EEM 308
SİNYALLER VE SİSTEMLER II

ÖRNEKLEME

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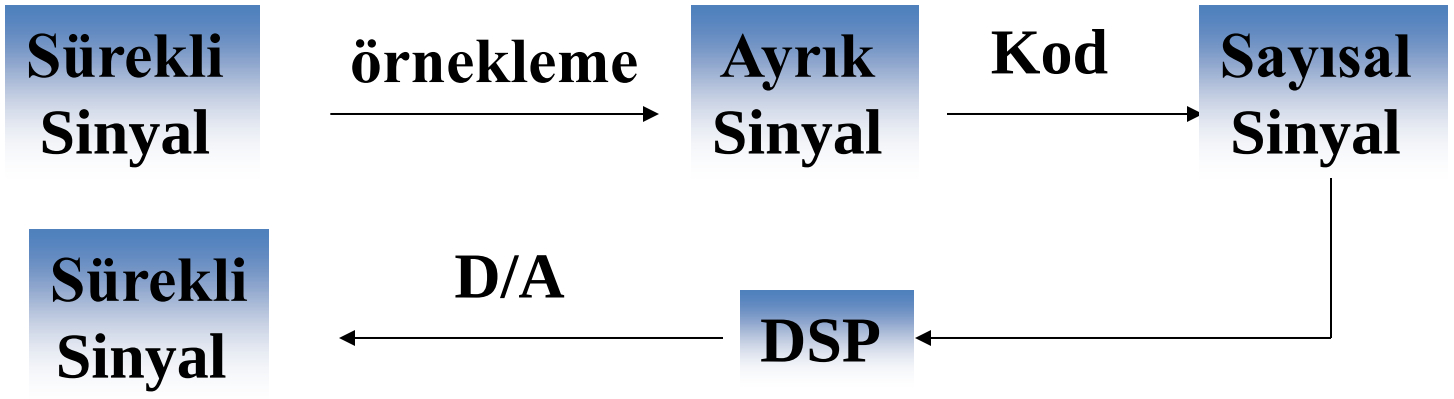
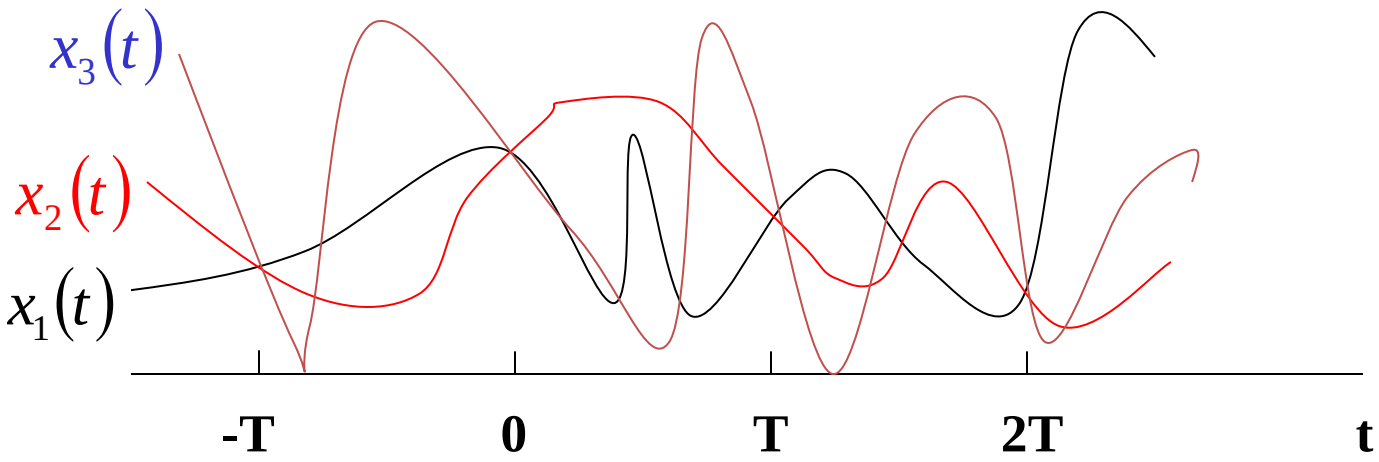
İÇERİK

- Örnekleme Teoremi
- Sinyalin Yeniden Oluşturulması
- Örtüşme (Aliasing)
- Sürekli Zaman Sinyallerinin Ayrık Zaman İşlemesi
- Ayrık Zaman Sinyallerinin Örneklenmesi

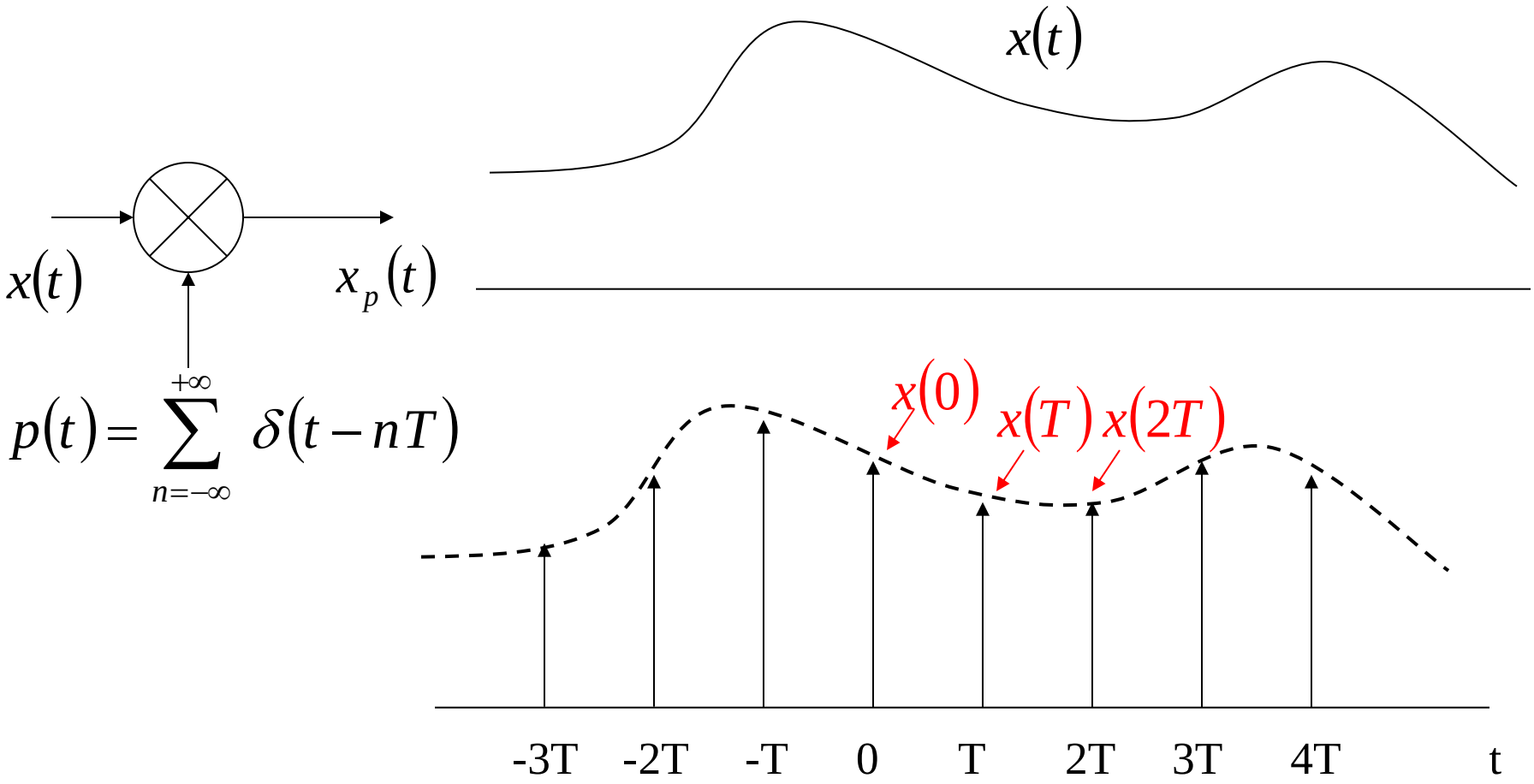




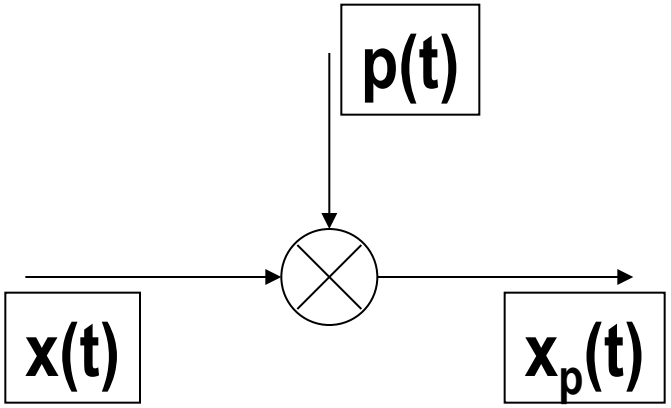
$$x_1(nT) = x_2(nT) = x_3(nT)$$

$$x_1(t) \neq x_2(t) \neq x_3(t)$$

1.1 Dürtü Katarı Örneklemesi



Örnekleme

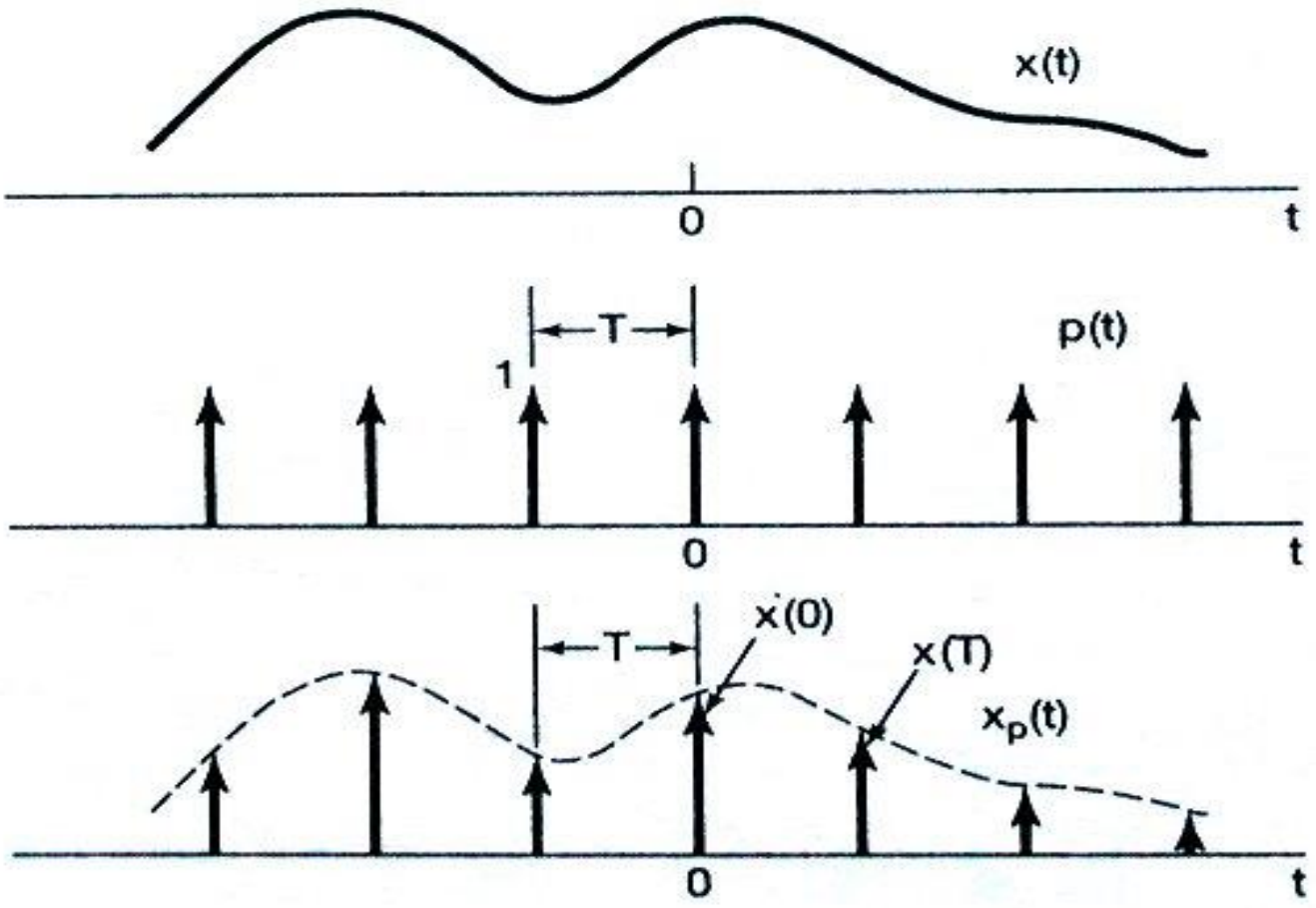


$$\begin{cases} x_p(t) = x(t)\,p(t) \\ X_p(j\omega) = \dfrac{1}{2\pi}[X(j\omega) * P(j\omega)] \end{cases}$$

$$p(t) = \delta_T(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

Zaman domeninde:

$$x_p(t) = x(t) \cdot \delta_T(t) = \sum_{n=-\infty}^{+\infty} x(nT)\delta(t - nT)$$



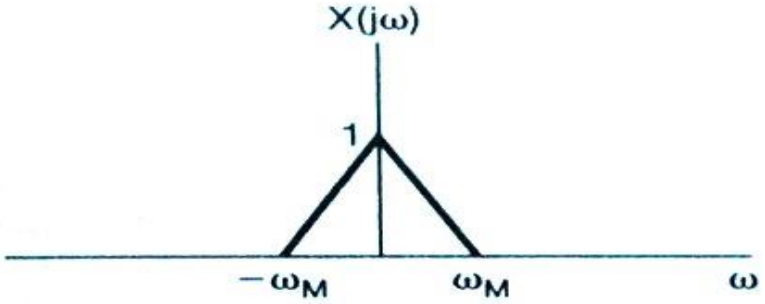
Frekans domeninde:

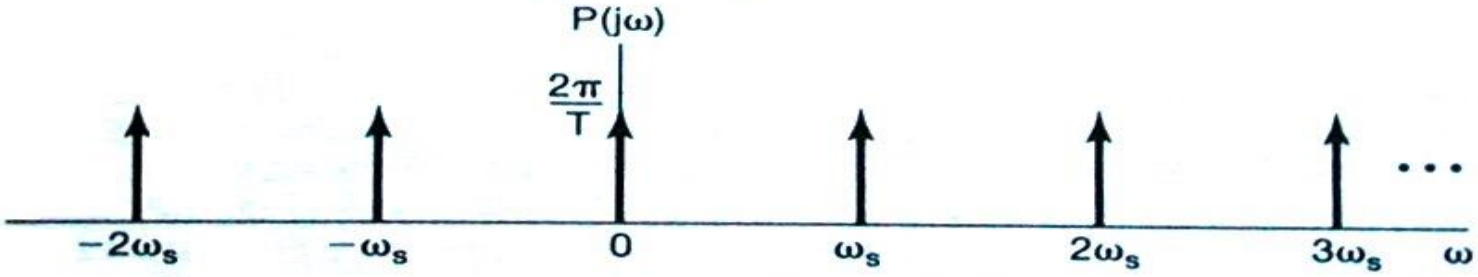
$$x(t) \overset{F}{\longleftrightarrow} X(j\omega)$$

$$p(t) = \delta_T(t) \qquad \textbf{(Periyodik Sinyal)}$$

$$p(t) \overset{F}{\longleftrightarrow} P(j\omega) = \sum_{k=-\infty}^{+\infty} \frac{2\pi}{T}\delta(\omega - k\omega_s) = \sum_{k=-\infty}^{+\infty} \omega_s \delta(\omega - k\omega_s)$$

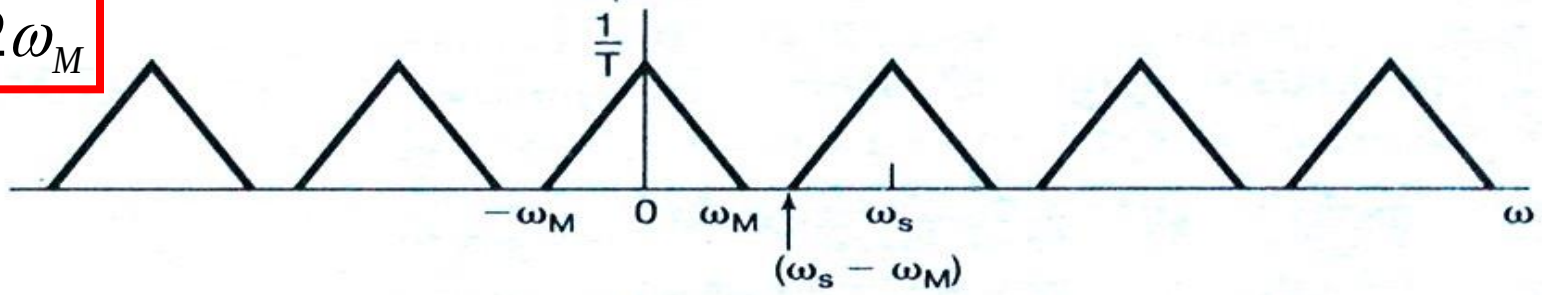
$$x_p(t) \overset{F}{\longleftrightarrow} X_p(j\omega) = \frac{\omega_s}{2\pi} \sum_{k=-\infty}^{+\infty} X(\omega - k\omega_s) = \frac{1}{T} \sum_{k=-\infty}^{+\infty} X(\omega - k\omega_s)$$



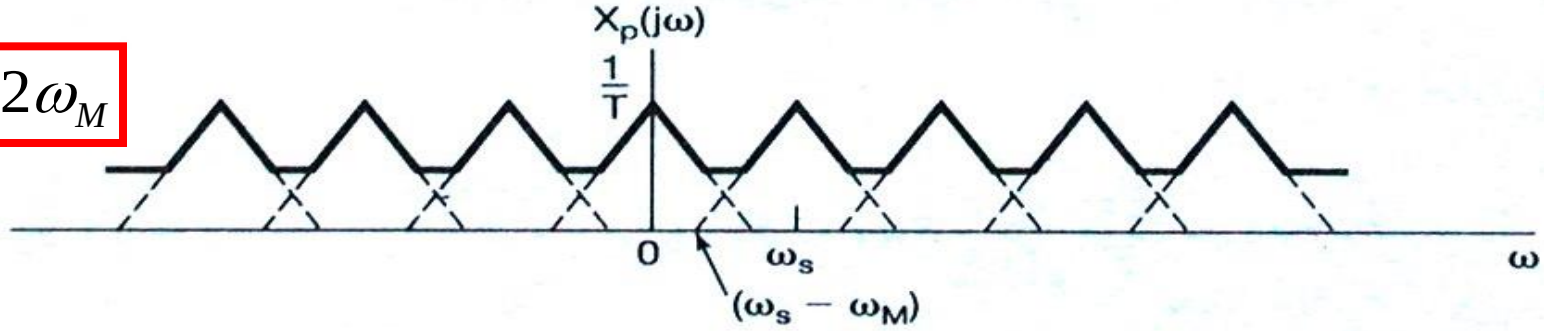


$$X_p(j\omega) = X(j\omega) * P(j\omega)$$

$\omega_s > 2\omega_M$



$\omega_s < 2\omega_M$



Örnekleme Teoremi:

$x(t)$ $X(j\omega)=0$, $|\omega|>\omega_M$ **bant sınırlı sinyal olsun.**

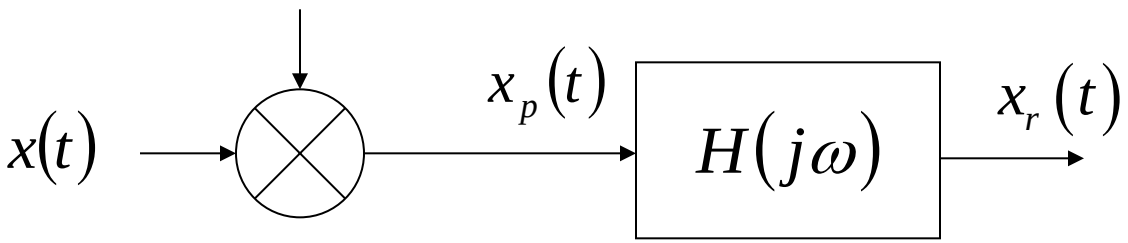
$\omega_s > 2\omega_M$ $\omega_s = \dfrac{2\pi}{T}$

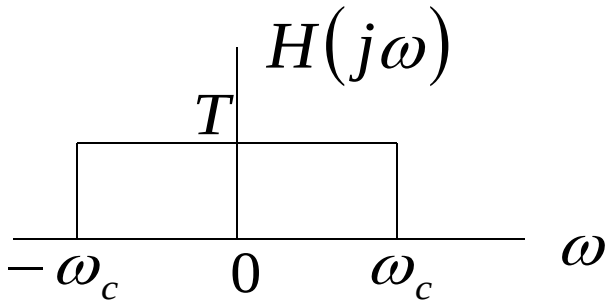
$x(t)$, $x(nT), n=0,\pm1,\cdots$ **örnekleri tarafından oluşturulabilir.**

$2\omega_M$: **Nyquist Hızı (Nyquist Frekansı)**

Sinyalin yeniden oluşturulması

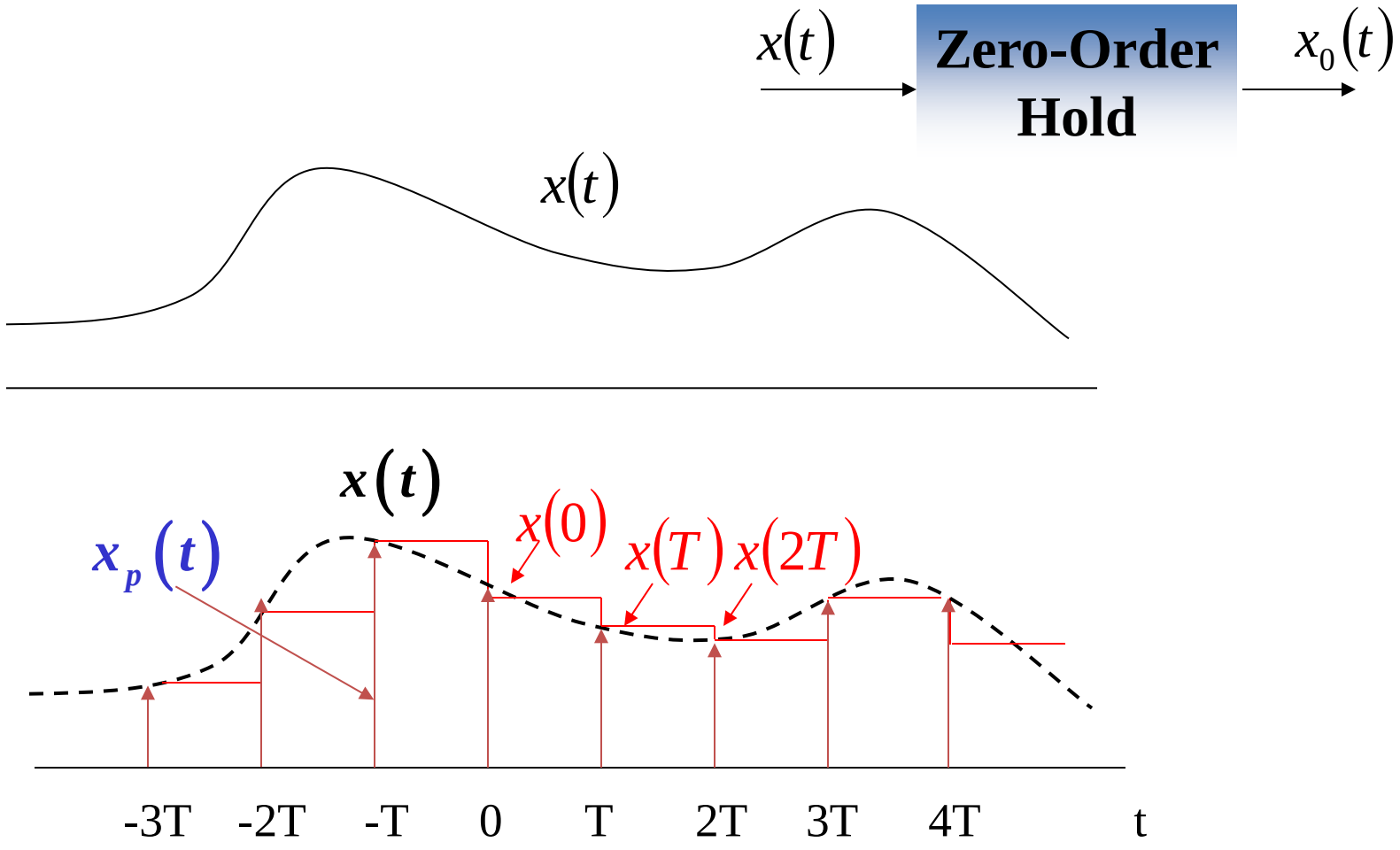
$$p(t)=\sum_{n=-\infty}^{+\infty}\delta(t-nT)$$

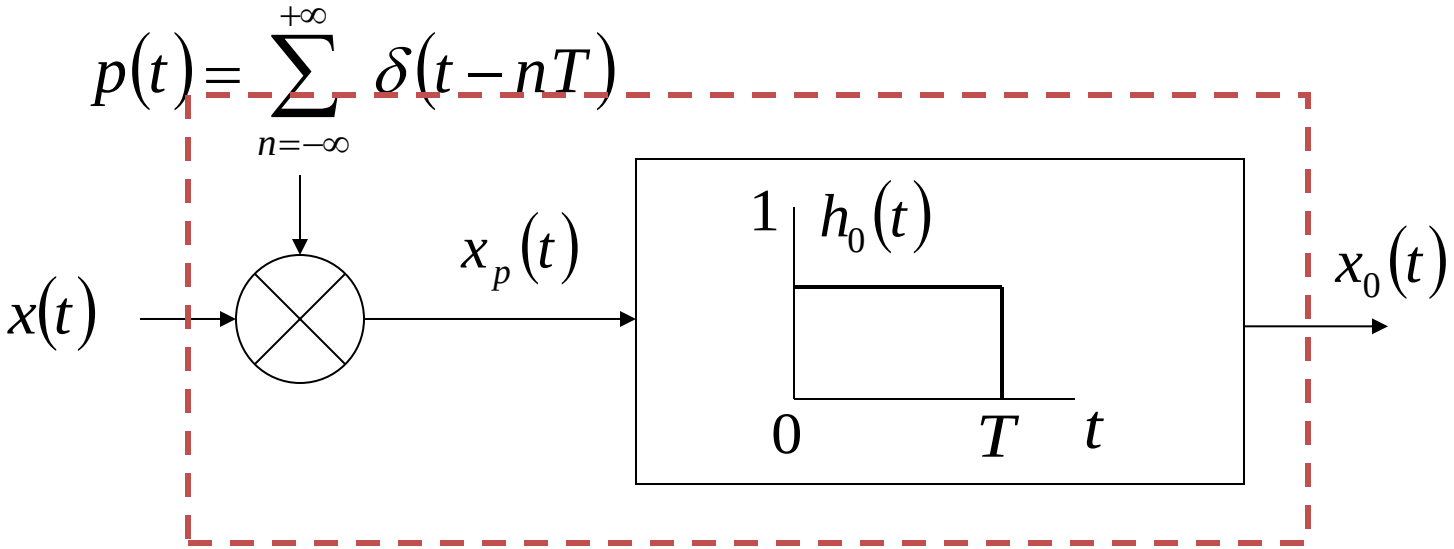




$$\omega_M < \omega_c < \omega_s - \omega_M$$

1.2. Sıfırıncı Dereceden Bir tutunmalı Örnekleme





Zero-Order Hold

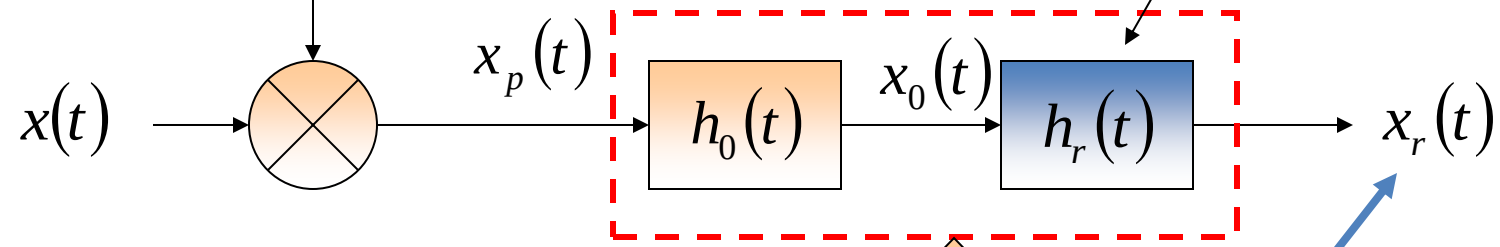
$$x_0(t)=x_p(t)*h_0(t)=\sum_{n=-\infty}^{+\infty}x(nT)h_0(t-nT)$$

$$H_o(j\omega)=e^{-j\omega T/2}\left[\frac{2\sin(\omega T/2)}{\omega}\right]$$

Zero-Order Hold

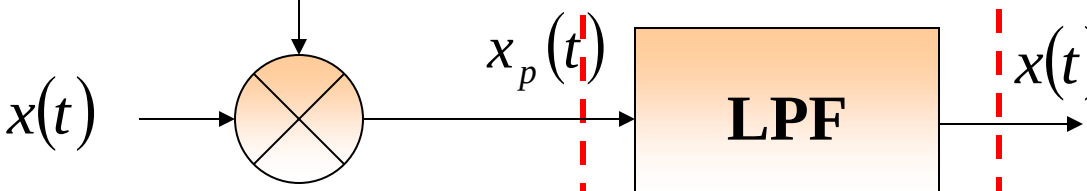
$$p(t)=\sum_{n=-\infty}^{+\infty}\delta(t-nT)$$

Yeniden Oluşturma Filtre



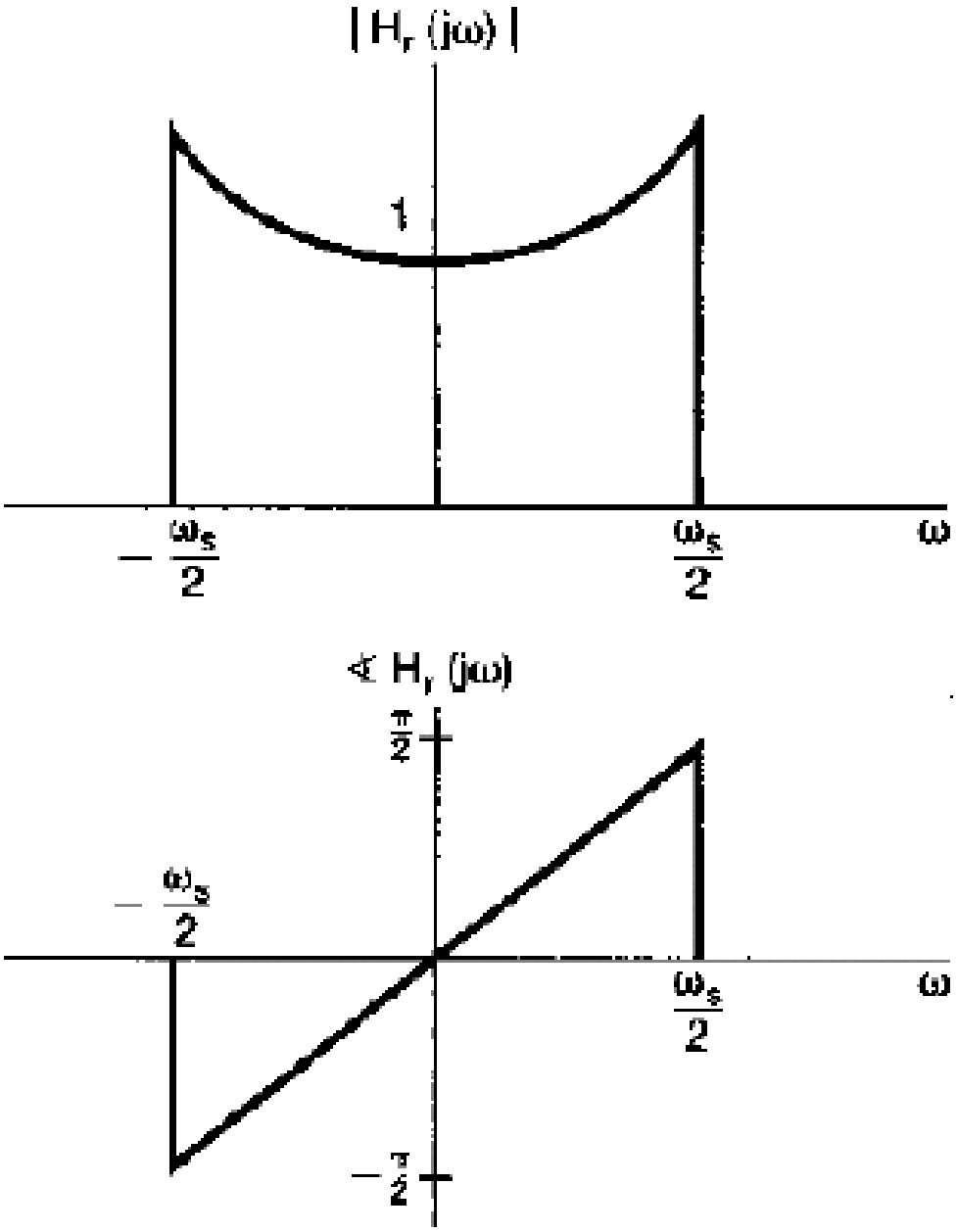
$$p(t)=\sum_{n=-\infty}^{+\infty}\delta(t-nT)$$

$$x_r(t)=x(t)$$



$$H_r(j\omega)=\frac{e^{j\omega T/2}H(j\omega)}{2\sin(\omega T/2)}{\omega}$$

$$\omega_M<\omega_c<\omega_s-\omega_M$$

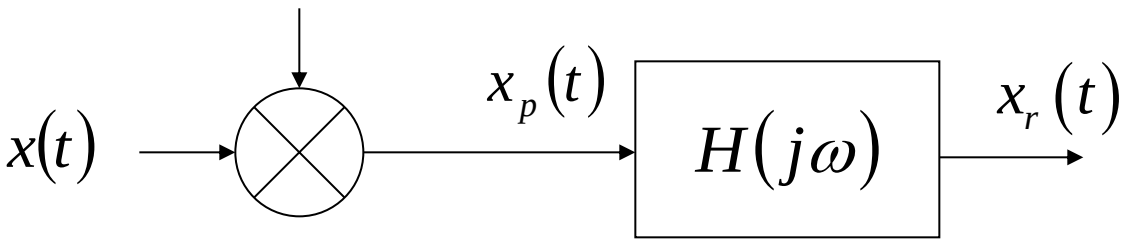


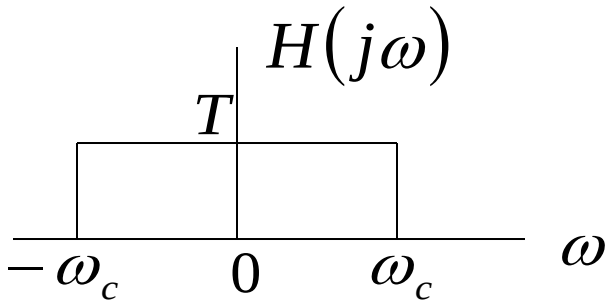
$$H_r(j\omega) = \frac{e^{j\omega T/2} H(j\omega)}{\dfrac{2\sin(\omega T/2)}{\omega}}$$

2. Sinyalin Yeniden Oluşturulması

Bant-sınırlı ara değerleme

$$p(t)=\sum_{n=-\infty}^{+\infty}\delta(t-nT)$$





$$\omega_M < \omega_c < \omega_s - \omega_M$$

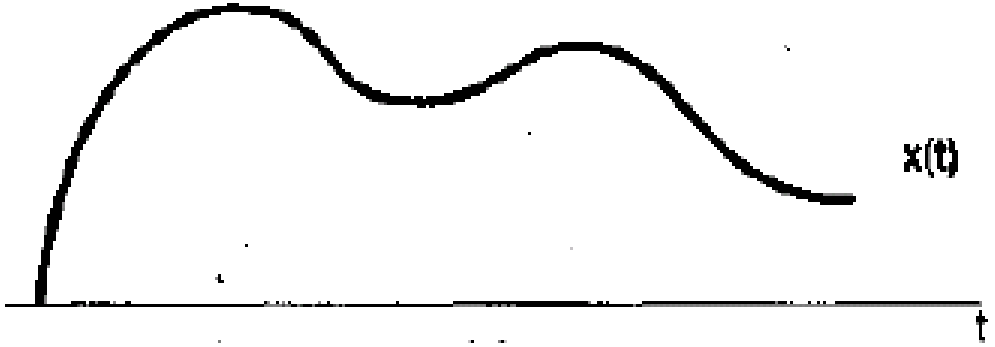
$$x_r(t) = x_p(t) * h(t) \qquad h(t) = \frac{T\sin(\omega_c t)}{\pi t}$$

$$= \sum_{n=-\infty}^{\infty}\left(x(nT)\delta(t-nT)\right) * h(t)$$

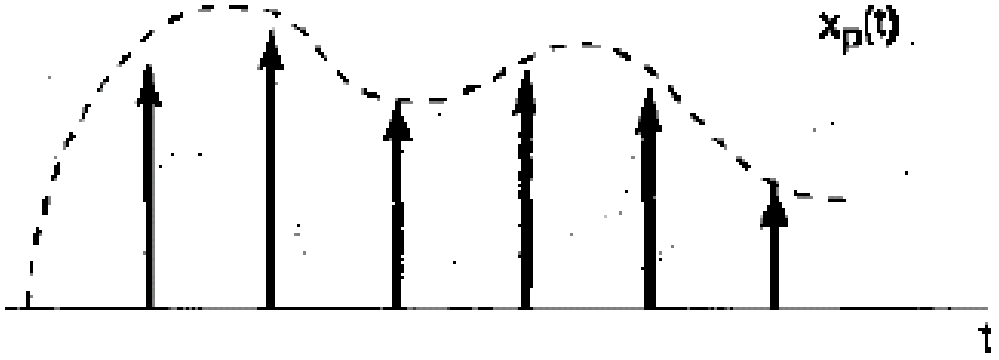
$$= \sum_{n=-\infty}^{\infty} x(nT)h(t-nT) \qquad = \sum_{n=-\infty}^{\infty} x(nT)\frac{T\sin[\omega_c(t-nT)]}{\pi(t-nT)}$$

Sinyalin Yeniden Oluşturulması

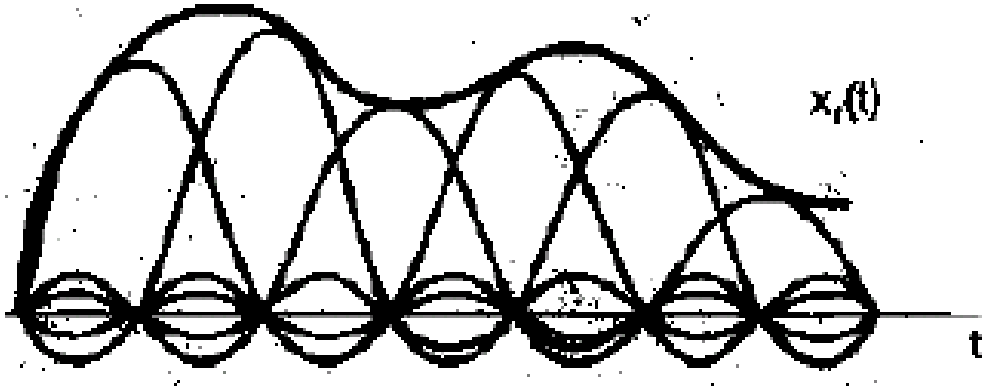
Orijinal Sinyal



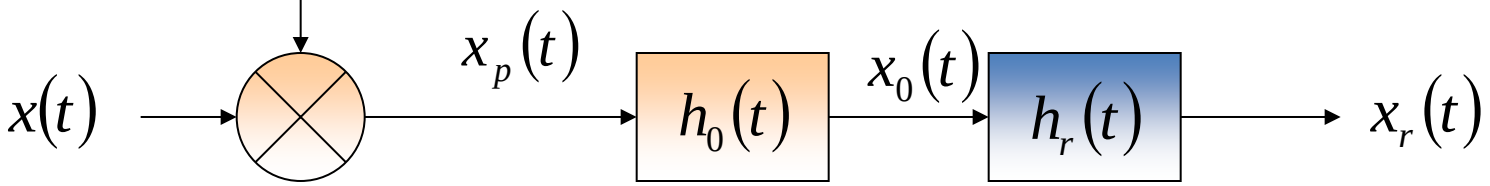
Örneklemeden Sonra



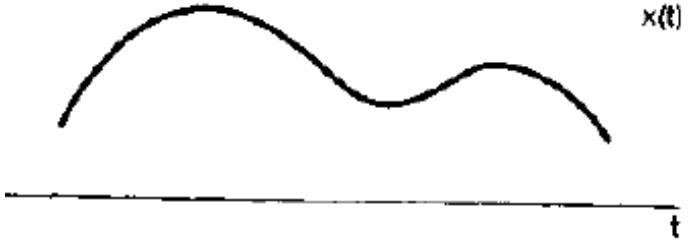
Alçak geçiren Süzgeçten geçtikten sonra



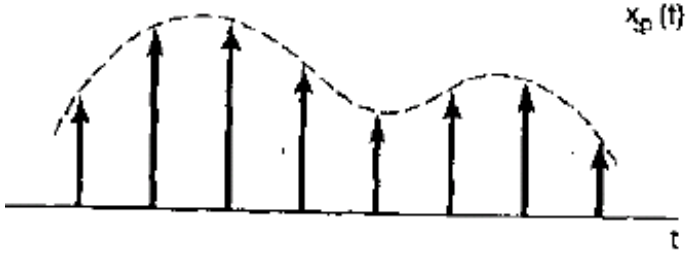
Zero-order hold $p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$



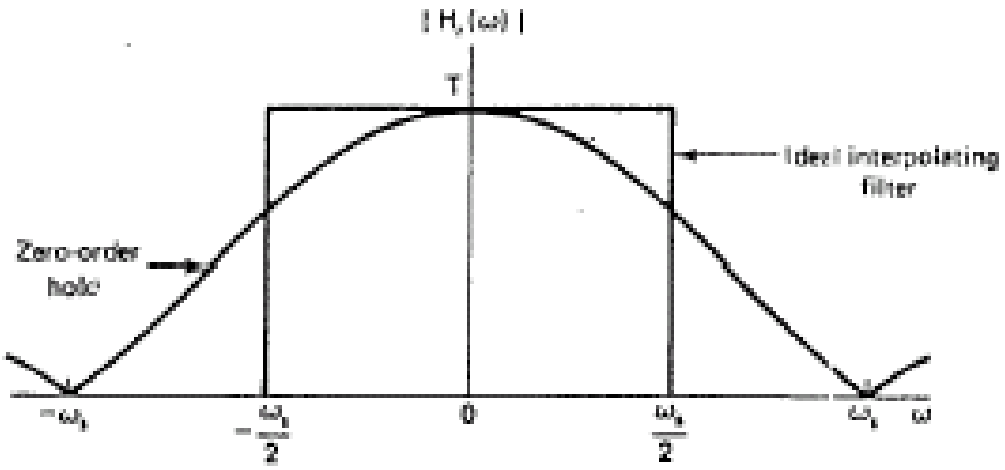
Orijinal Sinyal



Örneklemeden sonra



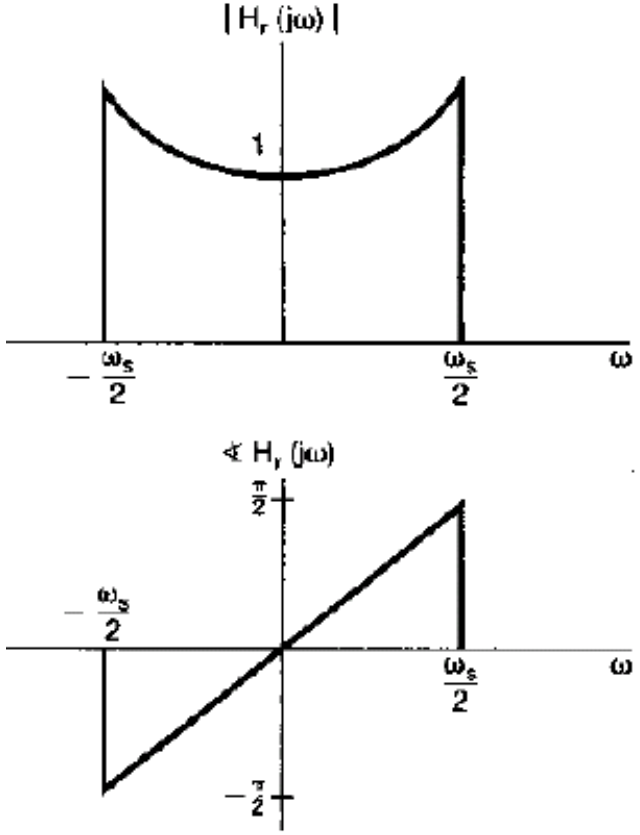
Sıfırıncı dereceden Tutma devresinden geçtikten sonra



Zero-Order Hold

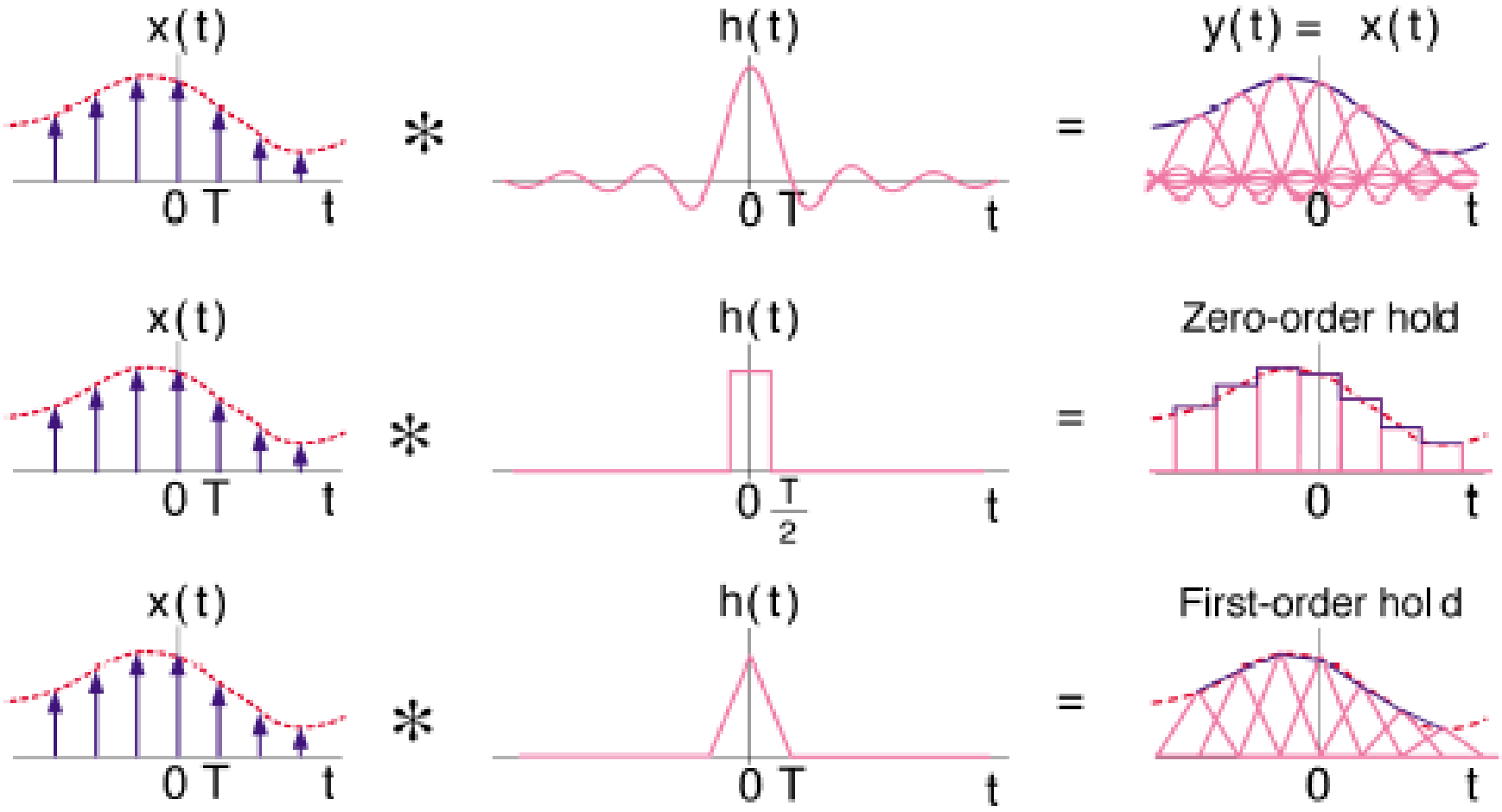
$$H_o(j\omega) = e^{-j\omega T/2}\left[\frac{2\sin(\omega T/2)}{\omega}\right]H(j\omega)$$

$$H_r(j\omega) = \frac{e^{j\omega T/2}H(j\omega)}{\dfrac{2\sin(\omega T/2)}{\omega}}$$



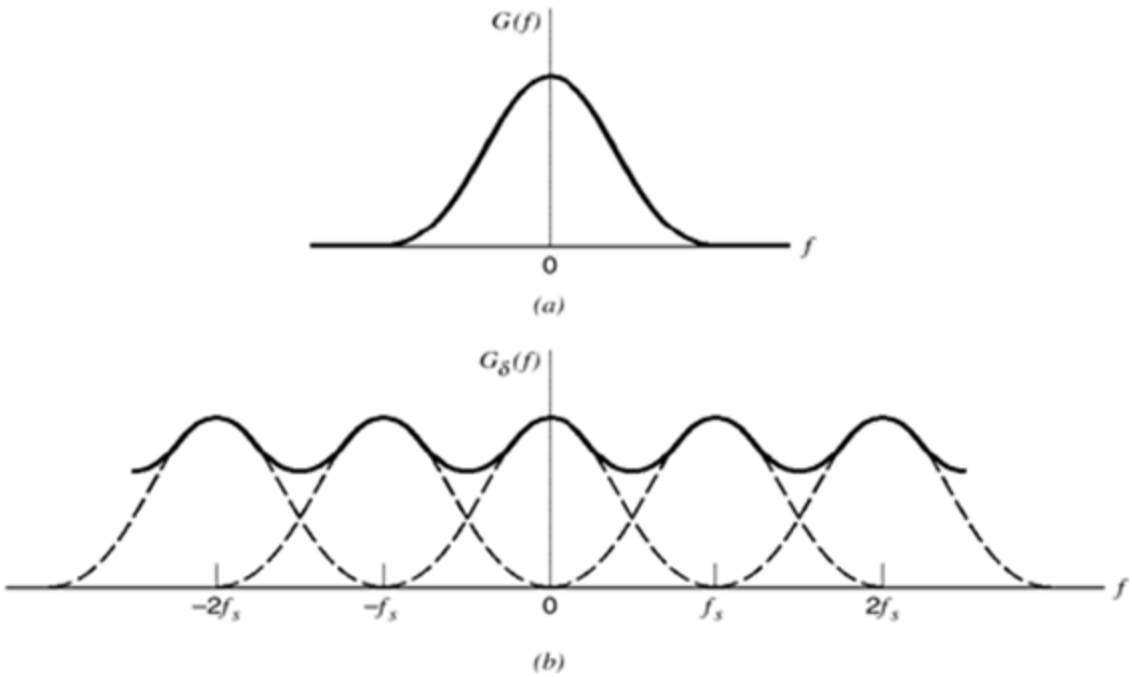
Zero-Order Hold Recover Filter

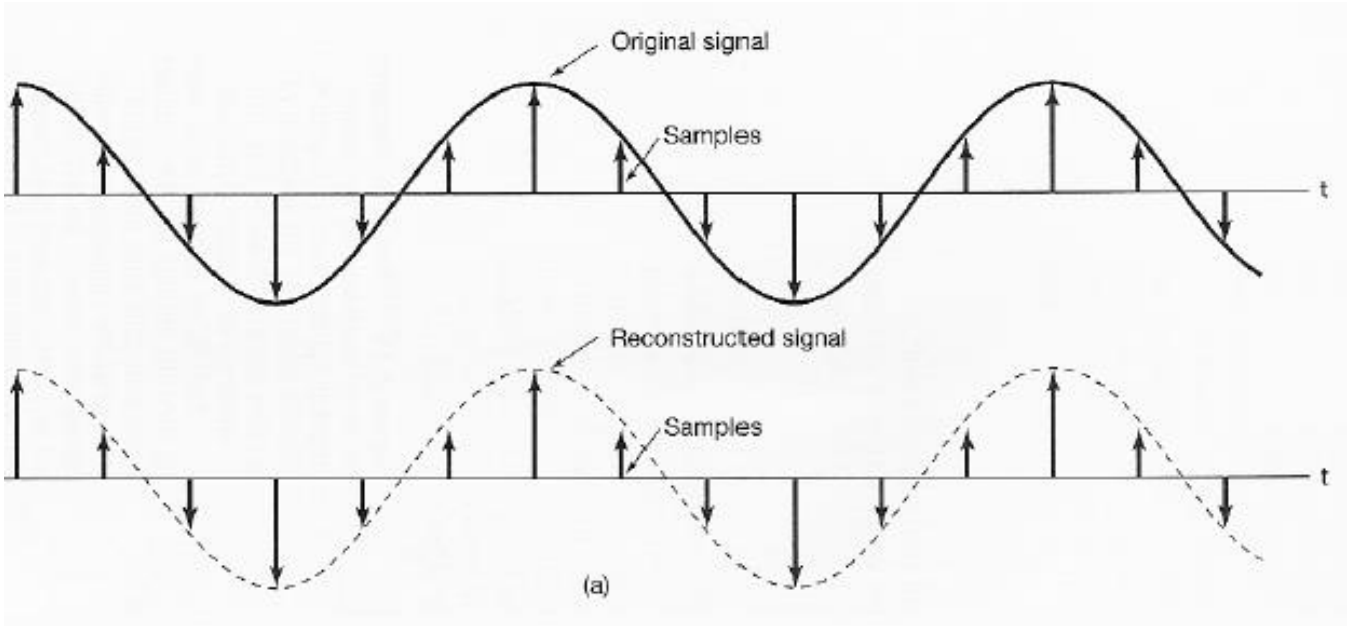
Sinyalin Yeniden Oluşturulması



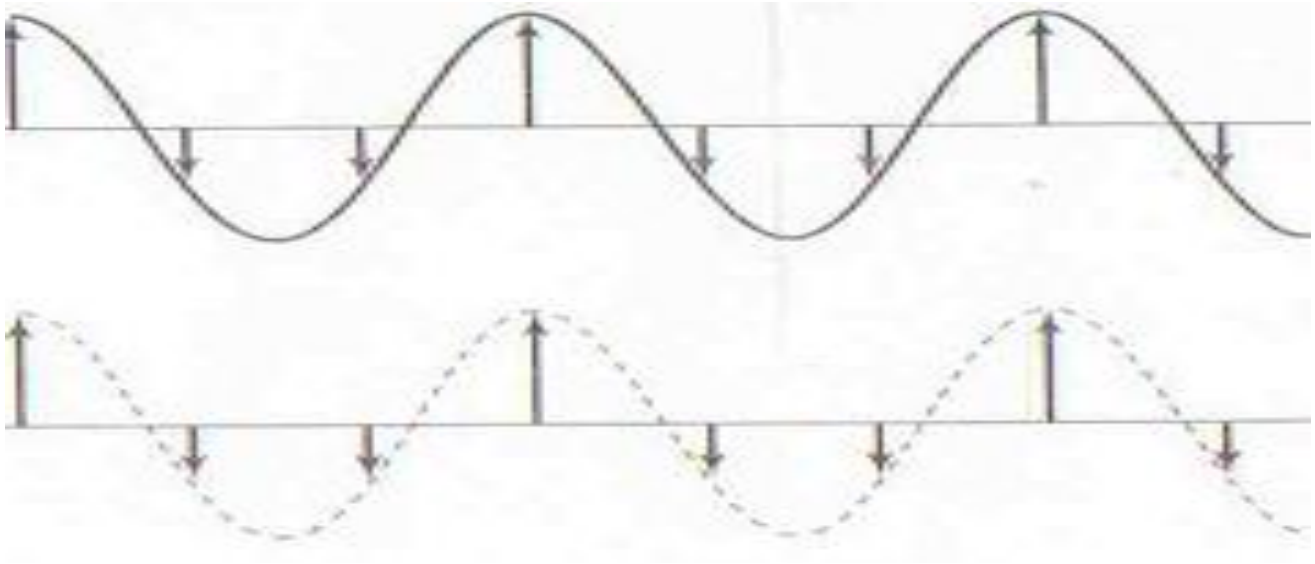
Örtüşme :

Eğer bir sinyal Nyquist hızından(frekansından) daha düşük bir hızda örneklenirse, spektral örtüşme meydana gelir.

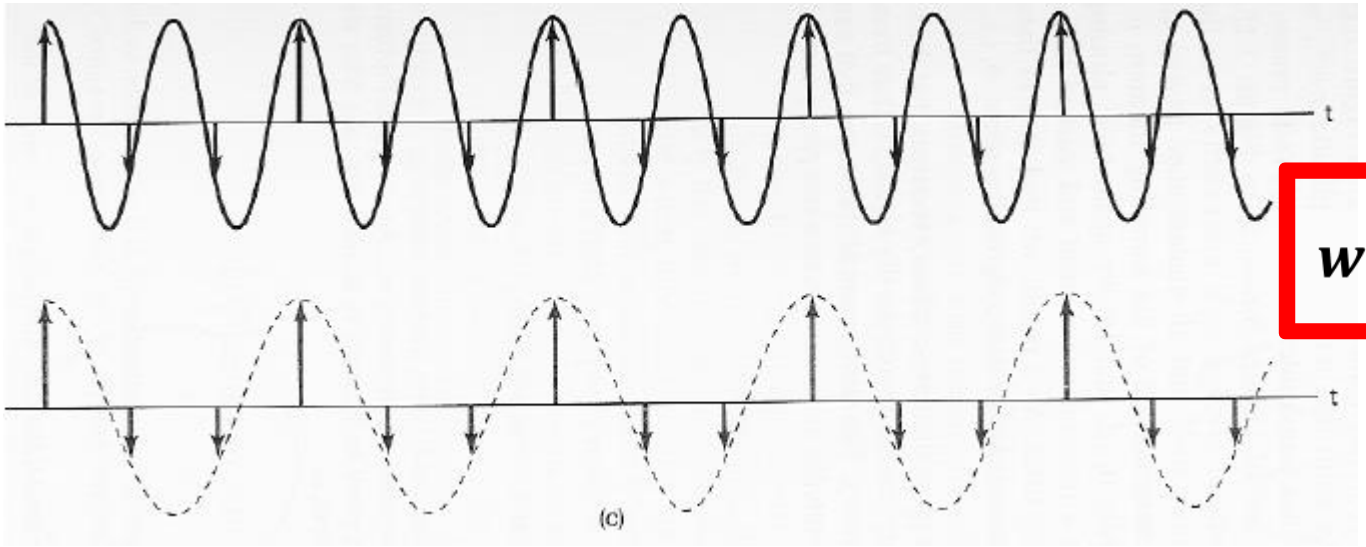




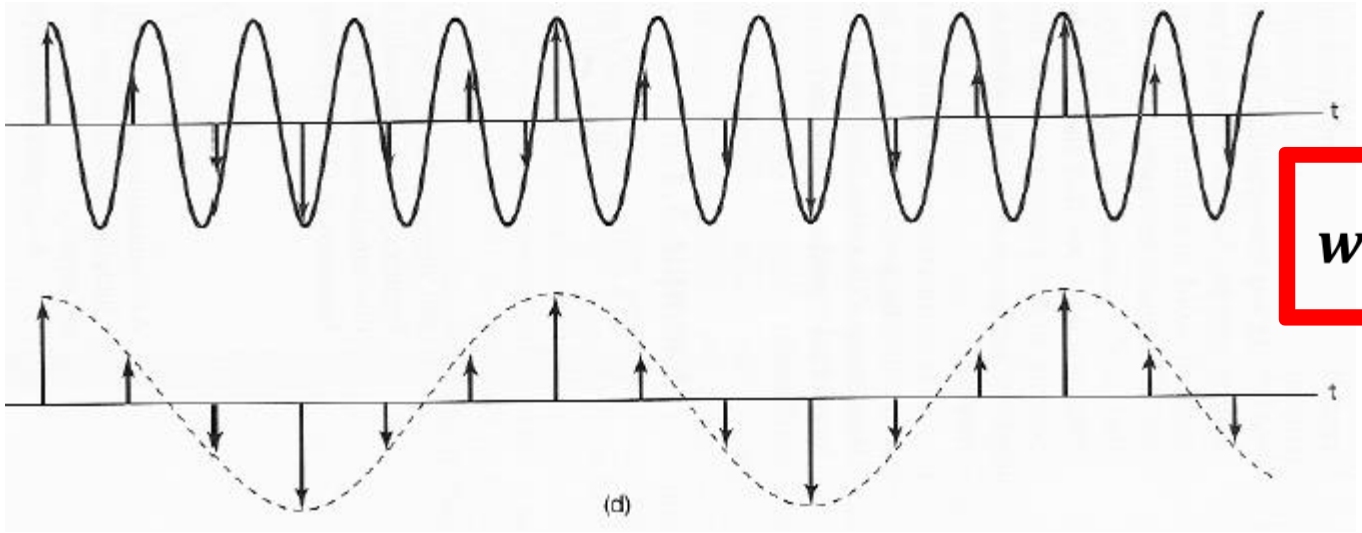
$$w_0 = \frac{w_s}{6}$$



$$w_0 = \frac{2w_s}{6}$$



$$w_0 = \frac{4w_s}{6}$$

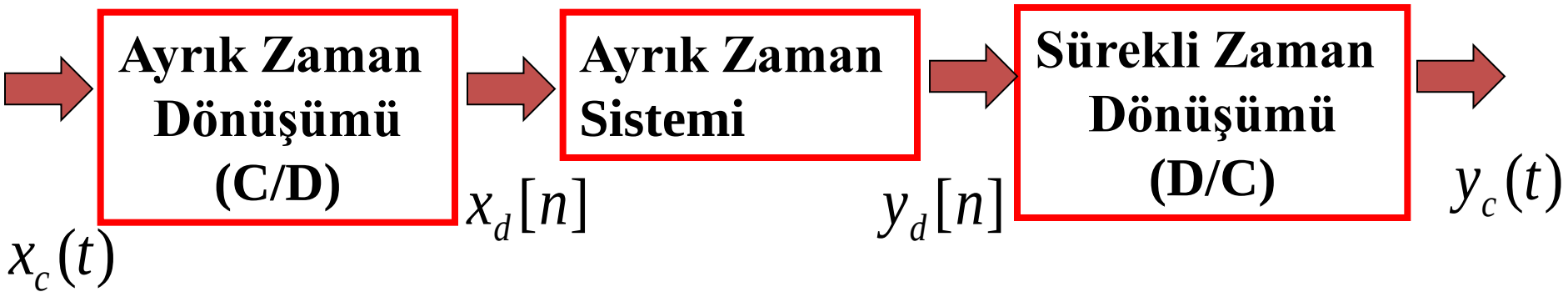


$$w_0 = \frac{5w_s}{6}$$

Sürekli Zaman Sinyallerinin Ayrık Zaman İşlemesi

$x_d[n] = x_c(nT)$

$y_d[n] = y_c(nT)$

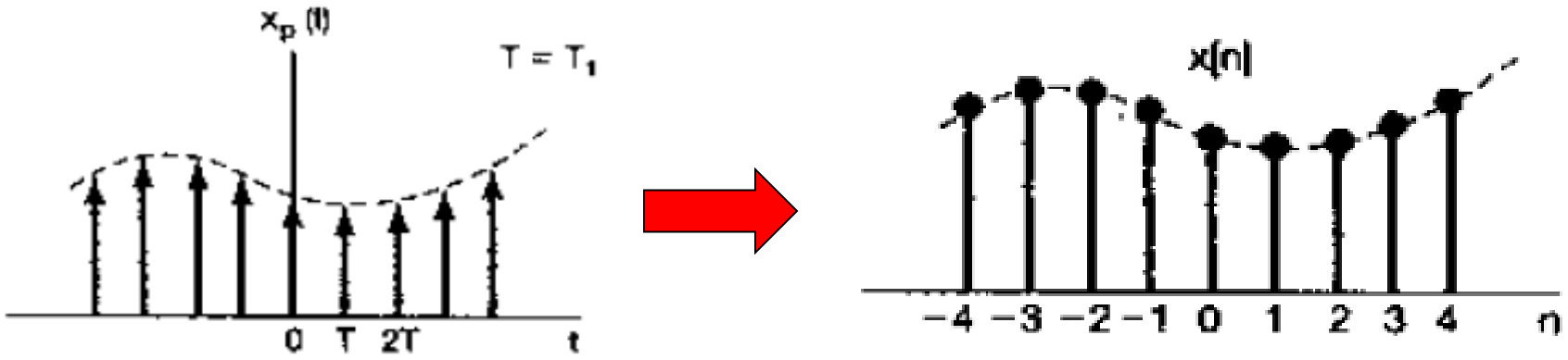


Sürekli Zaman Sinyallerinin Ayrık Zaman İşlemesi

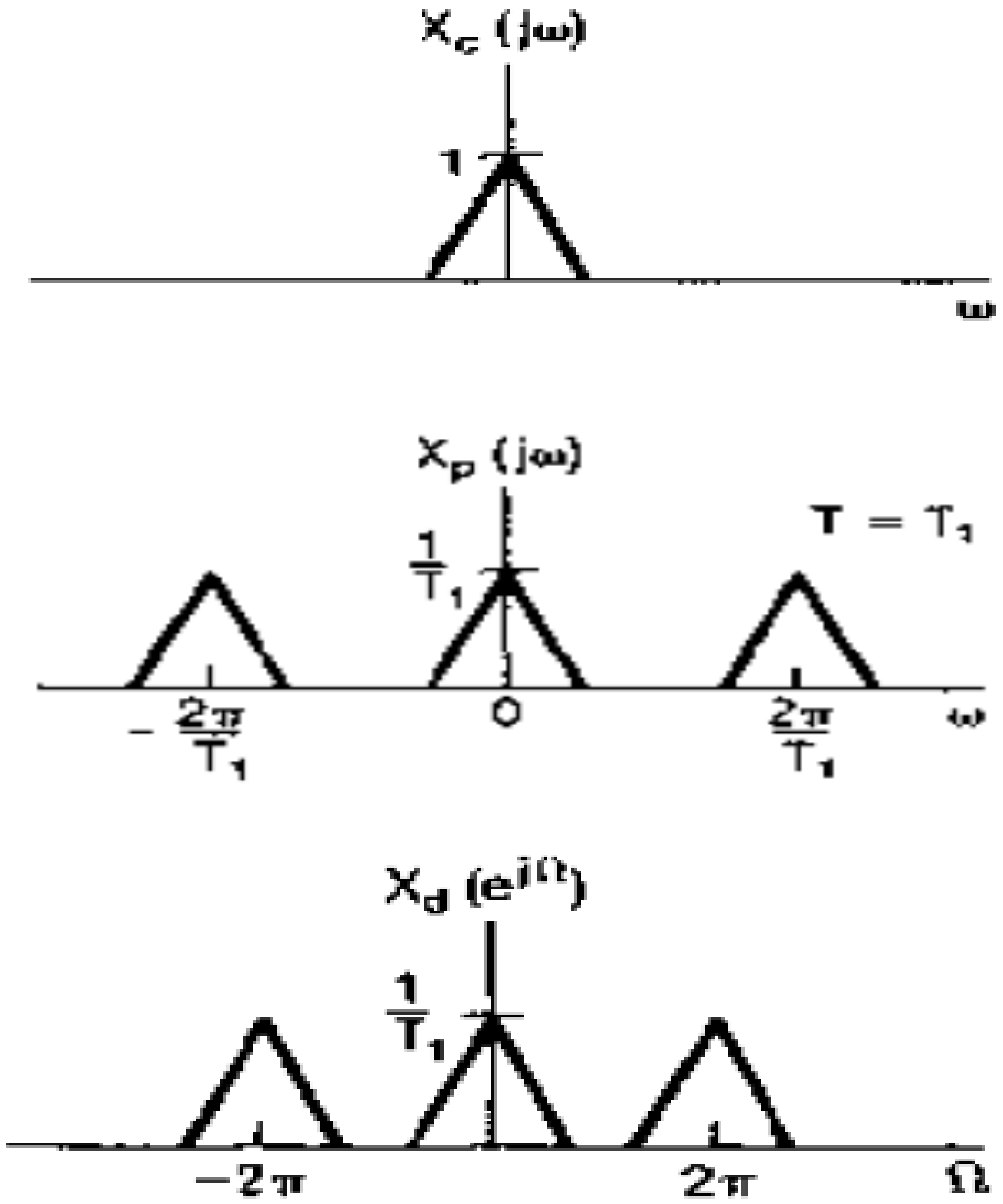
$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

$x_c(t)$ → $x_p(t)$ → **Zaman sinyalinin ayrık zaman dizisine dönüşümü** → $x[n]=x_c(nT)$

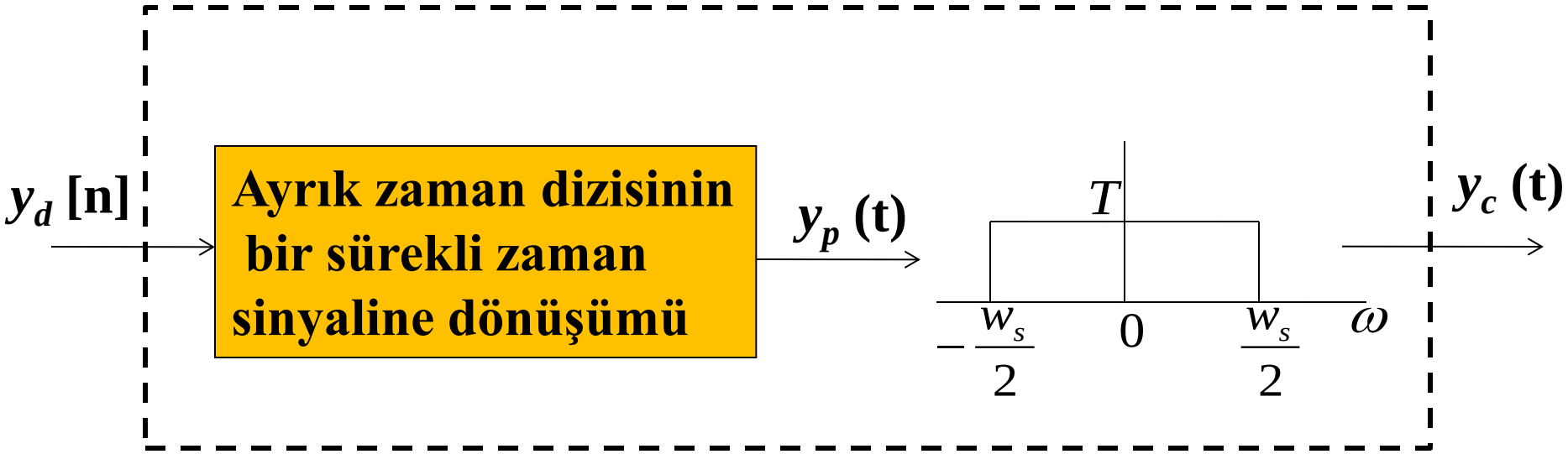
C/D dönüşümü



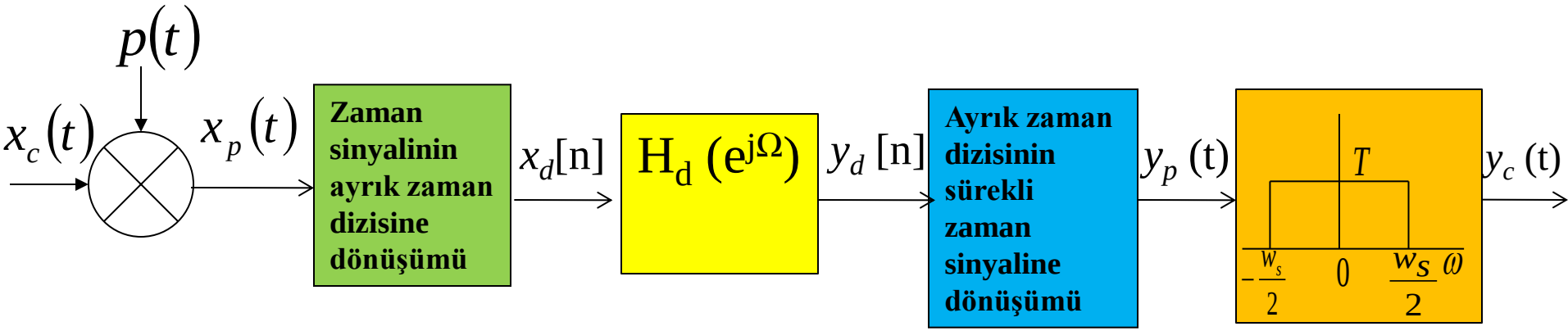
Sürekli Zaman Sinyallerinin Ayrık Zaman İşlemesi



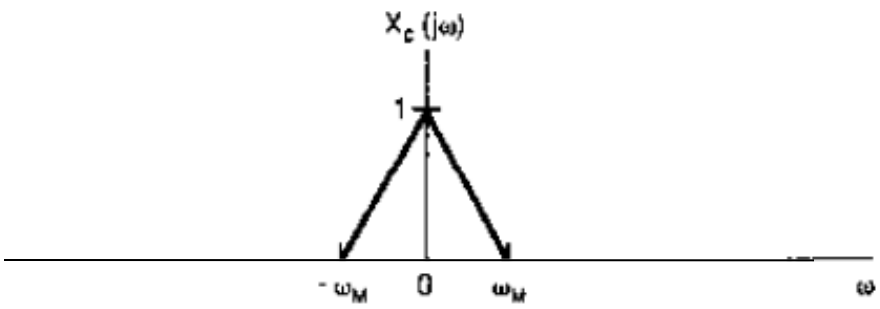
D/C Dönüşümü

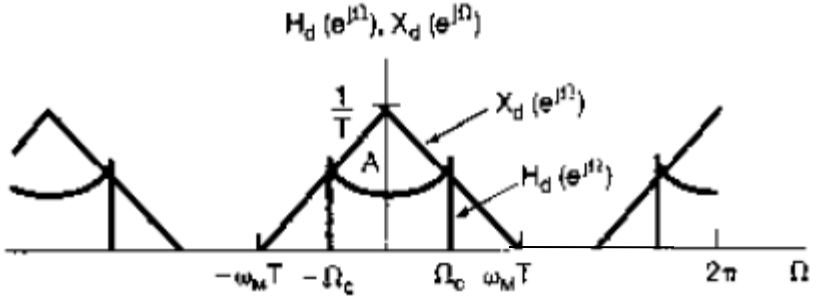

y_d [n]
Ayrık zaman dizisinin bir sürekli zaman sinyaline dönüşümü
y_p (t)
T
$-\frac{w_s}{2}$
0
$\frac{w_s}{2}$
ω
y_c (t)

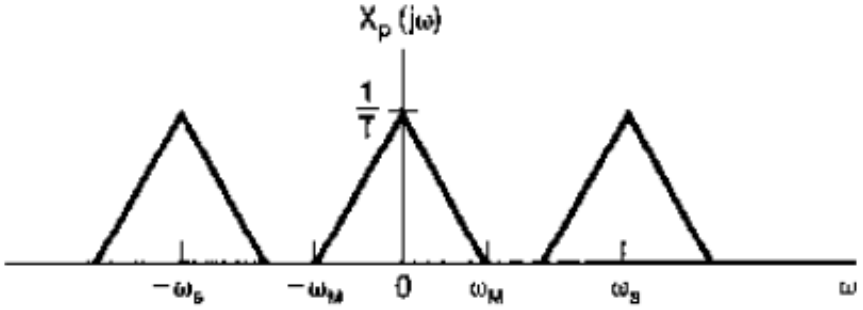
Sürekli Zaman Sinyallerinin Ayrık Zaman İşlemesi

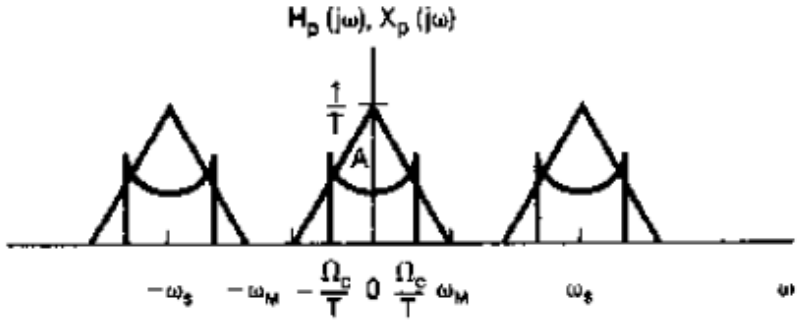


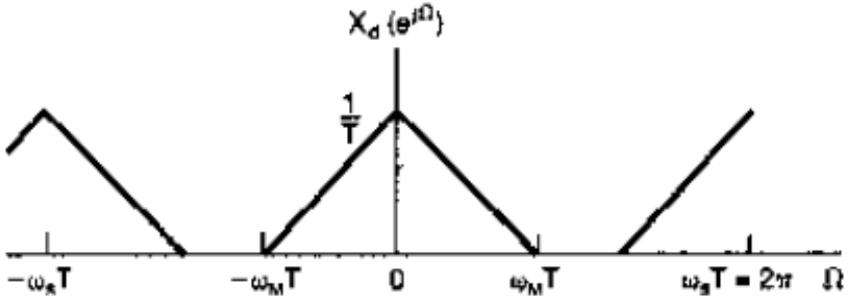
Sürekli Zaman Sinyallerinin Ayrık Zaman İşlemesi

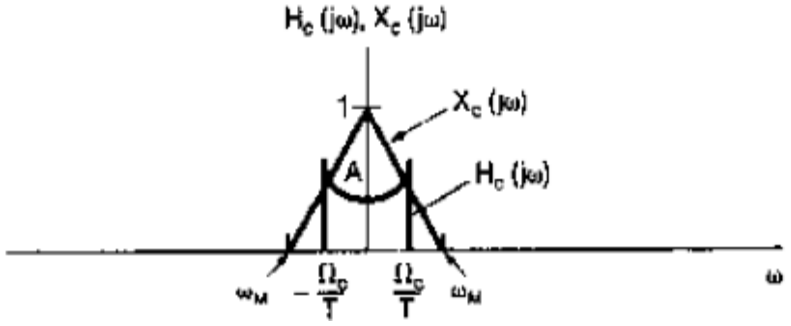


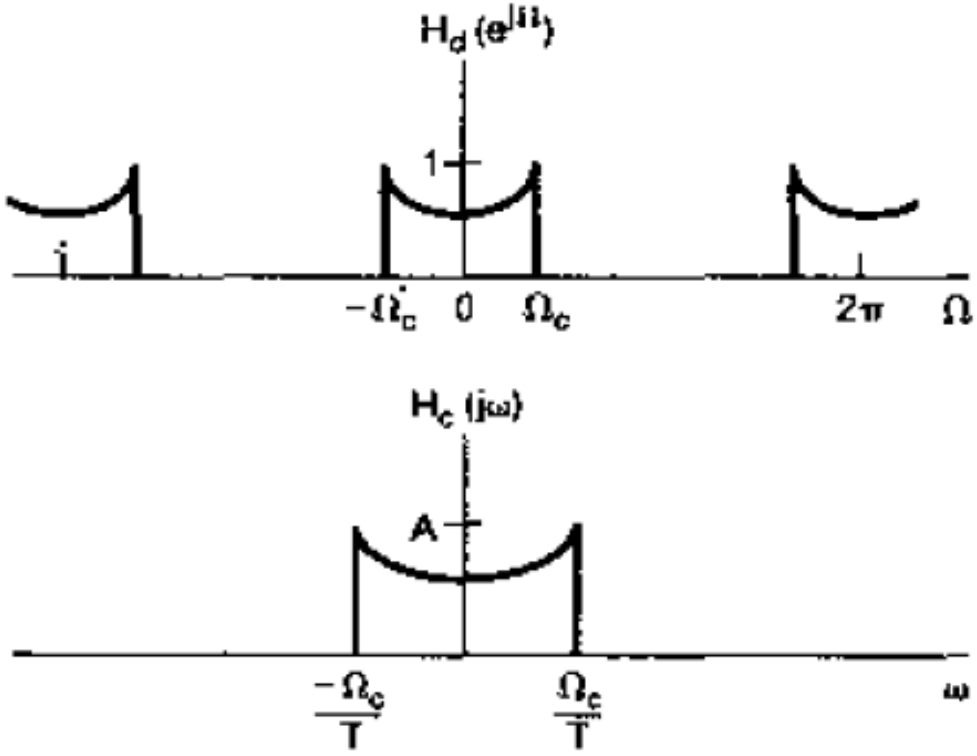




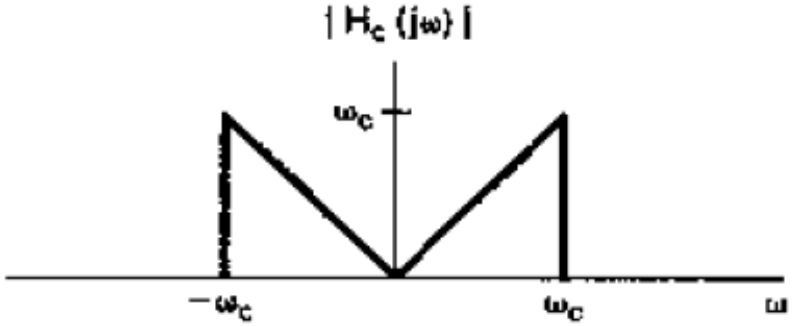


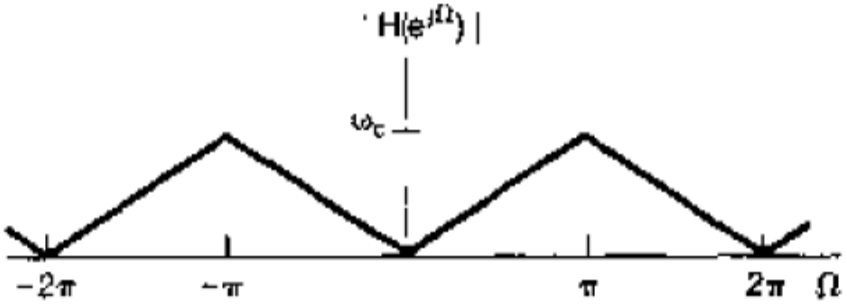


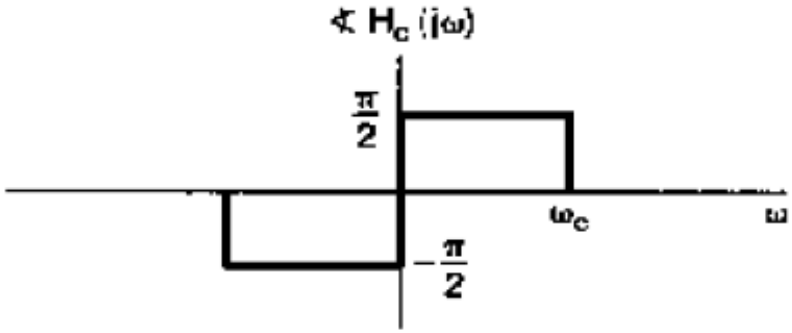


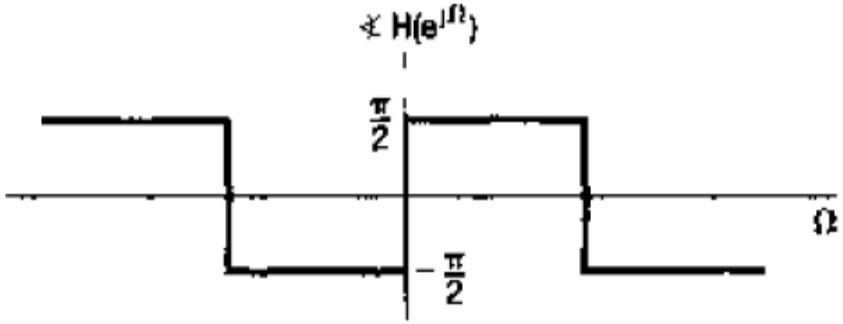

$H_d(e^{j\Omega})$
1
$-\Omega_c$
0
Ω_c
2π
Ω
$H_c(j\omega)$
A
$-\frac{\Omega_c}{T}$
$\frac{\Omega_c}{T}$
ω

Dijital Türev Alıcı

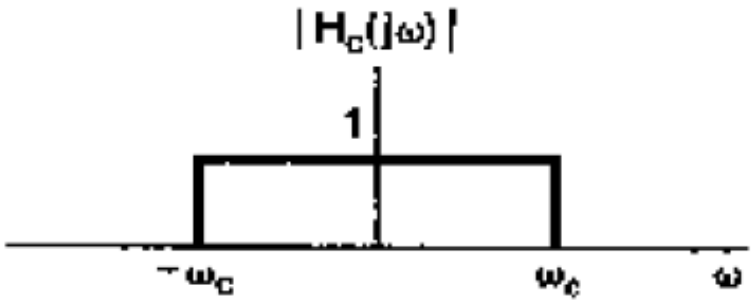


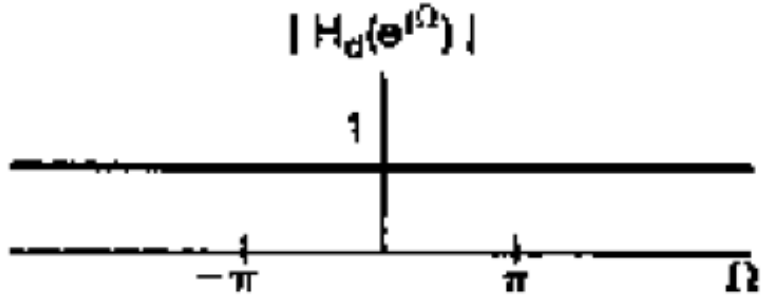


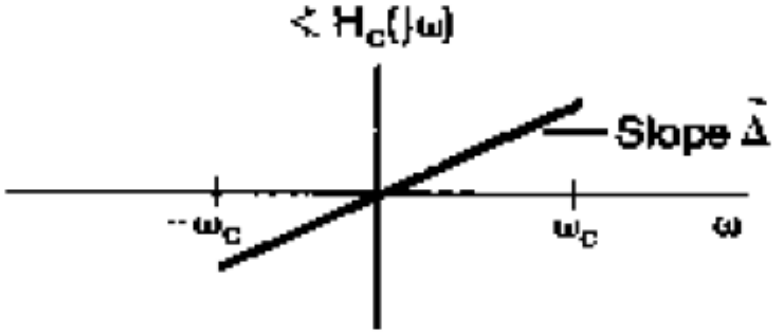


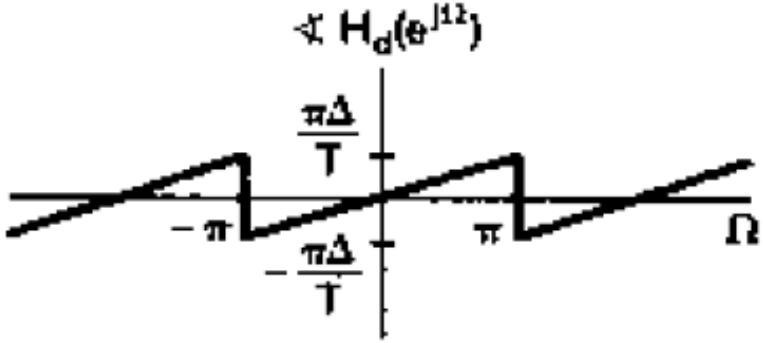


Yarı-Örneklem Gecikmesi

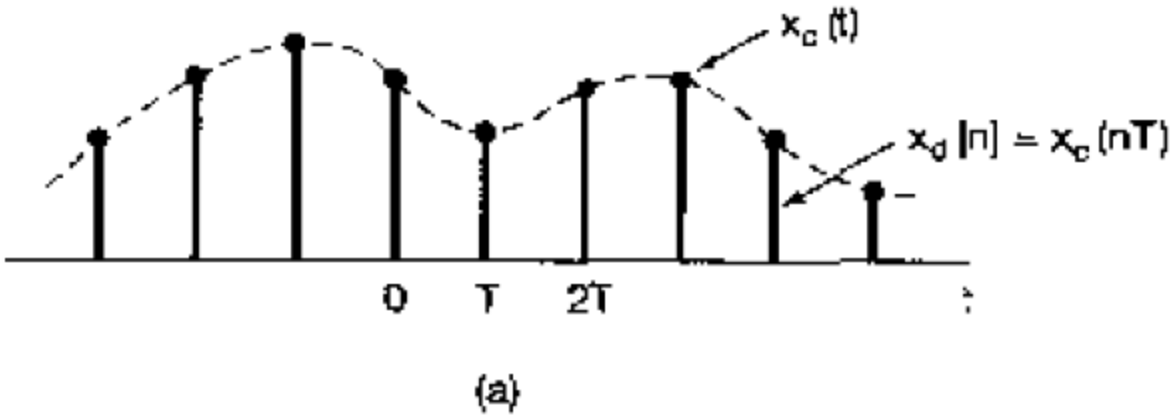




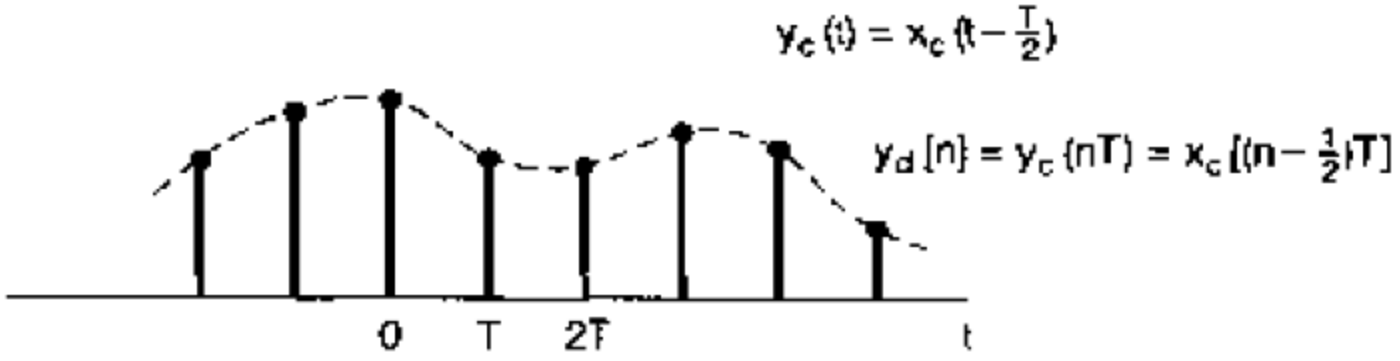




Yarı-Örneklem Gecikmesi

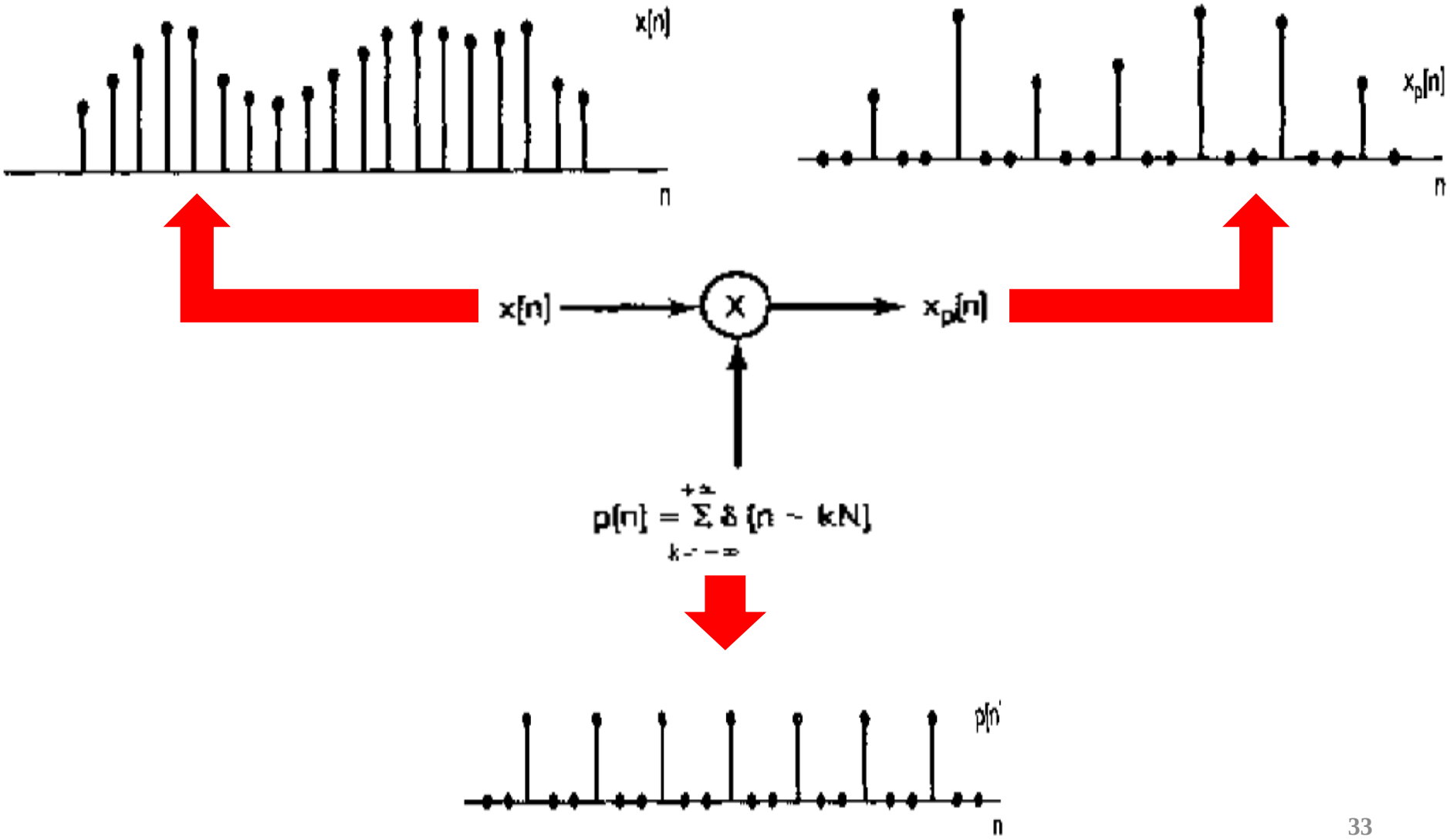


(a)

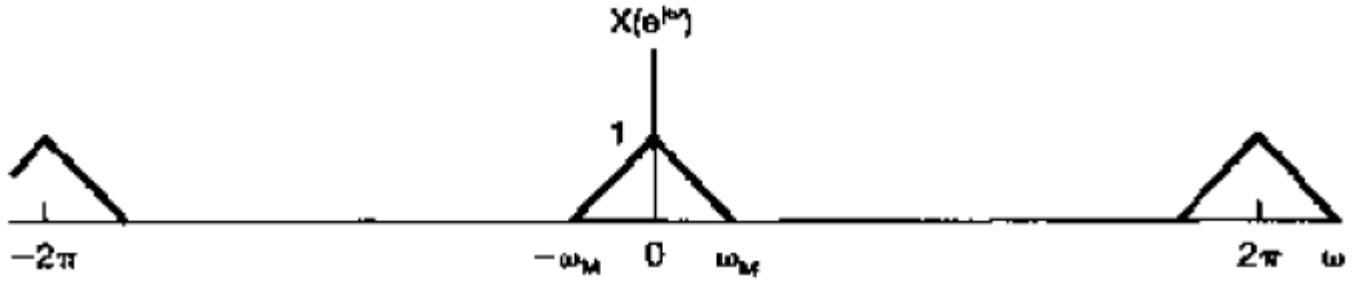


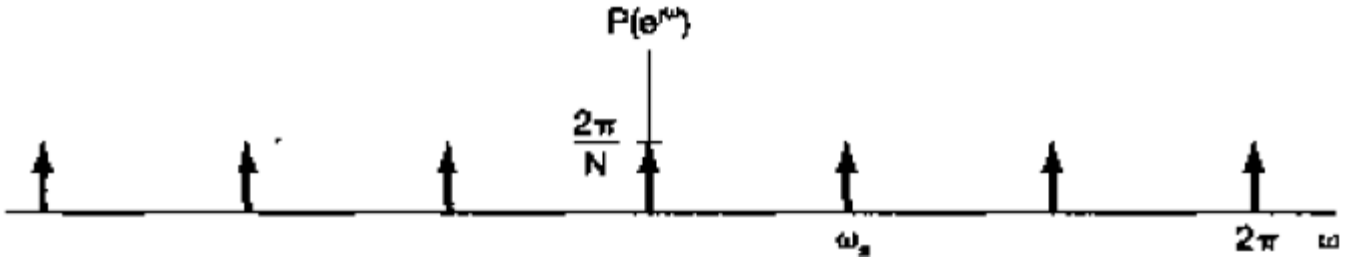
(b)

1. Dürtü Katarı Örneklemesi

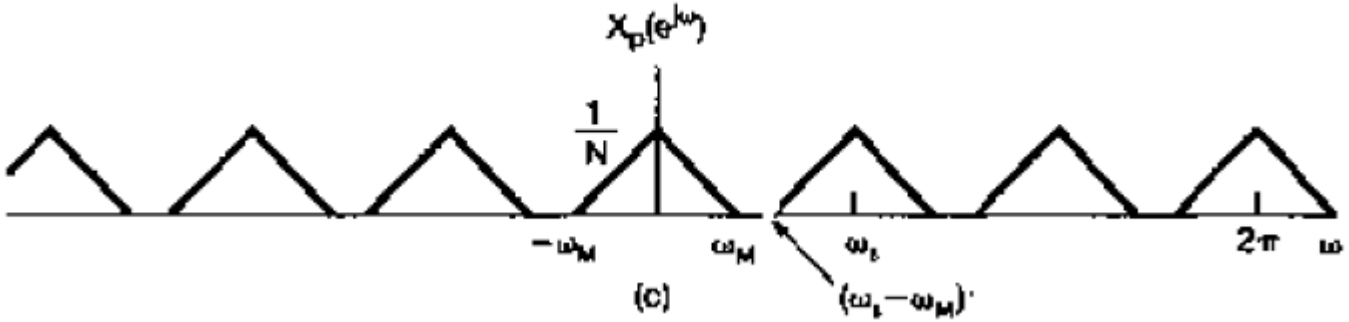


Dürtü Katarı Örneklemesi

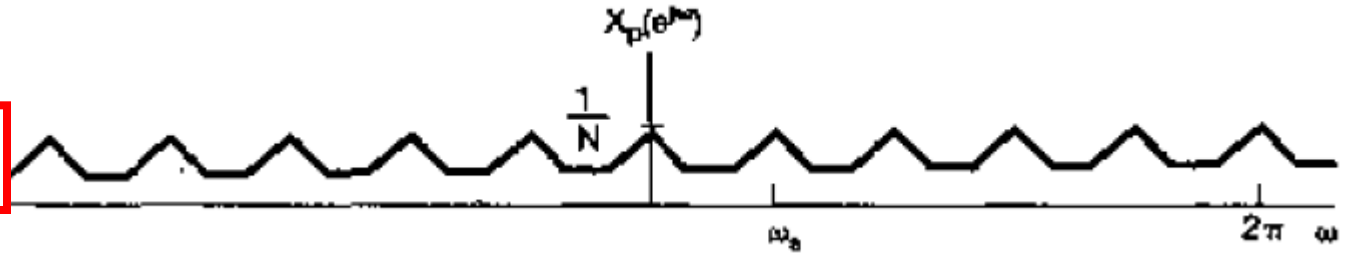




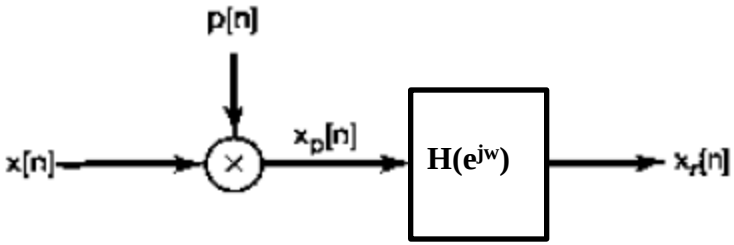
$\omega_s > 2\omega_M$



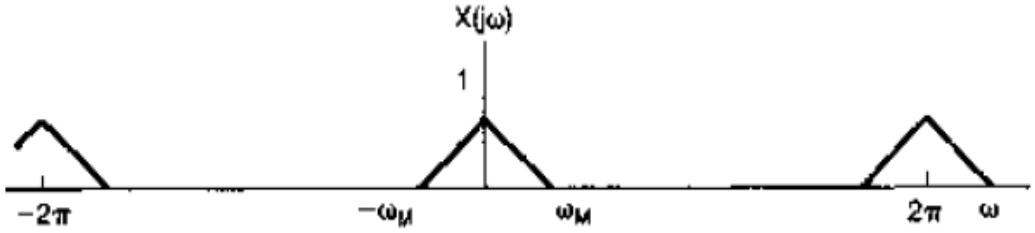
$\omega_s < 2\omega_M$

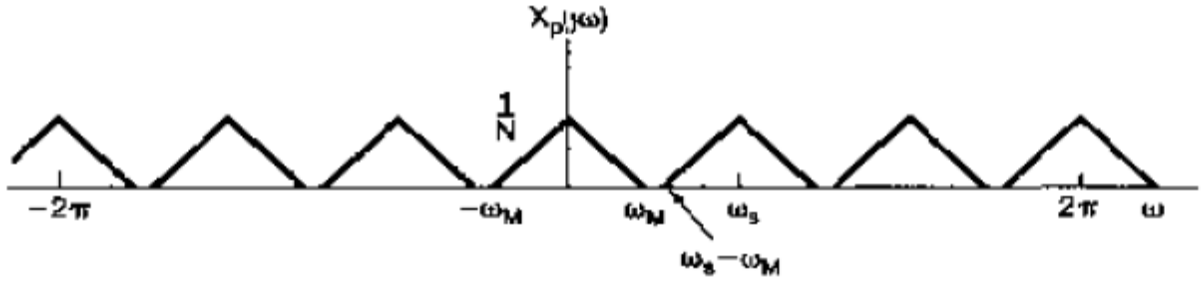


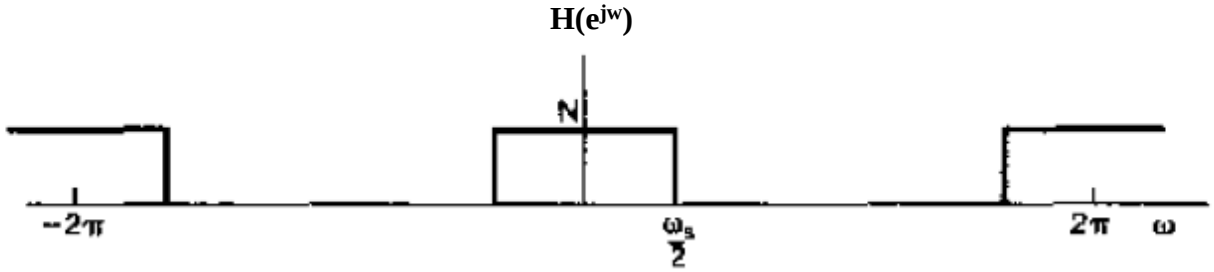
Dürtü Katarı Örneklemesi

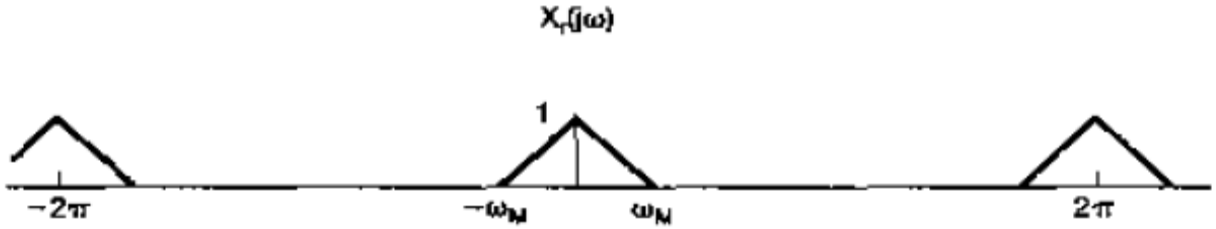


Bant sınırlı bir sinyalin örneklemlerinden geri çatılması ve Örneklenmesi için Blok Diagram

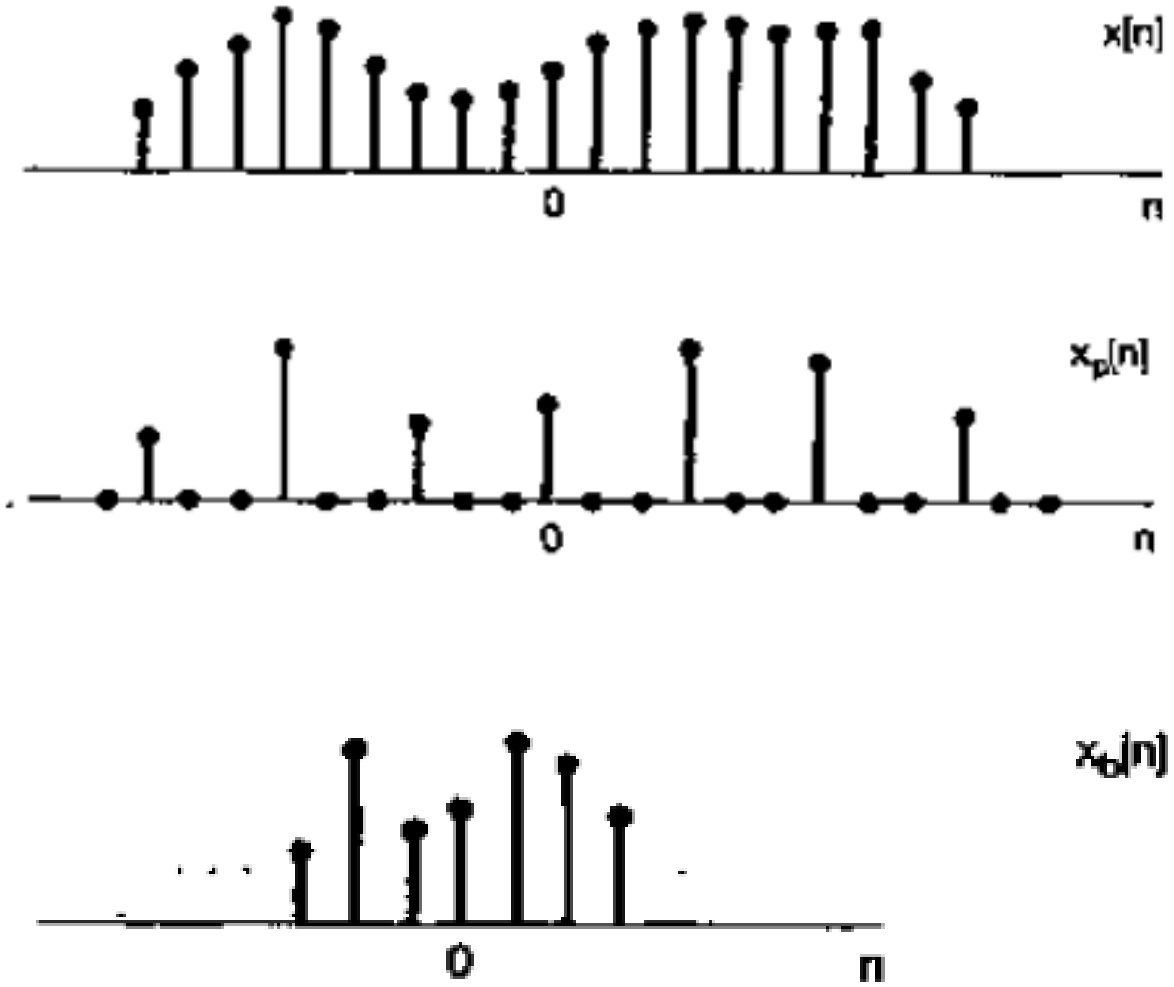




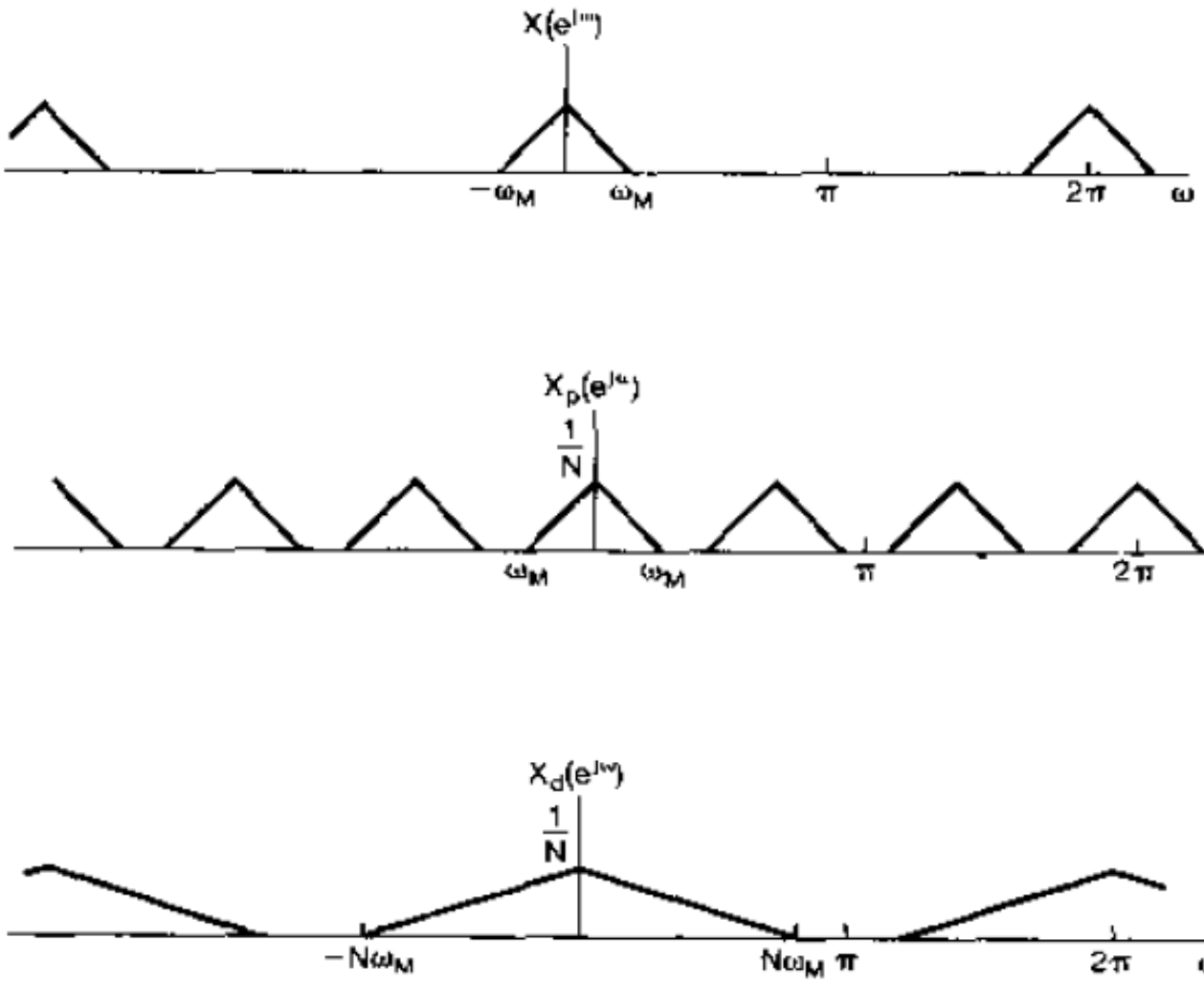




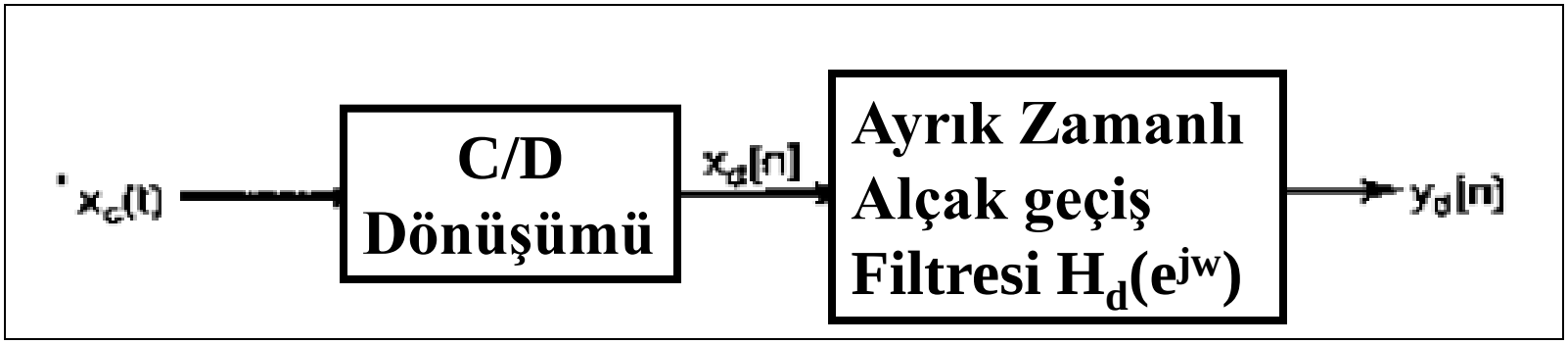
2. Ayrık Zamanlı Ondalıklama ve Ara Değerleme

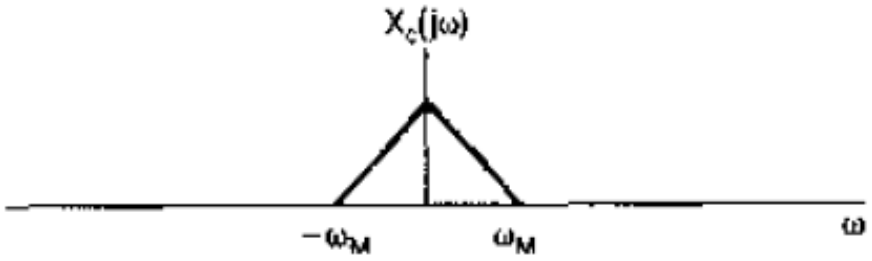


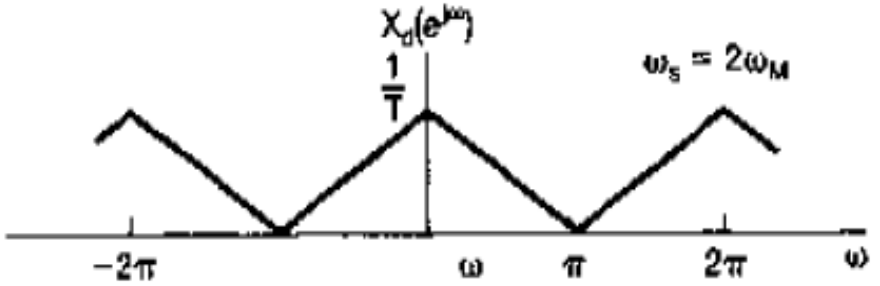
Ayrık Zamanlı Ondalıklama ve Ara Değerleme

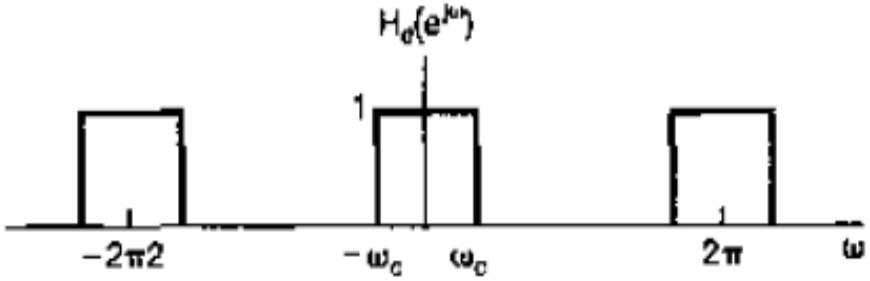


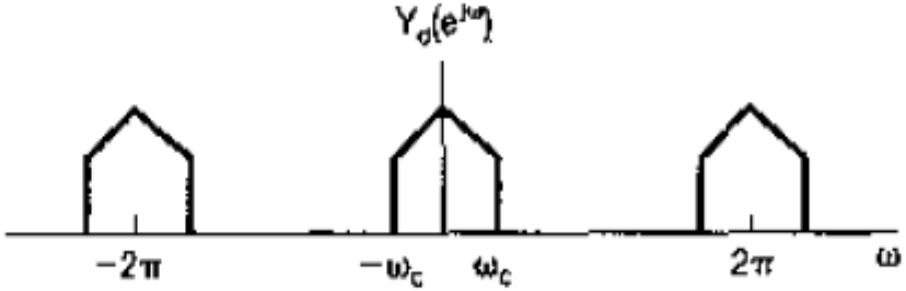
Ayrık Zamanlı Ondalıklama ve Ara Değerleme



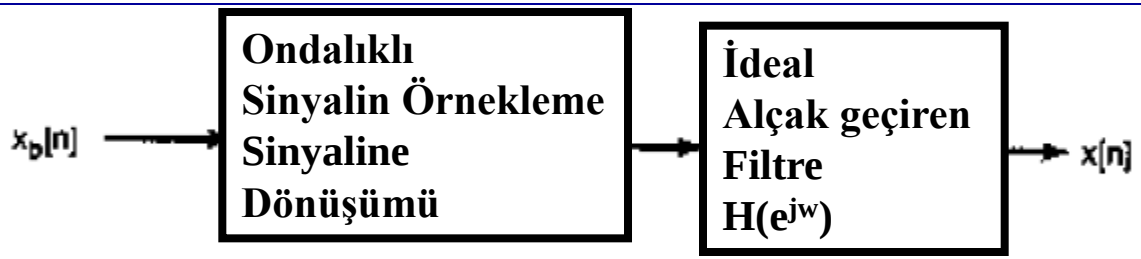


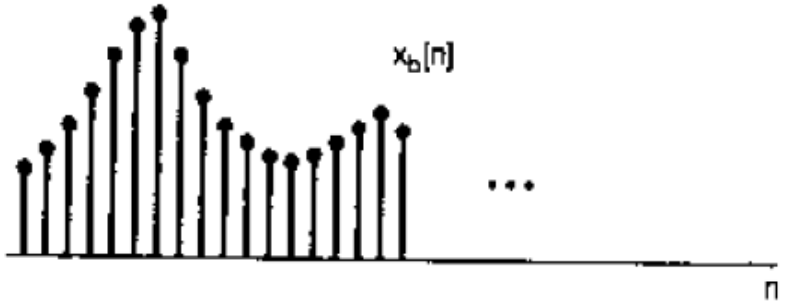


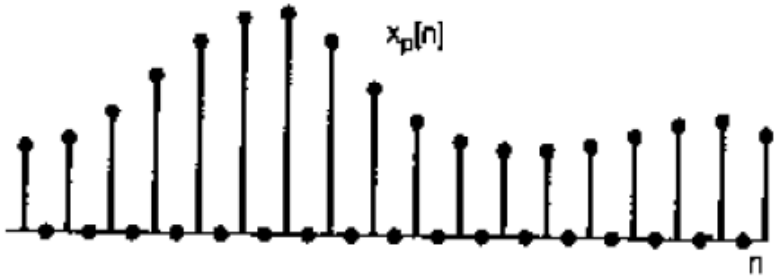


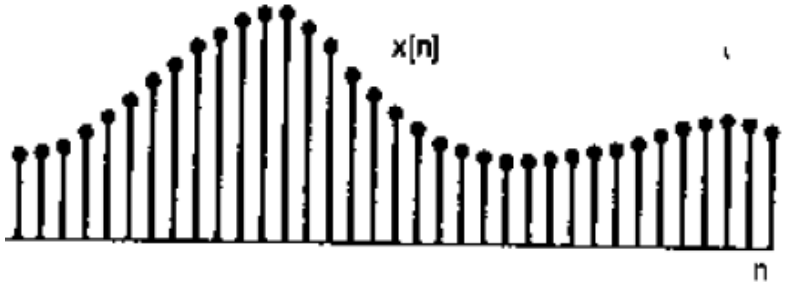


Ayrık Zamanlı Ondalıklama ve Ara Değerleme

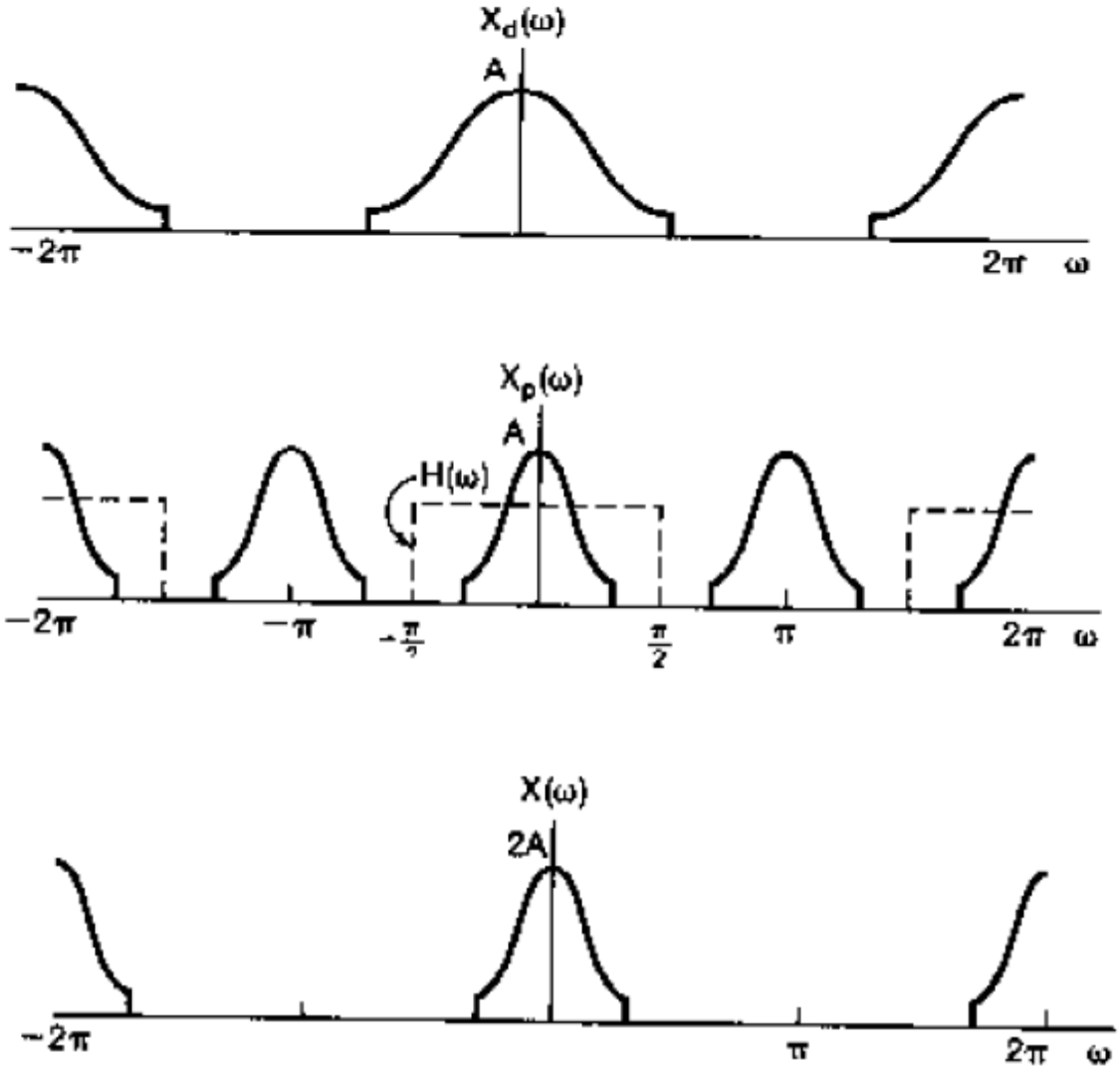




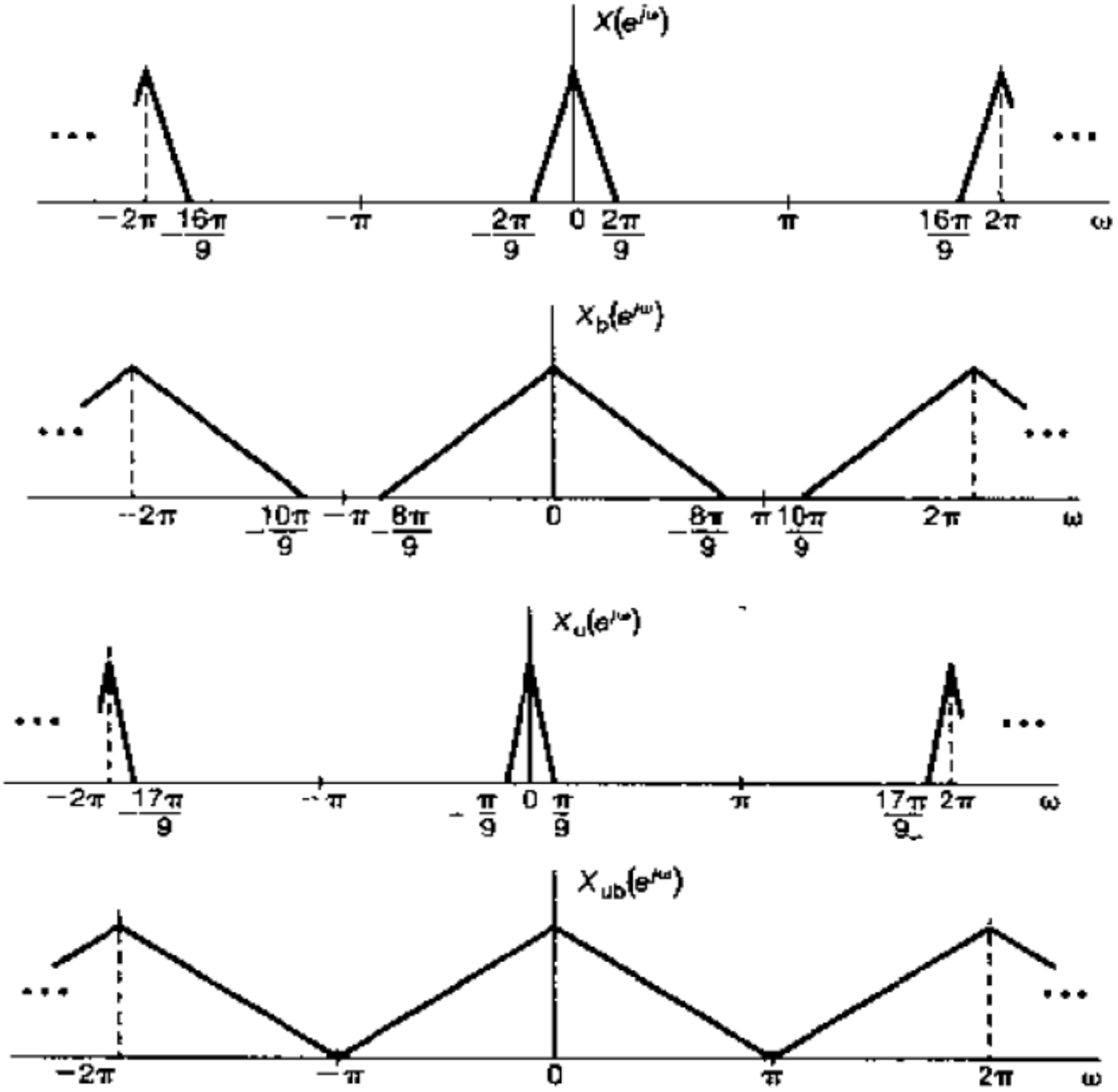




Ayrık Zamanlı Ondalıklama ve Ara Değerleme



Örnek :


$X(e^{j\omega})$
-2π $-\frac{16\pi}{9}$ $-\pi$ $-\frac{2\pi}{9}$ 0 $\frac{2\pi}{9}$ π $\frac{16\pi}{9}$ 2π ω
$X_b(e^{j\omega})$
-2π $-\frac{10\pi}{9}$ $-\pi$ $-\frac{8\pi}{9}$ 0 $\frac{8\pi}{9}$ π $\frac{10\pi}{9}$ 2π ω
$X_u(e^{j\omega})$
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$X_{ub}(e^{j\omega})$
-2π $-\pi$ 0 π 2π ω