MSRI Soergel bimodule workshop
June/July 2017

Week 2 Day 3 Morning: Basic Exercises

Jones and HOMFLY polynomials

1. (This question is surprisingly hard, which is why it is difficult to compute triply graded knot homology by elementary techniques.)

a) Consider the unknot, which is the closure of the identity braid on one strand. One should think of its triply graded knot homology as the total Hochschild cohomology of $R_1 = \mathbb{C}[x_1]$. Compute this triply graded vector space.

b) The unknot is also the closure of σ in the braid group on two strands. Compute its triply graded knot homology (this agrees with the previous part up to a normalization). Your base ring should be $R_2 = \mathbb{C}[x_1, x_2]$, acted on by S_2 in the usual way.

Cell theory

2. Why is the two-sided relation $<$ on indecomposable objects (i.e. $B_x < B_z$ if B_x is a summand of $M \otimes B_z \otimes N$ for some M and N) transitive? Why is the two-sided relation on a basis (i.e. $b_x < b_z$ if b_x appears with nonzero coefficient in the expansion of mb_zn for some m and n) not necessarily transitive?

3. a) Prove that, if b_x and b_y are in the same left cell of $\mathbf{H}$, then x and y have the same right descent set.

b) Find two different left cells with the same right descent set.

c) Prove that the non-identity elements with a unique reduced expression form a single two-sided cell. How does it split into left cells? (This is a good time to return to some supplementary exercises from the very first day, to explore this cell module.)

Robinson-Schensted

4. a) Prove that the number of rows in the partition λ associated to an element w is the length of the longest decreasing sequence: $i_1 < i_2 < \ldots < i_k$ such that $w(i_1) > w(i_2) > \ldots > w(i_k)$.

b) (Harder) The number of rows is also the size of the first column. Find a formula for the sum of the sizes of the first two columns. (Be careful, it is easy to get this slightly wrong!)

5. Explicitly prove Graham's theorem about the action of h_{w_0} on the KL basis, just for S_3.