NMIT1 - Praktikum 11 - ungerpet

Aufg2

a)

$$f(x_1, x_2) = \begin{pmatrix} 5x_1x_2 \\ x_1^2x_2^2 + x_1 + 2x_2 \end{pmatrix}, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$Df(x_1, x_2) = \begin{pmatrix} \frac{\partial f_1}{\partial x_1}(x_1, x_2) & \frac{\partial f_1}{\partial x_2}(x_1, x_2) \\ \frac{\partial f_2}{\partial x_1}(x_1, x_2) & \frac{\partial f_2}{\partial x_2}(x_1, x_2) \end{pmatrix}$$

$$\frac{\partial f_1}{\partial x_1}(x_1, x_2) = 5x_2$$

$$\frac{\partial f_1}{\partial x_2}(x_1, x_2) = 5x_1$$

$$\frac{\partial f_2}{\partial x_1}(x_1, x_2) = 2x_1 \cdot x_2^2 + 2x_2 + 1$$

$$\frac{\partial f_2}{\partial x_2}(x_1, x_2) = x_1^2 \cdot 2x_2 + x_1 + 2$$

$$Df(x_1, x_2) = \begin{pmatrix} 5x_2 & 5x_1 \\ 2x_1 \cdot x_2^2 \cdot 2x_2 + 1 & x_1^2 \cdot 2x_2 + x_1 + 2 \end{pmatrix}$$

$$Df(1, 2) = \begin{pmatrix} 10 & 5 \\ 33 & 7 \end{pmatrix}$$

b)

$$f(x_1, x_2, x_3) = \begin{pmatrix} ln(x_1^2 + x_2^2) + x_3^2 \\ exp(x_2^2 + x_3^2) + x_1^2 \\ \frac{1}{(x_3^2 + x_1^2)} + x_2^2 \end{pmatrix}, \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$Df(x_1, x_2, x_3) = \begin{pmatrix} \frac{\partial f_1}{\partial x_1}(x_1, x_2, x_3) & \frac{\partial f_1}{\partial x_2}(x_1, x_2, x_3) & \frac{\partial f_1}{\partial x_3}(x_1, x_2, x_3) \\ \frac{\partial f_2}{\partial x_1}(x_1, x_2, x_3) & \frac{\partial f_2}{\partial x_2}(x_1, x_2, x_3) & \frac{\partial f_2}{\partial x_3}(x_1, x_2, x_3) \\ \frac{\partial f_3}{\partial x_1}(x_1, x_2, x_3) & \frac{\partial f_3}{\partial x_2}(x_1, x_2, x_3) & \frac{\partial f_3}{\partial x_3}(x_1, x_2, x_3) \end{pmatrix}$$

$$\frac{\partial f_1}{\partial x_1}(x_1, x_2, x_3) = \frac{2x_1}{x_1^2 + x_2^2}$$

$$\frac{\partial f_1}{\partial x_2}(x_1, x_2, x_3) = \frac{2x_2}{x_1^2 + x_2^2}$$

$$\frac{\partial f_1}{\partial x_3}(x_1, x_2, x_3) = 2x_3$$

$$\frac{\partial f_2}{\partial x_1}(x_1, x_2, x_3) = 2x_1$$

$$\frac{\partial f_2}{\partial x_2}(x_1, x_2, x_3) = 2x_2 \cdot e^{(x_2^2 + x_3^2)}$$

$$\frac{\partial f_2}{\partial x_3}(x_1,x_2,x_3) = 2x_3 \cdot e^{(x_2^2+x_3^2)}$$

$$\frac{\partial f_3}{\partial x_1}(x_1,x_2,x_3) = -\frac{2x_1}{(x_1^2+x_3^2)^2}$$

$$\frac{\partial f_3}{\partial x_2}(x_1,x_2,x_3) = 2x_2$$

$$\frac{\partial f_3}{\partial x_1}(x_1,x_2,x_3) = -\frac{2x_3}{(x_1^2+x_3^2)^2}$$

$$Df(x_1,x_2,x_3) = \begin{pmatrix} \frac{2x_1}{x_1^2+x_2^2} & \frac{2x_2}{x_1^2+x_2^2} & 2x_3 \\ 2x_1 & 2x_2 \cdot e^{(x_2^2+x_3^2)} & 2x_3 \cdot e^{(x_2^2+x_3^2)} \\ -\frac{2x_1}{(x_1^2+x_3^2)^2} & 2x_2 & -\frac{2x_3}{(x_1^2+x_3^2)^2} \end{pmatrix}$$

$$Df(1,2,3) = \begin{pmatrix} \frac{2}{5} & \frac{4}{5} & 6 \\ 2 & 4 \cdot e^{(13)} & 6 \cdot e^{(13)} \\ -\frac{2}{121} & 4 & -\frac{6}{121} \end{pmatrix}$$