Lab 6

Propositional Logic

1

Statement:

- The car is either at John’s house or at Fred’s house.
- If the car is not at John’s house, then it must be at Fred’s house

1 (a)

Set of propositional letters which can be used to represent this statement:

X: car is at John’s house

Y: car is at Fred’s house

¬X: car is not at John’s house

1 (b)

- The car is either at John’s house or at Fred’s house:
  - $X \vee Y$
  - $(X \wedge \neg Y) \vee (\neg X \wedge Y)$ (car cannot be at John’s house and at Fred’s house at the same time)
- If the car is not at John’s house, then it must be at Fred’s house
  - $\neg X \Rightarrow Y$

1 (c)

- Can we determine where the car is?

X	Y	X∧¬Y	¬X∧Y	(X∧¬Y)∨(¬X∧Y)	¬X=>Y	(X∧¬Y)∨(¬X∧Y)) ∧ (¬X => Y)
T	T	F	F	F	T	F
T	F	T	F	T	T	T
F	T	F	T	T	T	T
F	F	F	F	F	F	T

No – the car can be either at John’s or at Fred’s house bbbbbb

2. ¬*((P v ¬Q) → R) → (P ^ R)

$\neg[((P \vee \neg Q) \rightarrow R) \rightarrow (P \wedge R)] \equiv$

Implication elimination: $(a \rightarrow b) \equiv (\neg a \vee b)$

2. ¬*((P v ¬Q) → R) → (P ^ R)

¬[((P v ¬Q) → R) → (P ^ R)] ≡ Implication elimination: (a → b) ≡ (¬a v b)

¬[¬((P v ¬Q) → R) v (P ^ R)] ≡ Implication elimination: (a → b) ≡ (¬a v b)

2. $\neg{}^*((P \vee \neg Q) \rightarrow R) \rightarrow (P \wedge R)$

$\neg[((P \vee \neg Q) \rightarrow R) \rightarrow (P \wedge R)] \equiv$ Implication elimination: $(a \rightarrow b) \equiv (\neg a \vee b)$

$\neg[\neg((P \vee \neg Q) \rightarrow R) \vee (P \wedge R)] \equiv$ Implication elimination: $(a \rightarrow b) \equiv (\neg a \vee b)$

$\neg[\neg(\neg(P \vee \neg Q) \vee R) \vee (P \wedge R)] \equiv$ De Morgan: $\neg(a \vee b) \equiv (\neg a \wedge \neg b)$

2. ¬*((P v ¬Q) → R) → (P ^ R)

¬[((P v ¬Q) → R) → (P ^ R)] ≡ Implication elimination: (a → b) ≡ (¬a ^ b)

¬[¬((P v ¬Q) → R) v (P ^ R)] ≡ Implication elimination: (a → b) ≡ (¬a ^ b)

¬[¬(¬(P v ¬Q) v R) v (P ^ R)] ≡ De Morgan: ¬ (a v b) ≡ (¬a ^ ¬b)

¬[¬((¬P ^ Q) v R) v (P ^ R)] ≡ De Morgan: ¬ (a v b) ≡ (¬a ^ ¬b)

2. ¬*((P v ¬Q) → R) → (P ^ R)

¬[((P v ¬Q) → R) → (P ^ R)] ≡ Implication elimination: (a → b) ≡ (¬a ^ b)

¬[¬((P v ¬Q) → R) v (P ^ R)] ≡ Implication elimination: (a → b) ≡ (¬a ^ b)

¬[¬(¬(P v ¬Q) v R) v (P ^ R)] ≡ De Morgan: ¬ (a v b) ≡ (¬a ^ ¬b)

¬[¬((¬P ^ Q) v R) v (P ^ R)] ≡ De Morgan: ¬ (a v b) ≡ (¬a ^ ¬b)

[((¬P ^ Q) v R) ^ ¬ (P ^ R)] ≡ De Morgan: ¬ (a ^ b) ≡ (¬a v ¬b)

2. ¬*((P v ¬Q) → R) → (P ^ R)

$\neg[((P \vee \neg Q) \rightarrow R) \rightarrow (P \wedge R)] \equiv$ Implication elimination: $(a \rightarrow b) \equiv (\neg a \wedge b)$

$\neg[\neg((P \vee \neg Q) \rightarrow R) \vee (P \wedge R)] \equiv$ Implication elimination: $(a \rightarrow b) \equiv (\neg a \wedge b)$

$\neg[\neg(\neg(P \vee \neg Q) \vee R) \vee (P \wedge R)] \equiv$ De Morgan: $\neg (a \vee b) \equiv (\neg a \wedge \neg b)$

$\neg[\neg((\neg P \wedge Q) \vee R) \vee (P \wedge R)] \equiv$ De Morgan: $\neg (a \vee b) \equiv (\neg a \wedge \neg b)$

$[((\neg P \wedge Q) \vee R) \wedge \neg (P \wedge R)] \equiv$ De Morgan: $\neg (a \wedge b) \equiv (\neg a \vee \neg b)$

$[((\neg P \wedge Q) \vee R) \wedge (\neg P \vee \neg R)] \equiv$ Distributivity of v over ^: $(a \wedge b) \vee c \equiv ((a \vee c) \wedge (b \vee c))$

2. $\neg{*}((P \lor \neg Q) \rightarrow R) \rightarrow (P \land R)$

$\neg[((P \lor \neg Q) \rightarrow R) \rightarrow (P \land R)] \equiv$	Implication elimination: $(a \rightarrow b) \equiv (\neg a \land b)$
$\neg[\neg((P \lor \neg Q) \rightarrow R) \lor (P \land R)] \equiv$	Implication elimination: $(a \rightarrow b) \equiv (\neg a \land b)$
$\neg[\neg(\neg(P \lor \neg Q) \lor R) \lor (P \land R)] \equiv$	De Morgan: $\neg(a \lor b) \equiv (\neg a \land \neg b)$
$\neg[\neg((\neg P \land Q) \lor R) \lor (P \land R)] \equiv$	De Morgan: $\neg(a \lor b) \equiv (\neg a \land \neg b)$
$[((\neg P \land Q) \lor R) \land \neg(P \land R)] \equiv$	De Morgan: $\neg(a \land b) \equiv (\neg a \lor \neg b)$
$[((\neg P \land Q) \lor R) \land (\neg P \lor \neg R)] \equiv$	Distributivity of v over ^: $(a \land b) \lor c \equiv ((a \lor c) \land (b \lor c))$
$[(\neg P \lor R) \land (Q \lor R) \land (\neg P \lor \neg R)] \equiv$	
$(\neg P \lor R) \land (Q \lor R) \land (\neg P \lor \neg R)$	

3. Use resolution Algorithm to solve the following problem

Given (KB):

$B \wedge C \rightarrow A$

B

$D \wedge E \rightarrow C$

$E \vee F$

$D \wedge \neg F$

Query:

A

KB $\models$ A iff (KB ^ ¬ A) is unsatisfiable

Can we entail the query from the knowledge base

Resolution rule:

$$\frac{a \lor b,\ \neg b \lor c}{a \lor c}$$

New clause contains all the literals of two original clauses except the two complementary literals (b and ¬b)

And-Elimination:

$$\frac{a \land b}{a} \qquad \frac{a \land b}{b}$$

From a conjunction any of the conjuncts can be inferred

Use resolution Algorithm to solve the following problem

Given (KB):

$B \wedge C \rightarrow A \equiv (\neg(B \wedge C) \vee A) \equiv (\neg B \vee \neg C \vee A)$

B

$D \wedge E \rightarrow C \equiv (\neg(D \wedge E) \vee C) \equiv (\neg D \vee \neg E \vee C)$

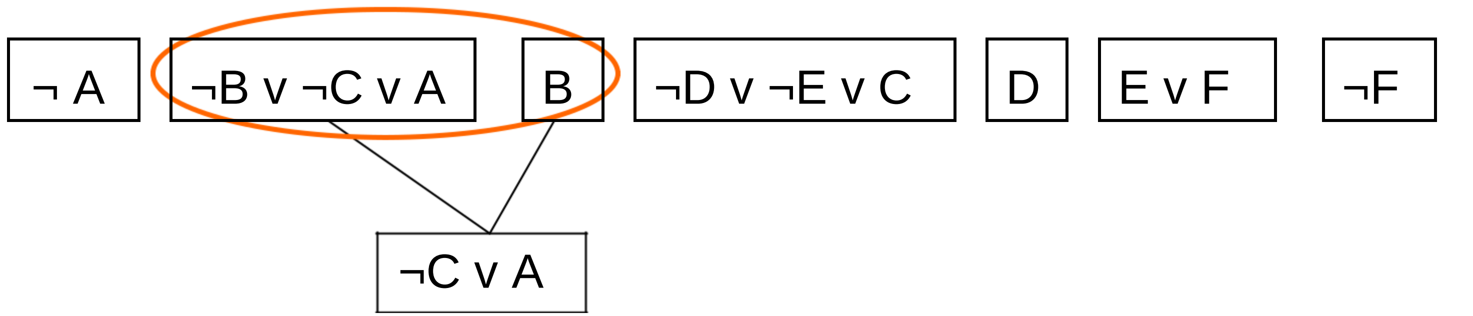
$E \vee F$

$D \wedge \neg F$

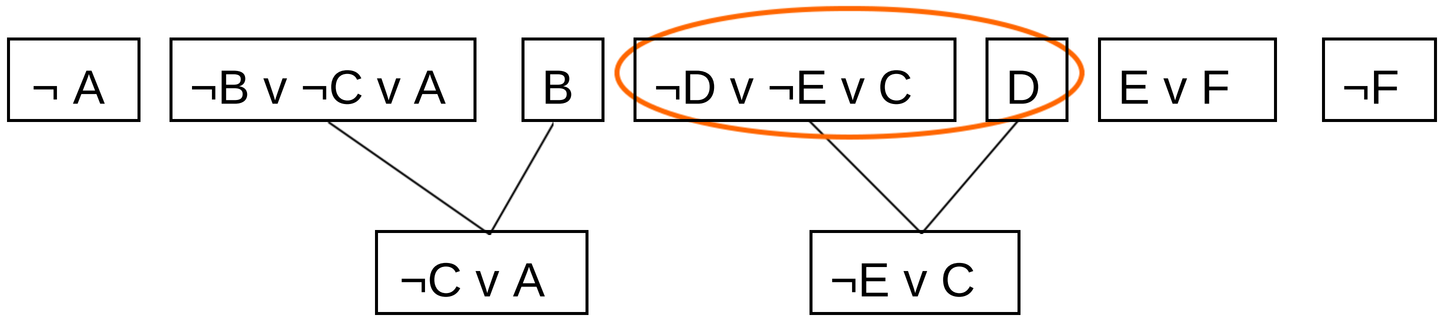
Use resolution Algorithm to solve the following problem

¬ A	¬B v ¬C v A	B	¬D v ¬E v C	D	E v F	¬F		

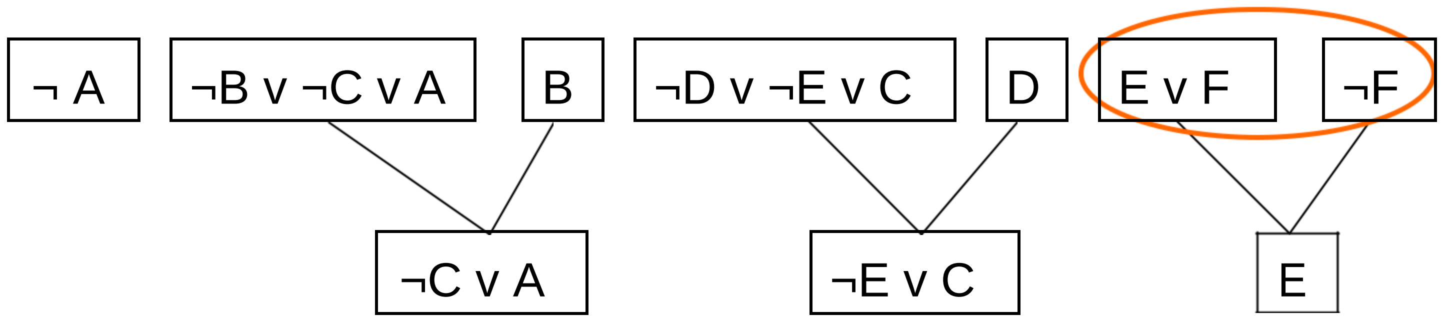
Use resolution Algorithm to solve the following problem



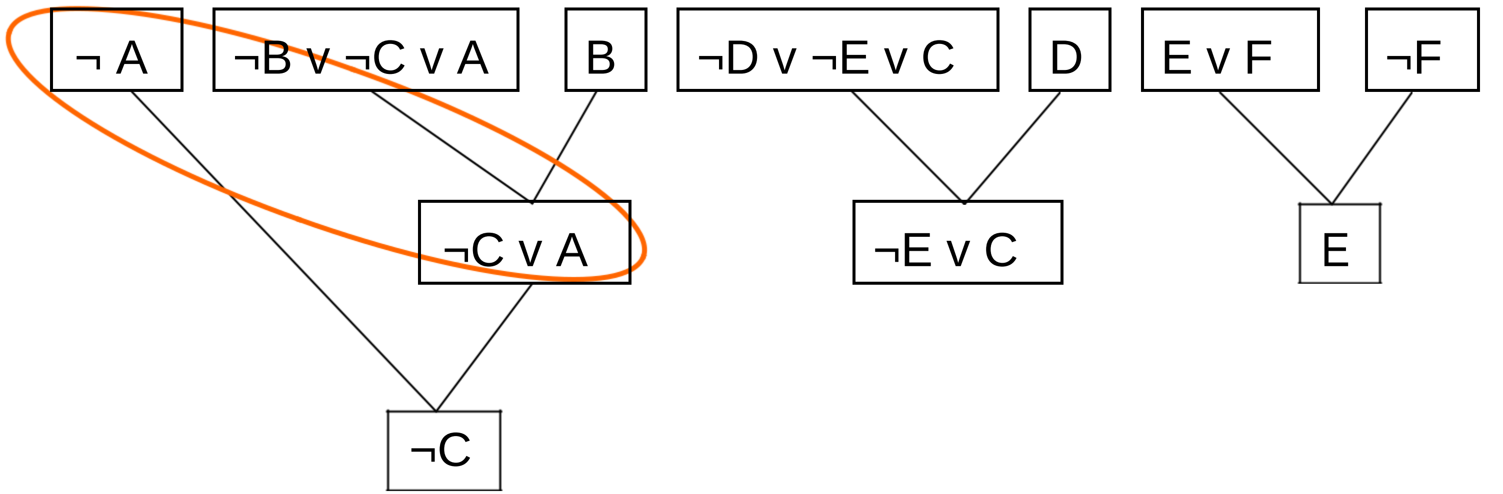
2. Use resolution Algorithm to solve the following problem



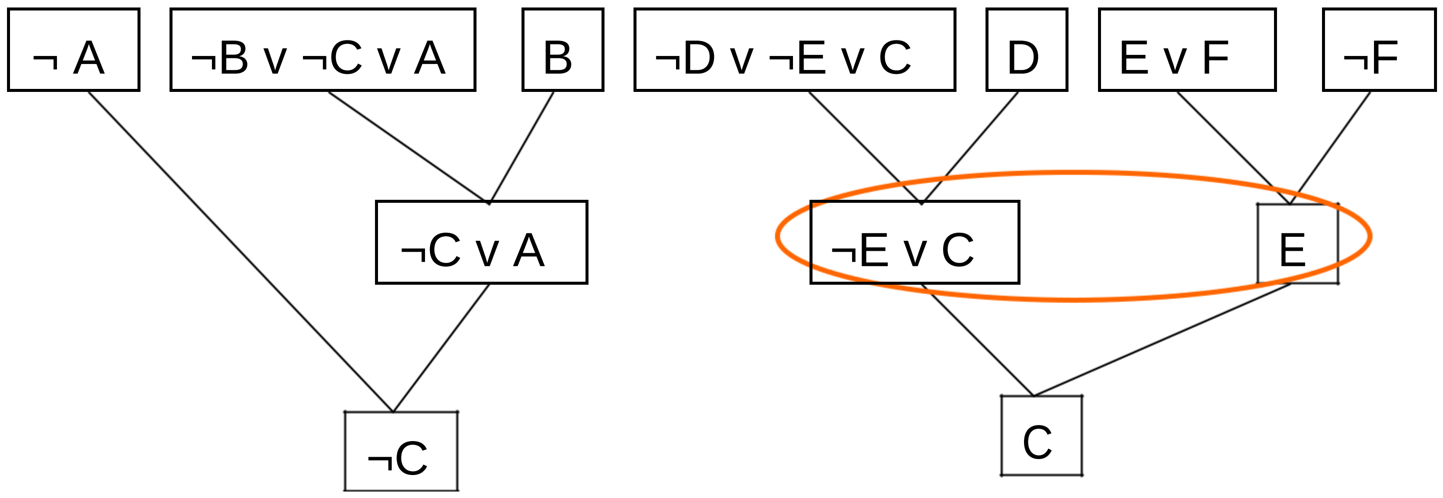
Use resolution Algorithm to solve the following problem



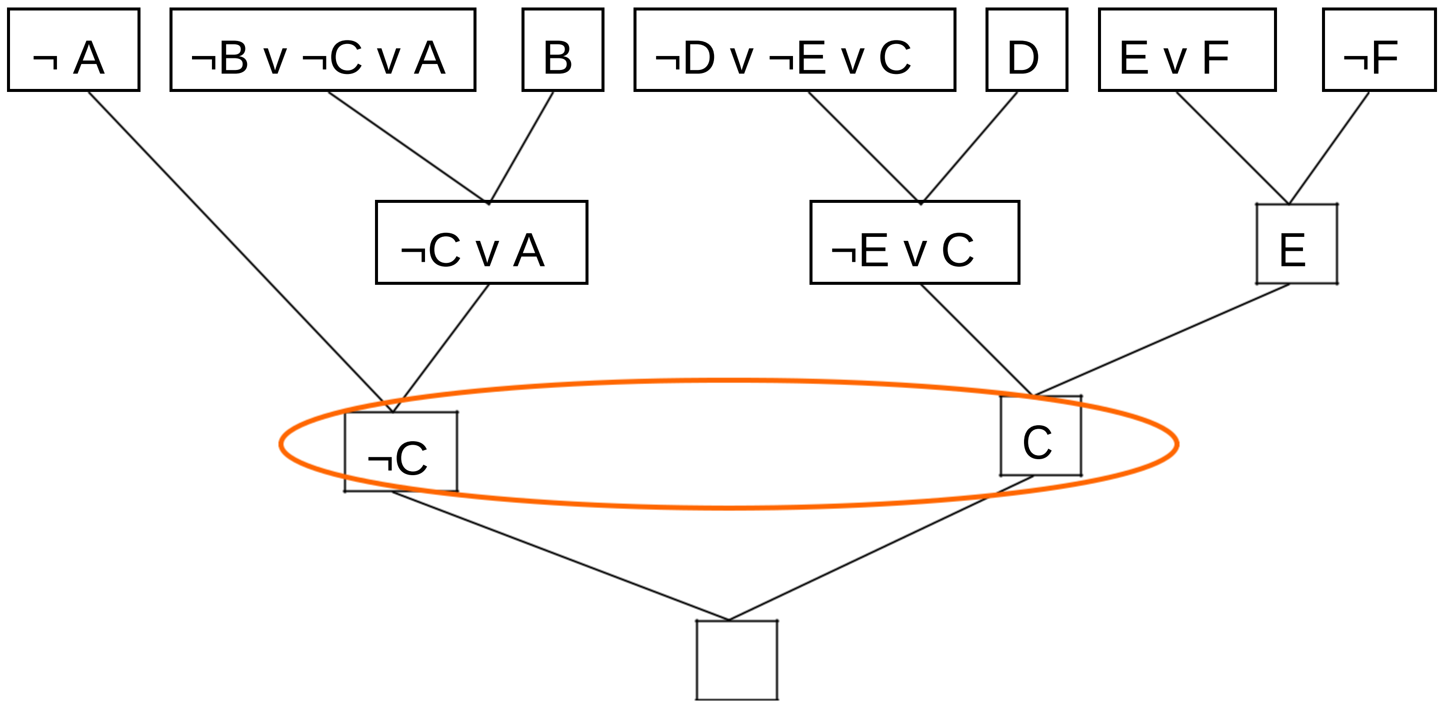
Use resolution Algorithm to solve the following problem



Use resolution Algorithm to solve the following problem



Use resolution Algorithm to solve the following problem



Thus, we can entail query A from the knowledge base KB

4.

Code filling

```
knowledge.add(
    Or(Symbol("GilderoyGryffindor"),
Symbol("GilderoyRavenclaw"))
)

knowledge.add(
    Not(Symbol("PomonaSlytherin"))
)
```

```
knowledge.add(
    Symbol("MinervaGryffindor")
)

for symbol in symbols:
    if model_check(knowledge, symbol):
        print(symbol)
```