

Used when a problem has **more than one** right answer Steps to Property-Based Testing:

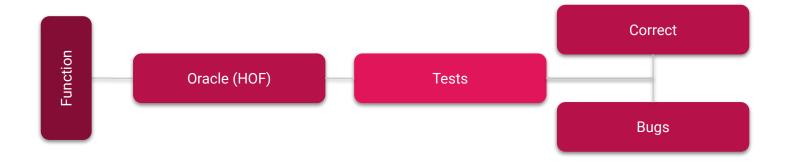
- 1. Start with a valid input to your problem
- 2. Run the algorithm on that input
- 3. Check that the result has all necessary characteristics

Property-Based Testing



Ex: Given a set of numbers $x = \{1,2,...,N-1,N\}$, generate a random subset of x. What are the properties of a correct output y to this problem?

- The number of elements in y must be less than or equal to N.
- 2. Every number in y must exist in x.
- 3. There must be no duplicates in y.



```
function oracle(subsetFunc) {
}
```

```
function oracle(subsetFunc) {
   let n = 100;
}
```

```
function oracle(subsetFunc) {
    let n = 100;
    for(let N = 0; N < n; ++N) {
    }
}</pre>
```

```
function oracle(subsetFunc) {
    let n = 100;
    for(let N = 0; N < n; ++N) {
        let x = arrayFrom1ToN(N); // x is {1,2,...,N-1,N}
    }
}</pre>
```

Oracle Functions

```
function oracle(subsetFunc) {
    let n = 100;
    for(let N = 0; N < n; ++N) {
        let x = arrayFrom1ToN(N); // x is {1,2,...,N-1,N}
        let y = subsetFunc(x);
    }
}</pre>
```

```
function oracle(subsetFunc) {
    let n = 100;
    for(let N = 0; N < n; ++N) {
        let x = arrayFrom1ToN(N); // x is {1,2,...,N-1,N}
        let y = subsetFunc(x);
        test('length of y less than or equal to N', function() {
            assert(y.length <= N);
        })
    }
}</pre>
```

```
function oracle(subsetFunc) {
    let n = 100;
    for(let N = 0; N < n; ++N) {
         let x = arrayFrom1ToN(N); // x is {1,2,...,N-1,N}
         let y = subsetFunc(x);
         test('length of y less than or equal to N', function() {
             assert(y.length <= N);</pre>
         })
         test('all elements in y are in x', function(){
             assert(y.every(d => x.includes(d));
         })
```

```
function oracle(subsetFunc) {
    let n = 100;
    for(let N = 0; N < n; ++N) {
        let x = arrayFrom1ToN(N); // x is {1,2,...,N-1,N}
        let y = subsetFunc(x);
        test('length of y less than or equal to N', function() {
             assert(v.length <= N);</pre>
         })
        test('all elements in y are in x', function(){
             assert(y.every(d => x.includes(d));
         })
        test('y does not contain duplicates', function(){
             assert(y.every(d => (y.indexOf(d) === y.lastIndexOf(d))));
         })
```

Exercise: Permutations



A function **genArray**(n: number): number[[[]] is supposed to generate an $n \times n$ array of numbers such that each row and column is a permutation of the numbers from 0 to n-1. Assume n is nonnegative.

Write an oracle that accepts the function genArray as input.

Use higher-order functions when appropriate. Try to write your implementation in $O(n^2)$.

```
function oracle(genArray) {
    let n = Math.floor(Math.random()*1000);
}
```

```
function oracle(genArray) {
    let n = Math.floor(Math.random()*1000);
    let a = genArray(n);
}
```

```
function oracle(genArray) {
    let n = Math.floor(Math.random()*1000);
    let a = genArray(n);
    const valid = x => x % 1 === 0 && 0 <= x && x < n;
}</pre>
```

Solution: Permutations

```
function oracle(genArray) {
    let n = Math.floor(Math.random()*1000);
    let a = genArray(n);
    const valid = x => x % 1 === 0 && 0 <= x && x < n;
    function isPerm(p) {
    }
}</pre>
```

Solution: Permutations

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```
function oracle(genArray) {
    let n = Math.floor(Math.random()*1000);
    let a = genArray(n);
    const valid = x => x % 1 === 0 && 0 <= x && x < n;
    function isPerm(p) {
        assert(p.length === n, 'length is n');
    }
}</pre>
```

```
function oracle(genArray) {
    let n = Math.floor(Math.random()*1000);
    let a = genArray(n);
    const valid = x => x % 1 === 0 && 0 <= x && x < n;
    function isPerm(p) {
        assert(p.length === n, 'length is n');
        assert(p.every(valid),'valid numbers');
    }
}</pre>
```

```
function oracle(genArray) {
    let n = Math.floor(Math.random()*1000);
    let a = genArray(n);
    const valid = x => x % 1 === 0 && 0 <= x && x < n;
    function isPerm(p) {
        assert(p.length === n, 'length is n');
        assert(p.every(valid),'valid numbers');
        const f = Array(n).fill(1);
        assert(p.every(e => --f[e] === 0), 'no duplicates');
}
```

Note: We're checking for duplicates in a single linear scan which takes O(n) time. This approach works for n = 0 and an empty array.

```
function oracle(genArray) {
    let n = Math.floor(Math.random()*1000);
    let a = genArray(n);
    const valid = x => x % 1 === 0 && 0 <= x && x < n;
    function isPerm(p) {
        assert(p.length === n, 'length is n');
        assert(p.every(valid),'valid numbers');
        const f = Array(n).fill(1);
        assert(p.every(e => --f[e] === 0), 'no duplicates');
    }
    assert(a.length === n, 'array has n rows');
}
```

```
function oracle(genArray) {
     let n = Math.floor(Math.random()*1000);
     let a = genArray(n);
     const valid = x => x % 1 === 0 && 0 <= x && x < n;
     function isPerm(p) {
           assert(p.length === n, 'length is n');
           assert(p.every(valid),'valid numbers');
           const f = Array(n).fill(1);
           assert(p.every(e => --f[e] === 0), 'no duplicates');
     assert(a.length === n, 'array has n rows');
     a.forEach(isPerm); // each row is a permutation
```

a is length n, and we call isPerm (an O(n) function) for each element of a. This will take O(n²) time

Solution: Permutations

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Write an oracle that accepts the function genArray as input.

```
function oracle(genArray) {
    let n = Math.floor(Math.random()*1000);
    let a = genArray(n);
    const valid = x => x % 1 === 0 && 0 <= x && x < n;
    function isPerm(p) {
        assert(p.length === n, 'length is n');
        assert(p.every(valid),'valid numbers');
        const f = Array(n).fill(1);
        assert(p.every(e => --f[e] === 0), 'no duplicates');
    }
    assert(a.length === n, 'array has n rows');
    a.forEach(isPerm); // each row is a permutation
    for (let k = 0; k < n; ++k) { isPerm(a.map(r => r[k])); } // each column is a permutation
}
```

Note: A different approach, where we check directly that every number from 0 to n-1 is included requires a linear scan for each number. That would be $O(n^2)$ per row/column and $O(n^3)$ overall. Do not do this!

With the left over time we'll go over exam questions that you all found difficult.

Suggestion: Q4 or Q6, but we can go over any you want.

Write a function Q4 trackTemp(val: number): (diff: number) => [temp: number, days: number] that takes as argument the noon temperature on day 0 and returns a closure that can be called repeatedly. A call represents a new day, with a change in the noon temperature by an amount diff (positive, negative or zero) from the day before. The closure should return a pair of numbers: the noon temperature on the current day, and the number of consecutive days (≥ 1) that have passed without the temperature change diff having switched sign (zero is not a sign switch).

What are some relevant test scenarios? Explain input, output, their purpose and what behavior they are testing.

An array represents employee wage increases in the years since their union was founded. There is one entry per year, representing the increase from the previous year as a ratio (assumed > 1). The first array entry is the increase in the year following the founding year. Write a function calclncrease(byYear: number[]): ((wage: number) => number)[] that takes such an array and returns an array of closures, with the same length. When called with a wage amount for the founding year, the closure for each year will return the adjusted wage in that year, after all increases. Use reduce, no loops. Avoid needless repeated computations.

```
type numFun = (x: number) => number

function calcIncrease(byYear: number[]): numFun[] {
  const res: numFun[] = [];
  byYear.reduce((acc, rate) => {
    acc *= rate;
    res.push(x => x * acc);
    return acc;
  }, 1);
  return res;
}
```

- Define an iterator class over an array.
- Define a class Polygon (defined by an array of points in 2D) which has a makelterator() method for its vertices.
- Extend the class to CenteredPolygon which has a method to compute its center.
- The center of weight has the average of the vertex coordinates.

```
class ArrayIterator<T> implements IterableIterator<T> {.....}

class Polygon {.....}

class CenteredPolygon extends Polygon {.....}
```

Exercise 2: Solution

```
class ArrayIterator<T> implements IterableIterator<T> {
   private array: T[];
   private index = 0;
   constructor(private array: T[]) {this.array = array}
   public next(): IteratorResult<T> {
        if (this.index < this.array.length) {
        return this.array[this.index++];
        } else {
        return null;
        }
   }
}</pre>
```

```
class CenteredPolygon extends Polygon {
   public Center(): number[] {
        const numVertices = this.vertices.length;
       let x = 0;
       let y = 0;
        const iterator = this.makeIterator();
        let current = iterator.next();
       while (current !== null) {
           x += current[0];
           y += current[1];
            current = iterator.next();
        return [x / numVertices, y / numVertices];
```

Exercise 2: Solution 2

```
interface Iterator<T> {
   next(): { value: T, done: boolean };
class Polygon {
    private vertices:[number, number][];
   constructor(vertices:[number, number][]) {
      this.vertices = vertices;
   makeIterator(): Iterator<number[]> {
      let index = 0:
      const vertices = this.vertices:
      return {
        next() {
          if (index < vertices.length) {</pre>
            return { value: vertices[index++], done: false };
          } else {
            return { value: null, done: true };
          }}};}}
```

```
class CenteredPolygon extends Polygon {
   private readonly sides: number;
   constructor(vertices: number[][]) {
     super(vertices);
     this.sides = vertices.length;
   const numVertices = this.vertices.length;
   center(): number[] {
     let x = 0;
     let v = 0;
     const iterator = this.makeIterator();
     let current = iterator.next().value;
     while (current !== null) {
       x += current[0];
       y += current[1];
        current = iterator.next().value;
     return [x / this.sides, y / this.sides];
```

```
class ArrayIterator<T> {
 private arr: T[];
 private idx: number;
 constructor(a: T[]) { this.arr = a; this.idx = -1;
 next(): T | null {
   return ++this.idx < this.arr.length ?</pre>
                     this.arr[this.idx] : null;
class Polygon {
 private vertices: [number, number][];
  constructor(vertices: [number, number][]) {
   this.vertices = vertices;
 makeIterator() {
   return new ArrayIterator(this.vertices);
```