Composite Hybrid Grids in the Overture Framework

Kyle Chand
Center for Applied Scientific Computing
Lawrence Livermore National Laboratory
Livermore, California
www.llnl.gov/CASC/Overture

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In the next 30 minutes:

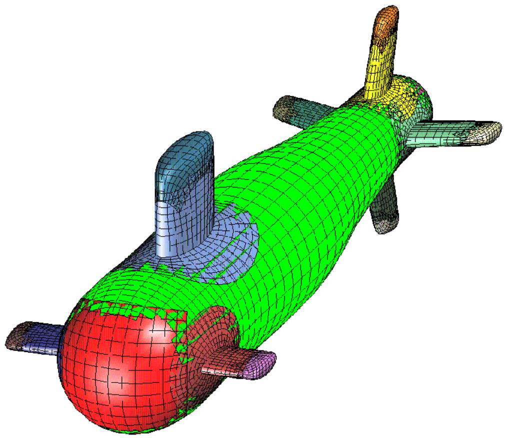
Overview of Overture

Hybrid mesh generation

Discrete operators for unstructuerd
and hybrid grids

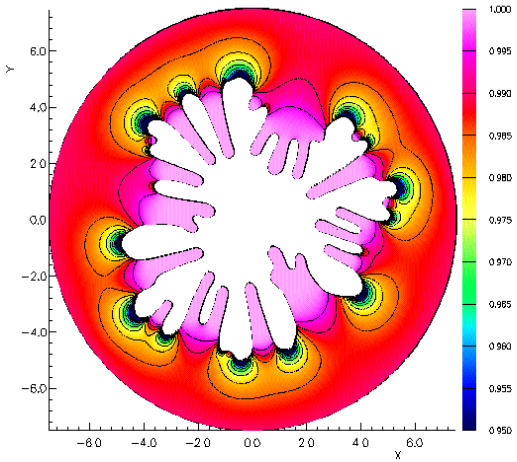
Byproducts of unstructured mesh support
in the Overture Framework

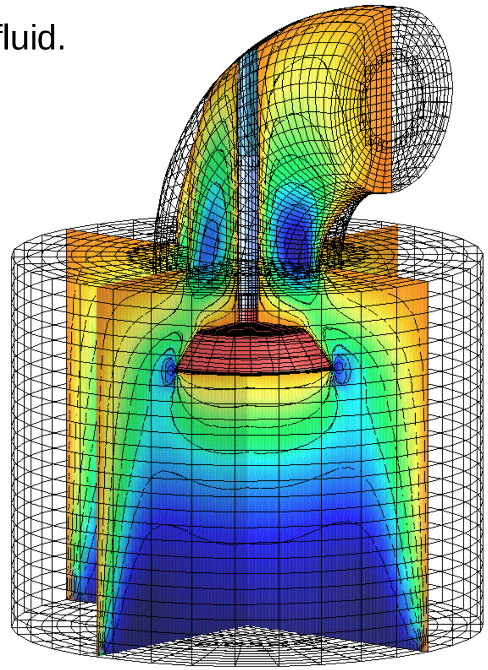
Overture: A Toolkit for Solving PDEs



Overlapping Grids
Fig: Bill Henshaw

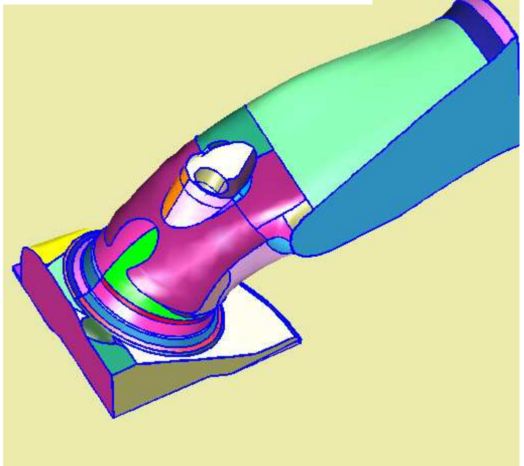
Hele Shaw flow of a non-Newtonian fluid.
Fig: Petri Fast.

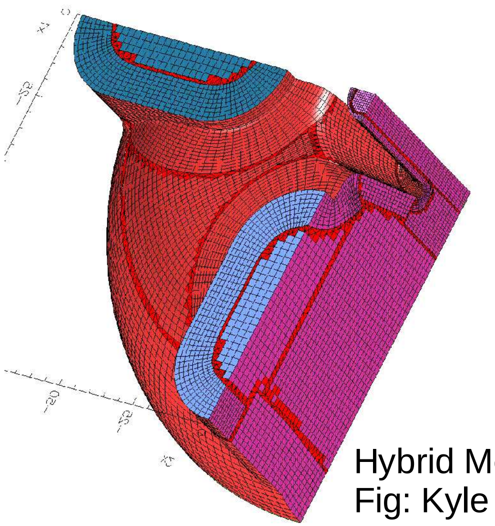




Moving Piston, Incompressible Navier-Stokes
Fig: Bill Henshaw.

CAD Geometry
Fig: Anders Petersson





Hybrid Meshes
Fig: Kyle Chand

PDE Solver Development

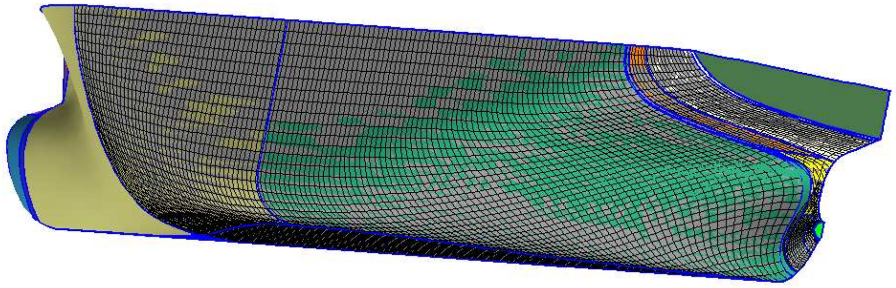
Grid Generation

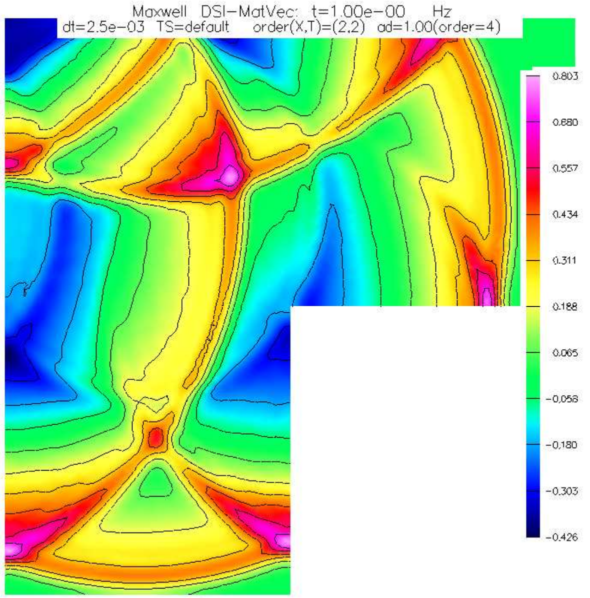
Geometry

Geometry, Mesh Generation, Discretization, Solvers

Geometry and grid generation:

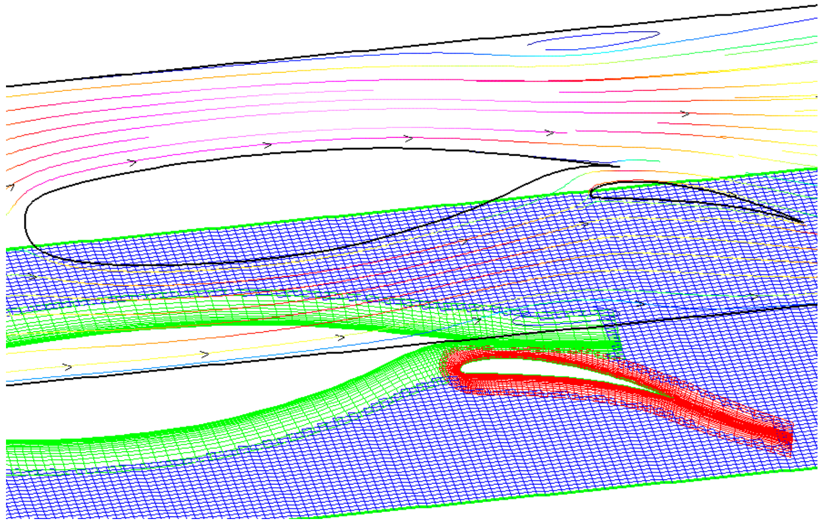
- Geometry creation and management
- CAD geometry import from IGES
- Structured grid generation tools
- Unstructured mesh generation
- Overlapping and hybrid grids





PDE discretization and solvers:

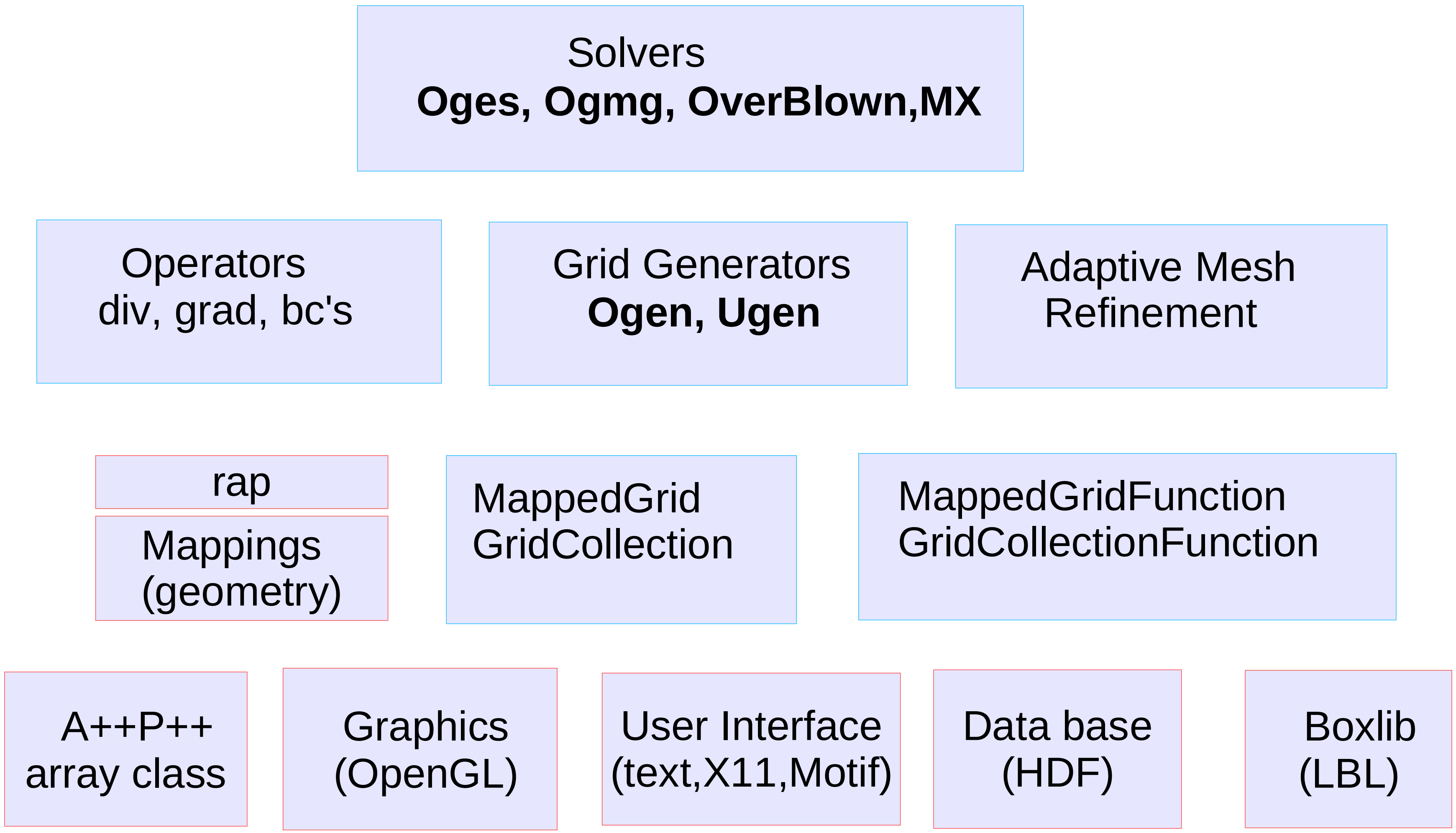
- Object-Oriented Field and Operator library
- Multiple levels of interface to balance
  convienence and performance
- Access to linear solvers and AMR
- High-order curvilinear grid operators



Application support:

- GUI and Visualization tools
- HDF Database interface

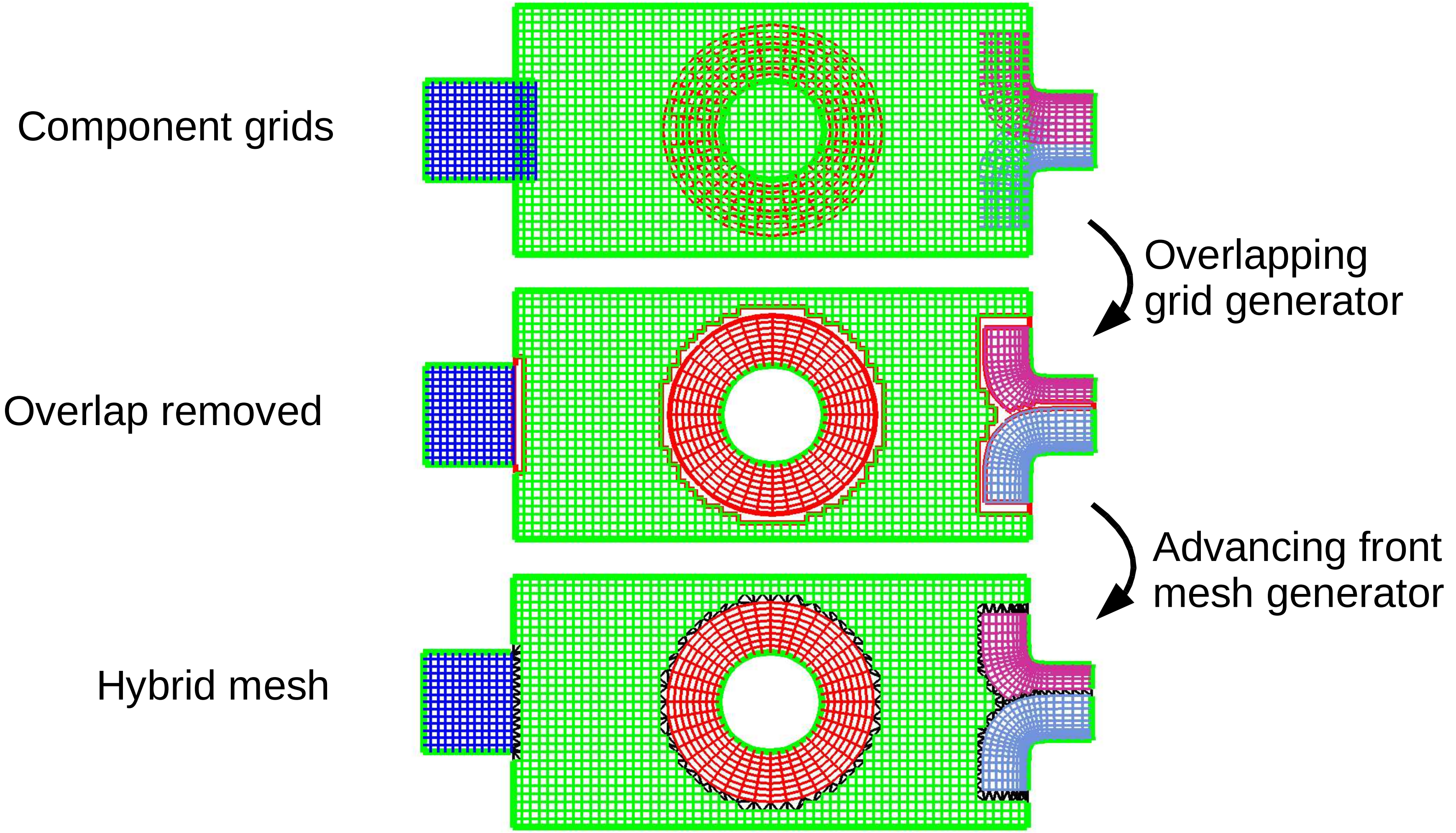
Overture: A toolkit for solving PDEs



Rapsodi : A geometry toolkit for mesh generation and discretization

Hybrid Mesh Generation

Hybrid meshes connect structured grids
with unstructured mesh.



Hybrid Mesh Algorithms and Software

- Overture Mapping classes --> component grids
- Overture Overlapping grid generator --> automatic hole cutting
- 2/3D Advancing front unstructured mesh generator
- UnstructuredMapping container class for the mesh
- Mesh optimization algorithms
- Visualization tools
- Discretizations on unstructured meshes

Similar work :

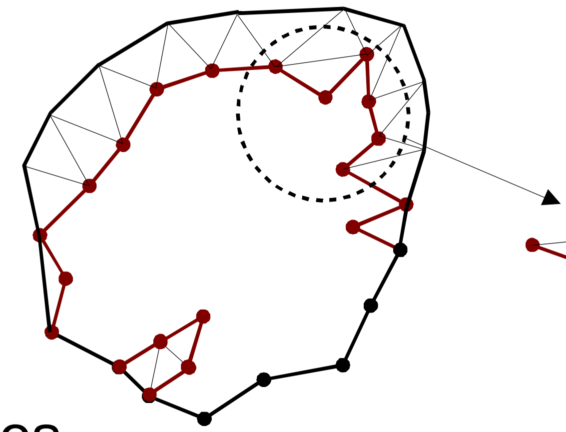
- Liou, Zheng and Civinskas --> DRAGON grids (1994)
- Shaw, Peace, Weatherill (1994)
- Weatherill gives a general discussion in *Numerical Grid Generation in CFD '88*

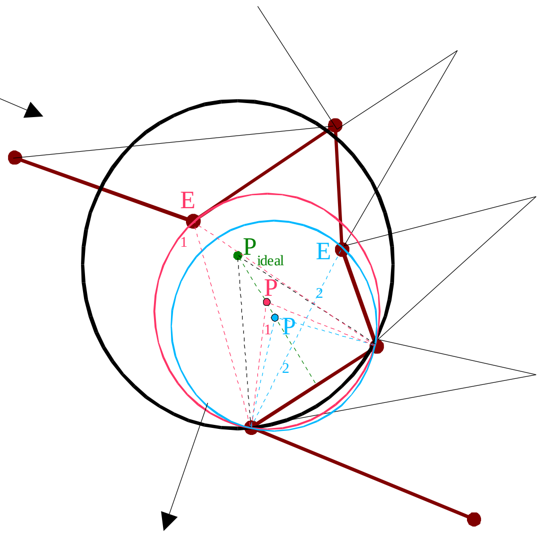
Advancing front sources :

- Lo (1985,1991)
- Peiró, Peraire, et al. (1987, 1992, ...)
- Löhner (1988, 1996)
- George, Seveno (1994)
- Jin, Tanner (1993)

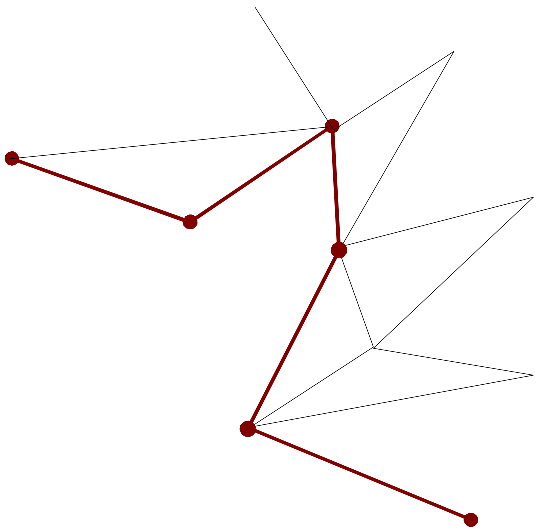
Advancing Front Algorithm

Begin with an initial front
- line segments
- triangles/quads



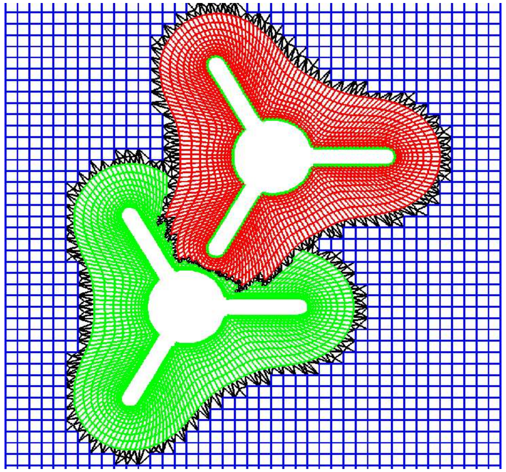


Select a face to advance
- search for existing vertices
- create candidate new vertices
- prioritize candidate elements
- select the first "consistent" element
  - no intersections
  - no enclosed vertices



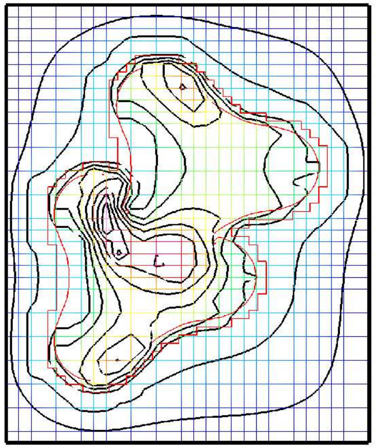
Delete old face(s) from the front
Add any new faces
Repeat until the front is empty

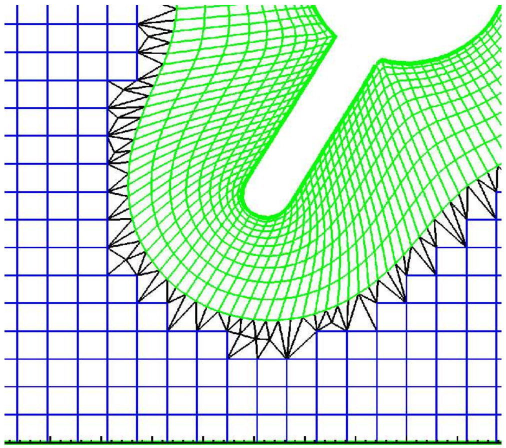
Mesh Spacing Control

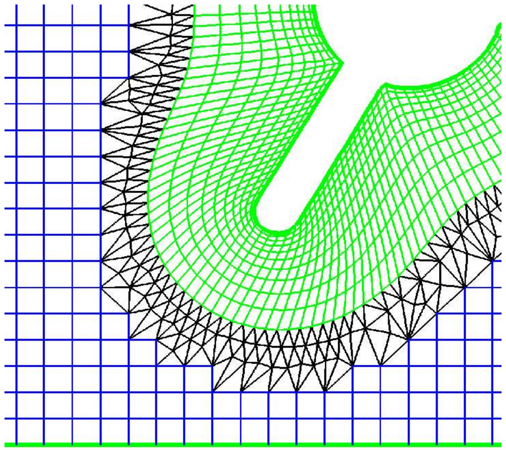


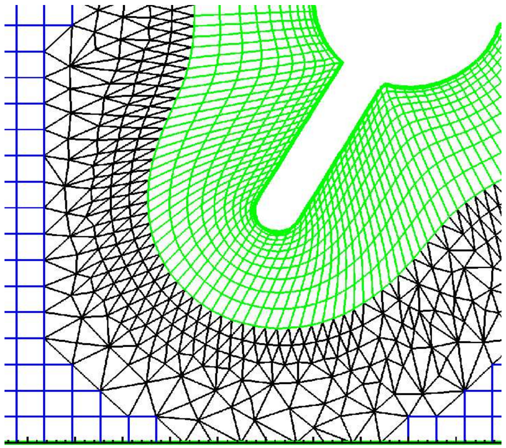
Unstructured mesh blends the spacings of the component grids

A background cartesian grid stores stretching information from the original component grids

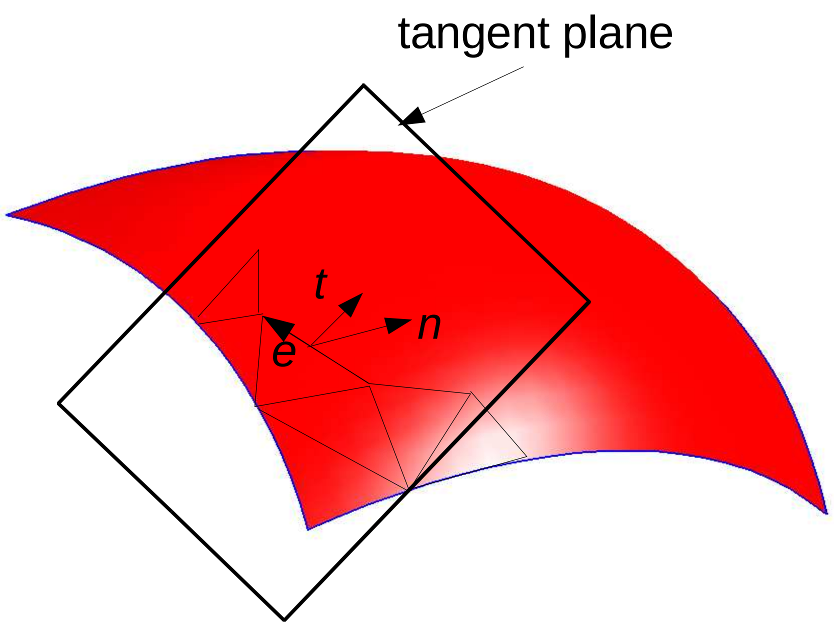








Surface mesh generation



e = edge vector pointing along the front
n = surface normal at midpoint of edge
t = advancement direction

$$t = e \times n$$

$$P^h_{ideal} = P^h_{midpoint} + d\,Tt$$

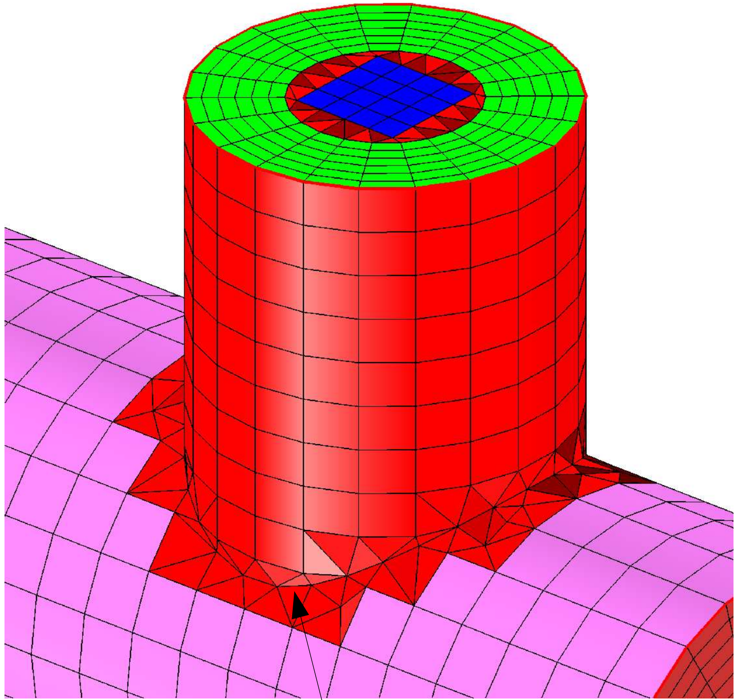
Surface normal n is computed from the geometry at the midpoint of the advancing face.

Points in the neighborhood of the advancing face are transformed by *T* and projected onto the plane defined by e^h and P^h_{ideal}.

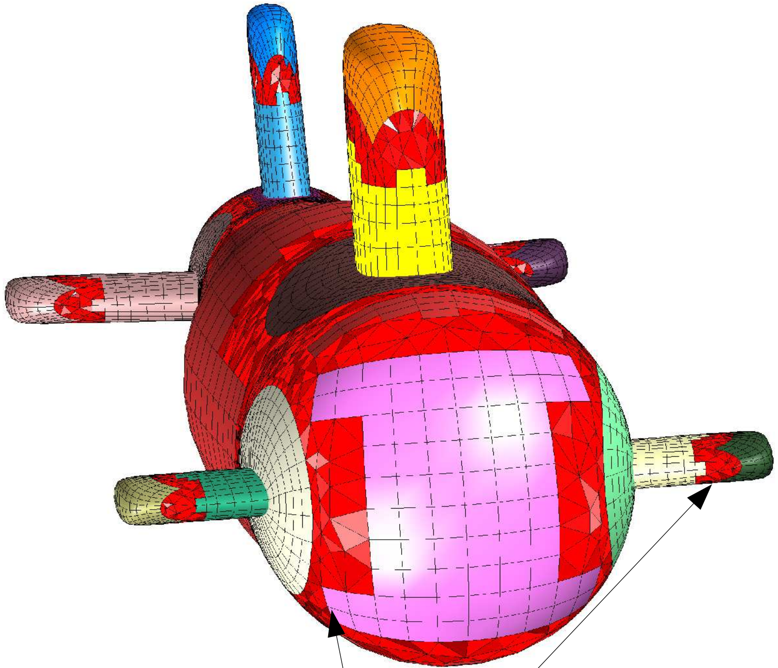
Validity tests are performed in the plane, essentially a 2D advancement.

High curvature surfaces are tricky:
during intersection checks, ignore faces that have surface normals differing by more than (say) 60 degrees from the normal at the current face midpoint.

Surface mesh generation



Intersecting surfaces

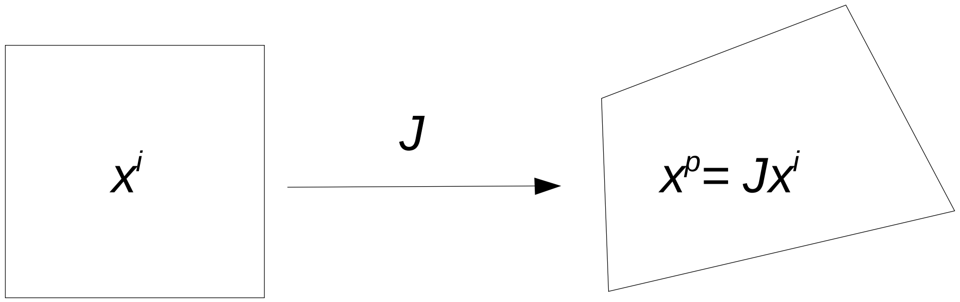


Overlapping surface grids

Mesh quality

Mesh quality assessement based on Pat Knupp's Algebraic Mesh Quality metrics (Knupp '99).

Metrics use properties of the Jacobian of the (linear) mapping between the actual and the "ideal" element:

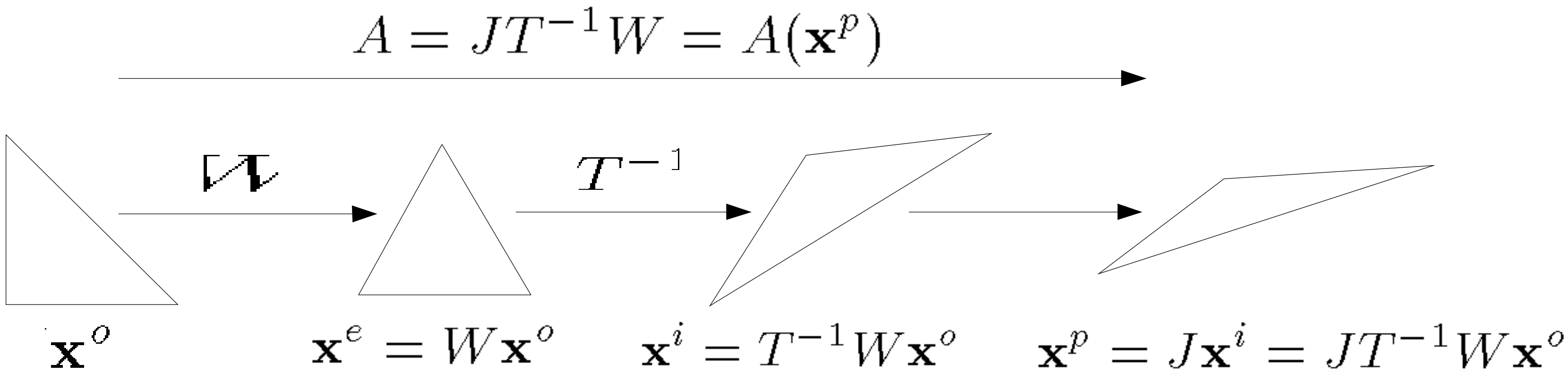


Useful metrics include :

det(J) – scaled size
K(J) - Condition number or C/K(J) - "shape" metric
min(det(J), 1/det(J)) C / K(J) – combined shape and size metric

Mesh quality

Computing the Jacobian between the "ideal"
element and the actual element (Pat Knupp) :



$$J = AW^{-1}T = AM$$

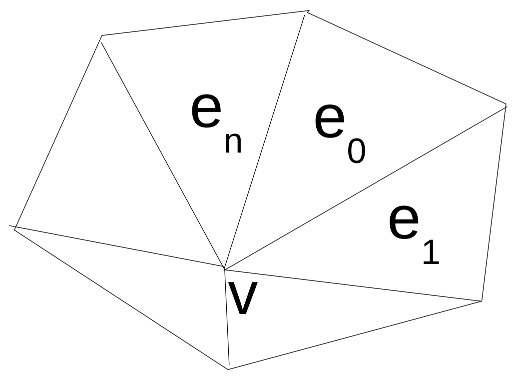
W is determined by the shape of the element, *T* by interpolation
from the spacing control grid and *A* from the actual element vertices

Mesh optimization

Local mesh improvement based on nonlinear optimization
of vertex locations (Lori Freitag, Pat Knupp '99, '00, ...)

Define : $f_v = f(x_v) = f(J_0(x_v), J_1(x_v) , ..., J_n(x_v))$
= the objective function at vertex v ($J_e = A_e M_e$)

Given a search direction d, iteratively search for
an optimal step size using a quadratic line search



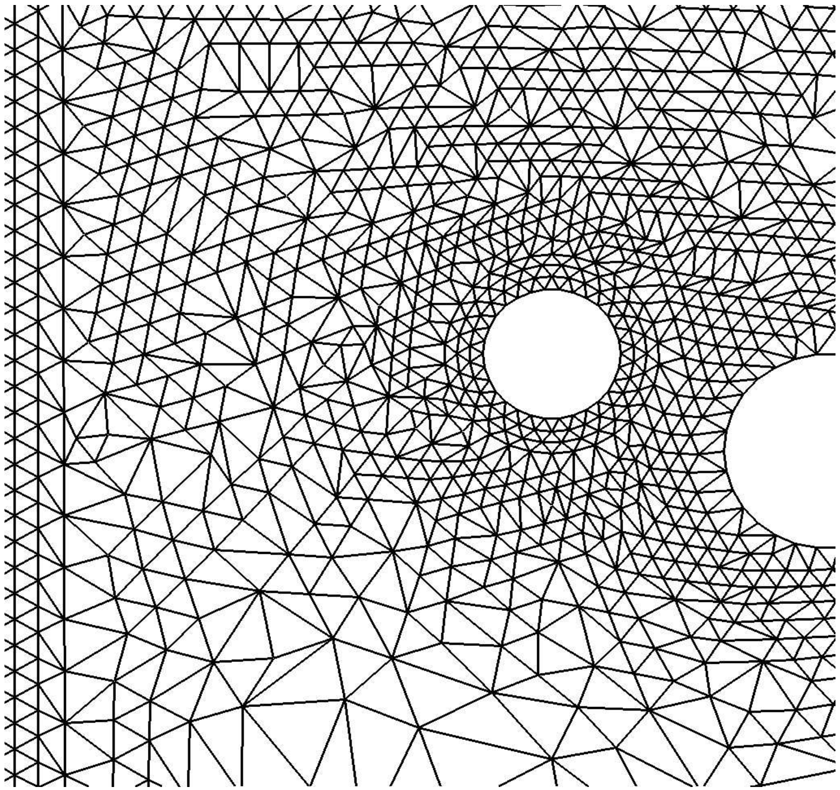
$$\mathbf{x}_v^{n+1} = \mathbf{x}_v^n + \mathbf{d}$$

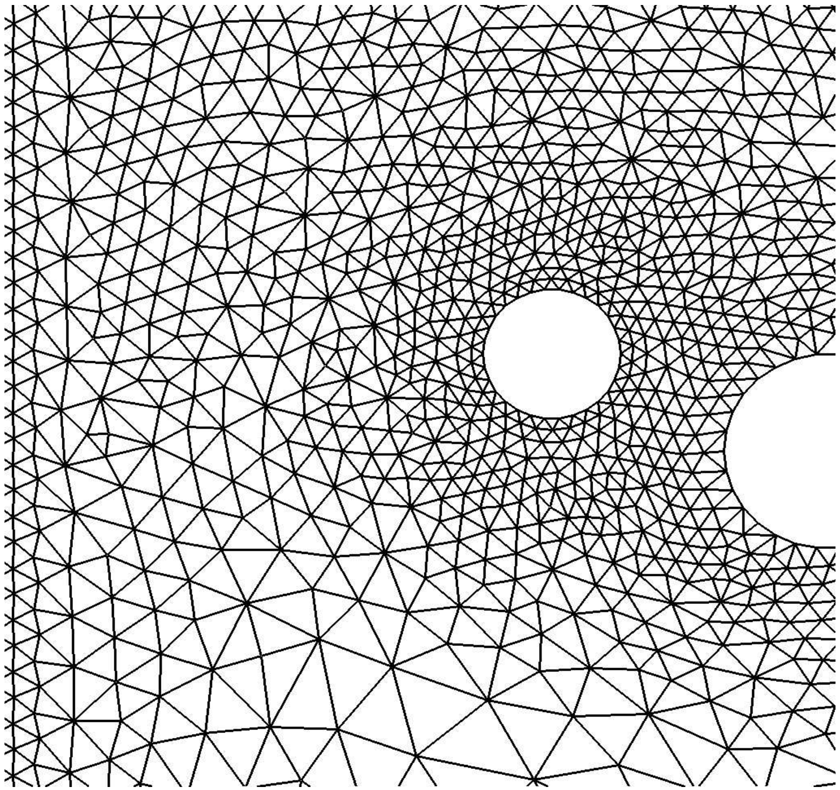
Steepest Descent :

$$f_v(\mathbf{x}_v) = \sum_{e=0}^{n} f_e(J_e(\mathbf{x}_v)) = \sum \kappa_e^2$$

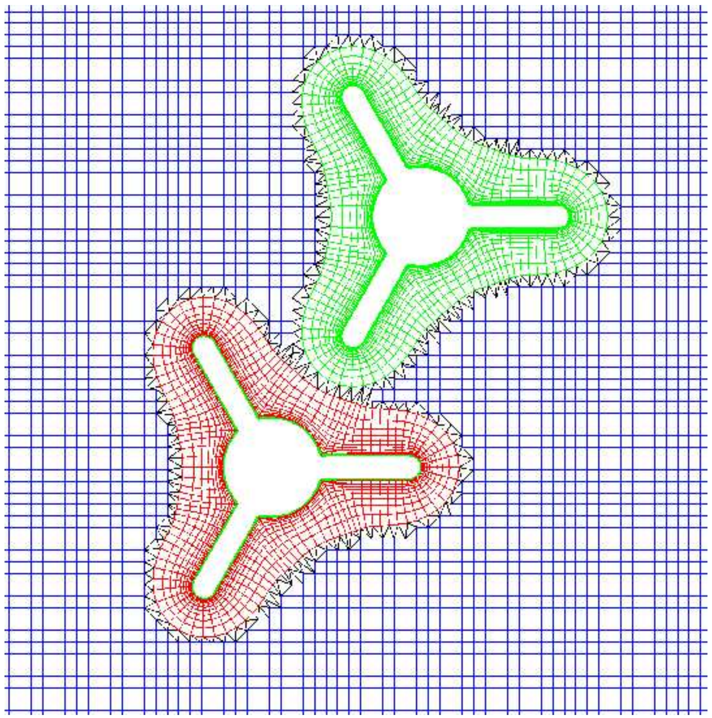
$$\frac{\partial f_e}{\partial x_v} = tr\left(\frac{\partial f_e}{\partial A}\frac{\partial A}{\partial x_v}^T\right) \longrightarrow \mathbf{d} = -d\nabla f_v$$

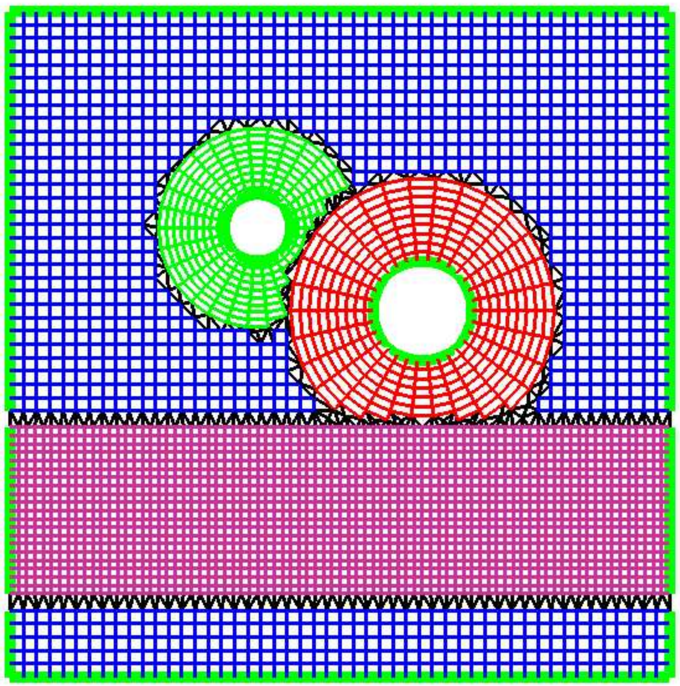
Mesh optimization example



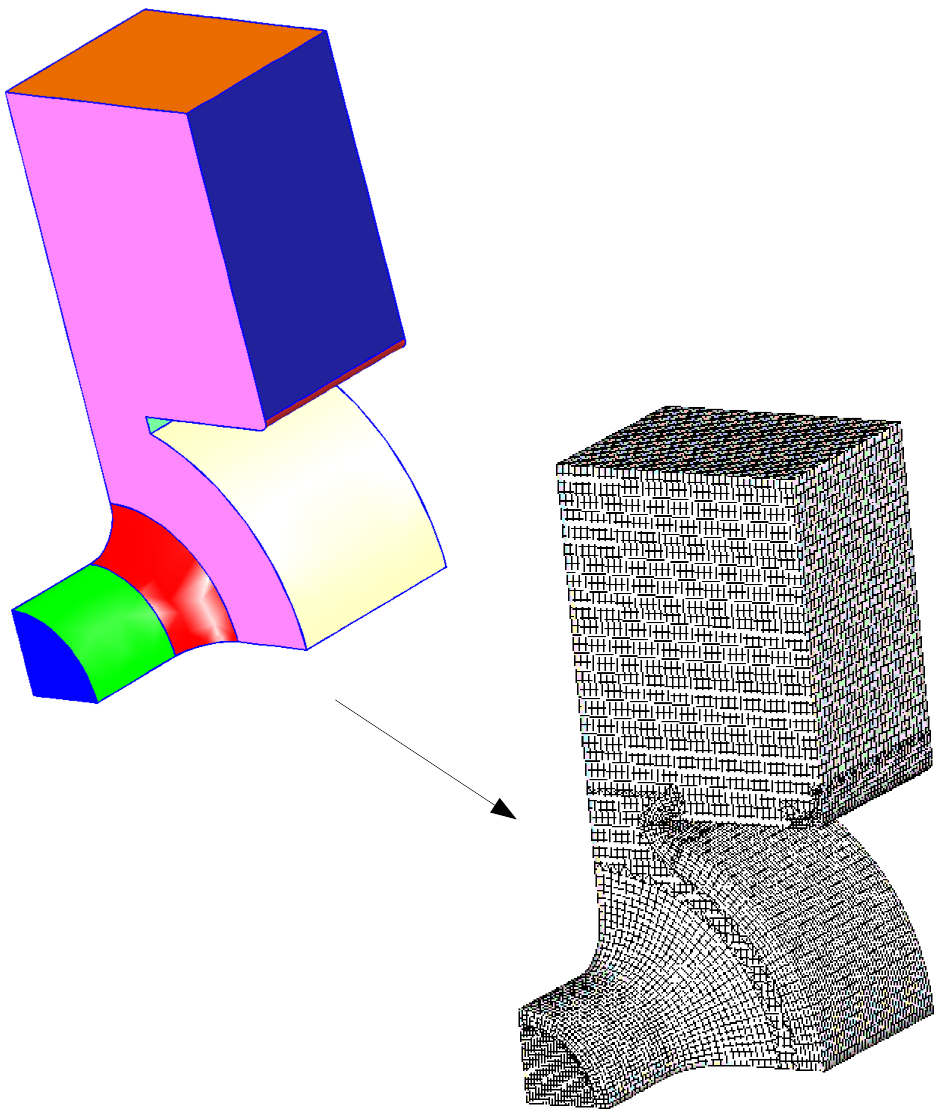


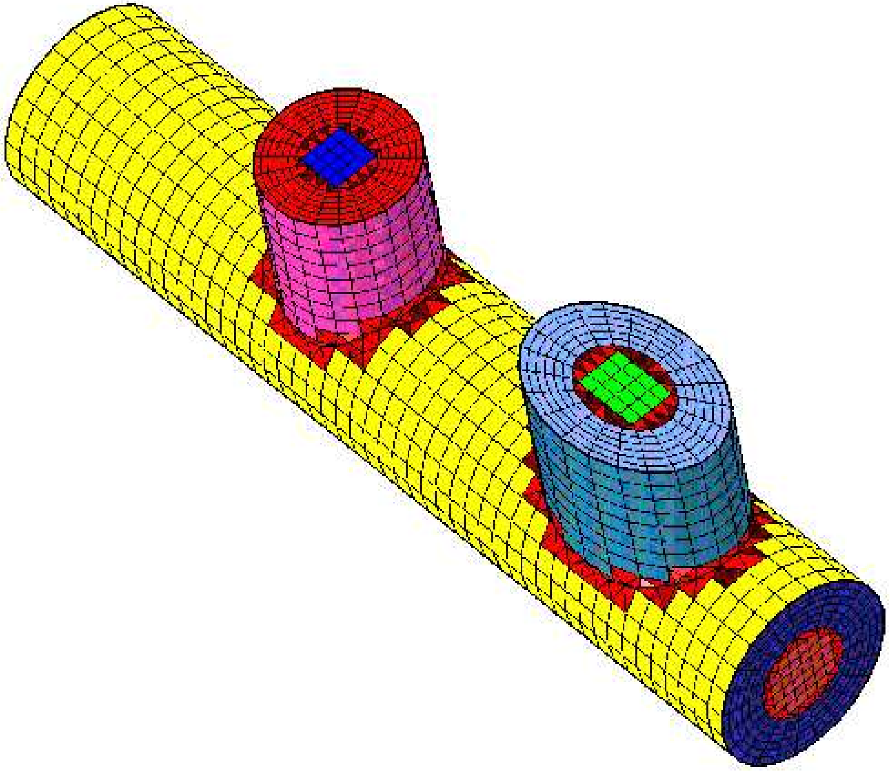
2D Hybrid grids



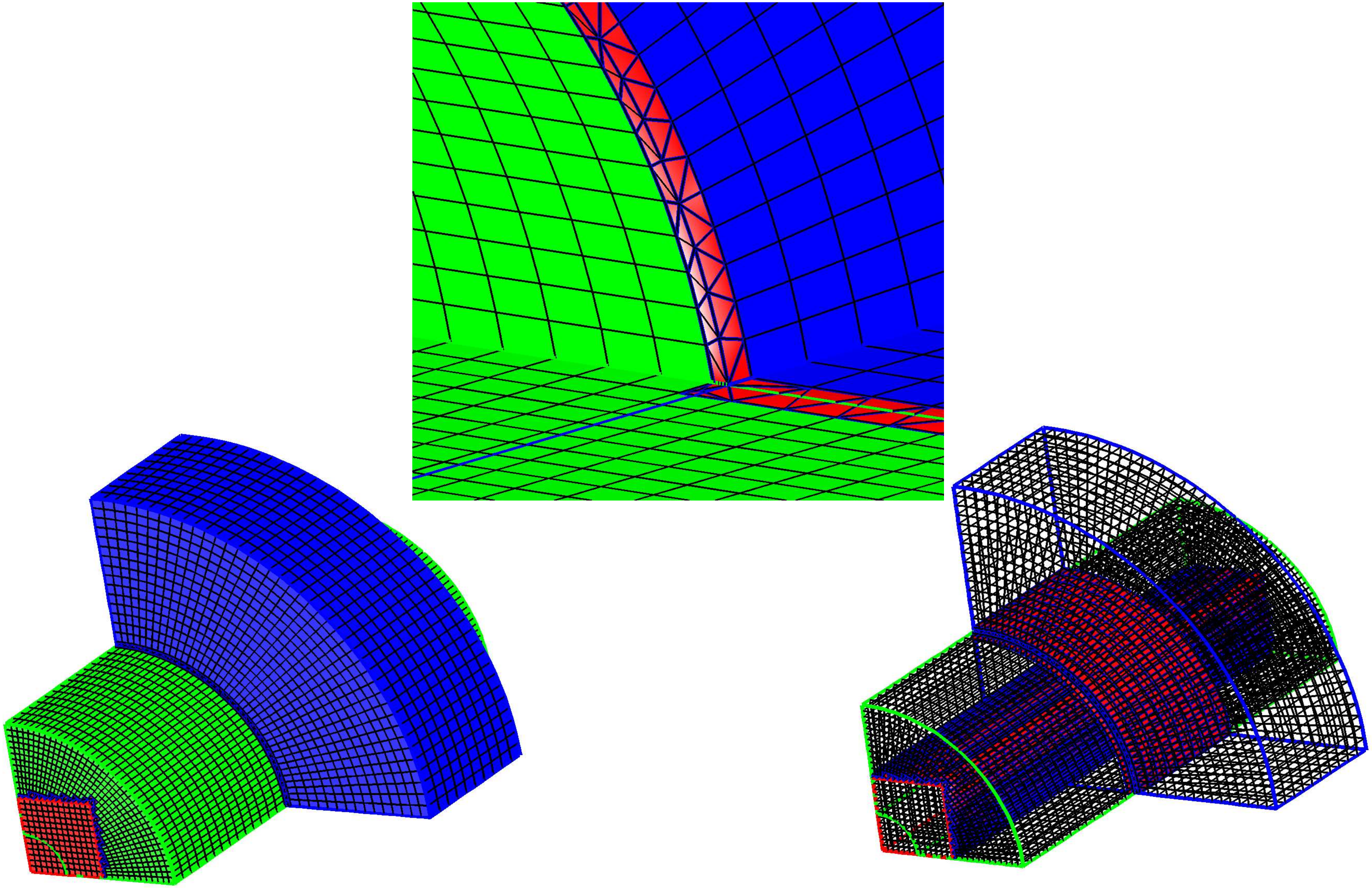


3D Hybrid Grids





3D Hybrid Grids

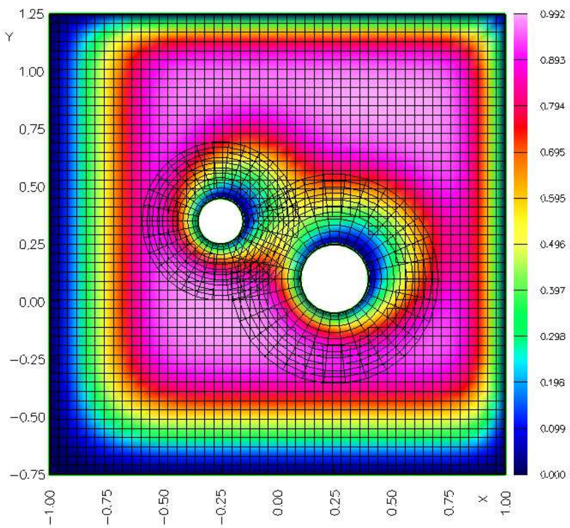


Interchangeable grids and discretizations: adding hybrid grids to Overture

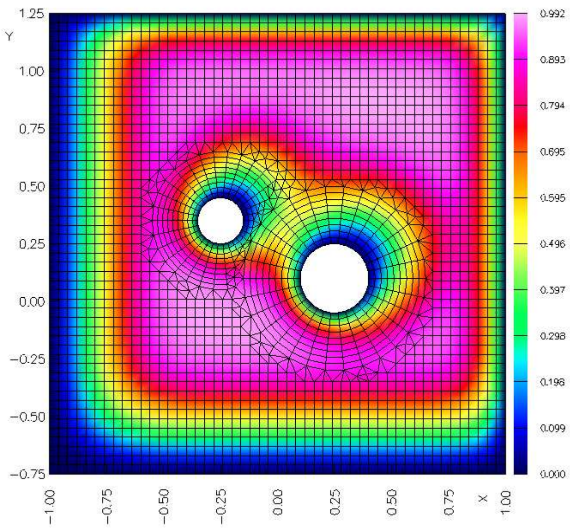
$$u_t = -a u_x - b u_y + \nu (u_{xx} + u_{yy})$$

```
u += dt*( -a*u.x() -b*u.y() + nu*(u.xx() + u.yy()) );
```

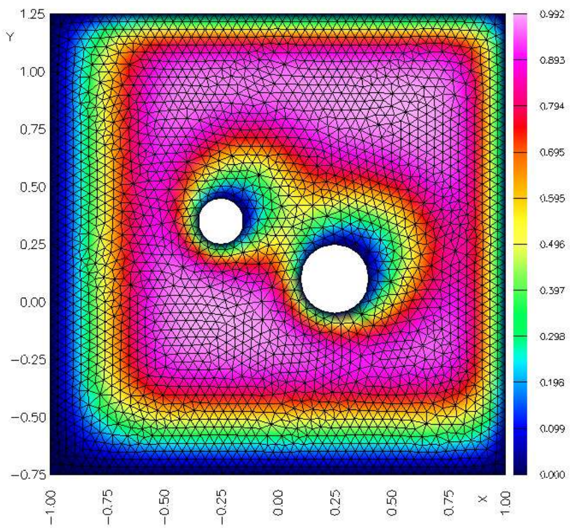
Overlapping



Hybrid



Unstructured



Unstructured Operators: Status

2nd Order finite volume operators implemented in 2 and 3D

Approximations for first & second derivatives, div, grad and Laplacian

High level operators are available for node and zone centered GridFunctions (e.g. u.x()...)

Mid-level interface to the operators provides different centerings and staggered grid support

Only dirichlet bounary condition operators are currently available (in progress)

Currently a work in progress...

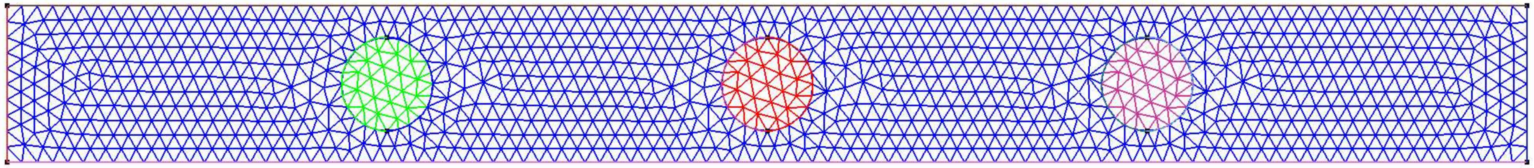
Some byproducts of the unstructured and hybrid grid support in Overture:

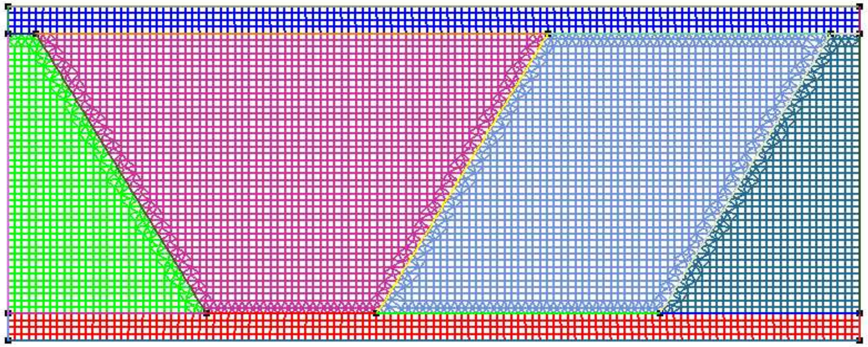
2D mixed element unstructured mesh generator, **`smesh`**
- advancing front mesh generator (2D planar)
- unstructured mesh optimization tools

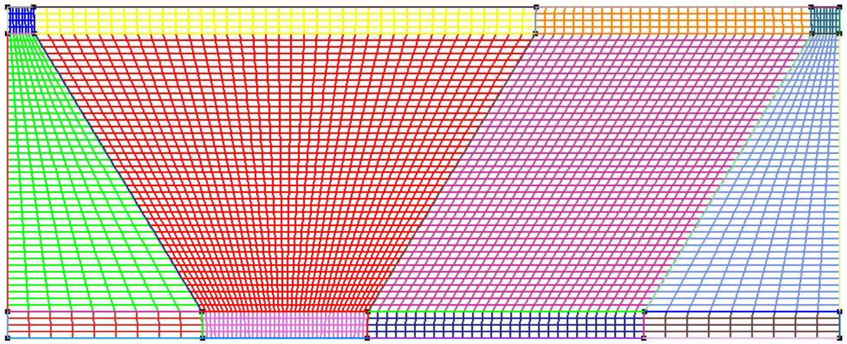
3D high order tet mesher for finite element applications
- advancing front mesh generator (3D volume)
- mesh optimization tools

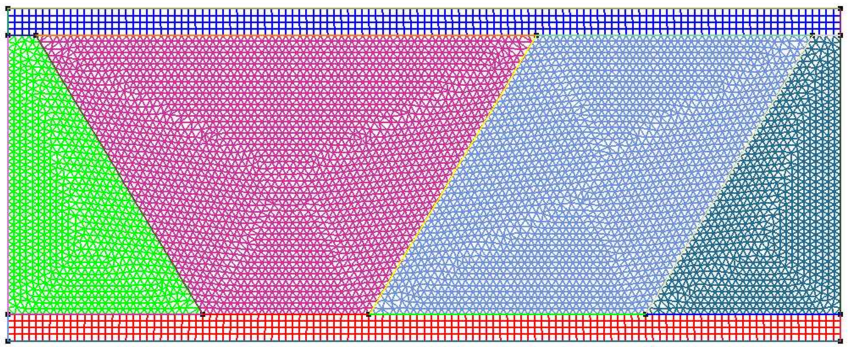
Surface stitching of overset grids for surface integrals
- surface mesh generator

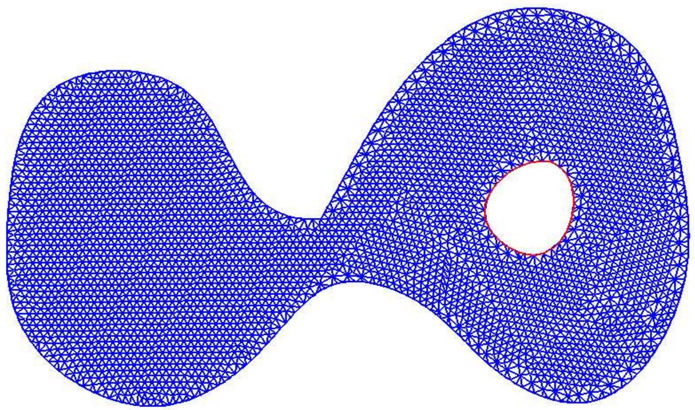
SMESH: Sample meshes





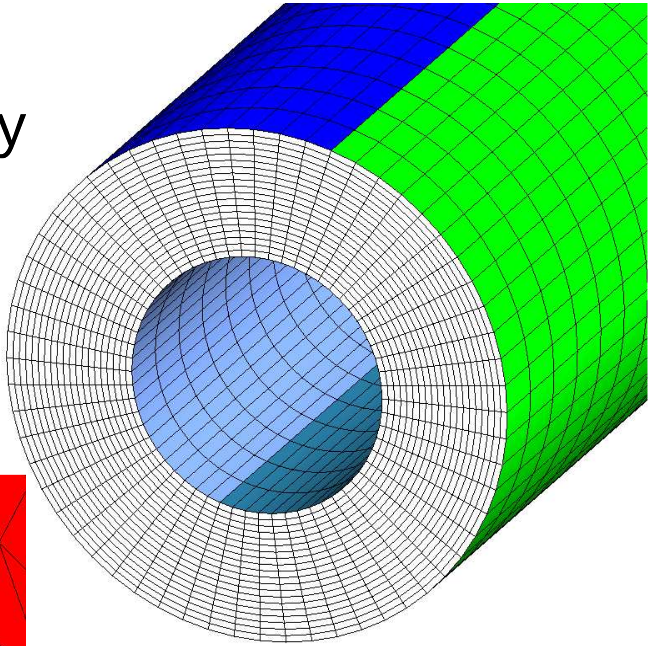


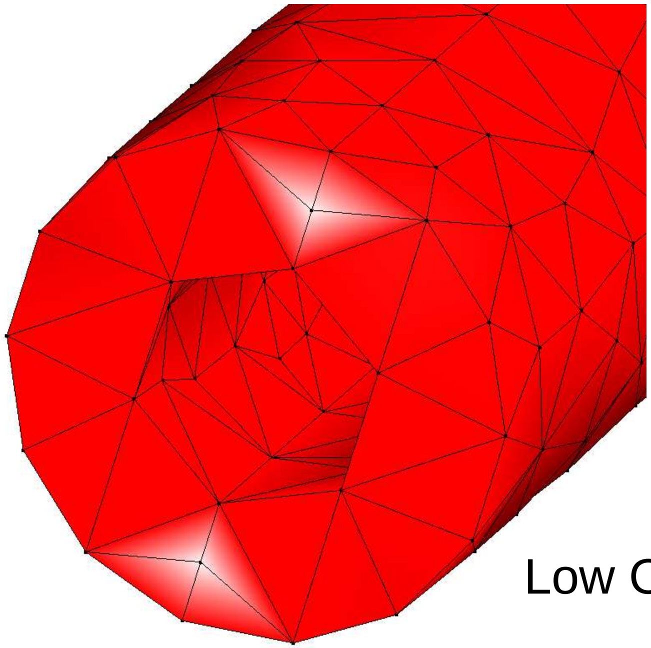


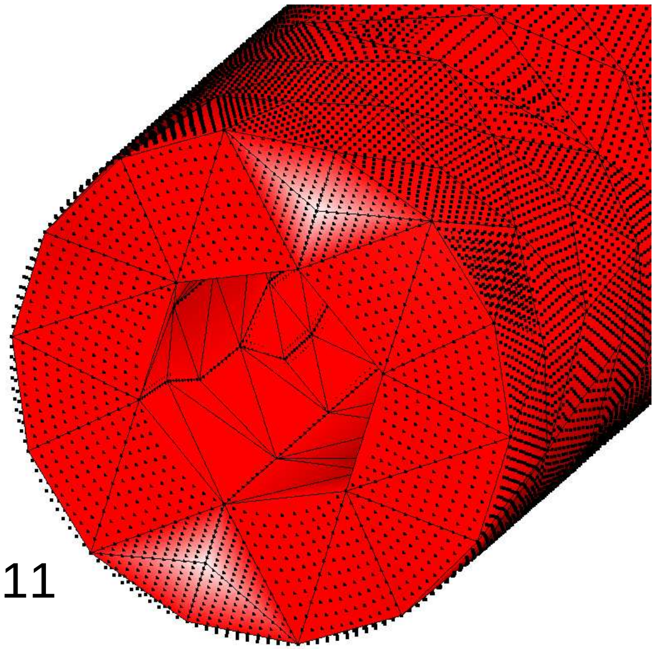


3DHOT: High order tetrahdral mesh generator

Geometry

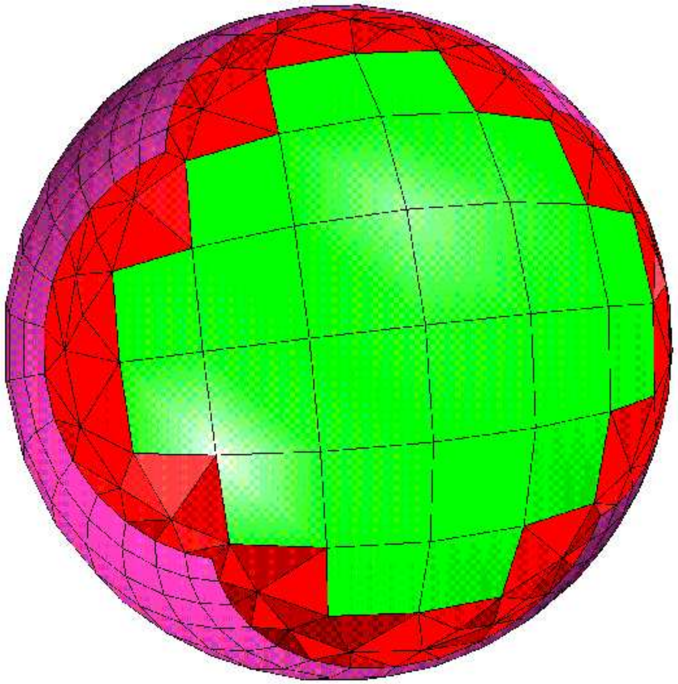


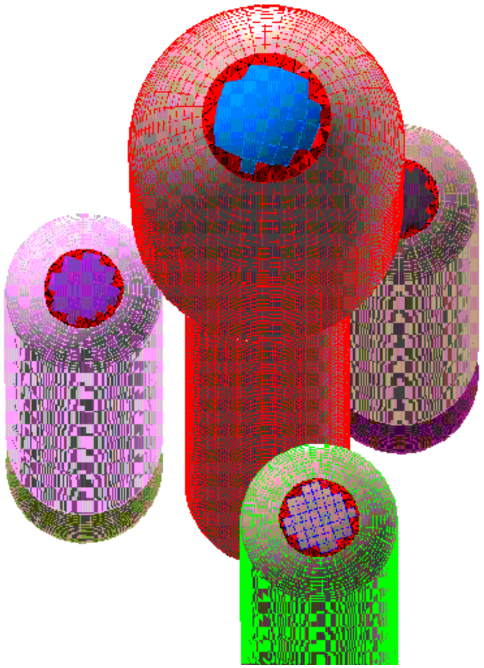




Low Order, p1 High Order, p11

Stitching overlapping surface grids





Obtaining Overture

Overture home page:
www.llnl.gov/CASC/Overture