or longitudinal) velocity derivative $\partial u/\partial x$ from different sources. All the data from Van Atta & Antonia (1980) have been replotted here using open squares without identifying the sources, for which their original paper should be consulted. The new data are explained in the inset.

thereafter, falling to about 0.22 at the highest R_λ. The flatness also started a downward trend from about 15 at $R_\lambda \approx 800$ to about 6.5 at 5000.

Tabeling et al inferred that, around an R_λ of 800, either some new transition to a different state of turbulence was occurring[4] or the behavior of skewness and flatness is nonuniversal. Some concerns about these measurements have been expressed (Sreenivasan 1995b, Emsellem et al 1996), and the issue has not been resolved satisfactorily. It would thus be desirable to have more corroboration of these trends for $R_\lambda \gg 1000$—especially for the derivative skewness whose rate of increase is rather slow. Measurements of velocity derivatives other than $\partial u/\partial x$ would be helpful in this regard. The normalized fifth-and sixth-order moments show an unequivocal and monotonic increase with R_λ (Antonia et al 1981).

[4]The possibility of a similar transition at high Rayleigh numbers (of the order 10^{11}) in thermal convection was suggested by Procaccia et al (1991) on the basis of observed changes in spectral characteristics, but Grossmann & Lohse (1993) have argued that these changes are due to the limited thermal response time of the boundary-layer fluid on the probe.

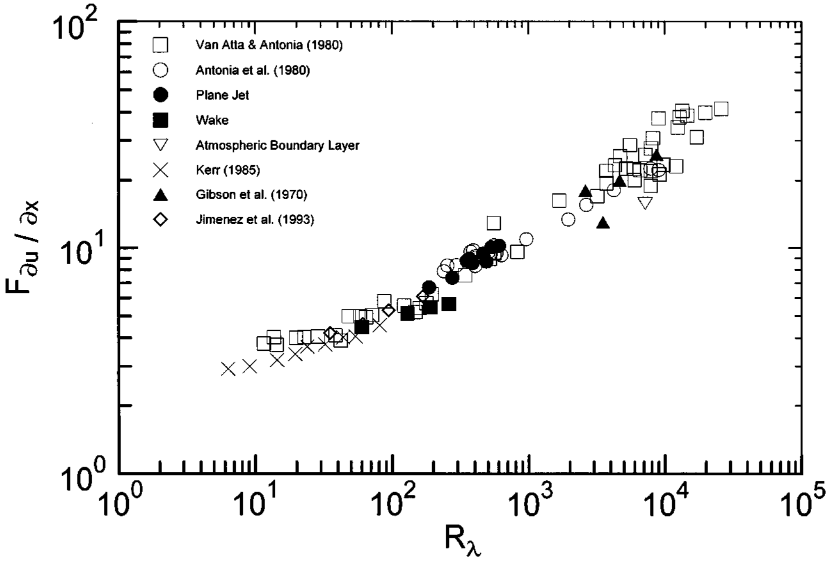

Figure 6 The Reynolds number variation of the flatness factor of the streamwise (or longitudinal) velocity derivative $\partial u/\partial x$, from different sources. All the data from Van Atta & Antonia (1980) have been replotted here using open squares without identifying the sources, for which their original paper should be consulted. The new data are explained in the inset.

3.6.2 THE PASSIVE SCALAR FIELD If one imposes large-scale inhomogeneities in both velocity and scalar fields, the skewness of the scalar derivative is finite even at the highest Reynolds numbers at which measurements are available today. The magnitude of the scalar derivative skewness is shown in Figure 7. The scatter in the data is rather large, but the skewness persists even for large Reynolds numbers. Note that the persistence of this skewness violates local isotropy (e.g. Monin & Yaglom 1995, Sreenivasan 1991a). The sign of the skewness depends on the product of the signs of mean shear and mean scalar gradient, and its magnitude for weak shear is rather complex (Sreenivasan & Tavoularis 1980). Holzer & Siggia (1994) showed that, for finite derivative skewness, it is sufficient to break the symmetry of the large scale of the scalar along the direction of the gradient for the skewness.

Figure 8, most of whose points were obtained by Antonia & Chambers (1980), shows the flatness data for $\partial\theta/\partial x$, $F_{\partial\theta/\partial x}$. Kerr's (1985) simulation data for a molecular Prandtl number *Pr* of unity have also been added.[5] The figure shows

[5]The simulations are particularly useful for assessing the effect of *Pr* on the temperature derivative statistics. Kerr found that $F_{\partial\theta/\partial x}$ increases with *Pr* for a given R_λ whereas the rate of increase

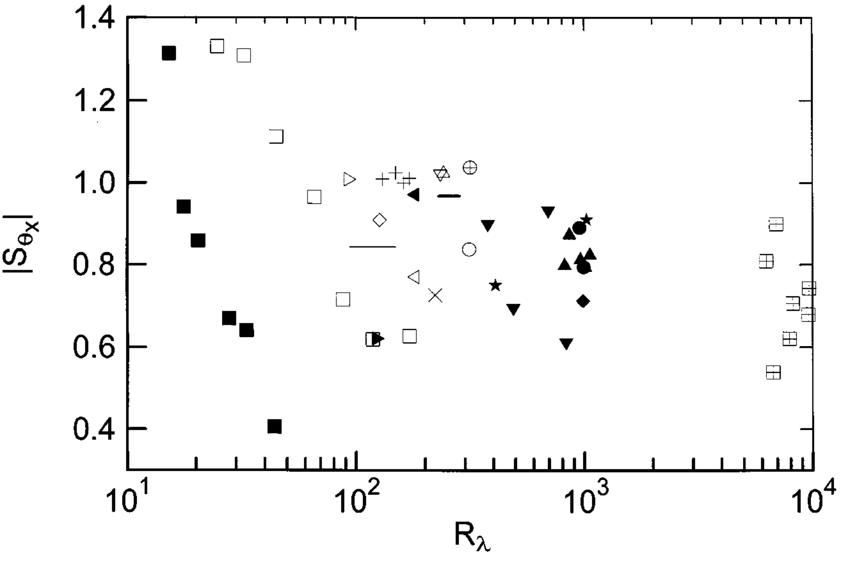

Figure 7 The Reynolds number variation of the skewness of the streamwise (or longitudinal) temperature derivative $\partial\theta/\partial x$ from different sources. Most of the data are replotted from Sreenivasan et al (1979), which should be consulted for the explanation of symbols. Some new data plotted are: thin line, Sreenivasan & Tavoularis (1980), homogeneous shear flow; thick line, Tavoularis & Corrsin (1981), homogeneous shear flow; filled right-pointing triangle, Zubair (1993), wake; filled left-pointing triangle, Browne et al (1983), wake; squares with crosses, Antonia et al (1979), atmospheric boundary layer.

that $F_{\partial\theta/\partial x}$ is significantly larger in magnitude than $F_{\partial u/\partial x}$. Also, $F_{\partial\theta/\partial x}$ has a stronger dependence on R_λ than $F_{\partial u/\partial x}$ does (Figure 6), again suggesting that the scalar field is more intermittent than is the velocity field.

In shear flows, where y is the direction of shear and z is transverse to x and y, skewness and flatness factors of $\partial\theta/\partial y$ and $\partial\theta/\partial z$ have been measured by Gibson et al (1977), Sreenivasan et al (1977), Mestayer (1982), Tavoularis & Corrsin (1981), Antonia et al (1986a), and others. These statistics have also been measured for decaying grid turbulence with and without a superimposed mean temperature gradient (Tong & Warhaft 1994, 1995) and have been obtained in simulations (Pumir 1994a,b). Although numerically different from those of $\partial\theta/\partial x$, these factors increase with R_λ at about the same rate.

of $F_{\partial\theta/\partial x}$ decreases with increasing *Pr*. Data at higher R_λ would be valuable (although experiments at different *Pr* would be difficult to conduct).

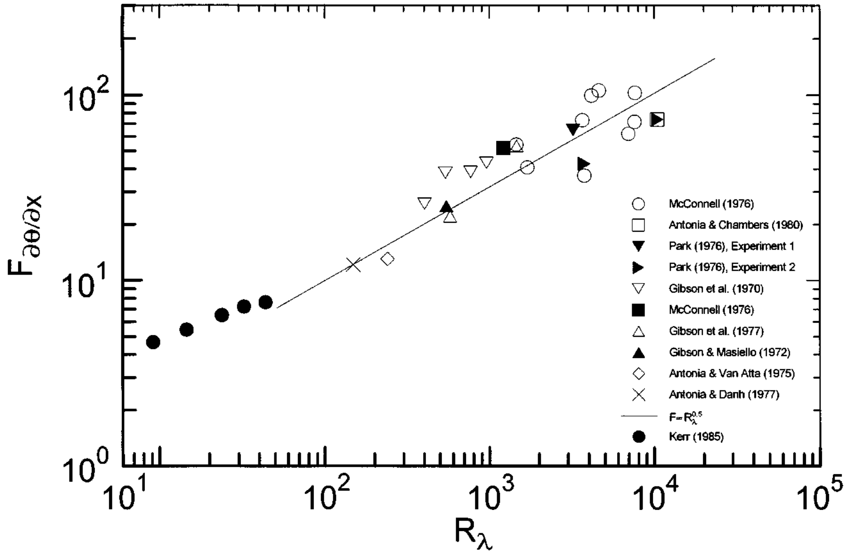

Figure 8 The Reynolds number variation of the flatness factor of the streamwise (or longitudinal) temperature derivative $\partial\theta/\partial x$, from different sources.

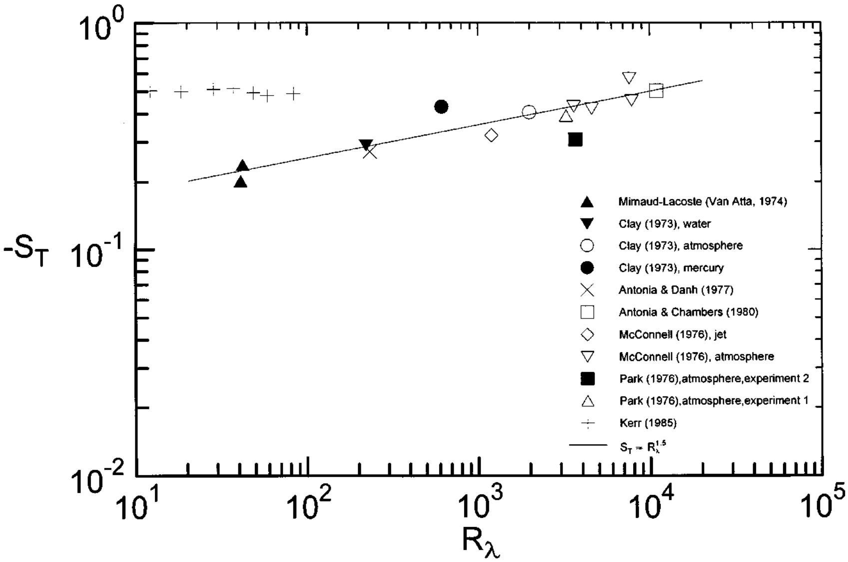

Figure 9 The Reynolds number variation of the mixed derivative skewness S_T, defined in Section 3.6.3.

3.6.3 MIXED DERIVATIVE STATISTICS Statistics of mixed derivatives are useful for testing various models (Antonia & Van Atta 1975, Meneveau et al 1990), and also for other reasons. For example, the mixed derivative skewness $S_T \equiv \langle(\partial u/\partial x)(\partial\theta/\partial x)^2\rangle/\langle(\partial u/\partial x)^2\rangle^{\frac{1}{2}}\langle(\partial\theta/\partial x)^2\rangle$ is of interest because $\langle(\partial u/\partial x)\ (\partial\theta/\partial x)^2\rangle$ represents the production rate of the mean-square temperature gradient (Wyngaard 1971). Most of the data in Figure 9 are from Antonia & Chambers (1980) for $Pr \simeq 0.7$; others are from Clay's (1973) measurements in water and mercury. Antonia & Chambers concluded that S_T increases slowly with R_λ and that the dependence on Pr is probably small; Kerr's simulations corroborate the weak dependence on Pr but indicate a negligible dependence on R_λ (Figure 9). The discrepancy in the magnitude of S_T between the experiments and Kerr's simulations needs to be resolved.

4. INTERMITTENCY MODELS

Intermittency models help explain some of the features discussed in previous sections, particularly the scaling exponents. One broad class of models assumes a local cascade of energy. These models are best cast, or recast, in terms of multifractal formalism (Mandelbrot 1974, Hentschel & Procaccia 1983, Parisi & Frisch 1985, Halsey et al 1986, Meneveau & Sreenivasan 1987), which provides a convenient superstructure; Kolmogorov's (1941a) original model is a degenerate case, as are Novikov & Stewart's (1964), Gurvich & Yaglom's (1967), and Novikov's (1971) models. Most of the models have been formulated in terms of either the moments of absolute values of velocity increments or the moments of dissipation. The following is an incomplete list of sources of cascade as well as noncascade models: Frisch et al (1978), Schertzer & Lovejoy (1983), Benzi et al (1984), Nakano & Nelkin (1985), Meneveau & Sreenivasan (1987, 1991), Yamazaki (1990), Hosokawa (1991), Hunt & Vassilicos (1991), Kraichnan (1991), She (1991), Chhabra & Sreenivasan (1992), Andrews et al (1994), and She & Leveque (1994).

Because the connection of these models to the Navier-Stokes equations is tenuous, we shall limit ourselves to making a few general remarks. The detailed physics of the models cannot be tested directly, and so their success can be evaluated chiefly on the basis of how well they agree with experiments. Within the range explored by experiments, several models work quite well. However, if different models agree within the domain of available data but depart outside of it, it is not possible to recommend one model over the other. Broad arguments based on internal consistency, positive definiteness of energy dissipation, and other mathematical constraints such as Hölder inequality can rule out some models, but this process is insufficient to establish the inevitability of any model. Consistency checks of this type have been suggested by Novikov

(1971, 1990) and others, and have been made by Meneveau & Sreenivasan (1991) and Borgas (1992, 1993). In particular, Borgas (1992) reviewed different models and concluded that a definitive model has yet to appear. Yet, several models can, in fact, make predictions (as opposed to mere "postdiction" of known aspects) and are of great pedagogical value. Particularly interesting, popular, and potentially powerful is the log-Poisson model of She & Leveque (1994). In practical terms, for example, the p-model (Meneveau & Sreenivasan 1987) has been used to generate stochastic fields that have essentially the same properties as real turbulence (Juneja et al 1994). Because these artificial data are statistically quite close to real turbulence, they can be used as initial conditions for DNS simulations (Juneja 1995).

Less attention has been given to the development of models for describing scalar intermittency. Korchaskhin (1970), Van Atta (1971), and Tennekes (1973) considered the influence of fluctuations in ε_r and χ_r for $Pr \simeq 0(1)$; Van Atta (1974) considered the effect of the fluctuations in ε_r and χ_r on the spectral predictions of Batchelor (1959) when $Pr \gg 1$ and of Batchelor et al (1959) when $Pr \ll 1$. For $Pr \sim 0(1)$, Van Atta (1971) extended the lognormal model by assuming that the ε_r and χ_r are lognormally distributed and that their joint *pdf* is bivariate lognormal with a correlation coefficient of $\rho(r) = \langle(\ln\varepsilon_r - \langle\ln\varepsilon_r\rangle)(\ln\chi_r - \langle\ln\chi_r\rangle)\rangle/(\sigma_{\ln\varepsilon_r}\sigma_{\ln\chi_r})$. Sreenivasan (1991b) noted a simple binomial model for the scalar dissipation. For other multifractal models of scalar dissipation, see Shivamoggi (1995).

5. SPATIAL STRUCTURE AND KINEMATICS

5.1 *Results from Direct Numerical Simulations*

5.1.1 PRELIMINARY REMARKS Most of the work mentioned above deals with a statistical description of the small scale. This description does not recognize explicitly that turbulence is a complex motion of fluids, containing vortex motions (Saffman 1992). Explicit models incorporating the geometry of vortex structures have been proposed by Townsend (1951), Corrsin (1962), Tennekes (1968), Saffman (1968), and Lundgren (1982). An interesting statistical mechanical perspective on turbulence has been developed by Chorin (1994), using vortex tubes as building blocks. The geometric structure of the small scale became better established with the ability to visualize it in the DNS data and (to a lesser extent) in experiments. These studies have made explicit the relations between fields of dissipation and vorticity, energy and scalar dissipations, and so forth, and shed light on shapes of vortex structures and (to a limited extent) on their Reynolds-number scaling.

Direct numerical simulations of homogeneous three-dimensional turbulence in a periodic box were first obtained by Orszag & Patterson (1972) and have since been performed by dozens of investigators. Advances in computational hardware and software have enabled rapid progress over the past fifteen years: On the average, the largest computable box size has doubled every three years or so, and the largest R_λ has doubled every five or six years. Bigger simulations are likely to be carried out in the near future, although the various bottlenecks in microelectronics technology suggest that the rate of growth will slow down. Currently, the largest box size used has 512^3 gridpoints (Chen et al 1993b, Jimenez et al 1993), and the highest R_λ reached is about 200. Interestingly, this Reynolds number exceeds those attained in all but the special grid experiments of Kistler & Vrebalovich (1966), for which R_λ was in the range 260–670. Simulations complement experiments in some respects and provide significantly new information on statistical as well as structural aspects of the small scale.

5.1.2 STATISTICAL INFORMATION COMPLEMENTING EXPERIMENTS On the whole, estimates of the velocity derivative skewness and flatness from DNS are consistent with the data compiled in Figures 4 and 5. The flatness of $\partial u/\partial y$ is larger and increases somewhat more rapidly than that of $\partial u/\partial x$. The experimentally observed scale dependence of the *pdf*s seems to be confirmed as well. In these respects, numerical simulations have come up with no surprises.

5.1.3 THE RELATION BETWEEN VORTICITY, STRAIN RATE, DISSIPATION, AND SCALAR GRADIENTS Using a 128^3 simulation of isotropic turbulence ($R_\lambda \leq 83$), Kerr (1985) was able to examine the spatial relationship between vorticity, strain rate, and scalar gradient. He concluded that (*a*) the vorticity is concentrated in tubes and sheets, with large concentrations of strain rate and scalar gradient nearby, (*b*) the largest principal strain rate is compressive and aligned perpendicular to the tube, (*c*) the stretching along the tube is small, whereas the largest stretching is perpendicular to the tube, and (*d*) large values of the scalar gradient are wrapped in sheets around the tube, and this gradient is in the direction of the compressive strain rate. In particular, the vorticity vector was shown to be aligned with the intermediate strain-rate eigenvector whereas the scalar gradient vector was aligned with the compressive strain rate. Ashurst et al (1987) and She et al (1990) have examined this aspect in more detail. She et al emphasized that the alignment is particularly strong for high strain rates. There is some experimental support for this observation in Dracos et al's (1989) and Tsinober et al's (1992) measurements.

Of special interest is the orientation of strain and vorticity fields because it bears directly on vortex stretching, tearing, and reconnection. Betchov's

(1956) work shows that the middle eigenvalue of the strain-rate tensor should be positive, as confirmed by Kerr (1987) and Ashurst et al (1987). These latter results show that different behaviors occur in low and high strain-rate regions. The histograms of the middle eigenvalue were roughly symmetric in regions of low strain-rate but skewed in high strain-rate regions. Ashurst et al found that the eigenvalues are in the ratios 3:1–4, and that the largest eigenvalue is compressive. Although there is no first-principle understanding of this observation, Jimenez (1991) has given a plausible interpretation based on numerical results.

5.1.4 THE GEOMETRY OF SPATIAL STRUCTURE: VORTEX TUBES Essentially all simulations show persistent and extended tubes, sheets, and blobs of small-scale vorticity, although their origins need to be clarified. In particular, intense vorticity is often concentrated in tubes. She et al (1990) computed vortex lines associated with high vorticity intensity and showed that they are in the form of long tube-like structures; although straight in the core region, they spiraled around the tube in the outer portion. The structures became more sheet-like at moderate vorticity amplitudes, but became patternless at lower amplitudes. Ruetsch & Maxey (1991) made comparable observations and found that moderate ε regions surround vortex tubes. Regions of moderate to intense scalar dissipation were wrapped around the tubes, but the highest intensities occurred in large flat sheets in the vicinity of moderate ω^2 sheets. Kida & Ohkitani (1992) reported that vorticity tends to be concentrated in long, thin, tubelike regions whereas the energy dissipation is double-peaked around a tube.

Siggia's (1981) and Kerr's (1985) simulations suggested that the tubes are long, with diameter of order η and length of order L (see also Vincent & Meneguzzi 1991, 1994; Jimenez et al 1993). The latter authors claimed that a circulation-based Reynolds number of vortex tubes is of the order $8R_\lambda^{1/2}$. Yamamoto & Hosokawa (1988) confirmed, in a 128^3 simulation of decaying isotropic turbulence ($R_\lambda \simeq 100$), the tube diameter to be of the order η but claimed their characteristic length to be on the order of the Taylor microscale. At the very least, these contradictions reflect the poor scale separation in relatively low-Reynolds-number simulations.

The vortex tubes exist also in nonhomogeneous flows, for example in the wall region of a low-Reynolds-number channel flow (Jimenez 1991) and in flows with homogeneous shear as well as various irrotational strains (Rogers & Moin 1987). The vortex filaments seem to be strengthened by the mean strain, and the high magnitude vorticity is especially aligned in the direction of expansive strain. Rogers & Moin also indicated that hairpin structures, which are formed by the roll-up of sheets of mean spanwise vorticity (described as a "scroll" by R. Narasimha—private communication), develop only when the mean shear is present. Some of these differences call for more detailed study. Soria et al (1994) observed the tendency for vortex tubes to align themselves with the eigenvector associated with the intermediate eigenvalue in simulations of incompressible and compressible mixing layers and wakes.

5.1.5 THE DYNAMICAL SIGNIFICANCE OF VORTEX TUBES Since the vortex tubes are responsible for high vorticity amplitudes, it might be thought that they play a major role in determining departures from Gaussianity of velocity derivatives and vorticity fluctuations. This aspect has been built into She & Leveque's (1994) intermittency model, which predicts the ESS scaling exponents of Benzi et al (1993) rather well. Yet this is no endorsement for the dynamical importance of the tubes because equally successful models can be constructed by incorporating singular structures other than vortex tubes. Furthermore, Jimenez et al (1993) raised the issue of the stability and survival, at high Reynolds numbers, of tubes of order L in length. Even at the R_λ of their simulations, the vorticity with intensity less than those of the tubes but greater than $\langle\omega^2\rangle^{1/2}$ was concentrated in large-scale vortex sheets (separating energy-containing eddies) and accounted for 80% of $\langle\epsilon\rangle$; they estimated, within a narrow range of R_λ, that the tubes had a volume fraction $<1\%$. Their conclusion was that the tubes are not especially important in the overall dynamics of turbulence. In contrast, as already noted, Chorin (1994) has developed a self-consistent statistical theory on the basis of self-avoiding random walk of vortex tubes.

An interesting question is whether the existence of the vortex tubes is consistent with the scaling presumed to exist in high-Reynolds-number turbulence (Section 3). For instance, Moffatt (1994) noted that the self-similarity implicit in the inertial power-law scaling ($\langle\Delta u_r^n\rangle \sim r^{\zeta_n}$) may be incompatible with the existence of vortex tubes on scales less than $0(\lambda)$. It is not yet clear that the tubes have greater dynamical significance than do vortex structures of other kinds and shapes. If the tubes are just a part of the hierarchy of structures, they probably have little effect on scaling, and one is left with the question of how to deal with the variety of structures. Multifractality of turbulence came to the fore largely for dealing with the complex geometry of many shapes with varying intensities. At this point, it may be helpful to look for qualitative guidance from other circumstances in hydrodynamics. It is known that the discrete vortex structures are quite important in transitional flows and in the viscous region of the turbulent wall-flows. It is also known that line dislocations in plastic flow determine the strength of solids. The Kosterlitz & Thouless (1973) theory of the X-Y model explicitly invokes vortex structures. However, there are many physical problems dealing with layered media in which the critical behavior is quite oblivious to the anisotropic structures. For example, anisotropic ferromagnets near the critical point have essentially the same critical indices as isotropic ferromagnets (Aharony 1976).

5.1.6 DEPLETION OF NONLINEARITY The persistence of coherent vortex tubes is often interpreted in terms of local depletion of nonlinear effects (e.g. Frisch & Orszag 1990). There is some support for this conclusion in the work of Pelz et al (1985), who found in simulations of a channel flow and a Taylor-Green vortex flow that the velocity and vorticity have a tendency to align (so-called "Beltramization") in regions of low dissipation rates (see also Kerr 1987, Rogers & Moin 1987). However, Kraichnan & Panda (1988) and Chen et al (1989) have pointed out that the depletion of nonlinearity is a property of a class of systems, depending broadly on how the interaction coefficients vary with the wavenumber.

5.2 *Experiments*

There have been several experimental attempts to visualize the small scale. Schwarz (1990), using tiny crystalline platelets in oscillatory grid turbulence, identified small-scale flow patterns with layered vortex sheets; these patterns occupy a significant fraction of the volume at the Reynolds number of the measurements, $R_\lambda \equiv 100$. Planar measurements of turbulent velocity fluctuations have now been made by several investigators (Adrian 1991). However, few of these measurements have been used for the study of small-scale turbulence (but see Sreenivasan et al 1995, who studied circulation properties). In Douady et al's (1991) experiment, high-vorticity (low-pressure) filaments were visualized in a turbulent flow enclosed between two counter-rotating parallel discs. The filaments were found to be thin and short lived; they first appeared in stretched shear regions and disintegrated relatively rapidly leading to larger structures with longer life. Cadot et al (1995) have presented more detailed statistics of the filaments. These authors drew attention to the general difficulty of selecting appropriate detection criteria and suggested that the largest depressions are associated with the longest filaments (radius $\sim \lambda$) but that filaments with the strongest vorticity are likely to be the thinnest and shortest. Such structures were seen even in a grid flow by Villermaux et al (1995), who noted that the filaments nucleate mostly at the surface of the tank housing the fluid.

The measurement of scalars has advanced to a fruitful stage (e.g. Dahm et al 1991), and various inferences can be drawn from them. For example, Sreenivasan (1991b) inferred that the scalar structure in a round jet was sheet-like (although this observation may be the result of the special nature of the three-dimensional large-scale motion in the jet). Using a rake of cold wires aligned in the spanwise direction, Antonia et al (1986a,b) noted that the temperature front was identified with the diverging separatrix (in the direction of the principal rate of strain) between adjacent vortical structures on the same side of the centerline of a plane jet. Some support for the quasi two-dimensionality of the front was provided by the joint *pdf* of $(\partial\theta/\partial x)^2$ and $(\partial\theta/\partial z)^2$, the largest values of $(\partial\theta/\partial x)^2$ occurring with highest probability when $(\partial\theta/\partial z)^2$ is small. Antonia et al's (1986a,b) observations are consistent with a description of the front as a scalar sheet. Statistics conditioned on the front (Antonia et al 1986a) indicated that virtually all of the skewness of $\partial\theta/\partial x$ was accounted for by the front, whereas the contribution of the front to the skewness of $\partial u/\partial x$ was negligible. At least at first sight, this result seems to be in accord with Sreenivasan's (1991b) claim that the small-scale scalar structure is influenced directly by the large-scale motion, and that the anisotropy is more pronounced in the small-scale scalar structure than in the small-scale velocity field (Sreenivasan 1991a, Antonia & Kim 1994).

6. CONCLUDING REMARKS

Space constraints preclude us from considering several other aspects of the small scale: Local isotropy, the scaling of pressure fluctuations, acceleration and high-order derivatives of velocity, renormalization group and applications, and multifractals and multiscaling are topics that could be reviewed separately. We have not covered compressible turbulence or turbulence in the presence of body forces such as buoyancy; nor have we considered small-scale mixing or small-scale models for large-eddy-simulations. It is, therefore, appropriate to make a few remarks here from a broader perspective.

Our present understanding of small-scale turbulence is based largely on the synthesis of experimental data in the light of K41 and its refinements. Theory continues to lag behind observations, and observations continue to be plagued by many undetermined effects such as those resulting from Taylor's hypothesis and the surrogacy assumption. The most serious shortcoming is the lack of data at high enough Reynolds numbers under controlled circumstances. Most high-Reynolds-number experiments focusing on small-scale turbulence have been made in the earth's atmosphere. Because atmospheric flows are uncontrolled and nonstationary over long times, the typical experimental strategy has been to obtain the same type of data in various flows and Reynolds numbers, and to examine these data for the existence (or otherwise) of a universal behavior. A complementary approach is to make multiple measurements in the same flow over a wide range of Reynolds numbers. Two principal efforts in this direction should be noted: the convection experiment of Libchaber and his collaborators (e.g. Wu 1991) and Tabeling et al's (1996) experiment on shear flow between rotating discs. Both experiments cover wide ranges of Reynolds numbers in the same apparatus by exploiting the variable properties of gaseous helium near the critical point. Libchaber's experiment forms the subject of Siggia's (1994) perceptive review. There seem to be several unresolved problems with respect to Tabeling et al's experiment but it is hoped that valuable results will be forthcoming.

Controlled experiments at high Reynolds numbers have been attempted using industrial-size facilities—most recently by Saddoughi & Veeravalli (1994) in the NASA Ames full-scale aerodynamic facility—and one has invariably learned something valuable from them. Faster progress might be expected if a dedicated high-Reynolds-number facility were available. The importance of the problem would seem to justify the expense involved. How high a Reynolds number should one seek, in order to resolve satisfactorily the status of small-scale turbulence in the asymptotic state (assuming one exists)? In addressing the question, it must be noted that the largeness of the Reynolds number in turbulence is appropriately measured by its logarithm. From an examination of the available scaling range, it appears that one ought to aim for Re $\sim O(10^8)$ or $R_\lambda \sim O(10^4)$. Such experiments are being discussed at present (Behringer et al 1994).[6] Despite the rapid progress being made on the computational front, it does not seem possible to attain this goal (though simulations have been quite valuable).

Because the precise determination of the scaling exponents is difficult at present, other aspects of small-scale turbulence may provide a fruitful relief. An interesting question is the manner in which the large scale influences the scaling regime: For instance, is this influence contained within the RSH in a benign way, or does it appear more explicitly in some other (yet unknown) way? Or, in general, what features of the large scale are sufficient to characterize their influence on the small scale? The effects of strong anisotropy need more serious consideration, as would the practical utilization of the concepts of scaling universality. Another question is, What, if any, are the implications of structures to scaling? A more careful study of *pdf*s of derivatives and increments of velocity and scalar would be worthwhile (even though the *pdf*s are bound to be affected by the large scale).

Unfortunately, the various limitations inherent in the experiments demand that, for the time being, we live with many uncertainties. Since a global theory of turbulence is unlikely to emerge in the near future, worthwhile theoretical advances will probably hinge on explaining consolidated experimental facts. Such explanations are a prerequisite to understanding the bigger picture. In this review, we have attempted to highlight some gaps in the experimental framework while presenting results as they are known today; to that extent, these

[6]The extent of the scaling regime is not a unique function of the Reynolds number. The nature, strength, and persistence of large-scale forcing is an important determining factor.

results demand a theory. But we can sympathize readily with a theorist who is frustrated by the uncertainty surrounding the experimental results. Definitive progress in the subject is linked inextricably to obtaining measurements of vastly improved quality. It would be presumptuous to relegate theory to the role of merely explaining empirical facts. On the contrary, we emphasize that theory should play a major part in posing the right questions. It is this interplay between theory and experiment that will eventually allow some headway to be made.

Wright's (1982) quotation at the beginning of this review might well have been made about small-scale turbulence: It reflects accurately both the fascination and the frustration of the field.

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