

## MSRI Soergel bimodule workshop

June/July 2017

### Week 1 Day 3 Afternoon: Basic Exercises

#### *Light leaves*

1. Describe all light leaves maps from  $ss \dots s$  ( $m$  times).
2. The diagram  $ss \rightarrow s$  of degree  $+1$ , which is a horizontal reflection of the light leaf for  $01$ , is not a light leaf. Rewrite this morphism as an  $R$ -linear combination of double leaves.
3. Let  $W$  be of type  $A_7$ , and let  $\underline{w}$  be the reduced expression

$$\underline{w} = 1357246352461357.$$

a) What braid relations of the form  $sts = tst$  can be applied to  $\underline{w}$ ?

b) Show that

$$\underline{e} = 1111010110100000$$

is the unique subexpression with defect zero and terminus

$$w_I = 13435437.$$

(Note that  $w_I$  is the longest element of the parabolic subgroup for  $I = \{1, 3, 4, 5, 7\}$ .)

- c) Draw the corresponding light leaf.
- d) Take this light leaf, and precompose it with the upside-down version of itself, to obtain a morphism  $\underline{w_I} \rightarrow \underline{w} \rightarrow \underline{w_I}$ . Compute this morphism, modulo terms lower than  $w_I$ . (A lengthy calculation, this is a supplemental exercise.)

#### *Zamolodchikov*

4. Let  $S = \{s, t, u\}$  be type  $A_3$ . Let  $\underline{w} = tstuts$  and let  $\underline{y} = utstut$  be two expressions for the longest element  $w_0 \in W$ . There are (essentially) two paths from  $\underline{w}$  to  $\underline{y}$  in the reduced expression graph of  $w_0$ . Find a reasonably quick proof that the two corresponding morphisms of Bott–Samelson bimodules are not equal. (Extra Credit: find the lower terms which express the difference of these two morphisms.)
5. Let  $S = \{s, t, u\}$  be type  $B_3$ , with  $m_{st} = 3$  and  $m_{tu} = 4$ . The “miraculous” reduced expression is  $\underline{w} = stustsutu$ . Draw the Zamolodchikov relation.