

MSRI Soergel bimodule workshop

June/July 2017

Week 1 Day 2 Afternoon: Supplementary/Advanced Exercises

Diagrammatics for monoidal categories

1. In class, we fixed a biadjunction between E and E^\vee , and a biadjunction between F and F^\vee . We demonstrated two ways to take a 2-morphism $\beta: E \rightarrow F$ and return a 2-morphism $F^\vee \rightarrow E^\vee$, known as the right mate β^\vee and the left mate ${}^\vee\beta$. One can think about these as “twisting” or “rotating” β by 180 degrees to the right or to the left. Visualize what it would mean to twist β by 360 degrees to the right, yielding another 2-morphism $\beta^{\vee\vee}: E \rightarrow F$. Verify that β is cyclic, i.e. $\beta^\vee = {}^\vee\beta$, if and only if $\beta = \beta^{\vee\vee}$.
2. Suppose that B is an object in a monoidal category with biadjoints, and $\phi: B \otimes B \otimes B \rightarrow \mathbb{1}$ is a cyclic morphism. What should it mean to “rotate” ϕ by 120 degrees? Suppose that $\text{Hom}(B \otimes B \otimes B, \mathbb{1})$ is one-dimensional over \mathbb{C} . What can you say about the 120 degree rotation of ϕ , vis a vis ϕ ?