

Used when a problem has **more than one** right answer Steps to Property-Based Testing:

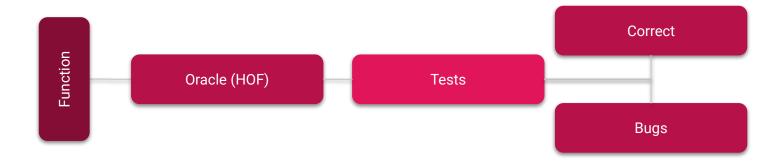
- 1. Start with a valid input to your problem
- 2. Run the algorithm on that input
- 3. Check that the result has all necessary characteristics

Property-Based Testing



Ex: Given a set of numbers $x = \{1,2,...,N-1,N\}$, generate a random subset of x. What are the properties of a correct output y to this problem?

- The number of elements in y must be less than or equal to N.
- 2. Every number in y must exist in x.
- 3. There must be no duplicates in y.



```
function oracle(subsetFunc) {
}
```

```
function oracle(subsetFunc) {
   let n = 100;
}
```

```
function oracle(subsetFunc) {
    let n = 100;
    for(let N = 0; N < n; ++N) {
    }
}</pre>
```

```
function oracle(subsetFunc) {
    let n = 100;
    for(let N = 0; N < n; ++N) {
        let x = arrayFrom1ToN(N); // x is {1,2,...,N-1,N}
    }
}</pre>
```

Oracle Functions

```
function oracle(subsetFunc) {
    let n = 100;
    for(let N = 0; N < n; ++N) {
        let x = arrayFrom1ToN(N); // x is {1,2,...,N-1,N}
        let y = subsetFunc(x);
    }
}</pre>
```

```
function oracle(subsetFunc) {
    let n = 100;
    for(let N = 0; N < n; ++N) {
        let x = arrayFrom1ToN(N); // x is {1,2,...,N-1,N}
        let y = subsetFunc(x);
        test('length of y less than or equal to N', function() {
            assert(y.length <= N);
        })
    }
}</pre>
```

```
function oracle(subsetFunc) {
    let n = 100;
    for(let N = 0; N < n; ++N) {
         let x = arrayFrom1ToN(N); // x is {1,2,...,N-1,N}
         let y = subsetFunc(x);
         test('length of y less than or equal to N', function() {
             assert(y.length <= N);</pre>
         })
         test('all elements in y are in x', function(){
             assert(y.every(d => x.includes(d));
         })
```

```
function oracle(subsetFunc) {
    let n = 100;
    for(let N = 0; N < n; ++N) {
        let x = arrayFrom1ToN(N); // x is {1,2,...,N-1,N}
        let y = subsetFunc(x);
        test('length of y less than or equal to N', function() {
             assert(v.length <= N);</pre>
         })
        test('all elements in y are in x', function(){
             assert(y.every(d => x.includes(d));
         })
        test('y does not contain duplicates', function(){
             assert(y.every(d => (y.indexOf(d) === y.lastIndexOf(d))));
         })
```

Exercise: Permutations



A function **genArray**(n: number): number[][] is supposed to generate an $n \times n$ array of numbers such that each row and column is a permutation of the numbers from 0 to n-1.

Write an oracle that accepts the function genArray as input.

Use higher-order functions when appropriate. Try to write your implementation in O(n²).

```
function oracle(genArray) {
    let n = Math.floor(Math.random()*1000);
}
```

```
function oracle(genArray) {
    let n = Math.floor(Math.random()*1000);
    let a = genArray(n);
}
```

```
function oracle(genArray) {
    let n = Math.floor(Math.random()*1000);
    let a = genArray(n);
    const valid = x => x % 1 === 0 && 0 <= x && x < n;
}</pre>
```

Solution: Permutations

```
function oracle(genArray) {
    let n = Math.floor(Math.random()*1000);
    let a = genArray(n);
    const valid = x => x % 1 === 0 && 0 <= x && x < n;
    function isPerm(p) {
    }
}</pre>
```

Solution: Permutations

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```
function oracle(genArray) {
    let n = Math.floor(Math.random()*1000);
    let a = genArray(n);
    const valid = x => x % 1 === 0 && 0 <= x && x < n;
    function isPerm(p) {
        assert(p.length === n, 'length is n');
    }
}</pre>
```

```
function oracle(genArray) {
    let n = Math.floor(Math.random()*1000);
    let a = genArray(n);
    const valid = x => x % 1 === 0 && 0 <= x && x < n;
    function isPerm(p) {
        assert(p.length === n, 'length is n');
        assert(p.every(valid),'valid numbers');
    }
}</pre>
```

```
function oracle(genArray) {
    let n = Math.floor(Math.random()*1000);
    let a = genArray(n);
    const valid = x => x % 1 === 0 && 0 <= x && x < n;
    function isPerm(p) {
        assert(p.length === n, 'length is n');
        assert(p.every(valid),'valid numbers');
        const f = Array(n).fill(1);
        assert(p.every(e => --f[e] === 0), 'no duplicates');
}
```

Note: We're checking for duplicates in a single linear scan which takes O(n) time. This approach works for n = 0 and an empty array.

```
function oracle(genArray) {
    let n = Math.floor(Math.random()*1000);
    let a = genArray(n);
    const valid = x => x % 1 === 0 && 0 <= x && x < n;
    function isPerm(p) {
        assert(p.length === n, 'length is n');
        assert(p.every(valid),'valid numbers');
        const f = Array(n).fill(1);
        assert(p.every(e => --f[e] === 0), 'no duplicates');
    }
    assert(a.length === n, 'array has n rows');
}
```

```
function oracle(genArray) {
     let n = Math.floor(Math.random()*1000);
     let a = genArray(n);
     const valid = x => x % 1 === 0 && 0 <= x && x < n;
     function isPerm(p) {
           assert(p.length === n, 'length is n');
           assert(p.every(valid),'valid numbers');
           const f = Array(n).fill(1);
           assert(p.every(e => --f[e] === 0), 'no duplicates');
     assert(a.length === n, 'array has n rows');
     a.forEach(isPerm); // each row is a permutation
```

a is length n, and we call isPerm (an O(n) function) for each element of a. This will take O(n²) time

Solution: Permutations

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Write an oracle that accepts the function genArray as input.

```
function oracle(genArray) {
    let n = Math.floor(Math.random()*1000);
    let a = genArray(n);
    const valid = x => x % 1 === 0 && 0 <= x && x < n;
    function isPerm(p) {
        assert(p.length === n, 'length is n');
        assert(p.every(valid),'valid numbers');
        const f = Array(n).fill(1);
        assert(p.every(e => --f[e] === 0), 'no duplicates');
    }
    assert(a.length === n, 'array has n rows');
    a.forEach(isPerm); // each row is a permutation
    for (let k = 0; k < n; ++k) { isPerm(a.map(r => r[k])); } // each column is a permutation
}
```

Note: A different approach, where we check directly that every number from 0 to n-1 is included requires a linear scan for each number. That would be $O(n^2)$ per row/column and $O(n^3)$ overall.

With the left over time we'll go over exam questions that you all found difficult.

Suggestion: Q3 or Q4, but we can go over any you want.

- Define an iterator class over an array.
- Define a class Polygon (defined by an array of points in 2D) which has a makelterator() method for its vertices.
- Extend the class to CenteredPolygon which has a method to compute its center.
- The center of weight has the average of the vertex coordinates.

```
class ArrayIterator<T> implements IterableIterator<T> {.....}

class Polygon {.....}

class CenteredPolygon extends Polygon {.....}
```

Exercise 2: Solution

```
class ArrayIterator<T> implements IterableIterator<T> {
   private array: T[];
   private index = 0;
   constructor(private array: T[]) {this.array = array}
   public next(): IteratorResult<T> {
        if (this.index < this.array.length) {
        return this.array[this.index++];
        } else {
        return null;
        }
   }
}</pre>
```

```
class CenteredPolygon extends Polygon {
   public Center(): number[] {
        const numVertices = this.vertices.length;
       let x = 0;
       let y = 0;
        const iterator = this.makeIterator();
        let current = iterator.next();
       while (current !== null) {
           x += current[0];
           y += current[1];
            current = iterator.next();
        return [x / numVertices, y / numVertices];
```

Exercise 2: Solution 2

```
interface Iterator<T> {
   next(): { value: T, done: boolean };
class Polygon {
    private vertices:[number, number][];
   constructor(vertices:[number, number][]) {
      this.vertices = vertices;
   makeIterator(): Iterator<number[]> {
      let index = 0:
      const vertices = this.vertices:
      return {
        next() {
          if (index < vertices.length) {</pre>
            return { value: vertices[index++], done: false };
          } else {
            return { value: null, done: true };
          }}};}}
```

```
class CenteredPolygon extends Polygon {
   private readonly sides: number;
   constructor(vertices: number[][]) {
     super(vertices);
     this.sides = vertices.length;
   const numVertices = this.vertices.length;
   center(): number[] {
     let x = 0;
     let v = 0;
     const iterator = this.makeIterator();
     let current = iterator.next().value;
     while (current !== null) {
       x += current[0];
       y += current[1];
        current = iterator.next().value;
     return [x / this.sides, y / this.sides];
```

```
class ArrayIterator<T> {
 private arr: T[];
 private idx: number;
 constructor(a: T[]) { this.arr = a; this.idx = -1;
 next(): T | null {
   return ++this.idx < this.arr.length ?</pre>
                     this.arr[this.idx] : null;
class Polygon {
 private vertices: [number, number][];
  constructor(vertices: [number, number][]) {
   this.vertices = vertices;
 makeIterator() {
   return new ArrayIterator(this.vertices);
```