

Used when a problem has **more than one** right answer Steps to Property-Based Testing:

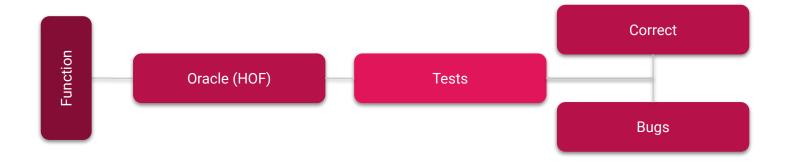
- 1. Start with a valid input to your problem
- 2. Run the algorithm on that input
- 3. Check that the result has all necessary characteristics

Property-Based Testing



Ex: Given a set of numbers $x = \{1,2,...,N-1,N\}$, generate a random subset of x. What are the properties of a correct output y to this problem?

- The number of elements in y must be less than or equal to N.
- 2. Every number in y must exist in x.
- 3. There must be no duplicates in y.



```
function oracle(subsetFunc) {
}
```

```
function oracle(subsetFunc) {
   let n = 100;
}
```

```
function oracle(subsetFunc) {
    let n = 100;
    for(let N = 0; N < n; ++N) {
    }
}</pre>
```

```
function oracle(subsetFunc) {
    let n = 100;
    for(let N = 0; N < n; ++N) {
        let x = arrayFrom1ToN(N); // x is {1,2,...,N-1,N}
    }
}</pre>
```

Oracle Functions

```
function oracle(subsetFunc) {
    let n = 100;
    for(let N = 0; N < n; ++N) {
        let x = arrayFrom1ToN(N); // x is {1,2,...,N-1,N}
        let y = subsetFunc(x);
    }
}</pre>
```

```
function oracle(subsetFunc) {
    let n = 100;
    for(let N = 0; N < n; ++N) {
        let x = arrayFrom1ToN(N); // x is {1,2,...,N-1,N}
        let y = subsetFunc(x);
        test('length of y less than or equal to N', function() {
            assert(y.length <= N);
        })
    }
}</pre>
```

```
function oracle(subsetFunc) {
    let n = 100;
    for(let N = 0; N < n; ++N) {
         let x = arrayFrom1ToN(N); // x is {1,2,...,N-1,N}
         let y = subsetFunc(x);
         test('length of y less than or equal to N', function() {
             assert(y.length <= N);</pre>
         })
         test('all elements in y are in x', function(){
             assert(y.every(d => x.includes(d));
         })
```

```
function oracle(subsetFunc) {
    let n = 100;
    for(let N = 0; N < n; ++N) {
        let x = arrayFrom1ToN(N); // x is {1,2,...,N-1,N}
        let y = subsetFunc(x);
        test('length of y less than or equal to N', function() {
             assert(v.length <= N);</pre>
         })
        test('all elements in y are in x', function(){
             assert(y.every(d => x.includes(d));
         })
        test('y does not contain duplicates', function(){
             assert(y.every(d => (y.indexOf(d) === y.lastIndexOf(d))));
         })
```

Exercise: Permutations



A function **genArray**(n: number): number[[[]] is supposed to generate an $n \times n$ array of numbers such that each row and column is a permutation of the numbers from 0 to n-1. Assume n is nonnegative.

Write an oracle that accepts the function genArray as input.

Use higher-order functions when appropriate. Try to write your implementation in $O(n^2)$.

```
function oracle(genArray) {
    let n = Math.floor(Math.random()*1000);
}
```

```
function oracle(genArray) {
    let n = Math.floor(Math.random()*1000);
    let a = genArray(n);
}
```

```
function oracle(genArray) {
    let n = Math.floor(Math.random()*1000);
    let a = genArray(n);
    const valid = x => x % 1 === 0 && 0 <= x && x < n;
}</pre>
```

Solution: Permutations

```
function oracle(genArray) {
    let n = Math.floor(Math.random()*1000);
    let a = genArray(n);
    const valid = x => x % 1 === 0 && 0 <= x && x < n;
    function isPerm(p) {
    }
}</pre>
```

Solution: Permutations

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```
function oracle(genArray) {
    let n = Math.floor(Math.random()*1000);
    let a = genArray(n);
    const valid = x => x % 1 === 0 && 0 <= x && x < n;
    function isPerm(p) {
        assert(p.length === n, 'length is n');
    }
}</pre>
```

```
function oracle(genArray) {
    let n = Math.floor(Math.random()*1000);
    let a = genArray(n);
    const valid = x => x % 1 === 0 && 0 <= x && x < n;
    function isPerm(p) {
        assert(p.length === n, 'length is n');
        assert(p.every(valid),'valid numbers');
    }
}</pre>
```

```
function oracle(genArray) {
    let n = Math.floor(Math.random()*1000);
    let a = genArray(n);
    const valid = x => x % 1 === 0 && 0 <= x && x < n;
    function isPerm(p) {
        assert(p.length === n, 'length is n');
        assert(p.every(valid),'valid numbers');
        const f = Array(n).fill(1);
        assert(p.every(e => --f[e] === 0), 'no duplicates');
}
```

Note: We're checking for duplicates in a single linear scan which takes O(n) time. This approach works for n = 0 and an empty array.

```
function oracle(genArray) {
    let n = Math.floor(Math.random()*1000);
    let a = genArray(n);
    const valid = x => x % 1 === 0 && 0 <= x && x < n;
    function isPerm(p) {
        assert(p.length === n, 'length is n');
        assert(p.every(valid),'valid numbers');
        const f = Array(n).fill(1);
        assert(p.every(e => --f[e] === 0), 'no duplicates');
    }
    assert(a.length === n, 'array has n rows');
}
```

```
function oracle(genArray) {
     let n = Math.floor(Math.random()*1000);
     let a = genArray(n);
     const valid = x => x % 1 === 0 && 0 <= x && x < n;
     function isPerm(p) {
           assert(p.length === n, 'length is n');
           assert(p.every(valid),'valid numbers');
           const f = Array(n).fill(1);
           assert(p.every(e => --f[e] === 0), 'no duplicates');
     assert(a.length === n, 'array has n rows');
     a.forEach(isPerm); // each row is a permutation
```

a is length n, and we call isPerm (an O(n) function) for each element of a. This will take O(n²) time

Solution: Permutations

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Write an oracle that accepts the function genArray as input.

```
function oracle(genArray) {
    let n = Math.floor(Math.random()*1000);
    let a = genArray(n);
    const valid = x => x % 1 === 0 && 0 <= x && x < n;
    function isPerm(p) {
        assert(p.length === n, 'length is n');
        assert(p.every(valid),'valid numbers');
        const f = Array(n).fill(1);
        assert(p.every(e => --f[e] === 0), 'no duplicates');
    }
    assert(a.length === n, 'array has n rows');
    a.forEach(isPerm); // each row is a permutation
    for (let k = 0; k < n; ++k) { isPerm(a.map(r => r[k])); } // each column is a permutation
}
```

Note: A different approach, where we check directly that every number from 0 to n-1 is included requires a linear scan for each number. That would be $O(n^2)$ per row/column and $O(n^3)$ overall.

Think back to lecture where you discussed the shapes classes. Before we start with this exercise, please familiarize yourself yourself for a few minutes with the code in the starter code. It should seem familiar to you.

Uncomment and run the three examples and make sure you understand!

Your TA will demonstrate this.

Implement a **class** Translate whose constructor takes a shape and a change in x and change in y value. When the draw method is called with a CanvasRenderingContext2D (ctx) and a color, shift the canvas by dx and dy using ctx.translate, draw the shape on the moved canvas, and move the canvas back to the starting position.

Now run 'npm run start'. You'll know your code is correct by the image.

Exercise: OOP

```
class Translate {
 dx: number;
 dy: number;
 shape: Shape;
 constructor(shape: Shape, dx: number, dy: number) {
   this.dx = dx;
   this.dy = dy;
   this.shape = shape;
 draw(ctx: CanvasRenderingContext2D, color: string) {
   ctx.translate(this.dx, this.dy);
   this.shape.draw(ctx, color);
   ctx.translate(-this.dx, -this.dy);
```

Live code this! Show students how the images show up. :)

- Define an iterator class over an array.
- Define a class Polygon (defined by an array of points in 2D) which has a makelterator() method for its vertices.
- Extend the class to CenteredPolygon which has a method to compute its center.
- The center of weight has the average of the vertex coordinates.

```
class ArrayIterator<T> implements IterableIterator<T> {.....}

class Polygon {.....}

class CenteredPolygon extends Polygon {.....}
```

Exercise 2: Solution

```
class ArrayIterator<T> implements IterableIterator<T> {
   private array: T[];
   private index = 0;
   constructor(private array: T[]) {this.array = array}
   public next(): IteratorResult<T> {
        if (this.index < this.array.length) {
        return this.array[this.index++];
        } else {
        return null;
        }
   }
}</pre>
```

```
class CenteredPolygon extends Polygon {
   public Center(): number[] {
        const numVertices = this.vertices.length;
       let x = 0;
       let y = 0;
        const iterator = this.makeIterator();
        let current = iterator.next();
       while (current !== null) {
           x += current[0];
           y += current[1];
            current = iterator.next();
        return [x / numVertices, y / numVertices];
```

Exercise 2: Solution 2

```
interface Iterator<T> {
   next(): { value: T, done: boolean };
class Polygon {
    private vertices:[number, number][];
   constructor(vertices:[number, number][]) {
      this.vertices = vertices;
   makeIterator(): Iterator<number[]> {
      let index = 0:
      const vertices = this.vertices:
      return {
        next() {
          if (index < vertices.length) {</pre>
            return { value: vertices[index++], done: false };
          } else {
            return { value: null, done: true };
          }}};}}
```

```
class CenteredPolygon extends Polygon {
   private readonly sides: number;
   constructor(vertices: number[][]) {
     super(vertices);
     this.sides = vertices.length;
   const numVertices = this.vertices.length;
   center(): number[] {
     let x = 0;
     let v = 0;
     const iterator = this.makeIterator();
     let current = iterator.next().value;
     while (current !== null) {
       x += current[0];
       y += current[1];
        current = iterator.next().value;
     return [x / this.sides, y / this.sides];
```

```
class ArrayIterator<T> {
 private arr: T[];
 private idx: number;
 constructor(a: T[]) { this.arr = a; this.idx = -1;
 next(): T | null {
   return ++this.idx < this.arr.length ?</pre>
                     this.arr[this.idx] : null;
class Polygon {
 private vertices: [number, number][];
  constructor(vertices: [number, number][]) {
   this.vertices = vertices;
 makeIterator() {
   return new ArrayIterator(this.vertices);
```