

#### **Weekly Lab Agenda**

- Go over reminders/goals
- Review past material
- Work in groups of 2-3 to solve a few exercises
  - Please sit with your group from last week.
- Discussion leaders will walk around and answer questions
- Solutions to exercises will be reviewed as a class
- Attendance taken at the end

#### **Reminders**

- Midterm 1 is tonight
  - Good luck everyone!:)

#### **Today's Goals**

- Midterm Review

#### **Exam Logistics + Advice**

- 1. Location & Time: **7-9pm**, see below for location (decided by *your labs start time!*)
  - a. Bartlett 65: **11:15, 1:25**
  - b. Thompson 102: **9:05**, **12:20**
- 2. Do not prioritize doing the exam in order!
  - a. The order of the questions means nothing. Move on to the next question if you are stuck.
- 3. Do not leave any question blank!
  - Don't write garbage or throw away comments. Write down notes of what you know, that might jog your memory.
- 4. Write notes to yourself or underline important statements!
  - a. What does this function take in? What it returns? What is being graded?
- 5. Ask questions!
  - a. If something doesn't make sense, raise your hand and a proctor will come over to help out.
- 6. Use a pencil! If you don't have any, you can get them free at the library!

What are the types of g, a and x?

```
const g = (a, x) => a.map(f => (x => f(f(x)))).map(f => f(x));
```

- Look at the smallest atomic parts of this function.
- Write down what you know immediately.
- Deduce the rest using a combination of what you figured out previously.

```
const g = (a, x) \Rightarrow a.map(f \Rightarrow (x \Rightarrow f(f(x)))).map(f \Rightarrow f(x));
```

- a is an Array.
  - For now we will say its a generic array, we might later figure out what it is a: T[]
- '.map' iterates over all elements of a
  - The f's in this expression '.map(f => (x => f(f(x))))' are the elements of a
  - They are being called -> they are functions
    - They take in the same type they return
  - a: ((x: T) => T)[] x: T
- 'a.map( $f \Rightarrow (x \Rightarrow f(f(x)))$ )' is an array of functions of type '(x: T)  $\Rightarrow T$ 
  - '.map(f => f(x))' must return an array, calling the functions made in the previous expression will result in T
- Answer: g<T>(a: ((x: T) => T)[], x: T): T[]

# Higher Order Functions

#### **Higher Order Functions (HOFs)**

```
function reducer(acc, e) {
    return {
        x: acc.y + e,
        y: acc.x
    };
} const pair = a => a.reduce(reducer, {x: "", y: ""});
What is pair(["that", "is", "a", "short", "text"])?
```

```
reducer switches the value of x and y AND concatenates
                                                  the current element e onto the value of x
function reducer(acc, e) {
                                                                                     initialValue
  return {
                                          index of | ["that", "is", "a", "short", "text"] {x:"", y:""}
     x: acc.y + e,
     y: acc.x
                                               acc={x:"",y:""} ~ {x: "that", y:""}
                                               acc={x: "that", y:""} {x: "is", y: "that"}
                                                e = "is"
                                               acc={x: "is", y: "that"}
                                                                         ~ {x:"that a", y:"is"}
                                                e = "a"
                                               acc= {x: "that a", y: "is"} ~ {x: "is short", y: "that a"}
                                                e = "short"
                                         4th
                                                acc={x: "is short ", y: "that a"}
                                                 e = "text"
                                                       {x:"that a text ", y:"is short "]
```

### Mental Models + Closures

For the line defining o1 below, state how many objects except { } are created when executing that line. State what values are printed. Explain your answers.

```
function f1(k) {
1.
     let o = {};
2.
3.
   while (--k >= 0) {
      o = {val: () => k, next: o}
4.
5.
      return o;
1.
    let o1 = f1(3);
2.
    console.log(o1.val());
3.
    console.log(o1.next.val());
```

For the line defining o1 below, state how many objects except { } are created when executing that line. State what values are printed. Explain your answers.

```
1.
    function f1(k) {
2.
      let o = {};
3.
      while (--k >= 0) {
        o = {val: () => k, next: o}
4.
5.
      return o:
    let o1 = f1(3);
1.
2.
    console.log(o1.val());
    console.log(o1.next.val());
3.
```

The loop at line 3 executes three times; three objects are created and linked, next references the previous created object. The three val closures share the same environment and refer to the same variable k, they are identical.

Calling the closure val evaluates k; by this time, f1 has completed and k is -1, this value is printed.

o1.next.val is an identical closure () => k. This prints -1, the value of k.

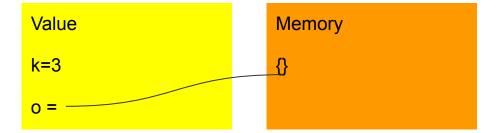
When f1 starts executing, k=3 is in the value map.

```
1.
    function f1(k) {
2.
     let o = {};
3.
   while (--k >= 0) {
      o = {val: () => k, next: o}
4.
5.
      return o;
1.
    let o1 = f1(3);
    console.log(o1.val());
2.
3.
    console.log(o1.next.val());
```

Value k=3 Memory

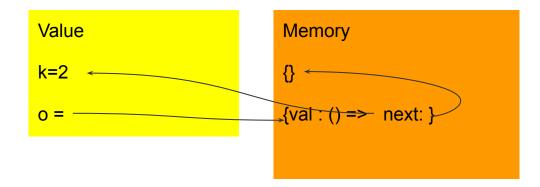
Line 2 creates an empty object is created in memory and stores its reference in o.

```
function f1(k) {
1.
2.
      let o = {};
3.
   while (--k >= 0) {
4.
      o = {val: () => k, next: o}
5.
      return o;
1.
    let o1 = f1(3);
    console.log(o1.val());
2.
3.
    console.log(o1.next.val());
```



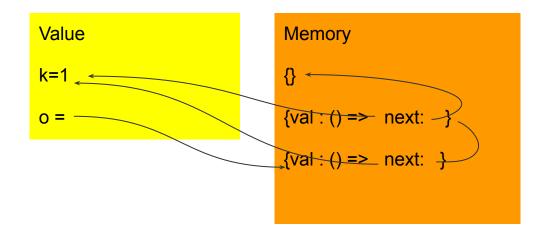
First iteration of the loop decreases value of k by 1 and creates a new object, next field of the newly created object copies reference in stored in o and o stores reference to the newly created object.

```
function f1(k) {
1.
      let o = {};
3.
      while (--k >= 0) {
        o = {val: () => k, next: o}
4.
5.
      return o;
    let o1 = f1(3);
1.
    console.log(o1.val());
2.
3.
    console.log(o1.next.val());
```



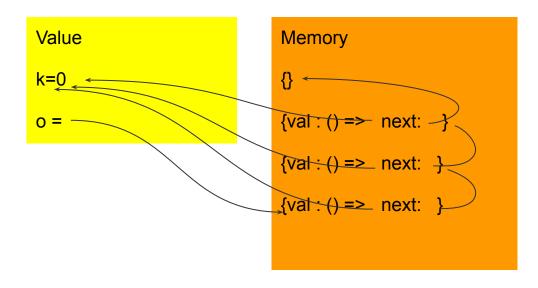
Second iteration of the loop another object is created

```
function f1(k) {
      let o = {};
3.
   while (--k >= 0) {
     o = {val: () => k, next: o}
4.
5.
      return o;
1.
    let o1 = f1(3);
    console.log(o1.val());
2.
3.
    console.log(o1.next.val());
```



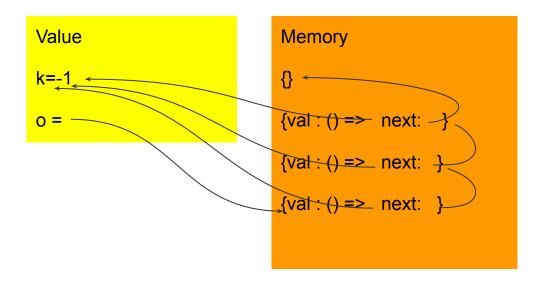
Third iteration of the loop. One more object created

```
function f1(k) {
      let o = {};
3.
   while (--k >= 0) {
      o = {val: () => k, next: o}
4.
5.
      return o;
1.
    let o1 = f1(3);
    console.log(o1.val());
3.
    console.log(o1.next.val());
```



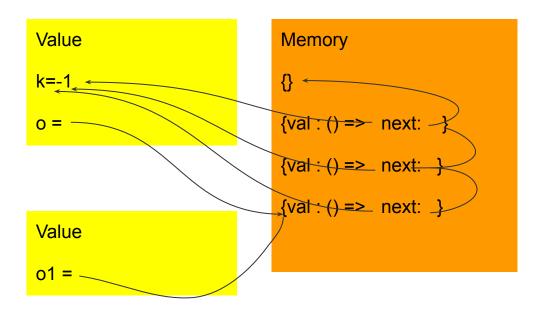
While condition is evaluated, which decreases k by 1. Body of the loop isn't executed.

```
function f1(k) {
      let o = {};
3.
   while (--k >= 0) {
      o = {val: () => k, next: o}
4.
5.
      return o;
1.
    let o1 = f1(3);
2.
    console.log(o1.val());
3.
    console.log(o1.next.val());
```



Upon return, o1 will have the reference value returned through o.

```
function f1(k) {
      let o = {};
3.
   while (--k >= 0) {
      o = {val: () => k, next: o}
4.
5.
      return o;
1.
    let o1 = f1(3);
    console.log(o1.val());
3.
    console.log(o1.next.val());
```



Write a function sumEveryK(lst: List<number>, k: number): List<number> that splits lst into groups of k adjacent elements (the last group may have fewer) and returns a list with the sum of each group (in the same order). Assume k is a positive natural. Try not to create other temporary lists.

#### **Example:**

For lst = 1, 0, 3, 5, 2, 3, 7 with k = 3

Return of sumEveryK(lst, k) should be list = 4, 10, 7

```
Iterative Solution:
function sumEveryK(lst: List<number>, k: number): List<number> {
    if (lst.isEmpty()) return lst;
    let sum = 0;
    for (let rem = k; --rem >= 0; ) {
        sum += lst.head();
        if ((lst = lst.tail()).isEmpty()) break;
    return node(sum, sumEveryK(lst, k));
```

#### **Recursive Solution:**

```
function sumEveryK(lst: List<number>, k: number): List<number> {
// rem: count remaining to make group of k, acc: accumulated sum
    return function sumRec(lst: List<number>, rem: number, acc: number) {
        if (lst.isEmpty()) return rem === k ? lst : node(acc, lst); // add last sum
        const newS = acc + lst.head(), tl = lst.tail(); // set up recursion
        return --rem === 0 ? node(newS, sumRec(tl, k, 0)) :
                                            sumRec(tl, rem, newS);
    } (lst, k, 0); \\ return sumRec(lst, k, 0);
```

# Testing

#### **Testing**

The findIndex function returns the index of the first element of array a for which f returns true, or -1 if no such element exists

```
// findIndex<T>(a: T[], f: T => boolean): number
function findIndex(a, f) {
    for (let i = 0; i < a.length; ++i) {
        if (f(a[i])) { return i; }
    }
    return -1;
}</pre>
```

Write three representative cases that you would use to test findIndex.
You need not write code, but clearly indicate inputs, output, and the purpose of the test

#### **Testing**

```
// findIndex<T>(a: T[], f: T => boolean): number
function findIndex(a, f) {
    for (let i = 0; i < a.length; ++i) {
        if (f(a[i])) { return i; }
    }
    return -1;
}</pre>
```

#### Three example tests:

```
a: [ ], f: x => x > 0. Output: -1. No element is found for an empty array.
```

a: [4, 3, 2, 5], f: x => x % 2 > 0. Output: 1. Test first index is returned for multiple matches.

a: ['hi', 'ho'], f: s => s.length > 2. Output: -1. Test case when no element matches.

### GOOD LUCK!!!



```
const g = (a, x) => a.map(f => (x => f(f(x)))).map(f => f(x));
                                                               map : ((T \Rightarrow T)[], (T \Rightarrow T) \Rightarrow T[]
                                                                                                        h \Rightarrow h(x)
                                 map: ((T \Rightarrow T)[], (T \Rightarrow T) \Rightarrow (T \Rightarrow T))
                                          => (T => T)[]
                                                                                                        : (T \Rightarrow T) \Rightarrow T
                                                             f : (T \Rightarrow T)
                                        f \Rightarrow (t: T \Rightarrow f(f(t)))
      a : A[]
                                       : A \Rightarrow (T \Rightarrow ?)
         : (T \Rightarrow T)[] \qquad : (T \Rightarrow T) \Rightarrow (T \Rightarrow T)
```

#### **More Higher Order Functions**

Write a function f: number[][]=>boolean that takes a 2D array of numbers and returns true if and only if every row contains at least one even number. Do not use loops or recursion.

let array = [[1,2,3], [4,5,6], [7,8], [9, 10]] //f(array)  $\rightarrow$  true let array2 = [[1,3], [4,5,6], [7,8], [9]] //f(array2)  $\rightarrow$  false

#### **More Higher Order Functions**

```
Write a function f: number[][]=>boolean that takes a 2D array of numbers and returns true if and only if every row contains at least one even number. Do not use loops or recursion. let array = [[1,2,3], [4,5,6], [7,8], [9, 10]] //f(array2) \rightarrow true let array2 = [[1,3], [4,5,6], [7,8], [9, ]] //f(array2) \rightarrow false
```

```
const f = (a) => a.every((row) => row.some((num) => num % 2 === 0))
const f2 = (a) => a.reduce((acc, e) => acc && e.some((num) => num % 2 === 0), true)
```

Consider the following code fragment working with lists as defined in class.

How many list nodes (created with node()) are no longer accessible at the end of this code fragment?

```
let lst1 = ... // create a list with 2 elements
const concat =
  (l1, l2) => l1.isEmpty()? l2 : node(l1.head(), concat(l1.tail(), l2));
lst1 = concat(concat(lst1, lst1), lst1)
// end of the code fragment
```

#### Hints:

node constructor creates an object of type List<T> and returns a reference to it.

Every call to the node constructor or to empty() creates a new object in memory.

At the end of the code, we have one variables in value map: lst1.

Objects that are not accessible through lst1 are no longer accessible.

Take a List<number> for example. Line 1 has two calls to the node constructor and one call to empty. This creates three objects of type List<number> in memory.

```
let lst1 = node(1, node(2, empty()));
const concat =
   (11, 12) => l1.isEmpty() ? 12 : node(l1.head(), concat(l1.tail(), 12));
lst1 = concat(concat(lst1, lst1), lst1);
```

```
Value Map

Ist1 =
```

```
Memory
{head: () => { throw new Error()}, tail: () => { throw new Error()}}
{head: () => 2; tail: () => }
{head: () => 1; tail: () => }
```

Line 2 is a function definition, memory remains the same.

```
let lst1 = node(1, node(2, empty());
const concat =
   (l1, l2) => l1.isEmpty() ? l2 : node(l1.head(), concat(l1.tail(), l2));
lst1 = concat(concat(lst1, lst1), lst1);
```

```
Value Map
```

```
Memory
{head: () => { throw new Error()}, tail: () => { throw new Error()}}
{head: () => 2; tail: () => }
{head: () => 1; tail: () => }
```

Line 2 is a function definition, memory remains the same.

```
let lst1 = node(1, node(2, empty());
const concat =
   (l1, l2) => l1.isEmpty() ? l2 : node(l1.head(), concat(l1.tail(), l2));
lst1 = concat(concat(lst1, lst1), lst1);
```

```
Value Map
{head: () => { throw new Error()}, tail: () => { throw new Error()}}
{head: () => 2; tail: () => }
{head: () => 1; tail: () => }
```

Line 3 has two function calls to concat which need to be evaluated to determine the reference that gets assigned to lst1.

```
let lst1 = node(1, node(2, empty());
const concat =
   (l1, l2) => l1.isEmpty() ? l2 : node(l1.head(), concat(l1.tail(), l2));
lst1 = concat(concat(lst1, lst1), lst1);
```

```
Value Map
{head: () => { throw new Error()}, tail: () => { throw new Error()}}
{head: () => 2; tail: () => }
{head: () => 1; tail: () => }
```

Inner call to concat calls the node constructor twice. Two more objects are created, after which l1.isEmpty() evaluates to true. Second object created references the old lst1 through the tail() call.

All objects are accessible through 1st1. Note that 1st1 hasn't changed.

```
let lst1 = node(1, node(2, empty());
const concat =
   (l1, l2) => l1.isEmpty() ? l2 : node(l1.head(), concat(l1.tail(), l2));
lst1 = concat(concat(lst1, lst1), lst1);
```

```
Value Map
{head: () => { throw new Error()}, tail: () => { throw new Error()}}
{head: () => 2; tail: () => }
{head: () => 2; tail: () => }
{head: () => 2; tail: () => }
{head: () => 1; tail: () => }

Copy of Ist1
```

Outer call to concat creates 4 more objects because the first argument concat(lst1, lst1) is a 4-element list. The tail to this 4 element list is set to the reference stored in lst1.

```
let lst1 = node(1, node(2, empty());
    const concat =
       (11, 12) => 11.isEmpty() ? 12 : node(l1.head(), concat(l1.tail(), l2));
    lst1 = concat(concat(lst1, lst1), lst1);
                  Memory
Value Map
                  {head: () => { throw new Error()}, tail: () => { throw new Error()}}
                  \{\text{head: () => 2 ; tail: () =>} \}
lst1 =
                  {head: () => 1; tail: () => }_
                                                                                     concat(lst1, lst1)
                  {head: () => 2; tail: () => }_
                  {head: () => 1; tail: () => }
                  {head: () => 2; tail: () => '
                  {head: () => 1; tail: () => }
                                                                copy of concat(lst1, lst1)
                  {head: () => 2; tail: () => }
                  {head: () => 1; tail: () => }___
```

After right hand side of the assignment is evaluated, 1st1 is updated. Two list nodes are no longer accessible.

```
let lst1 = node(1, node(2, empty());
    const concat =
       (11, 12) => 11.isEmpty() ? 12 : node(l1.head(), concat(l1.tail(), l2));
    lst1 = concat(concat(lst1, lst1), lst1);
                  Memory
Value Map
                  {head: () => { throw new Error()}, tail: () => { throw new Error()}}
                  \{\text{head: () => 2 ; tail: () =>} \}
Ist1 =
                  {head: () => 1; tail: () => }_
                  \{\text{head: () => 2 ; tail: () =>}\}
                                                  These two objects aren't reachable through lst1.
                  {head: () => 1; tail: () =>-}
                  {head: () => 2; tail: () => }
                  {head: () => 1; tail: () => }
                                                                copy of concat(lst1, lst1)
                  {head: () => 2; tail: () => }
                  -{head: () => 1 ; tail: () => }___
```

After right hand side of the assignment is evaluated, 1st1 is updated. Two list nodes are no longer accessible.

```
let lst1 = node(1, node(2, empty());
const concat =
   (l1, l2) => l1.isEmpty() ? l2 : node(l1.head(), concat(l1.tail(), l2));
lst1 = concat(concat(lst1, lst1), lst1);
```

