Assignment 4

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Question 1

M

$$\begin{bmatrix} 1/3 & 1/2 & 0 \\ 1/3 & 0 & 1/2 \\ 1/3 & 1/2 & 1/2 \end{bmatrix}$$

Ex 1.1

$$\begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}, \begin{bmatrix} 5/18 \\ 5/18 \\ 8/18 \end{bmatrix}, \begin{bmatrix} 25/108 \\ 34/108 \\ 49/108 \end{bmatrix}, \begin{bmatrix} 152/648 \\ 197/648 \\ 299/648 \end{bmatrix}$$

Ex 1.2

$$\begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}, \begin{bmatrix} 13/45 \\ 13/45 \\ 19/45 \end{bmatrix}, \begin{bmatrix} 175/675 \\ 211/675 \\ 289/675 \end{bmatrix}, \begin{bmatrix} 2641/10125 \\ 3109/10125 \\ 4375/10125 \end{bmatrix}$$

Ex 1.3

$$M = \begin{bmatrix} 0 & 1/n & 1/n & 1/n & 0 \\ 1/n & 0 & 1/n & 1/n & 0 \\ 1/n & 1/n & 0 & 1/n & 0 \\ 1/n & 1/n & 1/n & 0 & 0 \\ 1/n & 1/n & 1/n & 1/n & 0 \end{bmatrix} \qquad v = \begin{bmatrix} 1/n+1 \\ 1/n+1 \\ 1/n+1 \\ 1/n+1 \\ 1/n+1 \end{bmatrix}$$

$$Eq = \beta M_v + \frac{(1-\beta)e}{n+1}$$

$$Eq = \underbrace{\begin{bmatrix} n-1/n(n+1) \\ n-1/n(n+1) \\ \vdots \\ n/n+1 \end{bmatrix}}_{Mv} \beta + (1-\beta) \begin{bmatrix} 1/n+1 \\ 1/n+1 \\ 1/n+1 \\ 1/n+1 \\ 1/n+1 \end{bmatrix}$$

$$= \underbrace{\underbrace{\begin{bmatrix} \frac{n-1}{n(n+1)} \\ \vdots \\ n/n+1 \end{bmatrix}}_{Mv} \beta - \beta \begin{bmatrix} 1/n+1 \\ \vdots \end{bmatrix}}_{\text{taking } \frac{\beta}{n+1} \text{ common.}} + 1 \begin{bmatrix} 1/n+1 \\ \vdots \end{bmatrix}$$

$\because \frac{n-1}{n} - 1$

$\frac{n-1-n}{n} = -1/n$

$\because \frac{n}{n} - 1 = 0$

$$= \frac{\beta}{n+1} \begin{bmatrix} -1/n \\ -1/n \\ -1/n \\ -1/n \\ 0 \end{bmatrix} + \begin{bmatrix} 1/n+1 \\ 1/n+1 \\ 1/n+1 \\ 1/n+1 \\ 1/n+1 \end{bmatrix}$$

$\because \frac{-\beta}{n(n+1)} + \frac{1}{n+1}$

$\frac{1}{n+1}\left[1 - \frac{\beta}{n}\right]$

$\frac{1}{n+1}\left[\frac{n-\beta}{n}\right]$

$= \frac{n-\beta}{n(n+1)}$

$$= v' \begin{bmatrix} n-\beta/n(n+1) \\ n-\beta/n(n+1) \\ n-\beta/n(n+1) \\ n-\beta/n(n+1). \\ 1/n+1 \end{bmatrix}$$

Ex 1.4

$$\circlearrowleft A \not\to \cancel{B} \not\to \cancel{C} \not\to \cancel{D}$$

After deleting recursively dead-ends and ones we are left with

$$\circlearrowleft A$$

$M = [1] \quad v = [1]$

Iterate $[1], [1], [1] \cdot \cdot \cdot \cdot [1]$ PgRank for A

(Now bring back nodes in order we last deleted).

$\circlearrowleft A \to B$

B has 1 predecessor A

A has 2 successors A, B $(1/2, 1/2)$

So B contribution $= \frac{1}{2} \cdot \underbrace{1}_{\text{A pgrank}} = 1/2$

$B = 1/2$

Now C.

$\circlearrowleft A \to B \to C$

C has 1 predecessor B

B has 1 successor C (1)

So C contrib $= 1 \cdot \underbrace{\frac{1}{2}}_{\text{B pgrank}} = 1/2$

$C = 1/2$

And so on.

$D = 1/2$

Thus our page rank vector will look like

$$\begin{bmatrix} 1 \\ 1/2 \\ 1/2 \\ \vdots \\ 1/2 \end{bmatrix}$$

Ex 1-5

After deleting all dead-ends we again get

$M = [1] \quad v = [1]$

Iterate $Mv = [1], [1], [1] \cdots [1]$

Add last deleted.

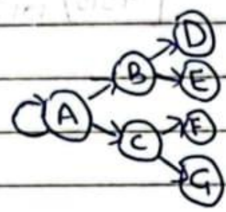
B has 1 pred A
A has 3 succ A, B, C (1/3)

$\boxed{B = 1 \cdot \frac{1}{3} = 1/3}$

C ″
A ″

$\boxed{C = 1 \cdot \frac{1}{3} = 1/3}$

Now next level



D has 1 pred B
B has 2 succ D, E (1/2)

$D = \frac{1}{3} \cdot \frac{1}{2} = 1/6$

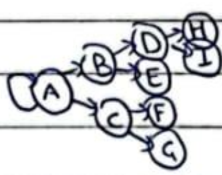
Same for E, F, G

$E = \frac{1}{3} \cdot 1/2 = 1/6$

$F = 1/6$

$G = 1/6$

Next level



H has 1 pred D
D has 2 succ H, I (1/2)

$H = \frac{1}{6} \cdot \frac{1}{2} = 1/12$

Similarly I & other on leve $= \frac{1}{6} \cdot \frac{1}{2} = 1/12$

- Through this we can see a pattern

$\left[1, \frac{1}{3} \cdot \frac{1}{2^{k-1}} \cdots \right]$ where k is the level of the tree.

$[1, 1/3, 1/3, 1/6, 1/6, 1/6, 1/6, 1/12, 1/12 \cdots]$

Question 4

4a) Prove $w(r') = w(r)$

Thus we need to show

$$\sum_{i=1}^{n} \sum_{j=1}^{n} M_{ij} r_j = \sum_{i=1}^{n} r_i$$

for no deadeneds $\sum_{i=1}^{n} M_{ij} = 1$

Thus

$$\sum_{j=1}^{n} (1) r_j = \sum_{i=1}^{n} r_i$$ ∵ Hence proved.

Prove $(w(r') = w(r))$. (under what circumstance).

4b) if $r_i' = \beta(\sum_{j=1}^{n} M_{ij} r_j) + (1-\beta)/n$

then

$$w(r') = \sum_{i=1}^{n} \beta \sum_{j=1}^{n} M_{ij} r_j + \sum_{i=1}^{\cancel{n}} \frac{(1-\beta)}{\cancel{n}}$$

for no dead-ends $\sum_{i=1}^{n} M_{ij} = 1$

thus

$$w(r') = \beta \sum_{j=1}^{n} r_j + 1 - \beta$$

$$w(r') = \beta\, w(r) + 1 - \beta$$

Assuming $w(r') = w(r)$

$$y = \beta y + 1 - \beta$$
$$y - \beta y = 1 - \beta$$
$$y(1-\beta) = 1 - \beta$$
$$y = \frac{1-\beta}{1-\beta}$$
$$y = 1$$

Thus when $w(r) = 1$ then under that circumstance $w(r') = w(r)$

∵ Hence proved

Ex 4c)

$$r' = \beta \sum_{j=1}^{n} M_{ij} r_j + (1-\beta)/n + (\beta/n) \sum_{j \in D} r_j$$

Again

$$w(r') = \sum_{i=1}^{n} \beta \sum_{j=1}^{n} M_{ij} r_j + \sum_{i=1}^{\cancel{n}} \frac{(1-\beta)}{\cancel{n}} + \sum_{i=1}^{\cancel{n}} \beta/\cancel{n} \sum_{j \in D} r_j$$

$\because \sum_{i=1}^{n} M_{ij} = 1$

$$w(r') = \beta \sum_{j=1}^{n} r_j + 1 - \beta + \beta \sum_{j \in D} r_j$$

$$w(r') = \beta \sum_{live} r_j + 1 - \beta + \beta \sum_{dead} r_j$$

$$\because \beta \sum_{Live} r_j + \beta \sum_{dead} r_j = \beta W(r)$$

Assuming $w(r) = 1$

$$w(r') = \beta w(r) + 1 - \beta$$

$$= \cancel{\beta(1)} + 1 - \cancel{\beta}$$

$w(r') = 1$ $\therefore$ Hence proved.

Question 5

Page_Rank_Algo.ipynb attached