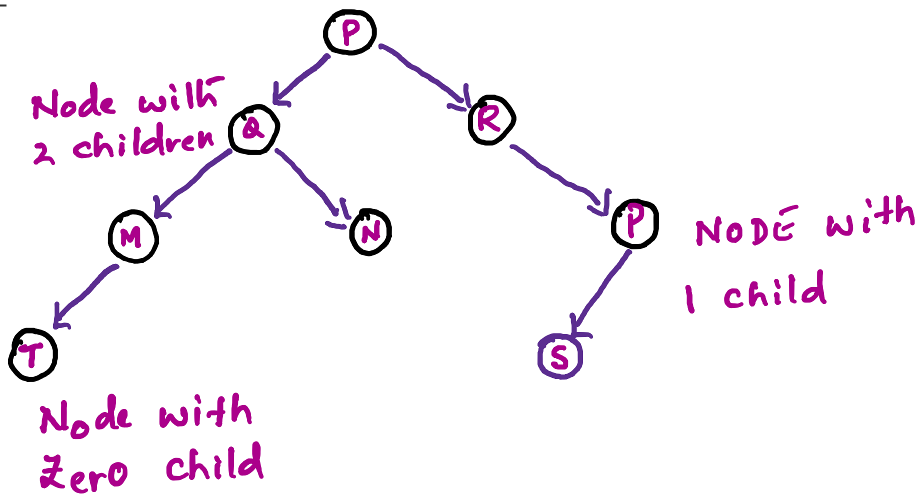
Binary Tree:

Binary tree is a special tree data structure in which each node can have at most 2 children.

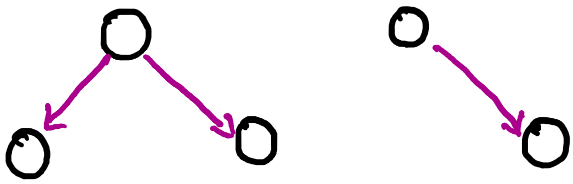
In binary tree, each node has either 0 child or 1 child or 2 children.

Example:



Unlabelled Binary Tree:

A binary tree is unlabelled if its nodes are not assigned any label



UNLABELED BINARY TREE

Number of Different Binary Trees possible with 'N' unlabeled nodes:

$$\boxed{= \frac{{}^{2n}C_{n}}{n+1}}$$

Example:

F ind nodes for a binary trees with 3 unlabeled nodes?

Using the above formula, Number of binary trees possible with 3 unlabeled nodes is

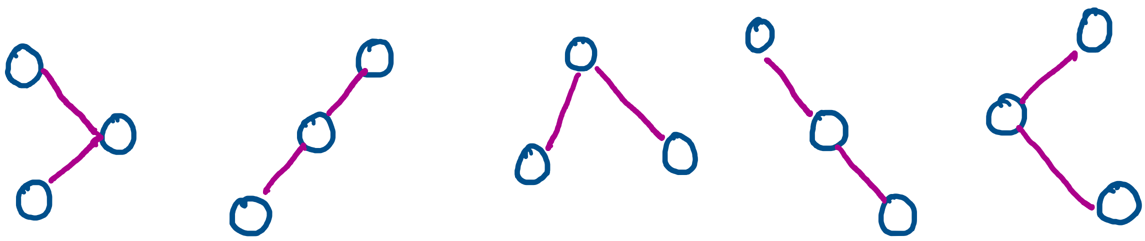
Number of binary trees $= {}^{2 \times 3}C_3 \,/\, (3 + 1)$

Number of binary trees $= {}^{6}C_3 \,/\, 4$

Number of binary trees $= 5$

Thus, With 3 unlabeled nodes, 5 unlabeled binary trees are possible.

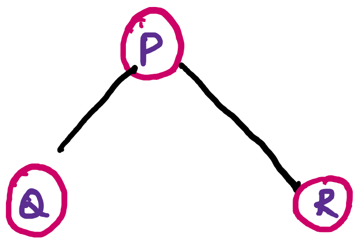
- These unlabeled binary trees are as follows-



BINARY TREES WITH 3 UNLABELED NODES

Labelled Binary Tree:

A binary tree is labelled if all its nodes are assigned a label.



NUMBER OF BINARY TREES WITH LABELED NODES

$$= \boxed{\frac{{}^{2n}C_n}{n+1} \times n!}$$

Example:

Draw all the binary trees possible with 3 labeled nodes ?

Using the above formula,

Number of binary trees = $\{ {}^{2 \times 3}C_3 / (3 + 1) \} \times 3!$

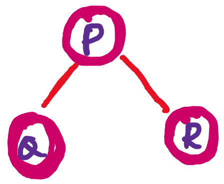
Number of binary trees = $\{ {}^{6}C_3 / 4 \} \times 6$

Number of binary trees = 5×6

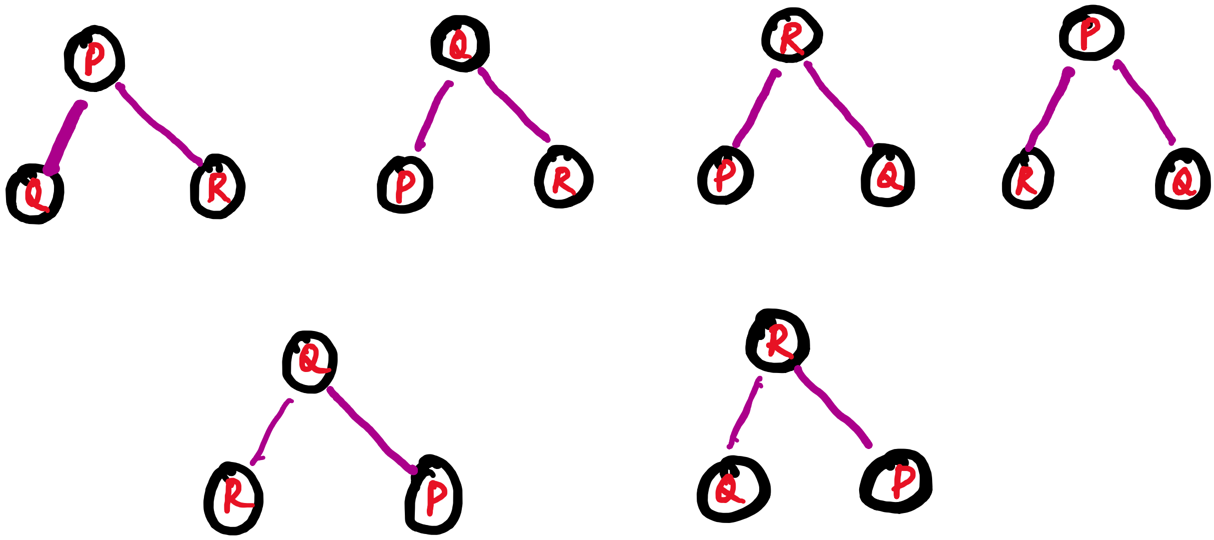
Number of binary trees = 30

with 3 labeled nodes, 30 labeled binary trees are possible.

- **Each unlabeled structure gives rise to 3! = 6 different labeled structures.**

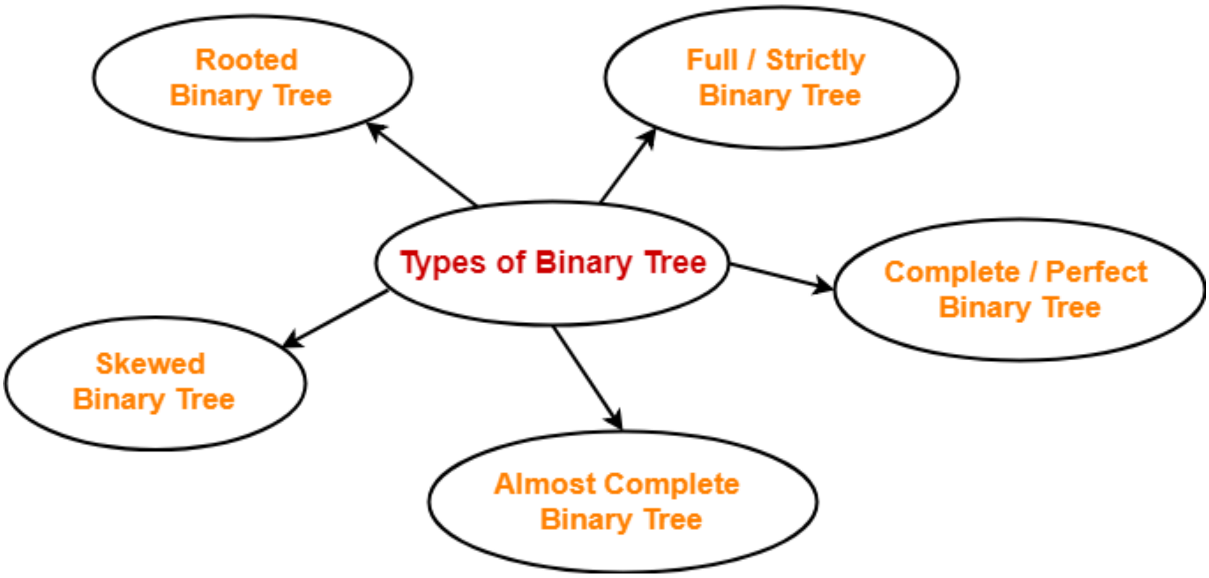


GIVES SIX LABELED STRUCTURES



- Every other unlabeled structure gives rise to 6 different labeled structures.
- Thus, in total 30 different labeled binary trees are possible.

<u>Types of Binary Trees:</u>



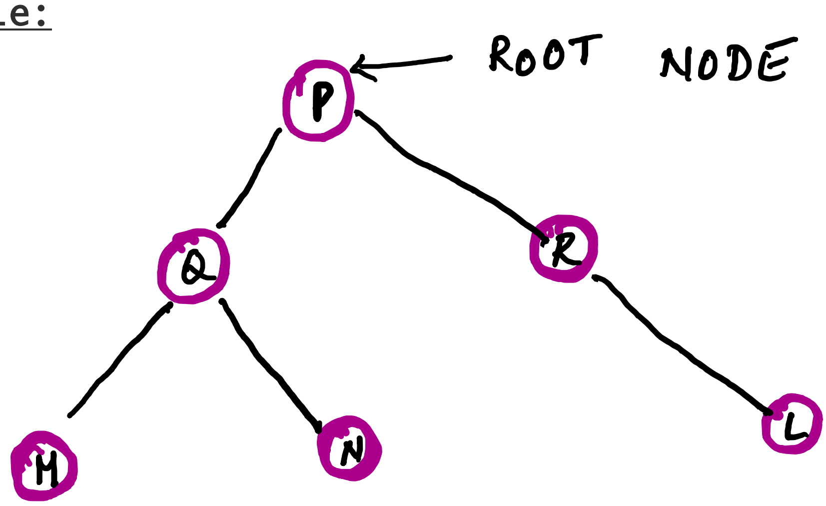
1. Rooted Binary Tree
2. Full / Strictly Binary Tree
3. Complete / Perfect Binary Tree
4. Almost Complete Binary Tree
5. Skewed Binary Tree

1. Rooted Binary Tree:

A **rooted binary tree** is a binary tree that satisfies the following 2 properties-

- It has a root node.
- Each node has at most 2 children.

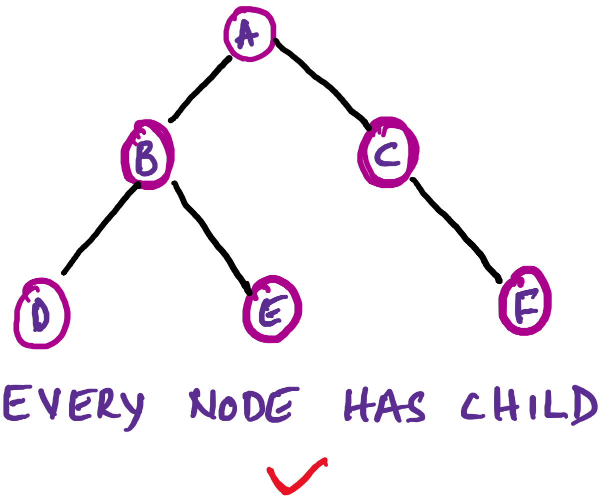
<u>Example:</u>

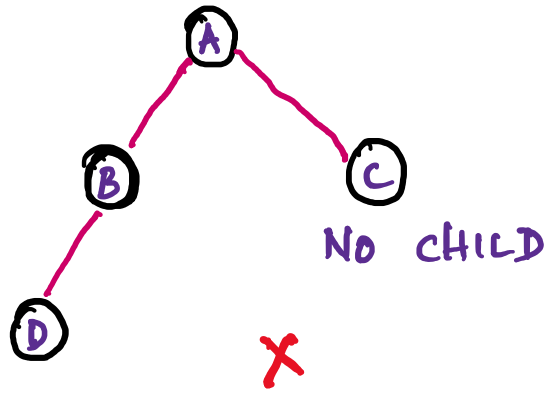


2. Full / Strictly Binary Tree:

- A binary tree in which every node has either 0 or 2 children is called as a **Full binary tree**.
- Full binary tree is also called as **Strictly binary tree**.

Example:





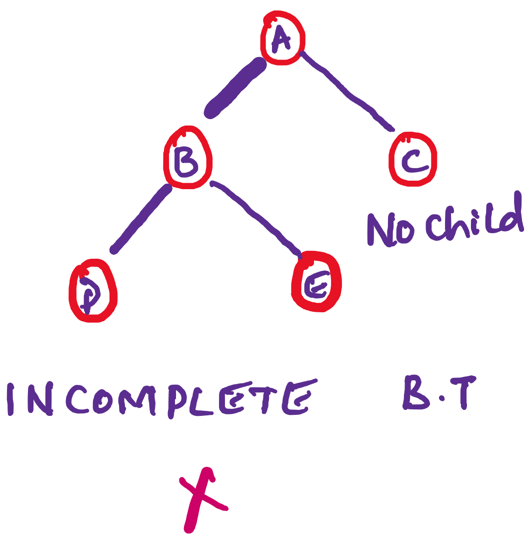
Binary tree is not a full binary tree.This is because node C has only 1 child.

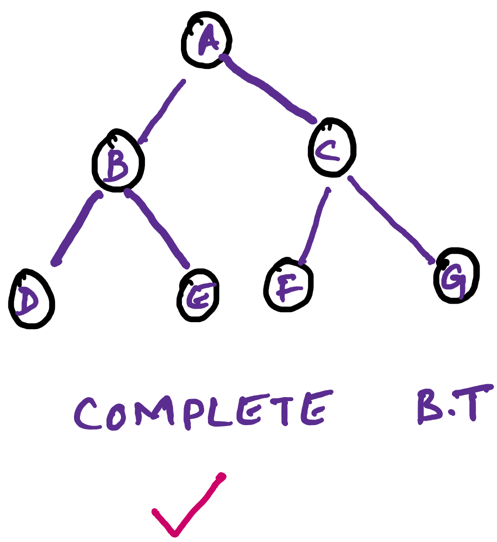
3. Complete / Perfect Binary Tree:

Complete binary tree is also called as **Perfect binary tree**.A **complete binary tree** is a binary tree has 2 properties as shown below-

- Every internal node has exactly 2 children.
- All the leaf nodes are at the same level.

Example:





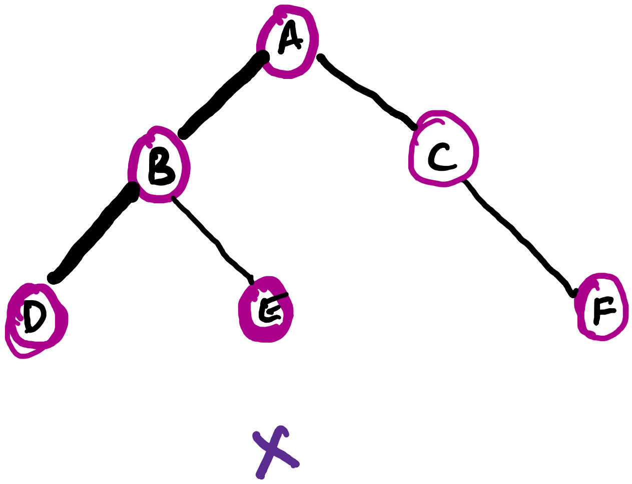
- Binary tree is not a complete binary tree.
- This is because all the leaf nodes are not at the same level.

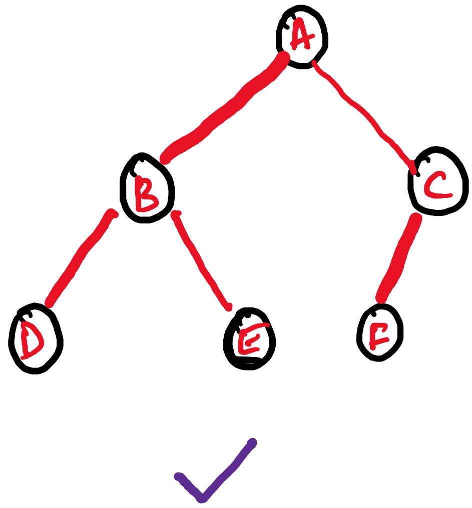
4. Almost Complete Binary Tree:

An **almost complete binary tree** is a binary tree has 2 properties-

- All the levels are completely filled except possibly the last level.
- The last level must be strictly filled from left to right.

Example:





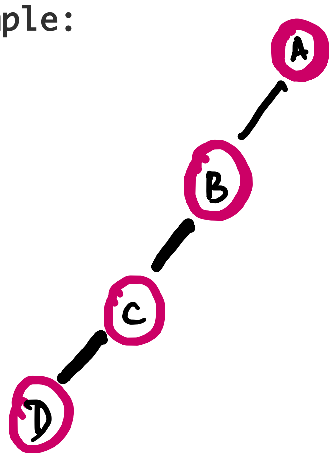
Binary tree is not an almost complete binary tree because the last level is not filled from left to right.

5. Skewed Binary Tree:

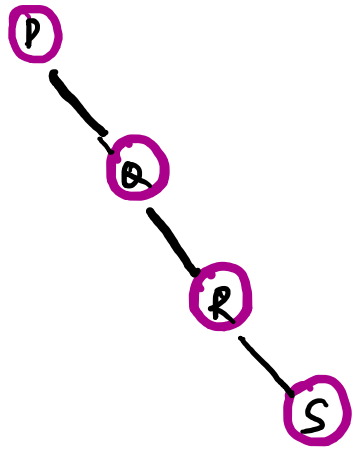
A **skewed binary tree** is a binary tree that satisfies has 2 properties-

- All the nodes except one node has one and only one child.The remaining node has no child.
- A **skewed binary tree** is a binary tree of n nodes such that its depth is (n-1).

Example:

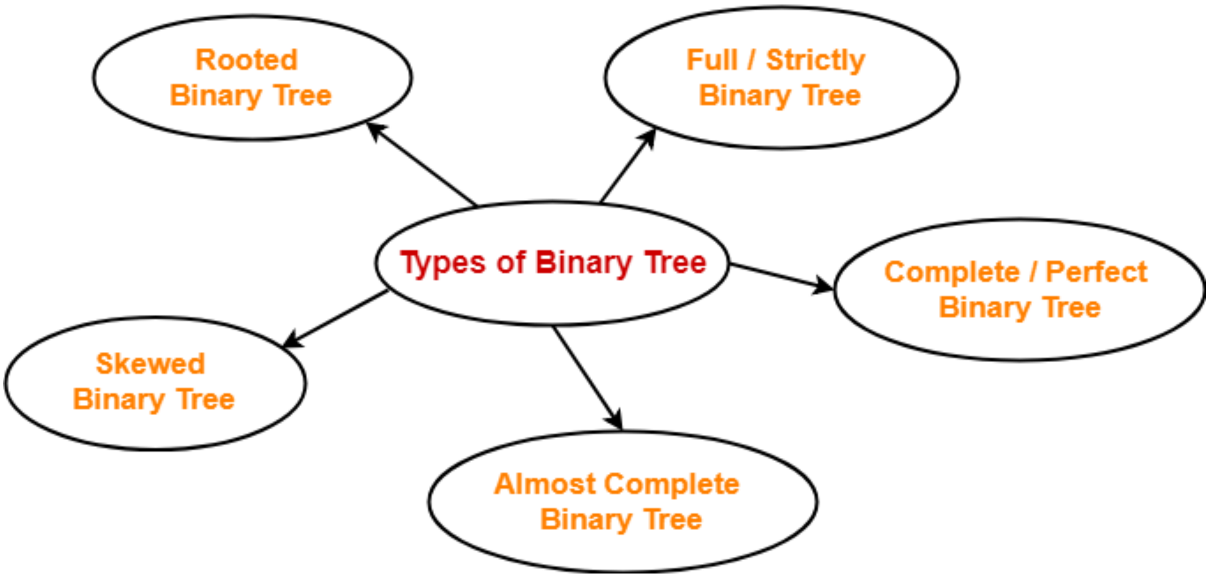


LEFT SKEWED B.T



RIGHT SKEWED B.T

Summary : Types of binary trees:

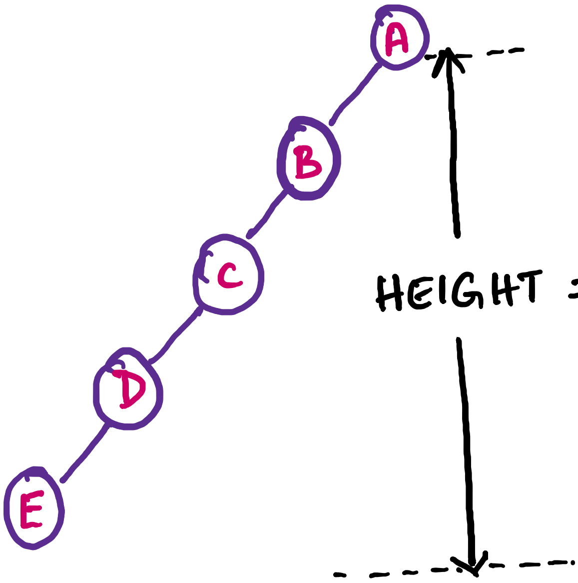


Binary Tree Properties:

Property-01: Minimum number of nodes in a binary tree of height H = H + 1

Example:

To construct a binary tree of height = 4, we need at least 4 + 1 = 5 nodes.



Property-02:

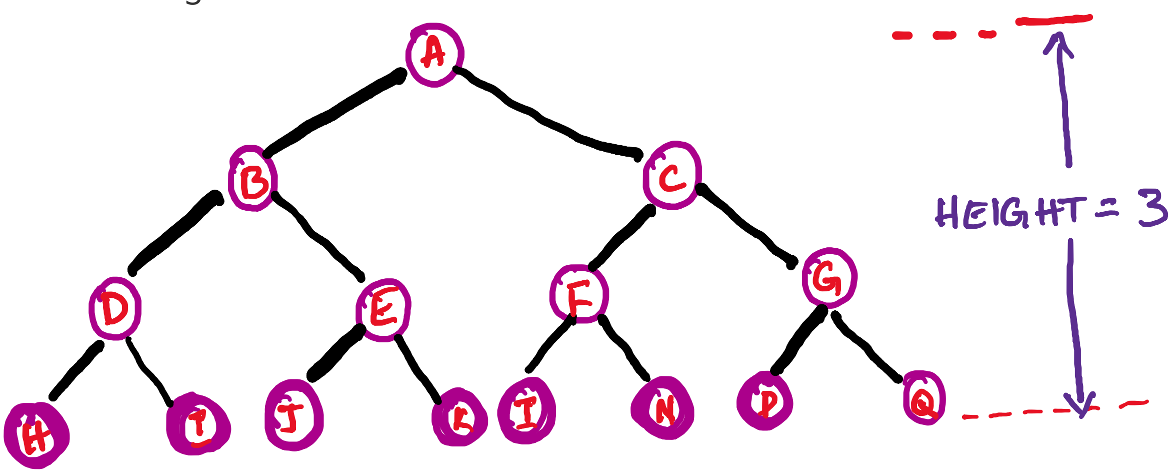
Maximum number of nodes in a binary tree of height H = $2^{H+1} - 1$

Example:

Maximum number of nodes in a binary tree of height 3

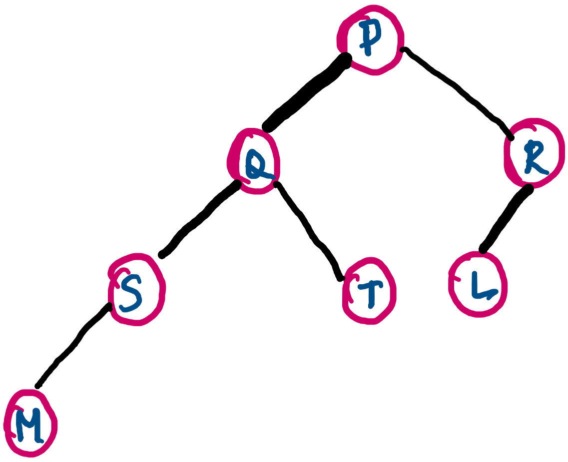
= $2^{3+1} - 1 == 16 - 1 = 15$ nodes

Thus, maximum number of nodes that can be inserted in a binary tree of height = 3 is 15.



Note :inserting more number of nodes in this binary tree.

Property-03: Total Number of leaf nodes in a Binary Tree = Total Number of nodes with 2 children + 1
Example:



Number of leaf nodes = 3

Number of nodes with 2 children = 2.

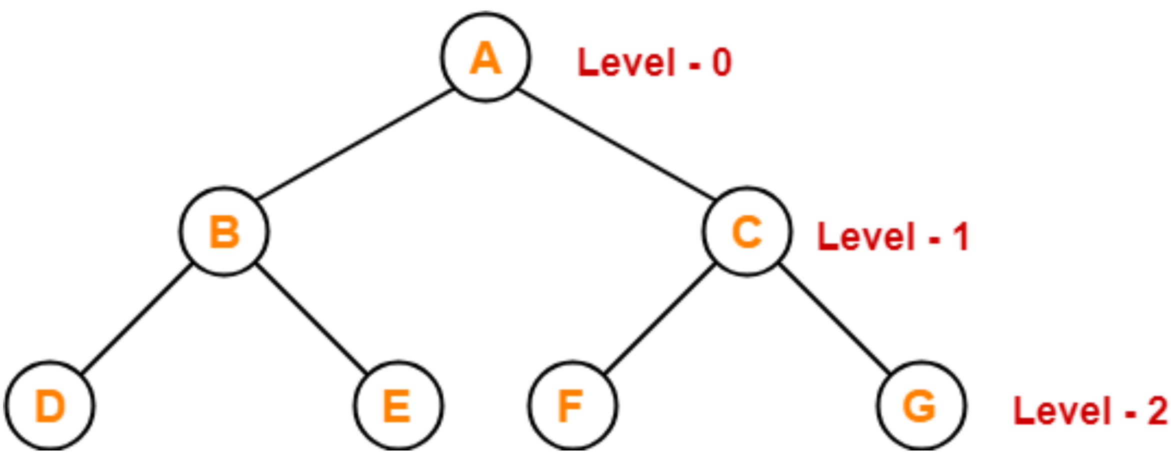
number of leaf nodes is one greater than number of nodes with 2 children in this Tree Structure..

NOTE:_Number of leaf nodes in any binary tree depends only on the number of nodes with 2 children.

Property-04:
Maximum number of nodes at any level 'L' in a binary tree = 2^L
Example:

Maximum number of nodes at level-2 in a binary tree = 2^2 = 4

In a binary tree, maximum number of nodes exists at level-2 = 4.



PRACTICE PROBLEMS :

Problem-01:

A binary tree T has n leaf nodes. The number of nodes of degree-2 in T is ______?

1. $\log_2 n$
2. **n-1. --- Correct**
3. n
4. 2^n

Solution: Using property-3,

Number of degree-2 nodes = Number of leaf nodes – 1

= n – 1

Problem-02

In a binary tree, for every node the difference between the number of nodes in the left and right subtrees is at most 2. If the height of the tree is h > 0, then the minimum number of nodes in the tree is ______?

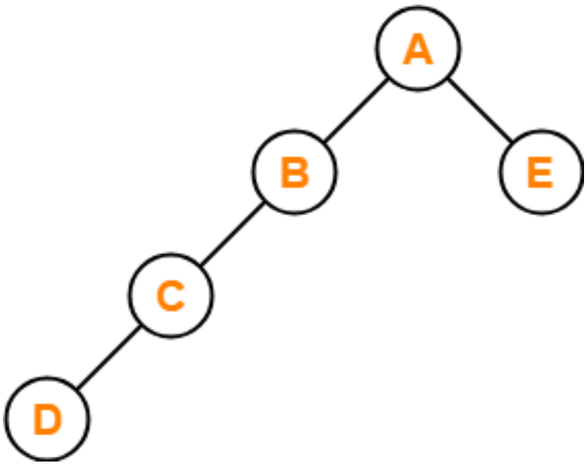
1. 2^{h-1}
2. $2^{h-1} + 1$
3. $2^h - 1$
4. 2^h

Solution:

Let us assume any random value of h. Let h = 3. Then the given options reduce to-

1. 4
2. 5 **is correct.**
3. 7
4. 8

consider the following binary tree with height h = 3-



- This binary tree satisfies the question constraints.
- It is constructed using minimum number of nodes.

Problem-03:

In a binary tree, the number of internal nodes of degree-1 is 5 and the number of internal nodes of degree-2 is 10. The number of leaf nodes in the binary tree is ______?

1. 10
2. **11** **is correct**
3. 12
4. 15

Solution:

Using property-3, Number of leaf nodes in a binary tree = Number of degree-2 nodes + 1

= 10 + 1 = 11

Problem-04:

The height of a binary tree is the maximum number of edges in any root to leaf path. The maximum number of nodes in a binary tree of height h is ______?

1. 2^h
2. $2^{h-1} - 1$
3. **$2^{h+1} - 1$.** **is correct**
4. 2^{h+1}

Solution: Use property-2.

Problem-05:

A binary tree T has 20 leaves. The number of nodes in T having 2 children is ______?

Solution:

Use property-3, correct answer is 19.