EEM 308
SİNYALLER VE SİSTEMLER II

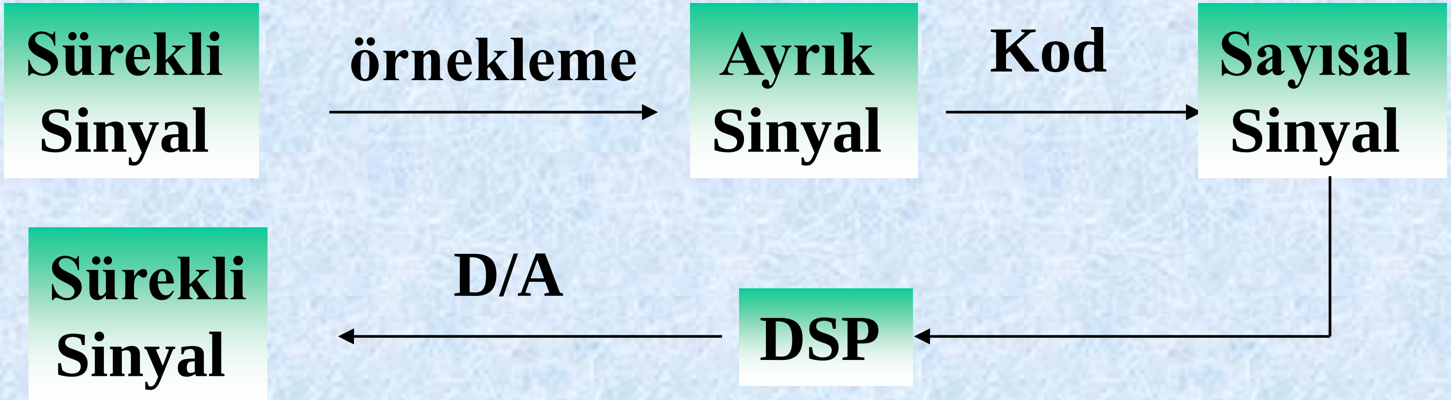
ÖRNEKLEME

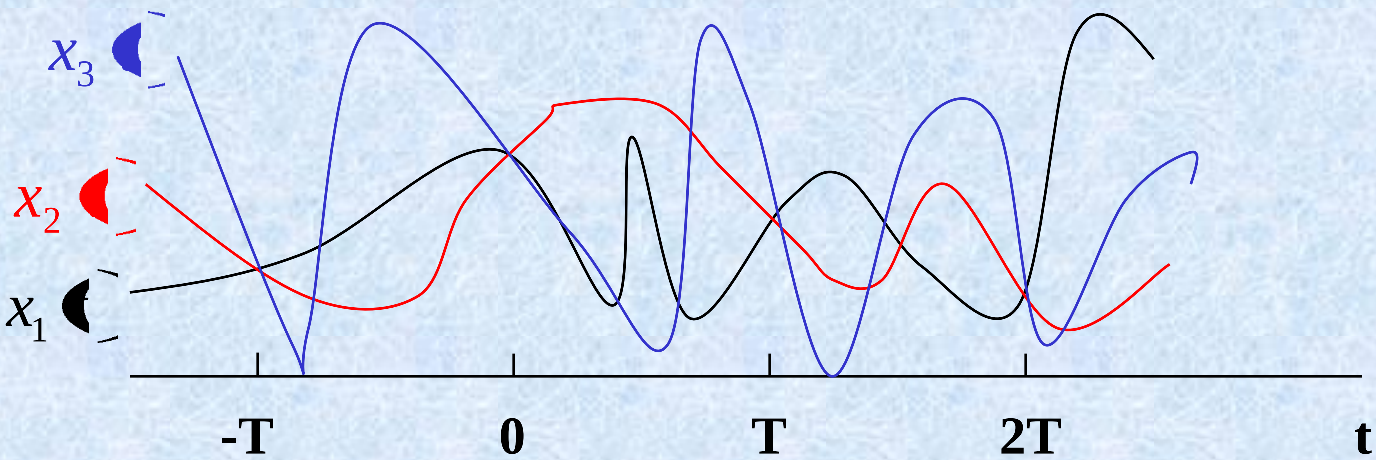
Yrd. Doç. Dr. Selda GÜNEY

İÇERİK

- Örnekleme Teoremi
- Sinyalin Yeniden Oluşturulması
- Örtüşme (Aliasing)
- Sürekli Zaman Sinyallerinin Ayrık Zaman İşlemesi
- Ayrık Zaman Sinyallerinin Örneklenmesi

Örnekleme Teoremi

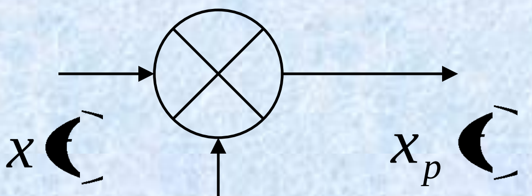




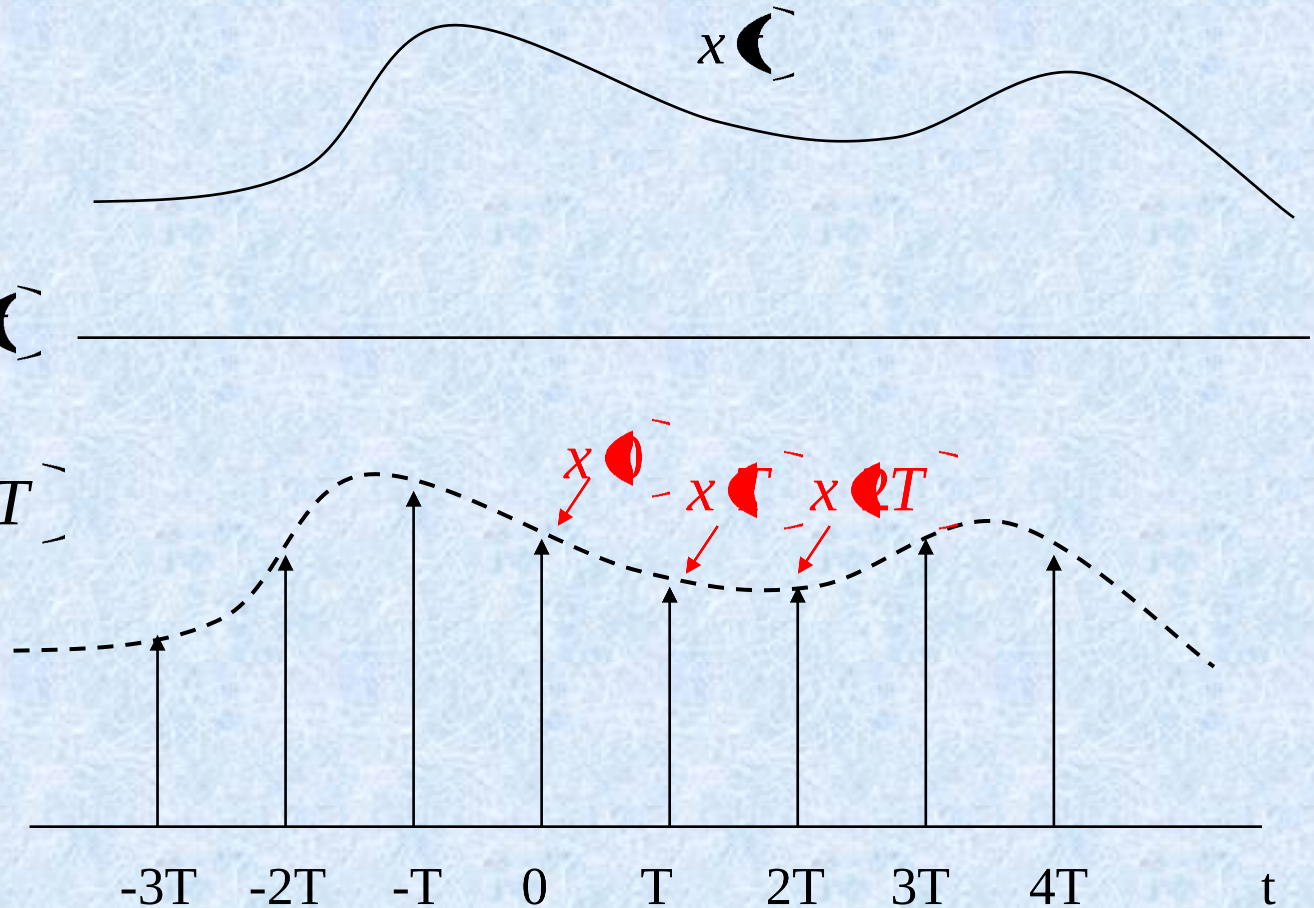
$$x_1(nT) = x_2(nT) = x_3(nT)$$

$$x_1(t) \neq x_2(t) \neq x_3(t)$$

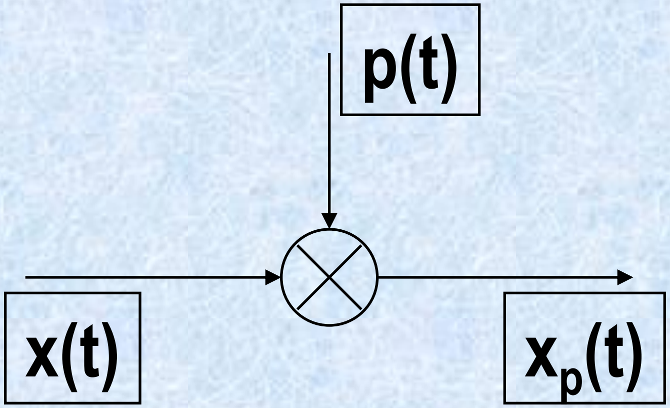
1.1 Dürtü Katarı Örneklemesi



$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$



Örnekleme

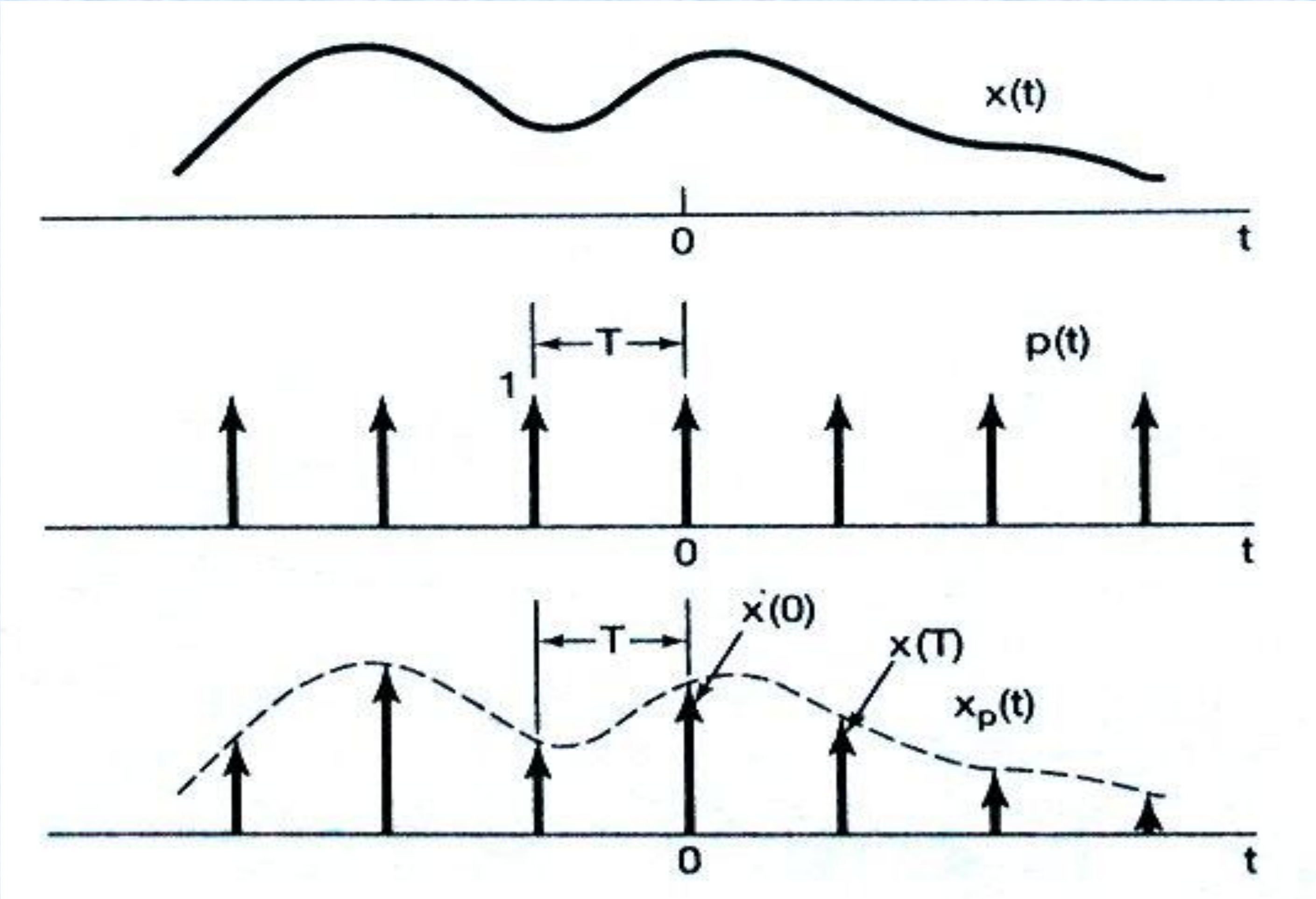


$$\begin{cases} x_p(t) = x(t)\,p(t) \\ X_p(j\omega) = \dfrac{1}{2\pi}[X(j\omega) * P(j\omega)] \end{cases}$$

$$p(t) = \delta_T(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

Zaman domeninde:

$$x_p(t) = x(t)\cdot\delta_T(t) = \sum_{n=-\infty}^{+\infty} x(nT)\delta(t-nT)$$



Frekans domeninde:

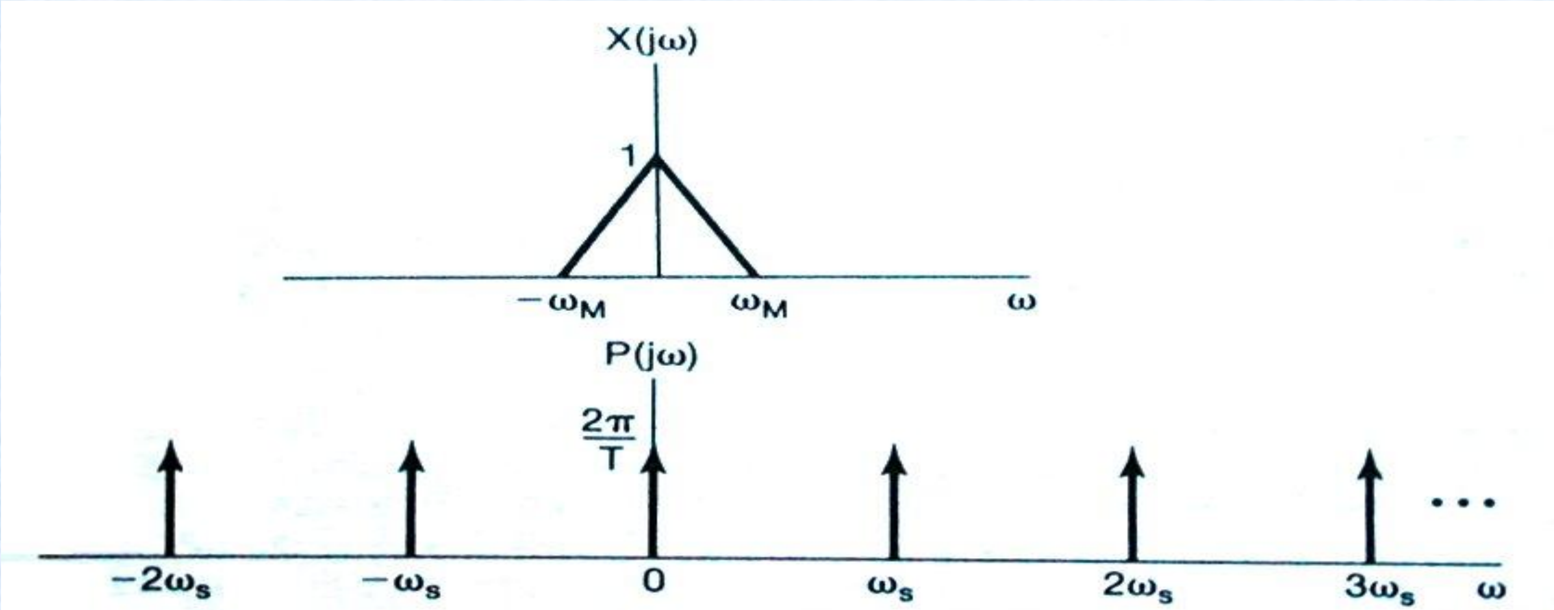
$$x(t) \xleftrightarrow{F} X(j\omega)$$

$$p(t) = \delta_T(t)$$ **(Periyodik Sinyal)**

$$p(t) \xleftrightarrow{F} P(j\omega) = \sum_{k=-\infty}^{+\infty} \frac{2\pi}{T} \delta(\omega - k\omega_s) = \sum_{k=-\infty}^{+\infty} \omega_s \delta(\omega - k\omega_s)$$

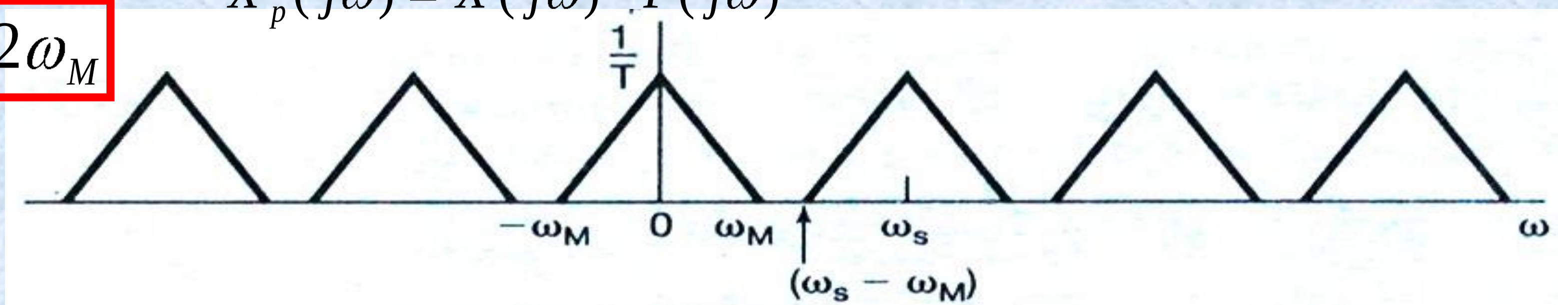
$$x_p(t) \xleftrightarrow{F} X_p(j\omega) = \frac{\omega_s}{2\pi} \sum_{k=-\infty}^{+\infty} X(\omega - k\omega_s) = \frac{1}{T} \sum_{k=-\infty}^{+\infty} X(\omega - k\omega_s)$$

Örnekleme Teoremi

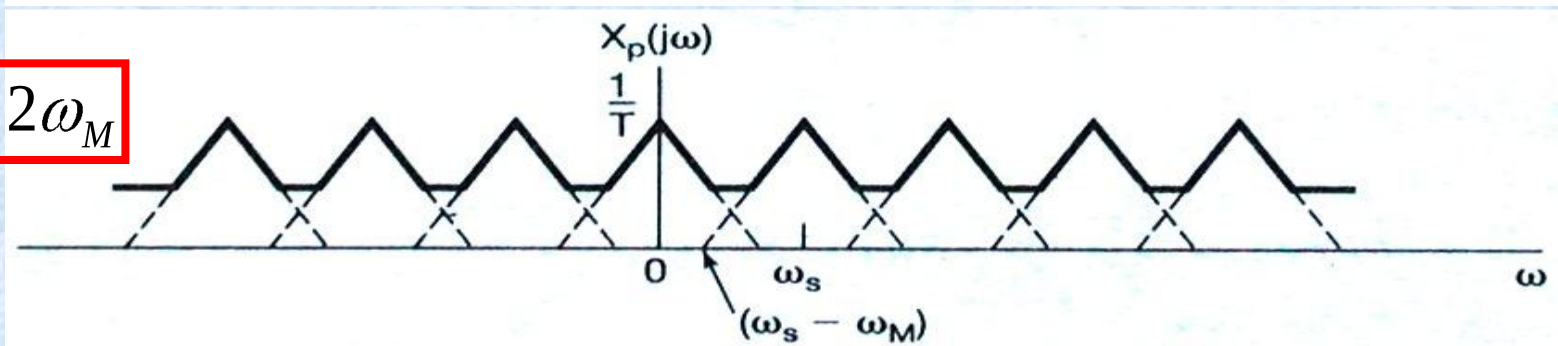


$$X_p(j\omega) = X(j\omega) * P(j\omega)$$

$\boxed{\omega_s > 2\omega_M}$



$\boxed{\omega_s < 2\omega_M}$



Örnekleme Teoremi:

$x(t) \qquad X(j\omega) = 0 \ , \quad |\omega| > \omega_M$ **bant sınırlı sinyal olsun.**

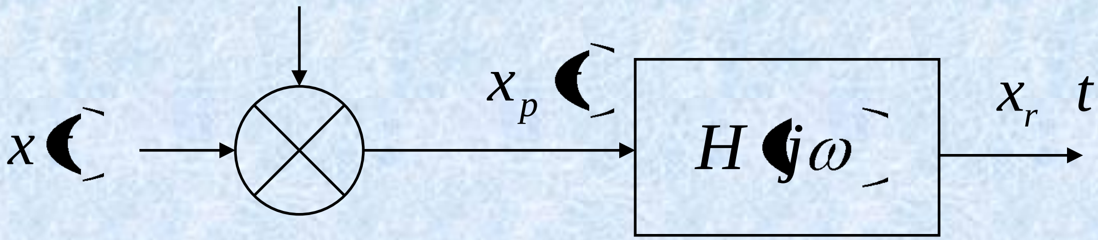
$$\omega_s > 2\omega_M \qquad \omega_s = \frac{2\pi}{T}$$

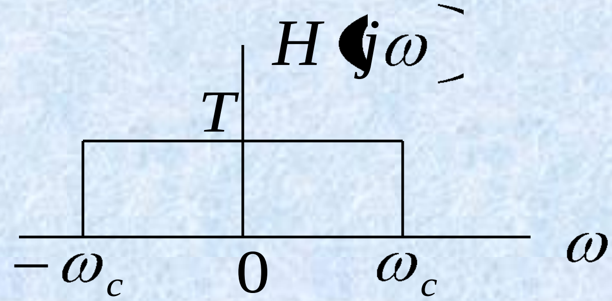
$x(t), \qquad x(nT), n = 0, \pm 1, \cdots$ **örnekleri tarafından oluşturulabilir.**

$2\omega_M$: **Nyquist Hızı (Nyquist Frekansı)**

Sinyalin yeniden oluşturulması

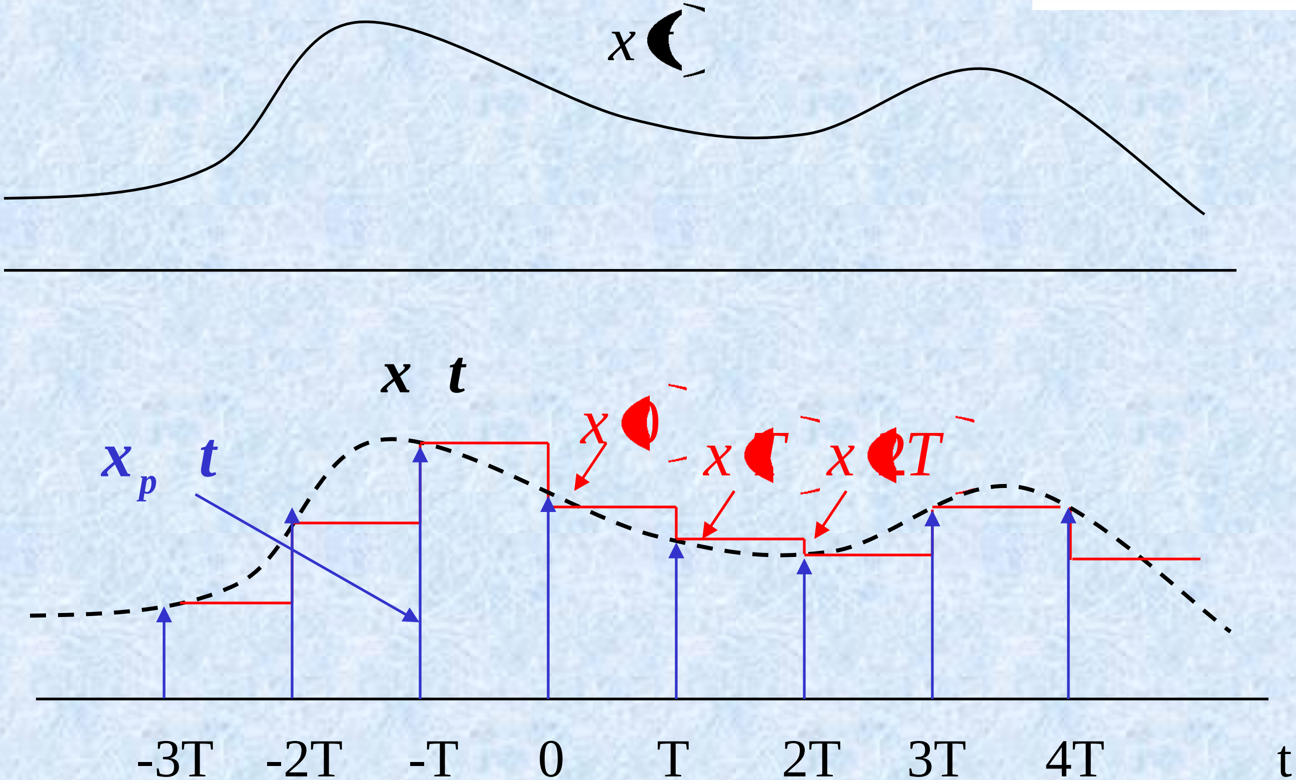
$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$



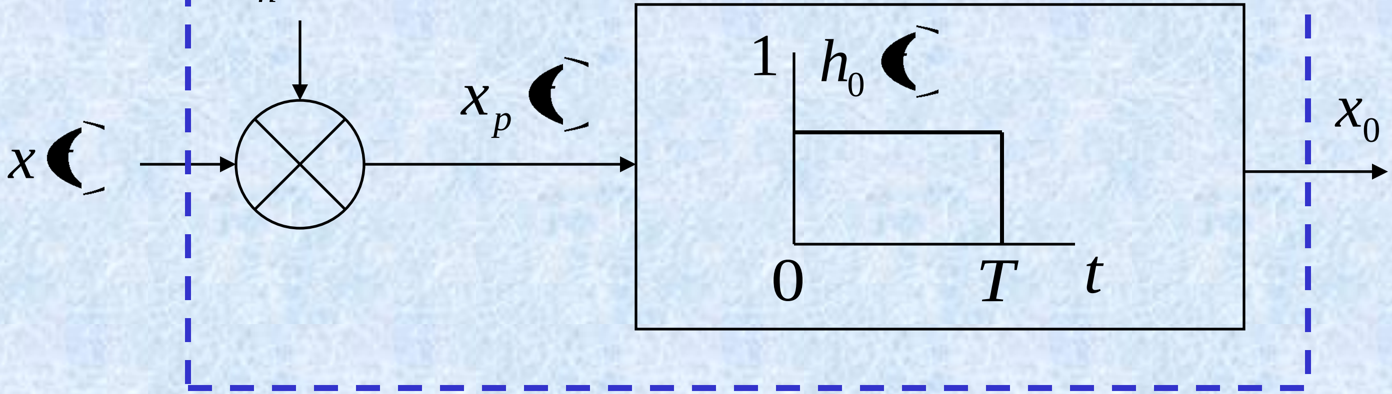


$$\omega_M < \omega_c < \omega_s - \omega_M$$

1.2. Sıfırıncı Dereceden Bir tutunmalı Örnekleme



$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$



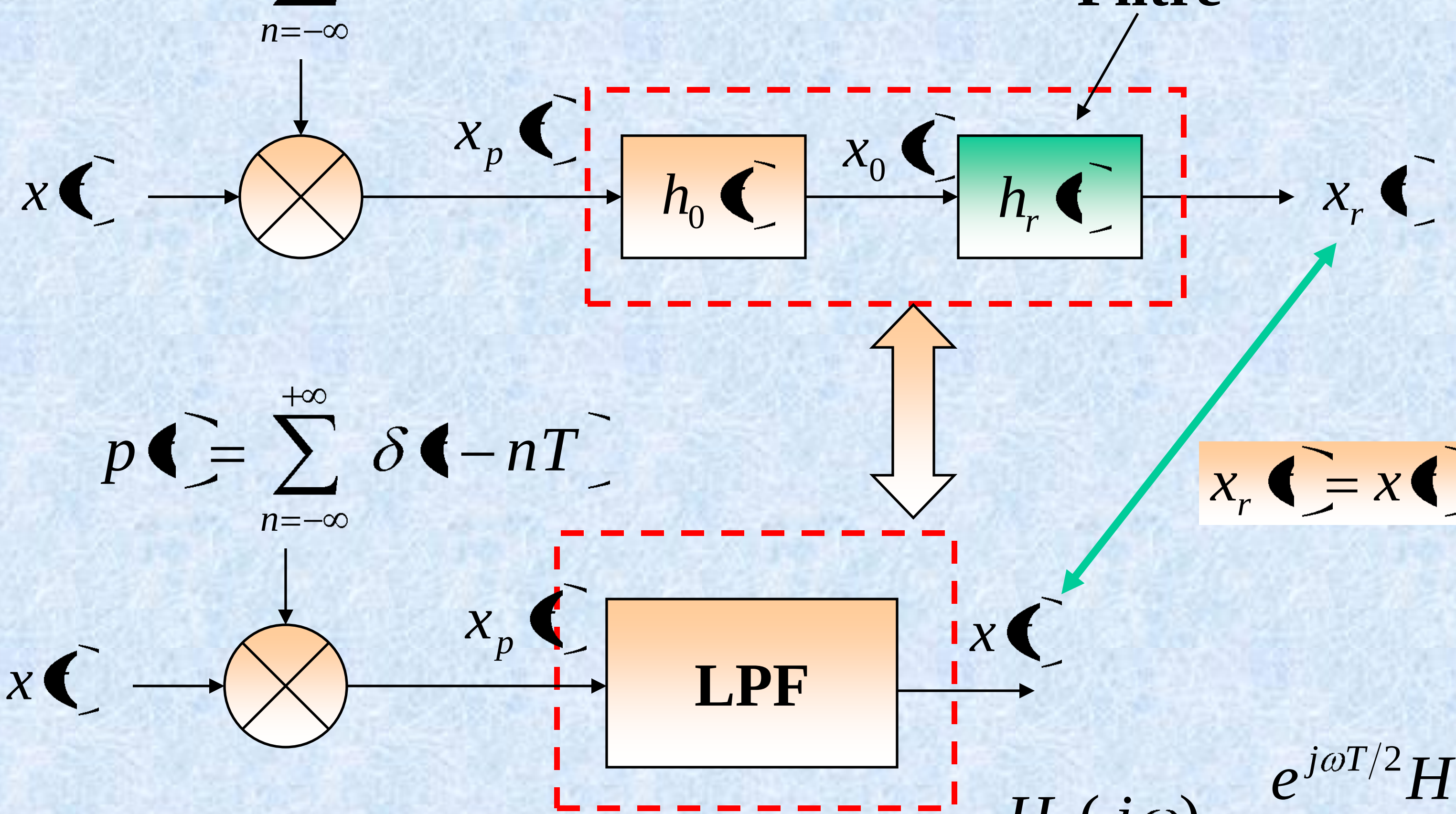
Zero-Order Hold

$$x_0(t) = x_p(t) * h_0(t) = \sum_{n=-\infty}^{+\infty} x(nT)\, h_0(t - nT)$$

$$H_o(j\omega) = e^{-j\omega T/2}\left[\frac{2\sin(\omega T/2)}{\omega}\right]$$

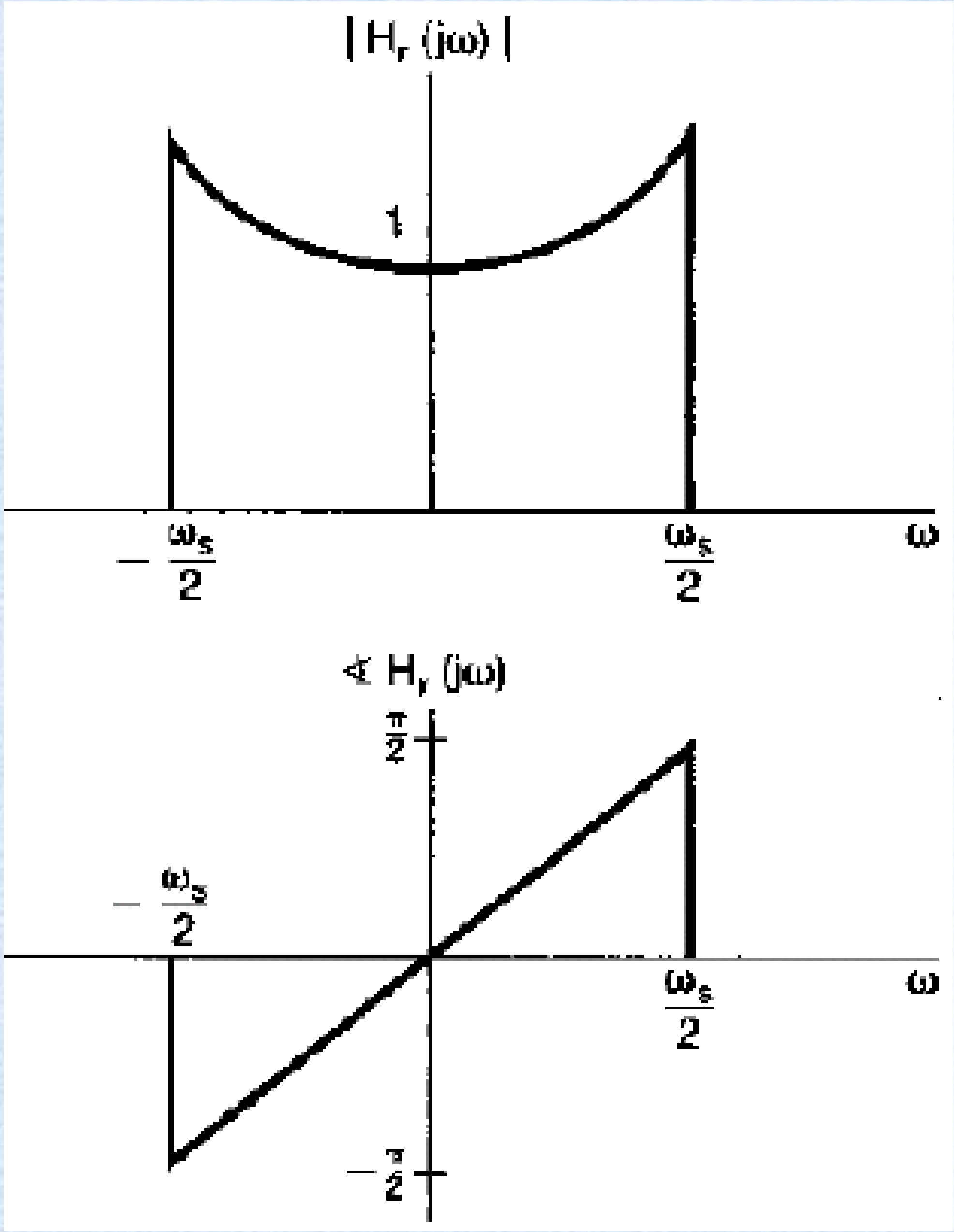
Zero-Order Hold

$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$



$$H_r(j\omega) = \frac{e^{j\omega T/2} H(j\omega)}{\frac{2\sin(\omega T/2)}{\omega}}$$

$$\omega_M < \omega_c < \omega_s - \omega_M$$

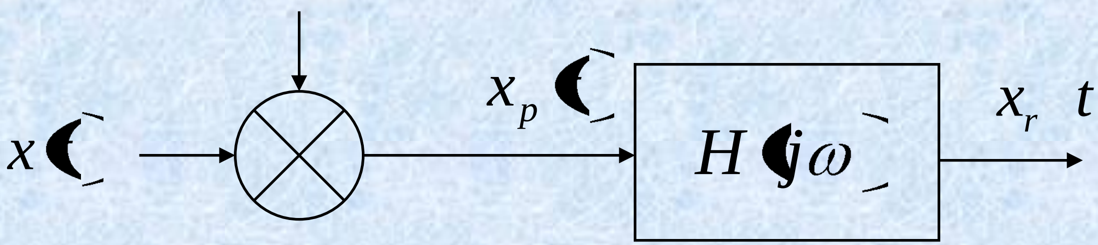


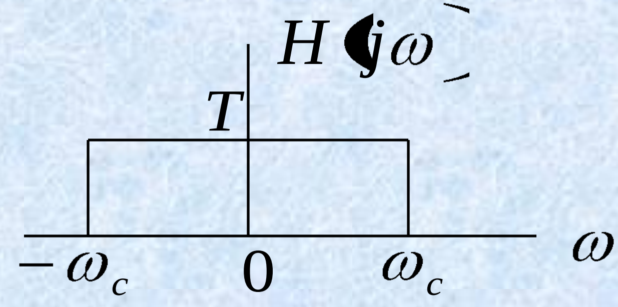
$$H_r(j\omega) = \frac{e^{j\omega T/2} H(j\omega)}{\dfrac{2\sin(\omega T/2)}{\omega}}$$

2. Sinyalin Yeniden Oluşturulması

Band-sınırlı ara değerleme

$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$





$$\omega_M < \omega_c < \omega_s - \omega_M$$

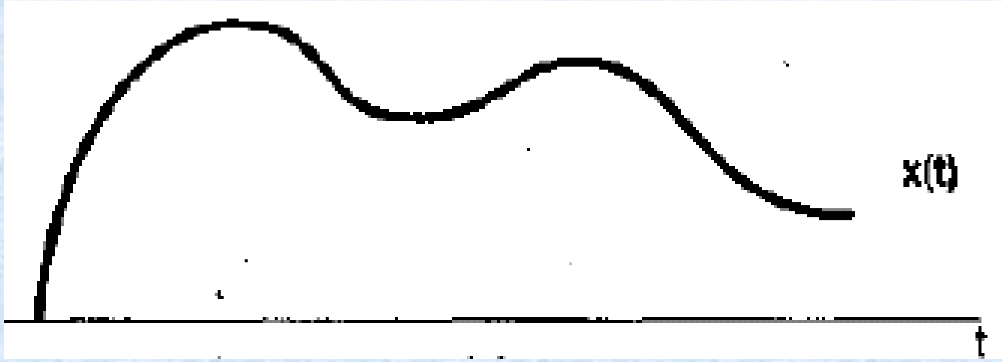
$$x_r(t) = x_p(t) * h(t) \qquad h(t) = \frac{T\sin(\omega_c t)}{\pi t}$$

$$= \sum_{n=-\infty}^{\infty} \left(x(nT)\delta(t - nT) \right) * h(t)$$

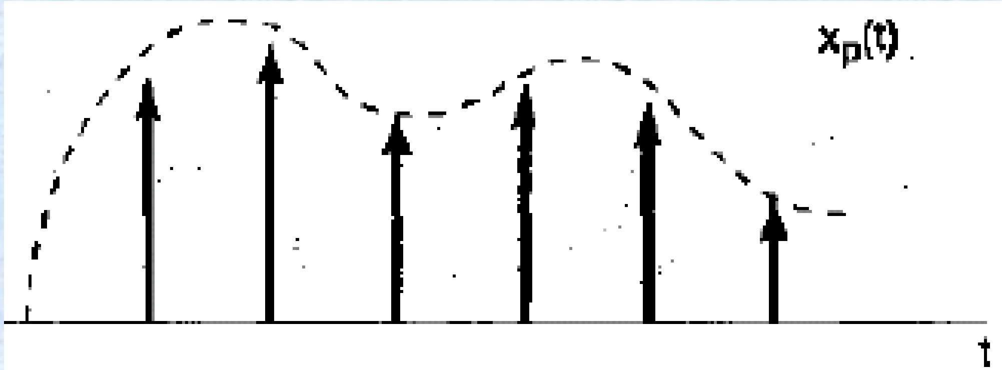
$$= \sum_{n=-\infty}^{\infty} x(nT)h(t - nT) \qquad = \sum_{n=-\infty}^{\infty} x(nT)\frac{T\sin[\omega_c (t - nT)]}{\pi (t - nT)}$$

Sinyalin Yeniden Oluşturulması

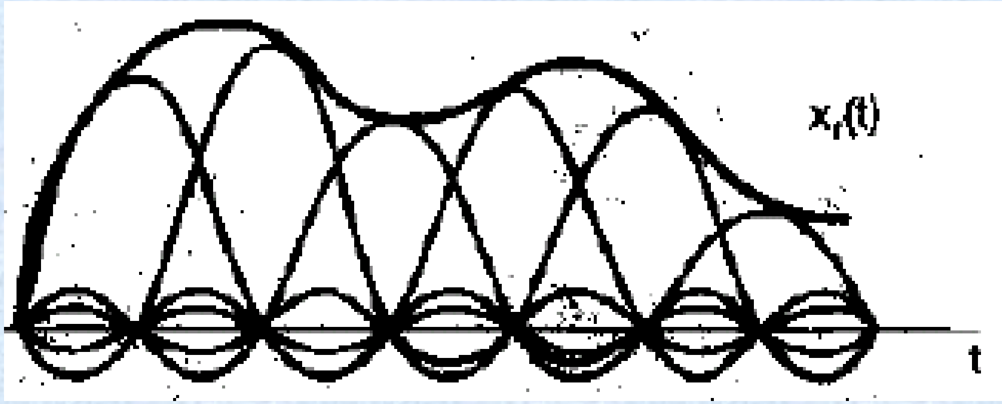
Orijinal Sinyal



Örneklemeden Sonra

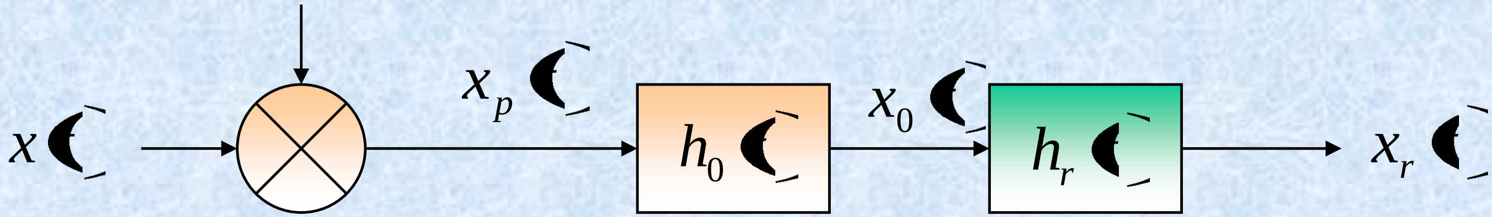


Alçak geçiren Süzgeçten geçtikten sonra

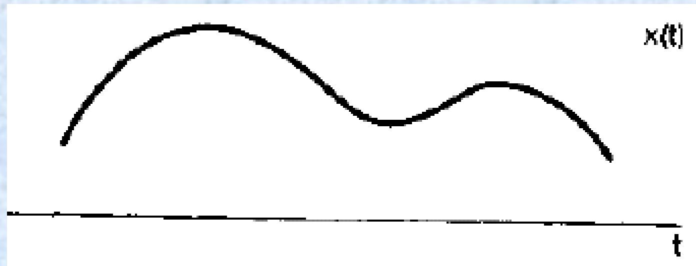


Sinyalin Yeniden Oluşturulması

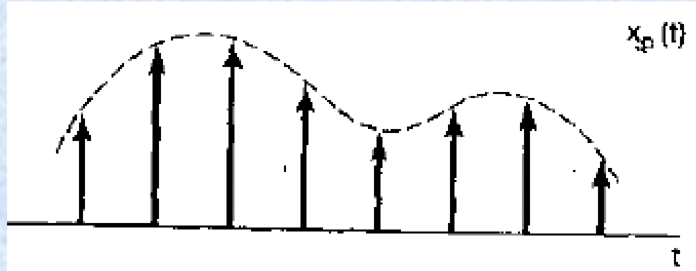
Zero-order hold $p(t)=\sum_{n=-\infty}^{+\infty}\delta(t-nT)$



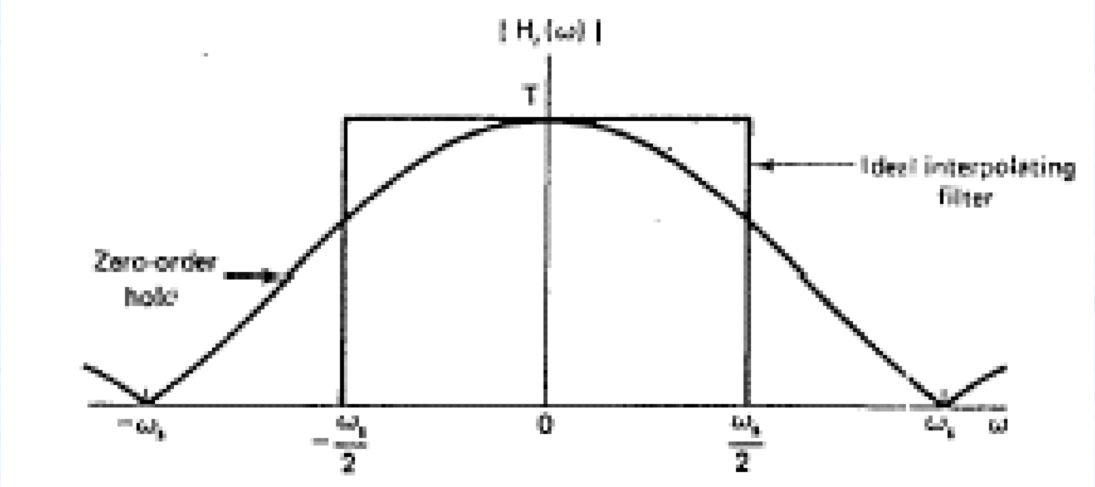
Orijinal Sinyal



Örneklemeden sonra



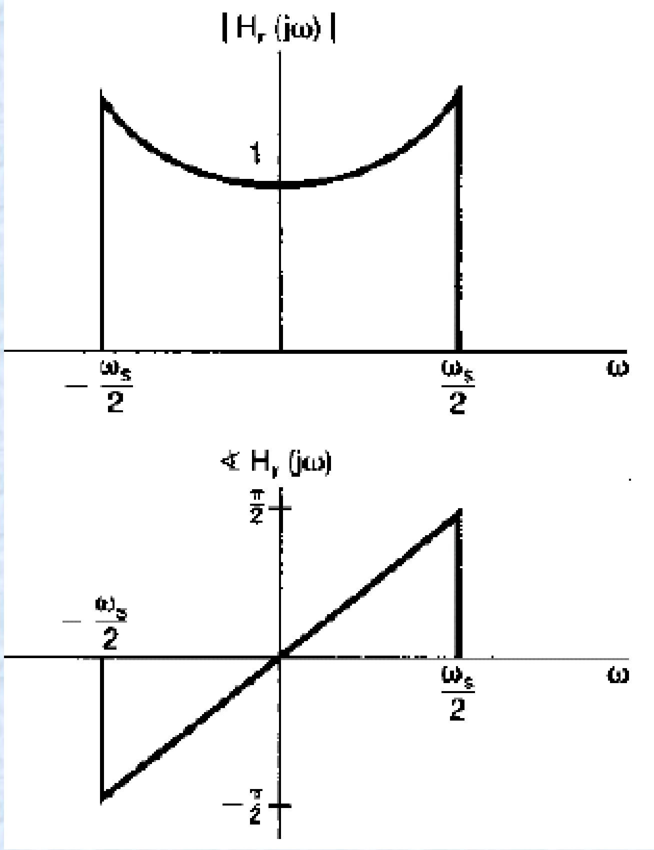
Sıfırıncı dereceden Tutma devresinden geçtikten sonra



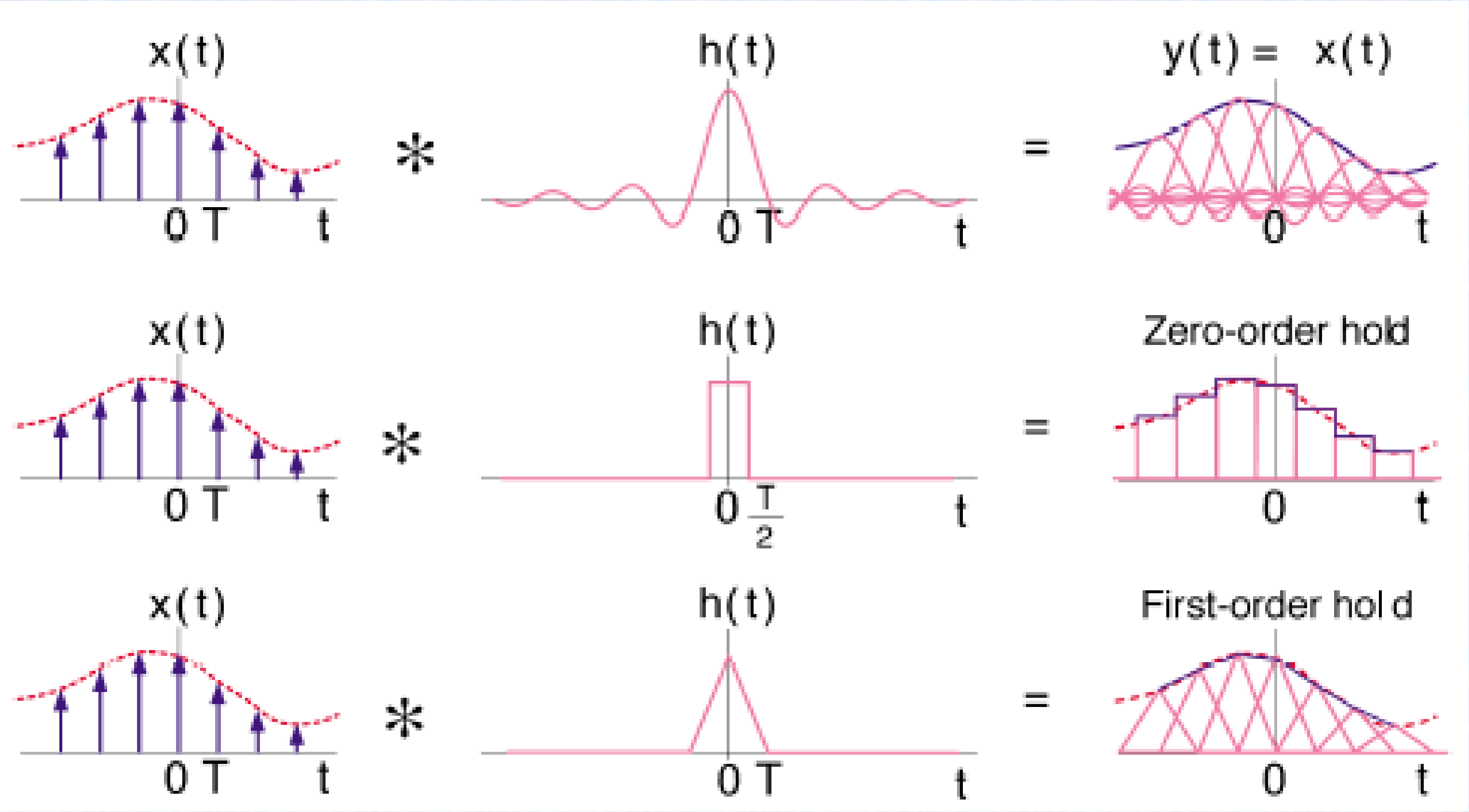
Zero-Order Hold

$$H_o(j\omega) = e^{-j\omega T/2}\left[\frac{2\sin(\omega T/2)}{\omega}\right]H(j\omega)$$

$$H_r(j\omega) = \frac{e^{j\omega T/2}H(j\omega)}{\dfrac{2\sin(\omega T/2)}{\omega}}$$

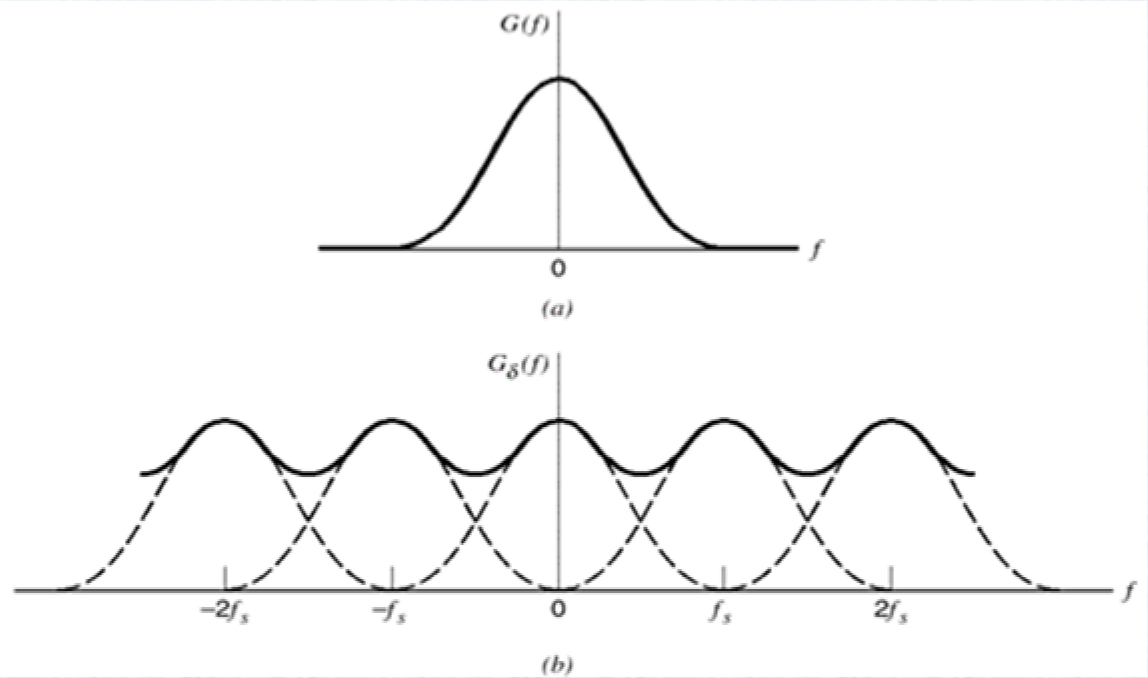


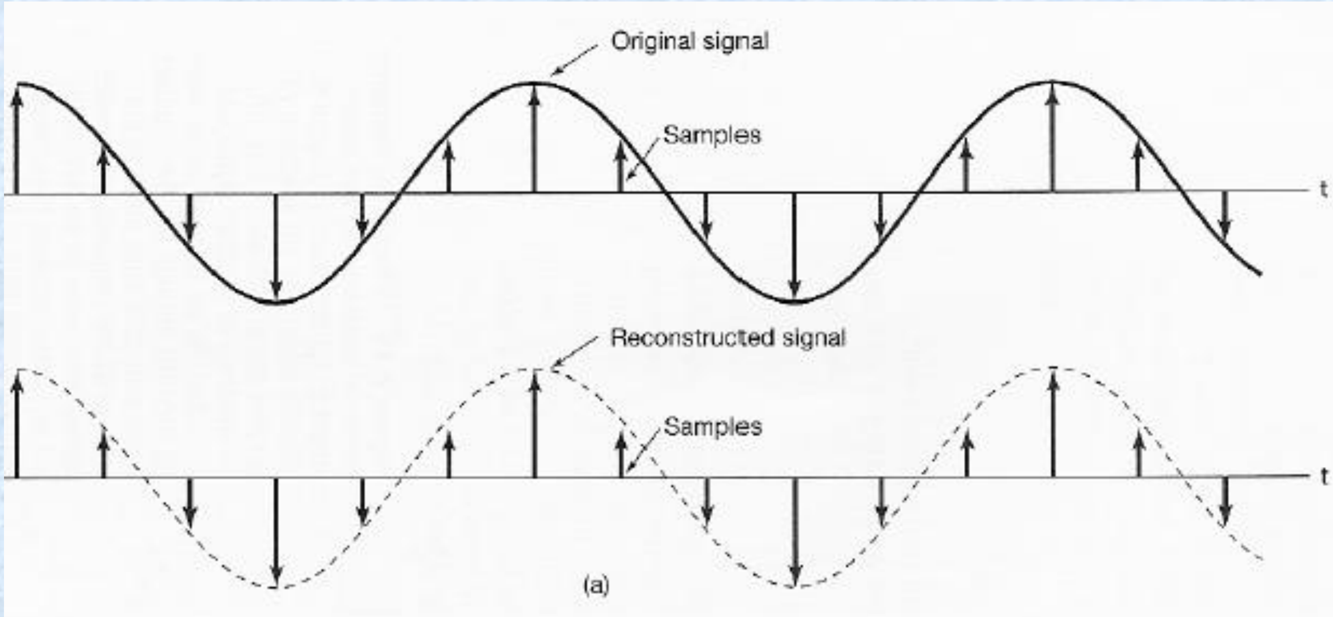
Zero-Order Hold Recover Filter


x(t)
0 T
t
*
h(t)
0 T
t
=
y(t) = x(t)
0
t
x(t)
0 T
t
*
h(t)
0 T/2
t
=
Zero-order hold
0
t
x(t)
0 T
t
*
h(t)
0 T
t
=
First-order hol d
0
t

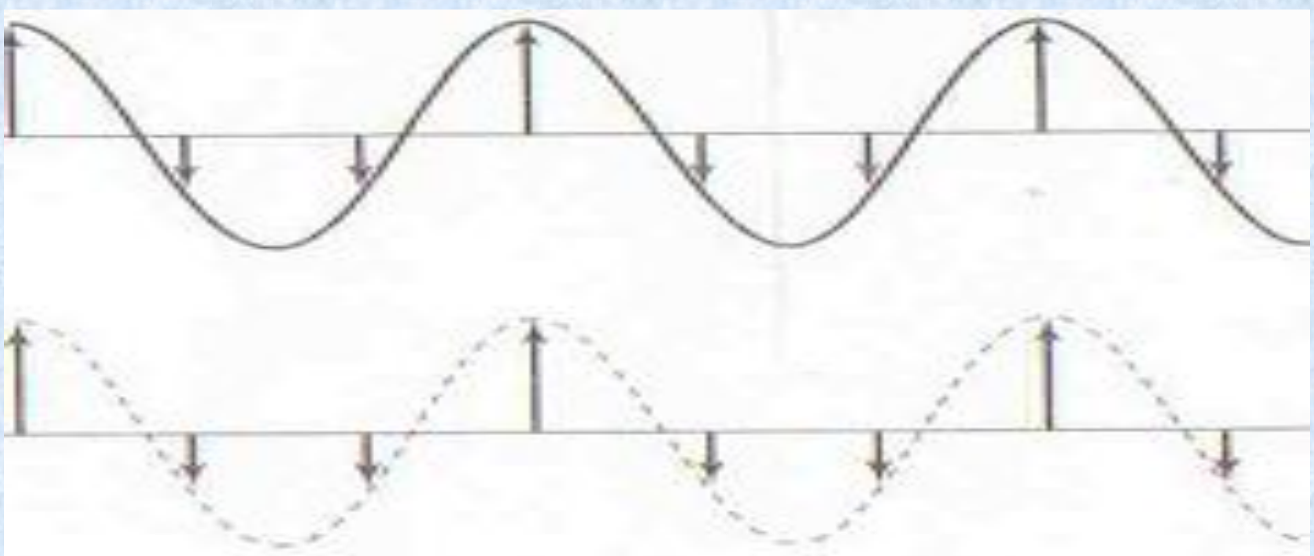
Örtüşme :

Eğer bir sinyal Nyquist hızından(frekansından) daha düşük bir hızda örneklenirse, spektral örtüşme meydana gelir.

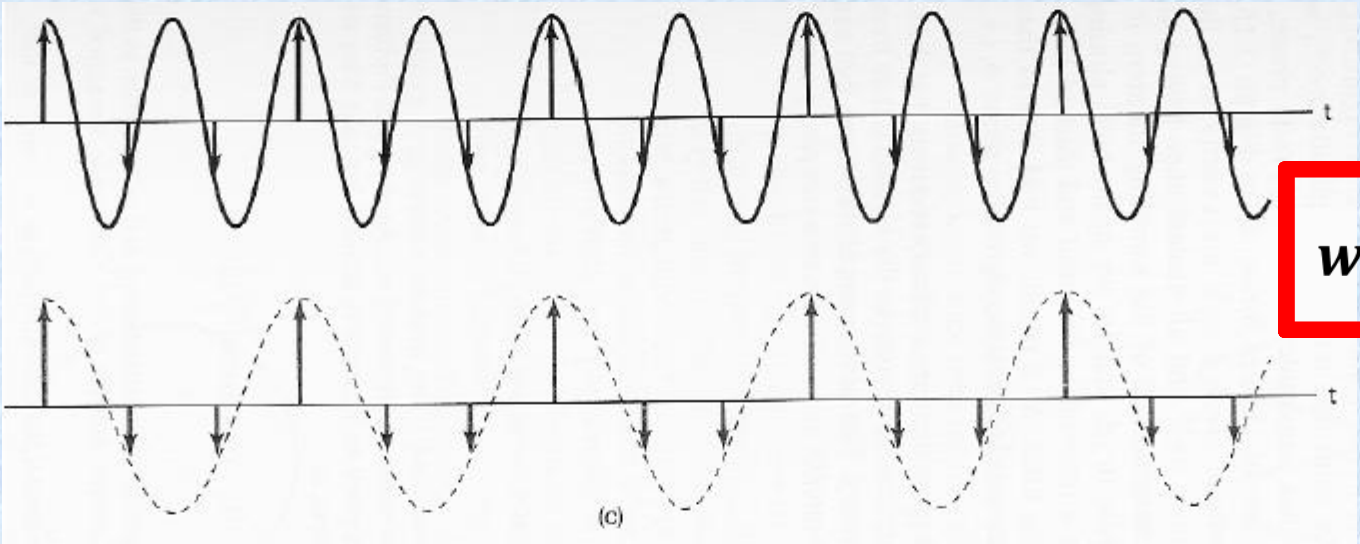




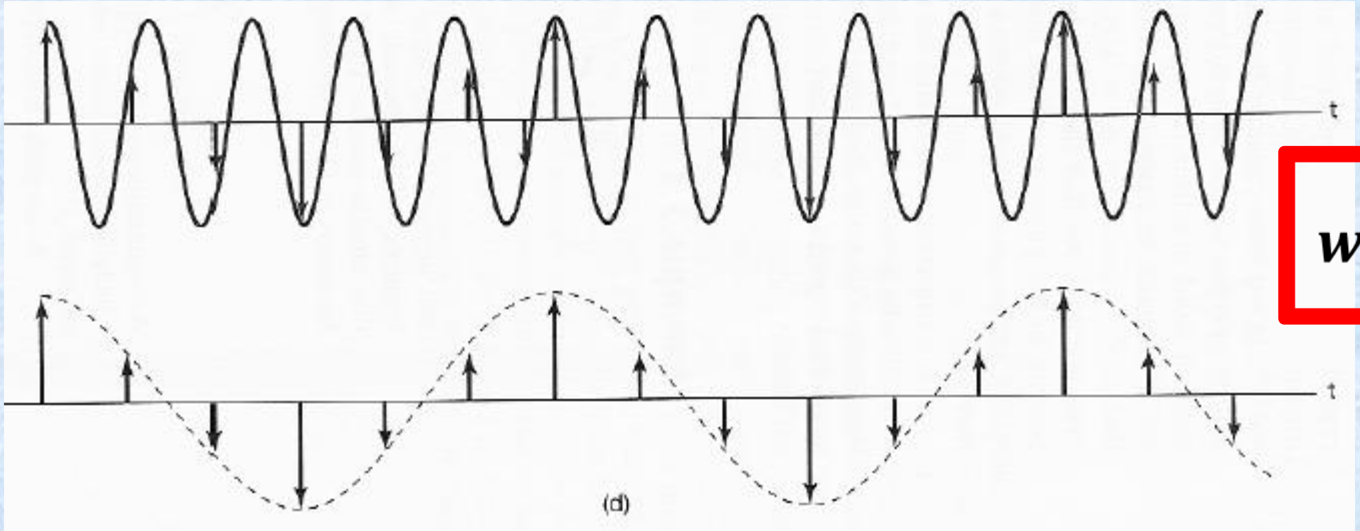
$$\boxed{w_0 = \frac{w_s}{6}}$$



$$\boxed{w_0 = \frac{2w_s}{6}}$$



$$w_0 = \frac{4w_s}{6}$$



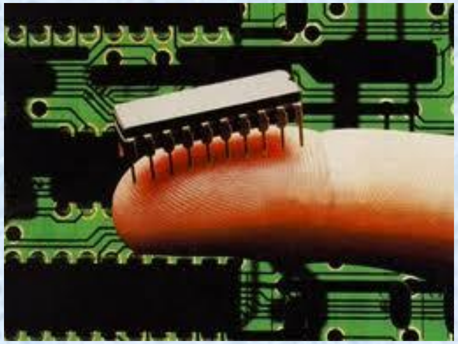
$$w_0 = \frac{5w_s}{6}$$

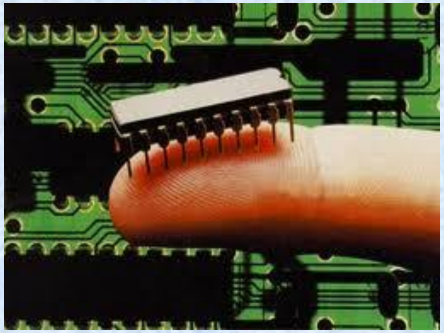
Sürekli Zaman Sinyallerinin Ayrık Zaman İşlemesi

$x_d[n] = x_c(nT)$

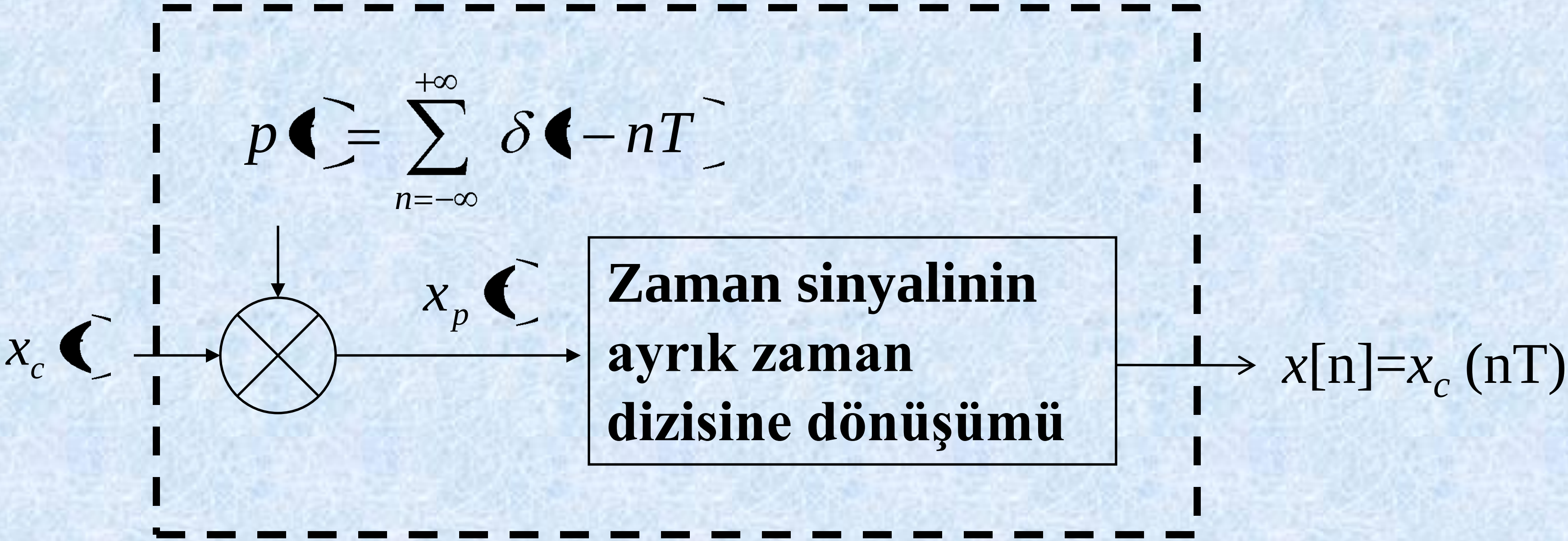
$y_d[n] = y_c(nT)$

$x_c(t)$ → **Ayrık Zaman Dönüşümü (C/D)** → $x_d[n]$ → **Ayrık Zaman Sistemi** → $y_d[n]$ → **Sürekli Zaman Dönüşümü (D/C)** → $y_c(t)$

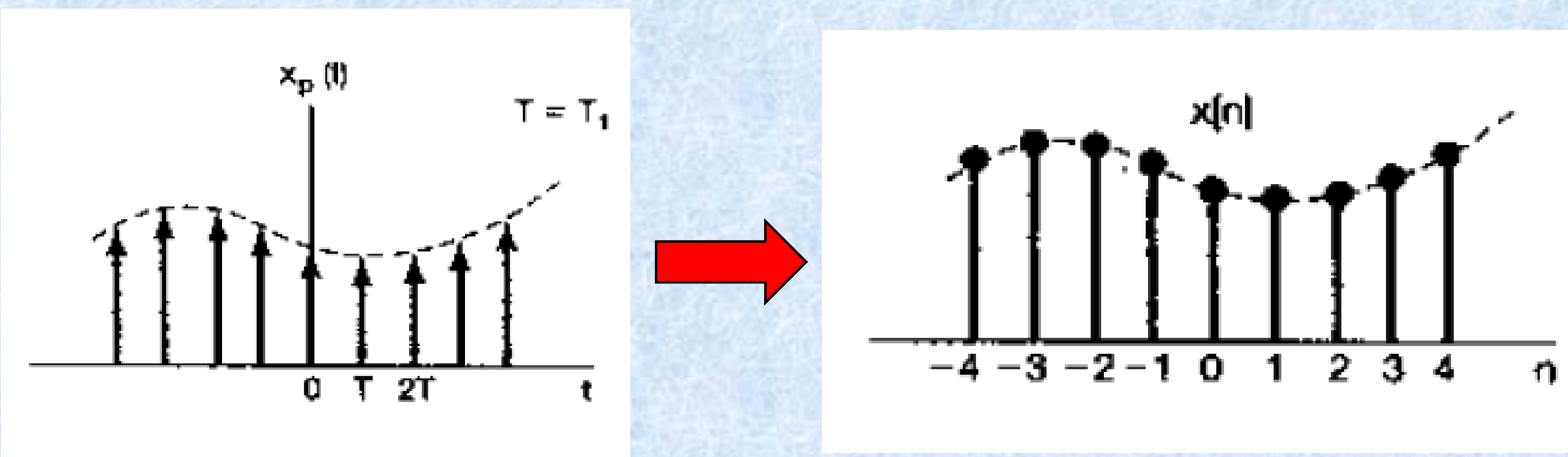




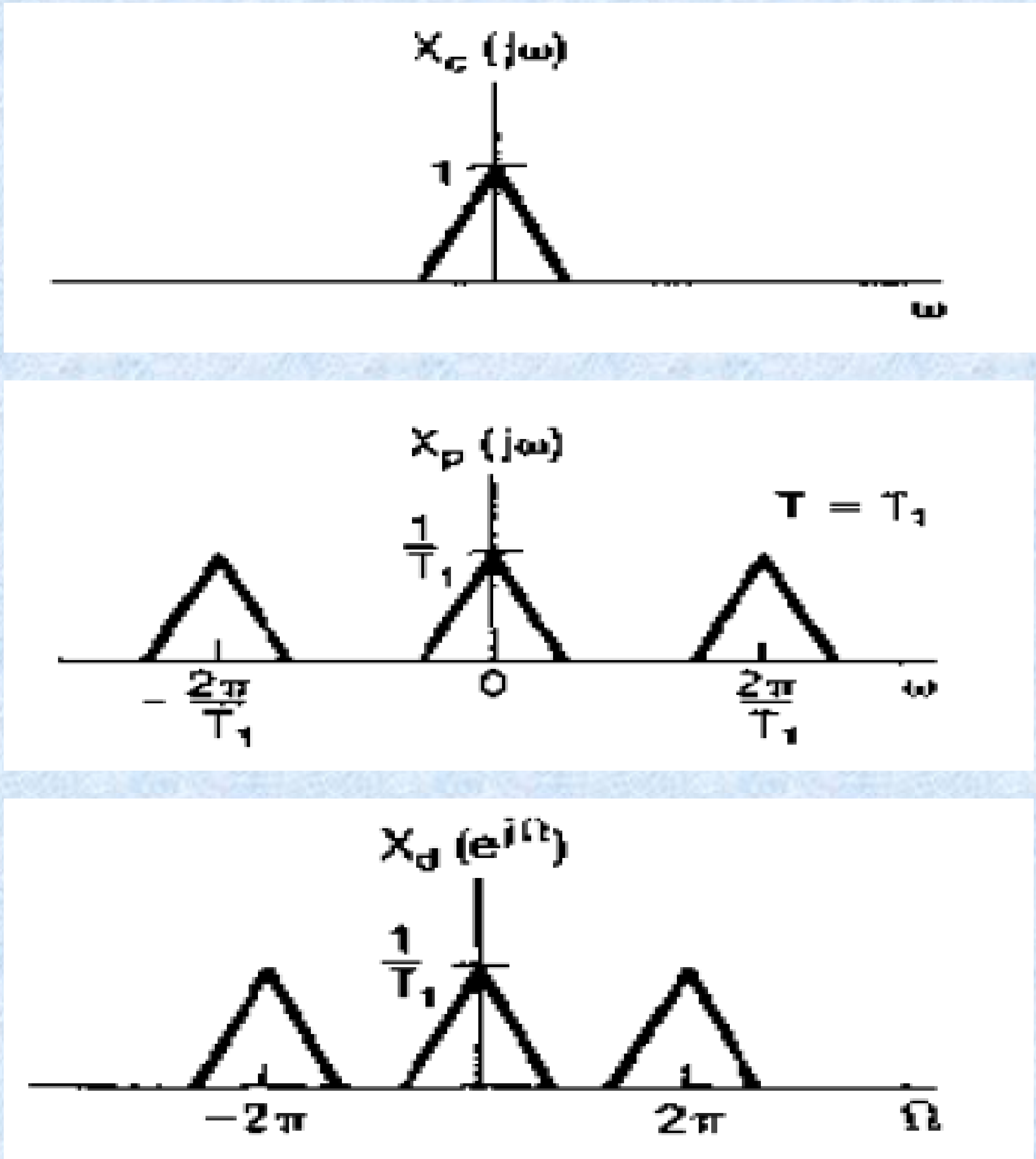
Sürekli Zaman Sinyallerinin Ayrık Zaman İşlemesi



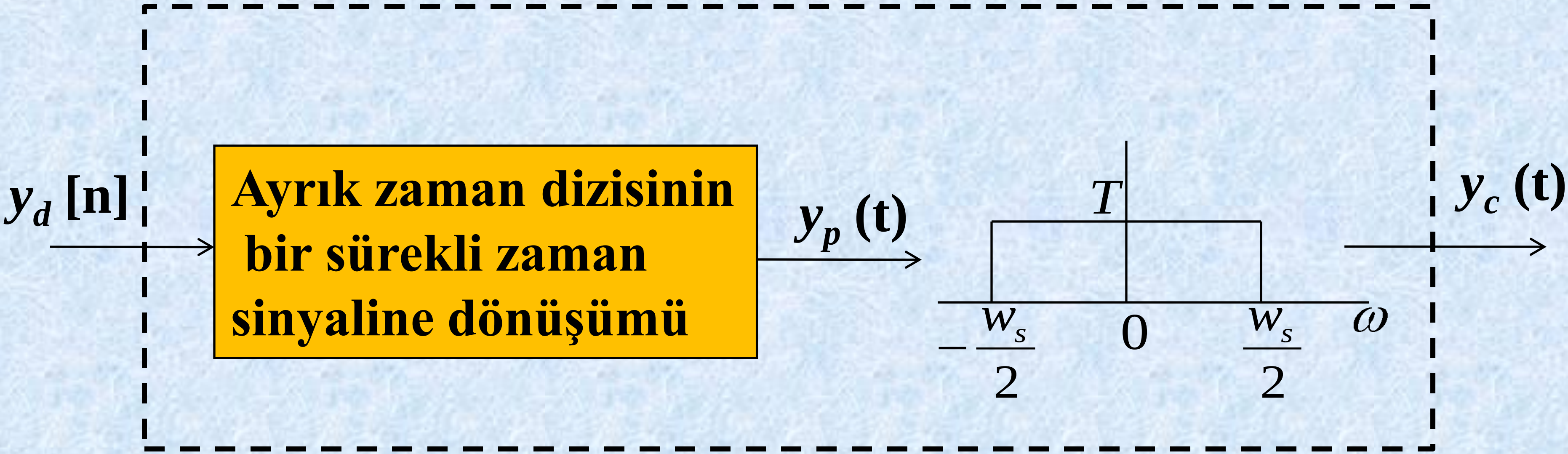
C/D dönüşümü

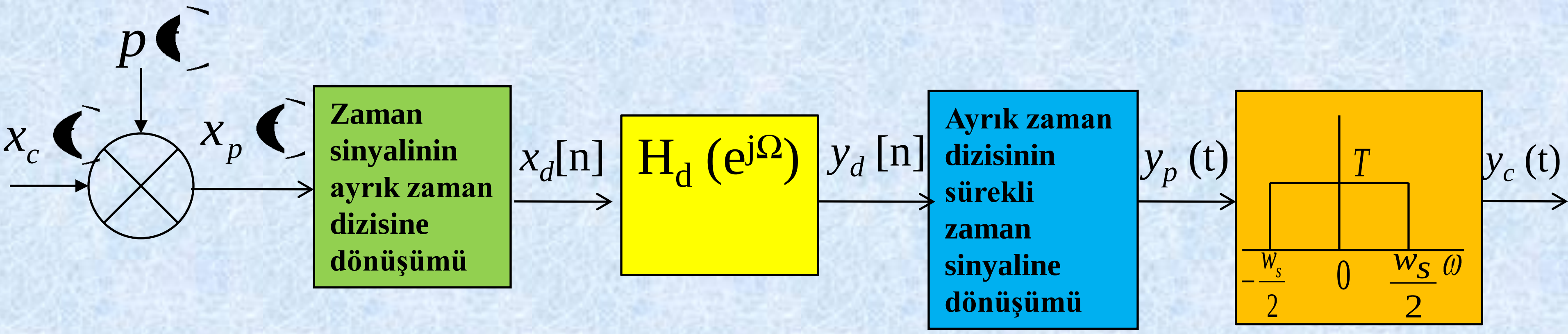


Sürekli Zaman Sinyallerinin Ayrık Zaman İşlemesi

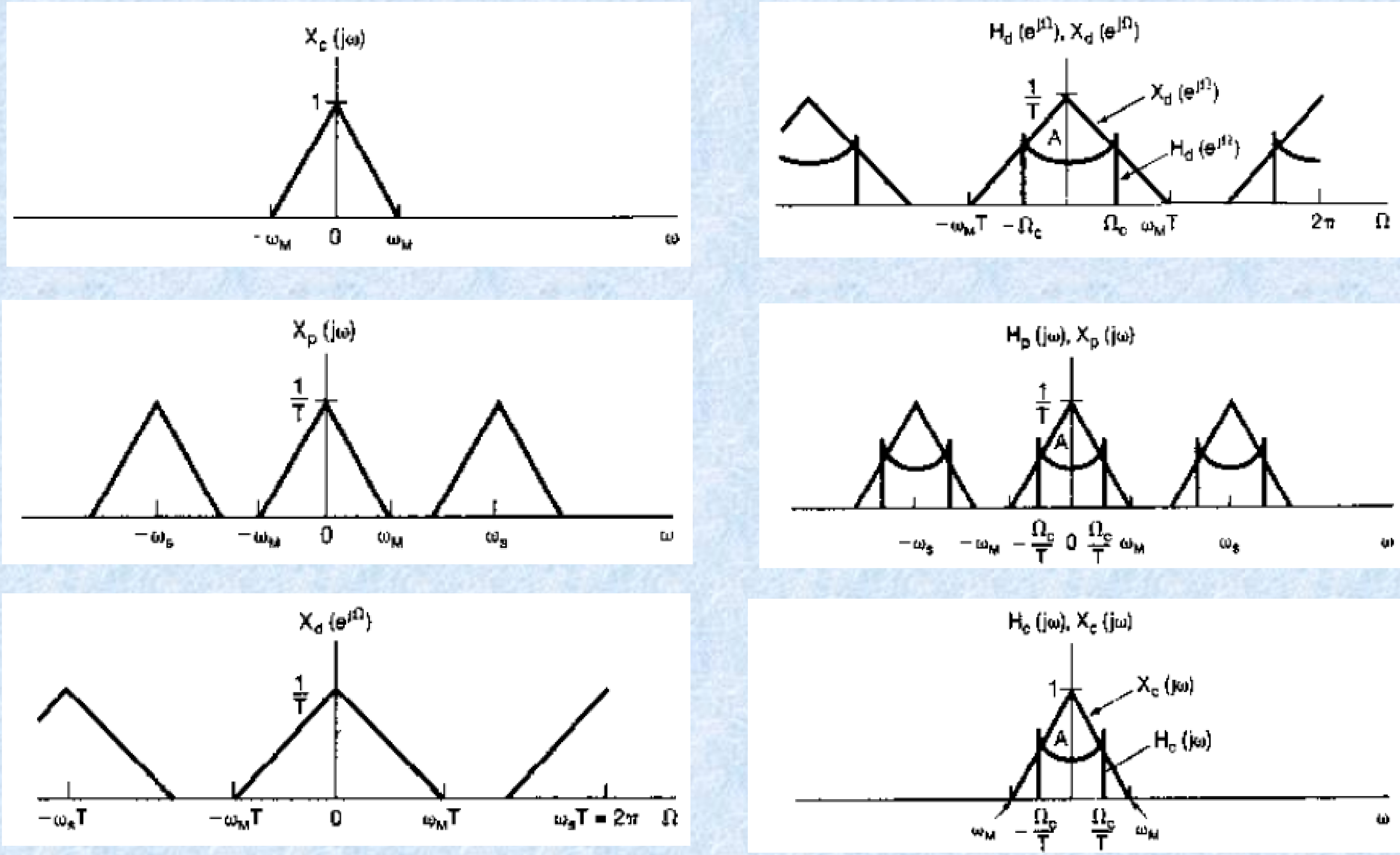


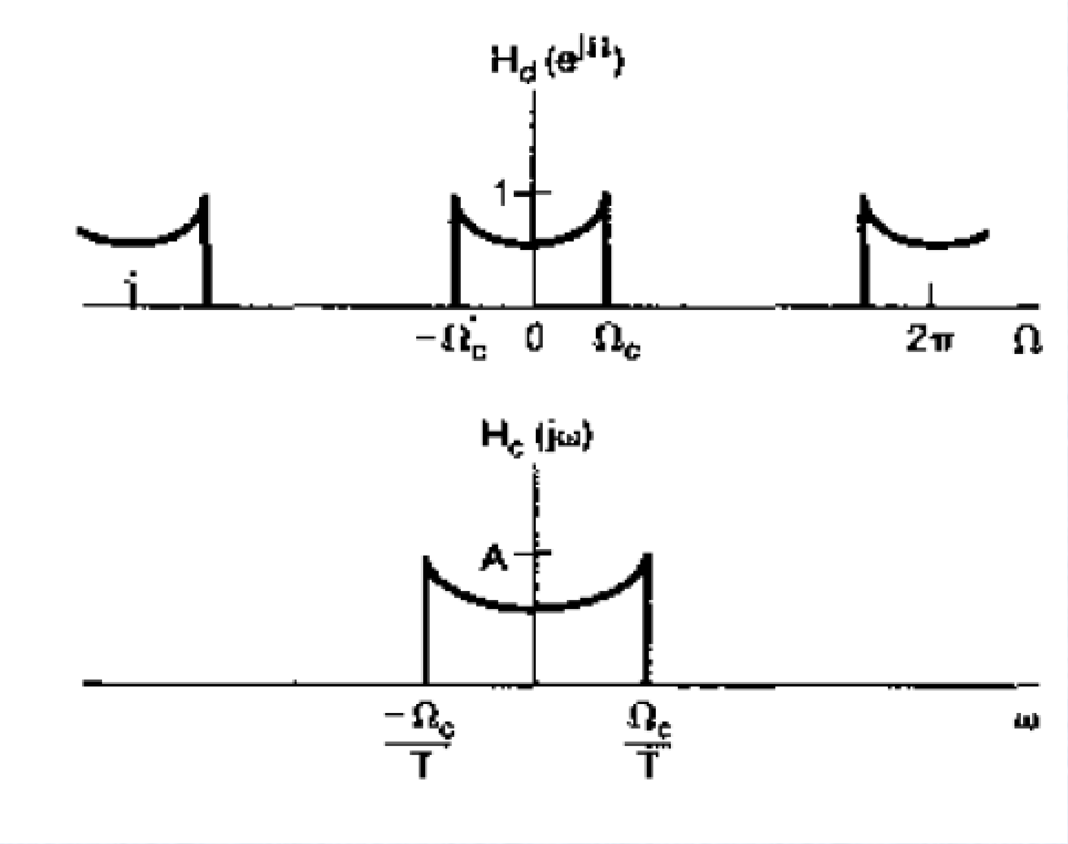
D/C Dönüşümü


$y_d\,[n]$
Ayrık zaman dizisinin
bir sürekli zaman
sinyaline dönüşümü
$y_p\,(t)$
T
$-\frac{w_s}{2}$
0
$\frac{w_s}{2}$
ω
$y_c\,(t)$

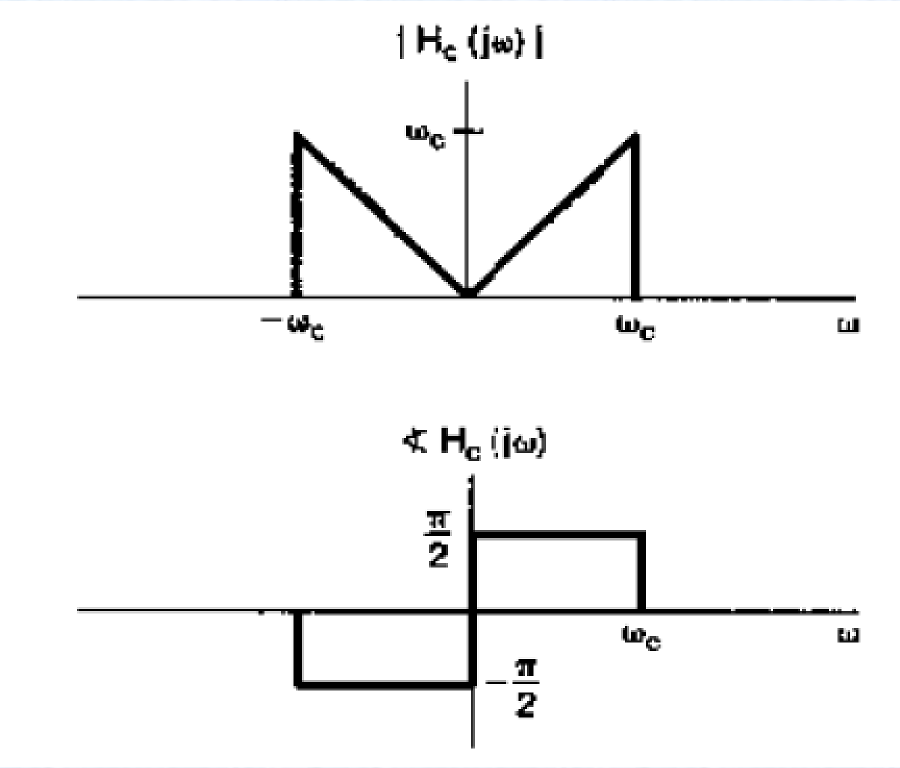

$p(t)$
$x_c(t)$
$x_p(t)$
Zaman sinyalinin ayrık zaman dizisine dönüşümü
$x_d[n]$
$H_d(e^{j\Omega})$
$y_d[n]$
Ayrık zaman dizisinin sürekli zaman sinyaline dönüşümü
$y_p(t)$
T
$-\frac{w_s}{2}$
0
$\frac{w_s}{2}$
ω
$y_c(t)$

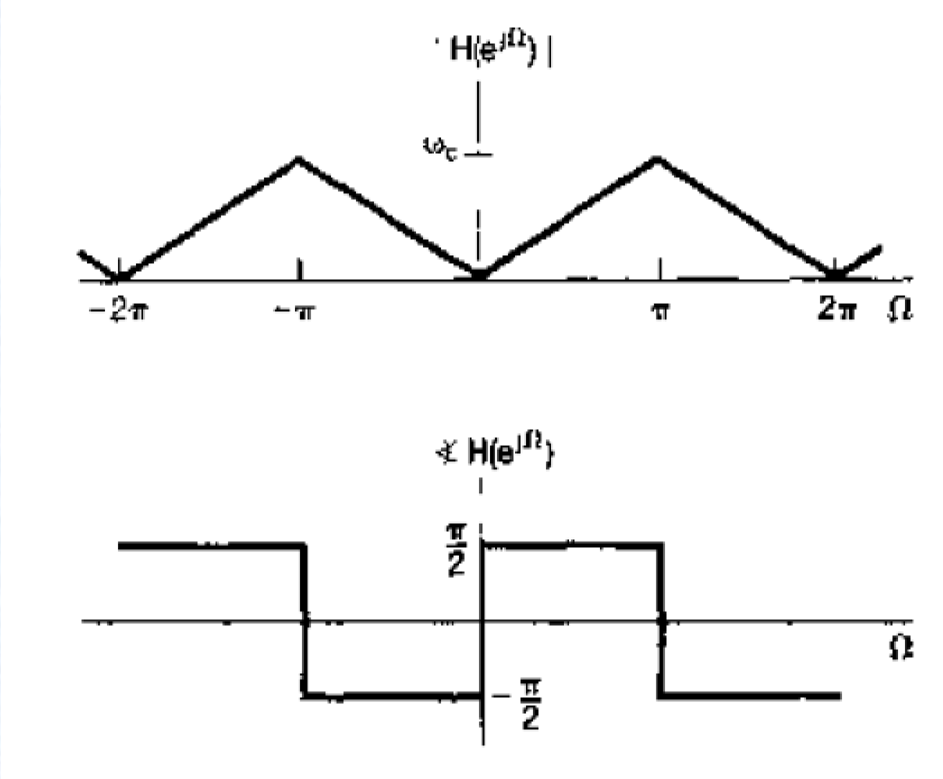
Sürekli Zaman Sinyallerinin Ayrık Zaman İşlemesi



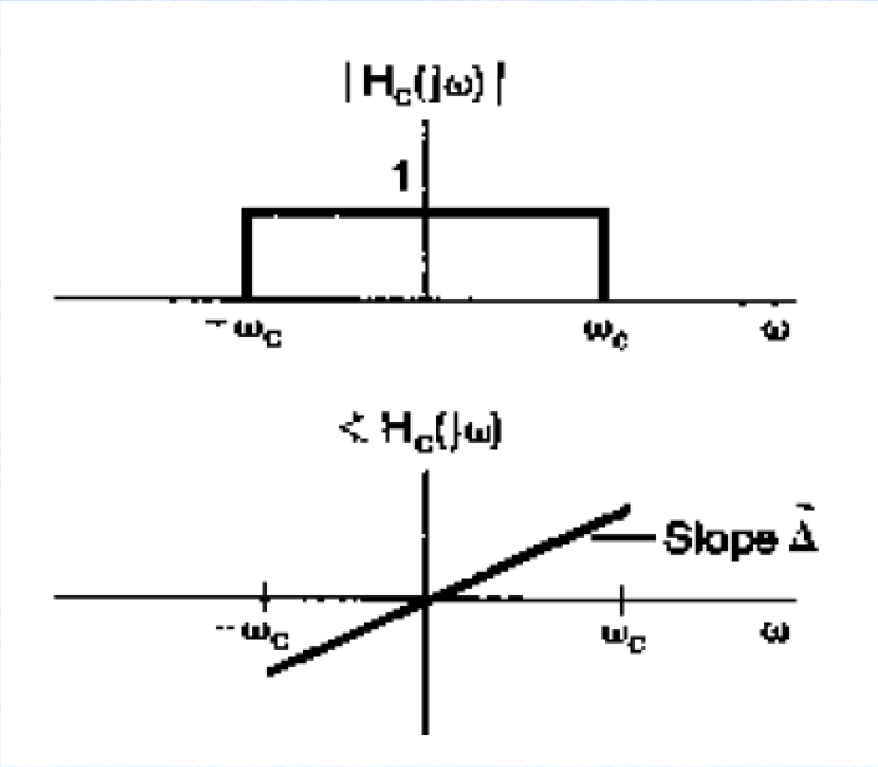

$H_d(e^{j\Omega})$
1
$-\Omega_c$
0
Ω_c
2π
Ω
$H_c(j\omega)$
A
$\frac{-\Omega_c}{T}$
$\frac{\Omega_c}{T}$
ω

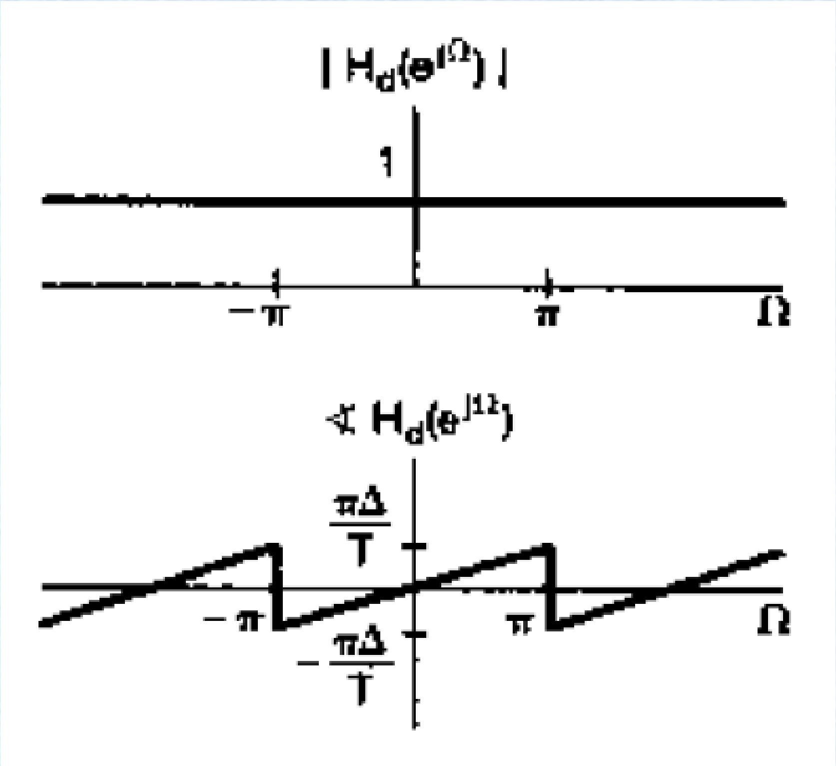
Dijital Türev Alıcı



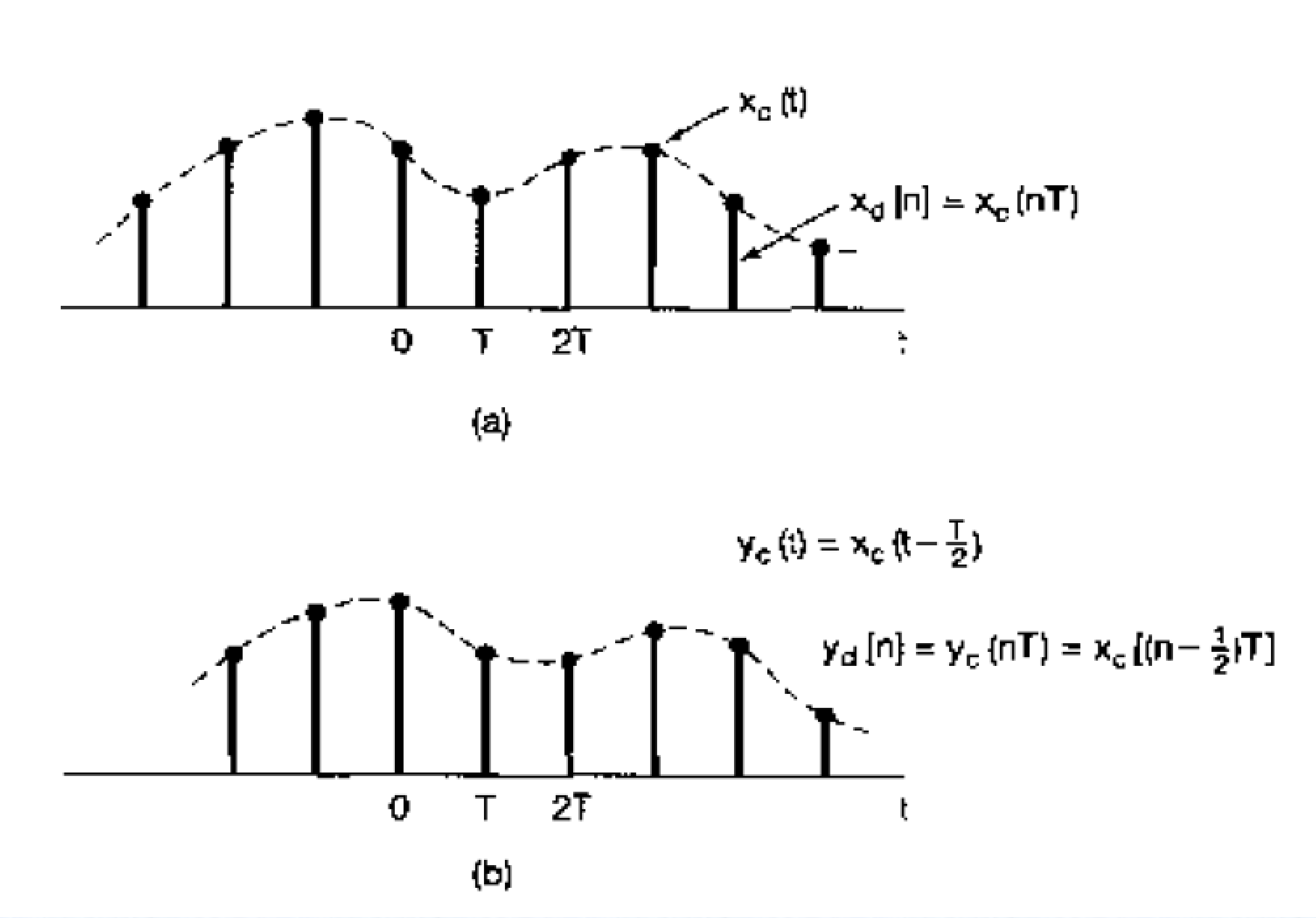


Yarı-Örneklem Gecikmesi

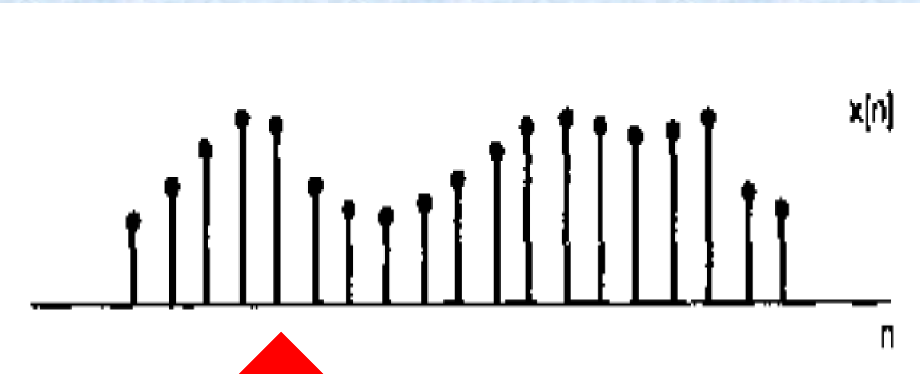


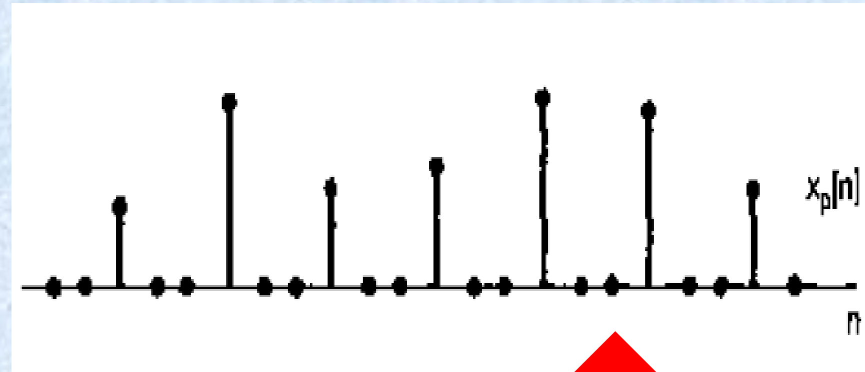


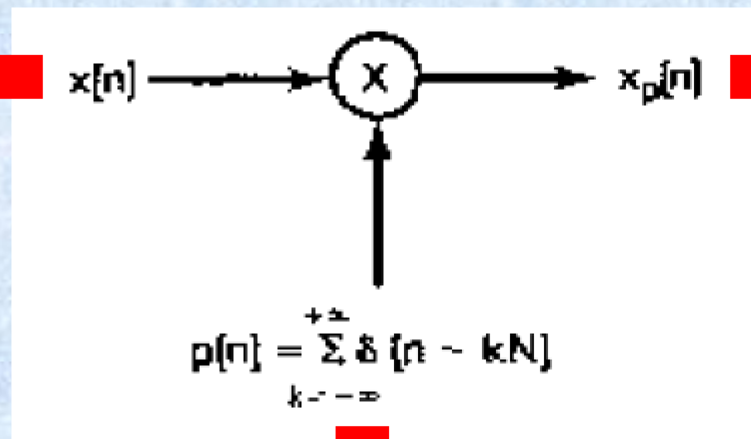
Yarı-Örneklem Gecikmesi

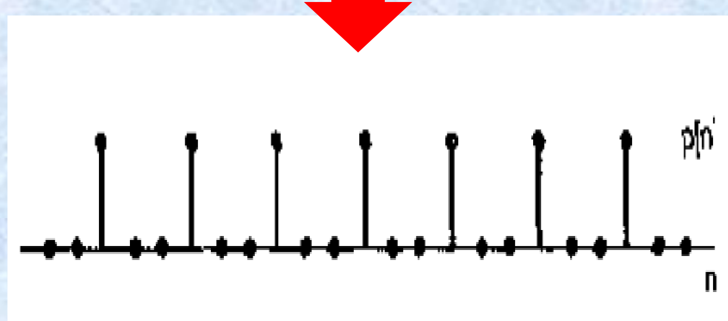


1. Dürtü Katarı Örneklemesi

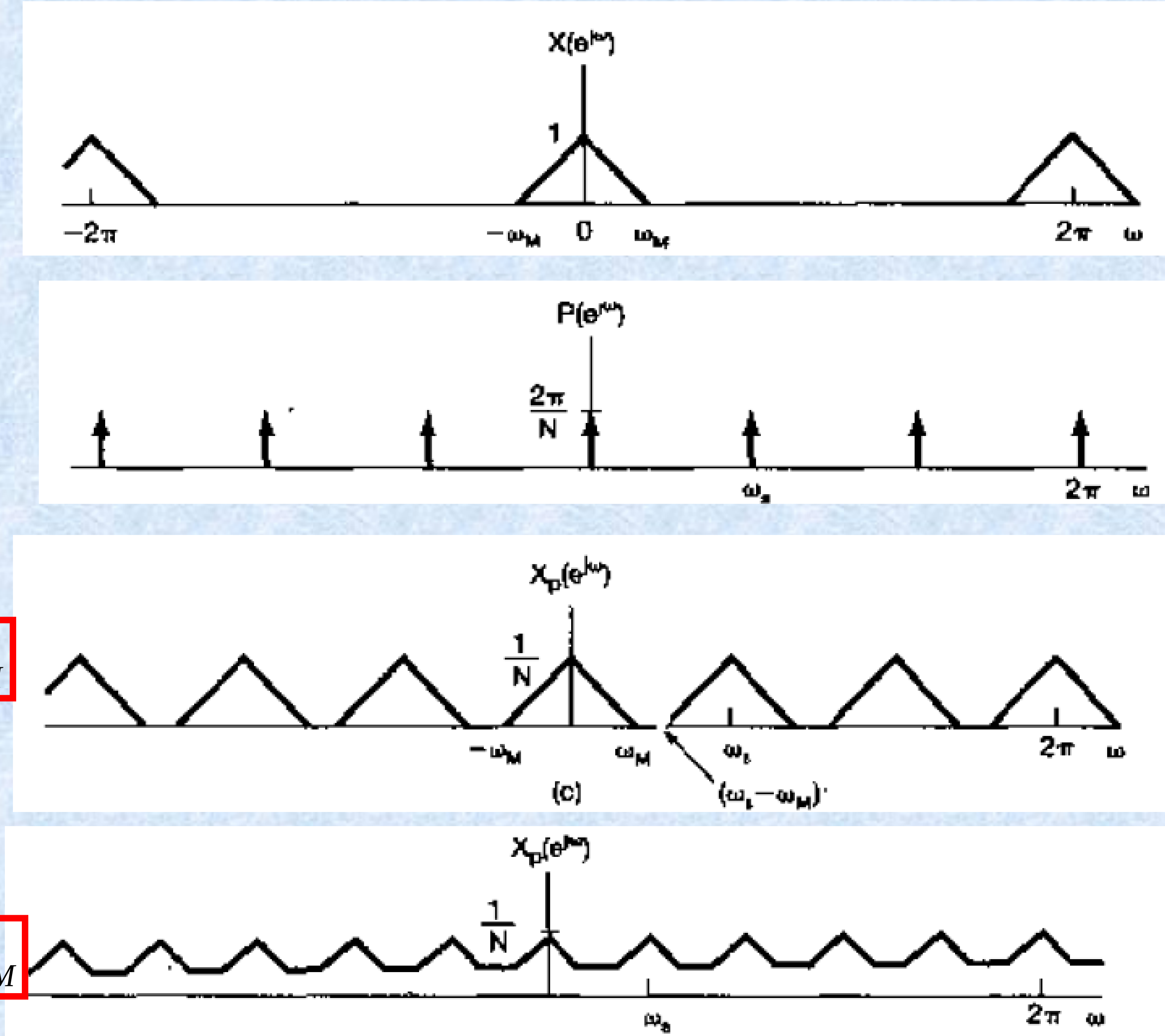








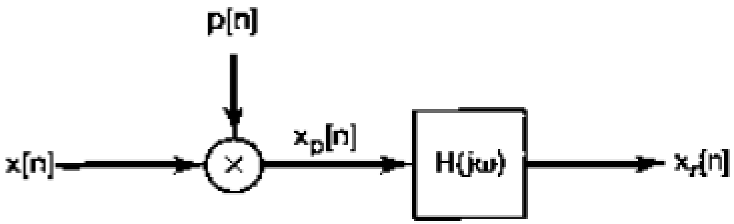
Dürtü Katarı Örneklemesi



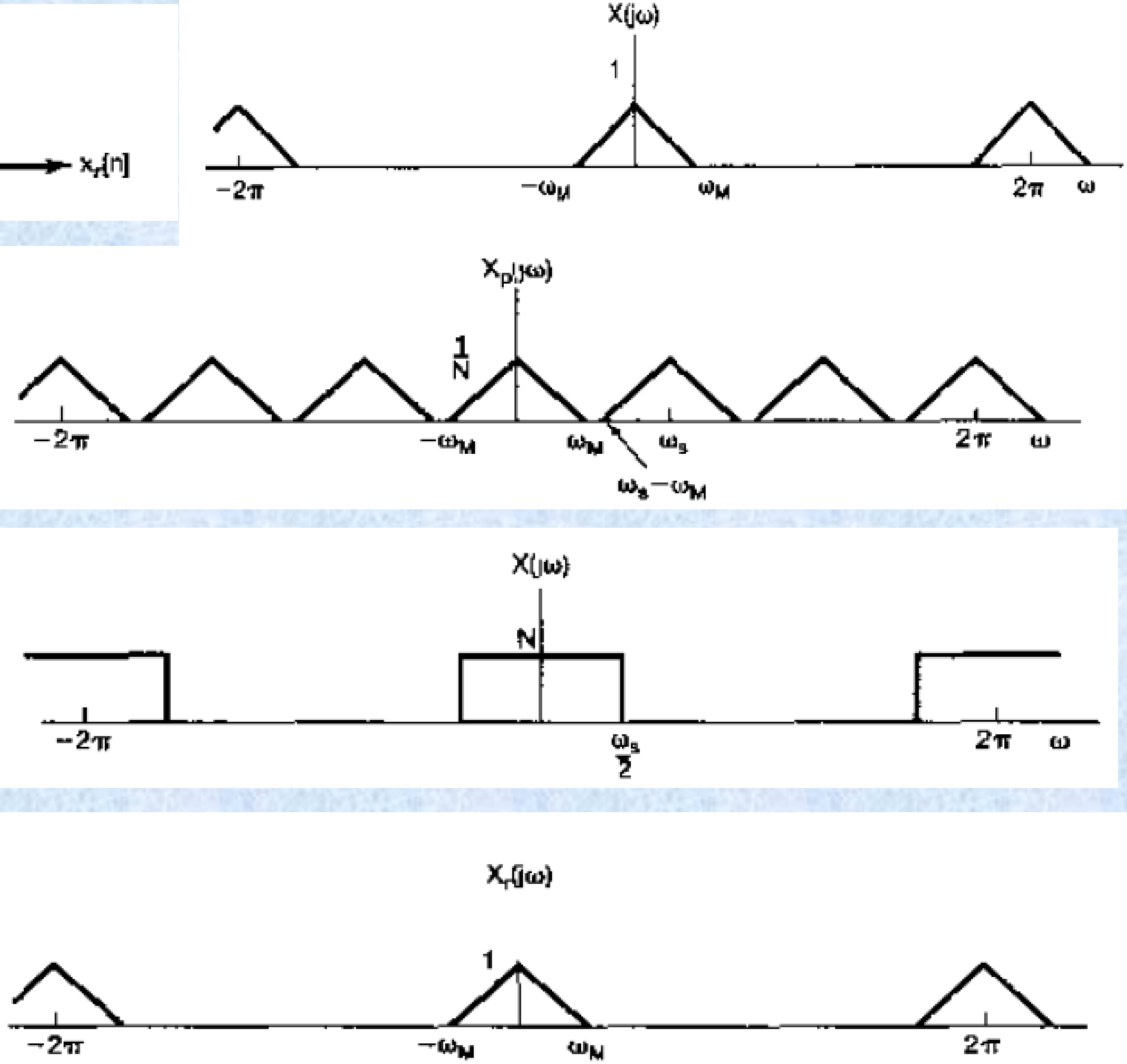
$\boxed{\omega_s > 2\omega_M}$

$\boxed{\omega_s < 2\omega_M}$

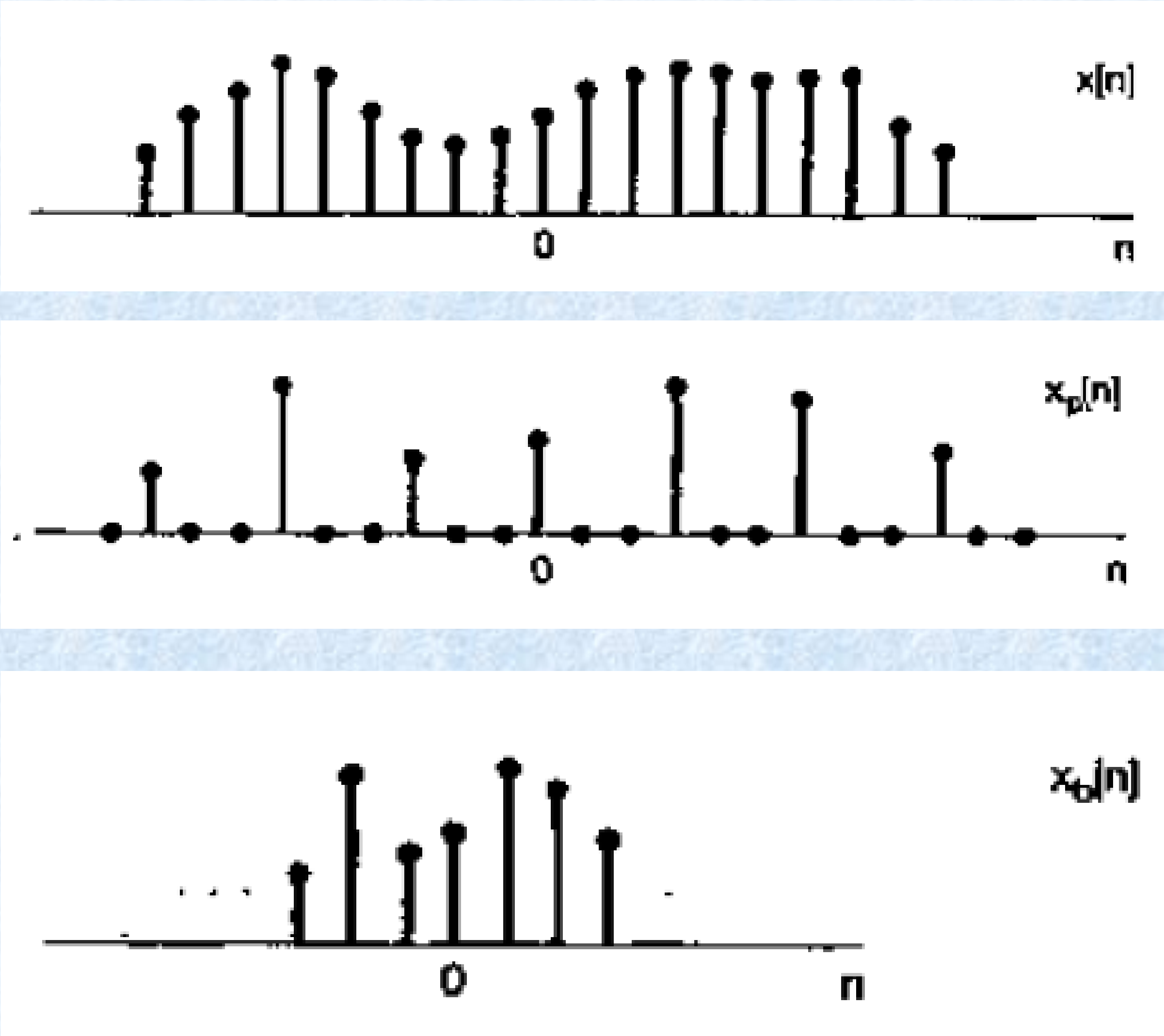
Dürtü Katarı Örneklemesi

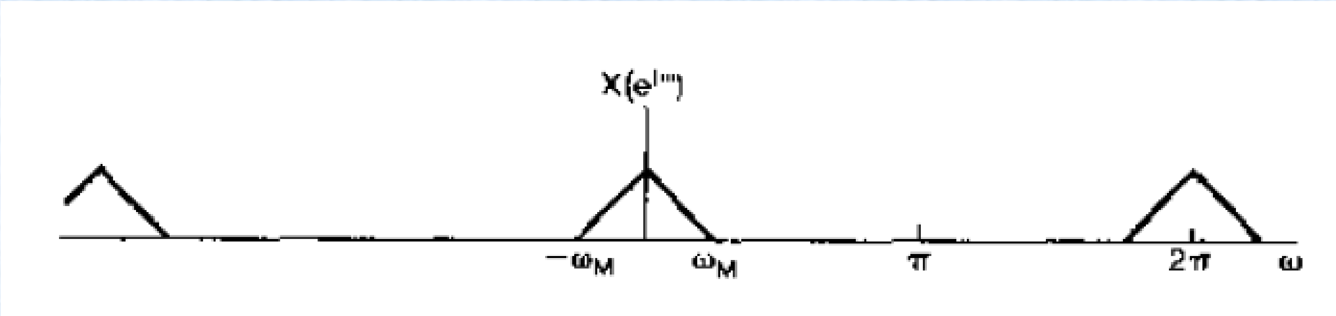


Bant sınırlı bir sinyalin örneklemlerinden geri çatılması ve Örneklenmesi için Blok Diagram

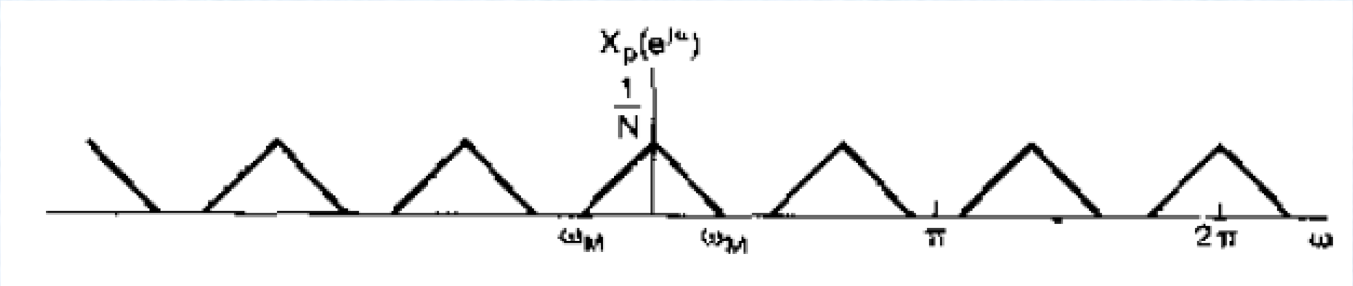


2. Ayrık Zamanlı Ondalıklama ve Ara Değerleme

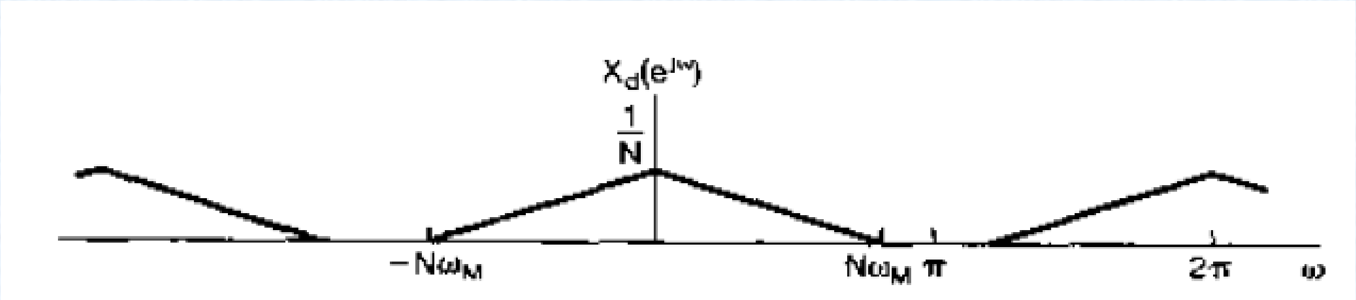




$X(e^{j\omega})$
$-\omega_M$
ω_M
π
2π
ω

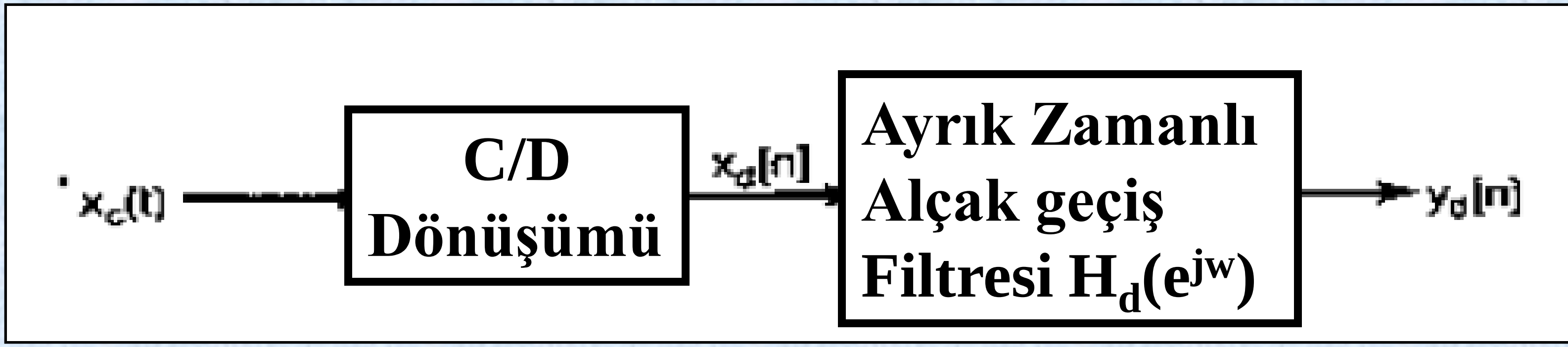


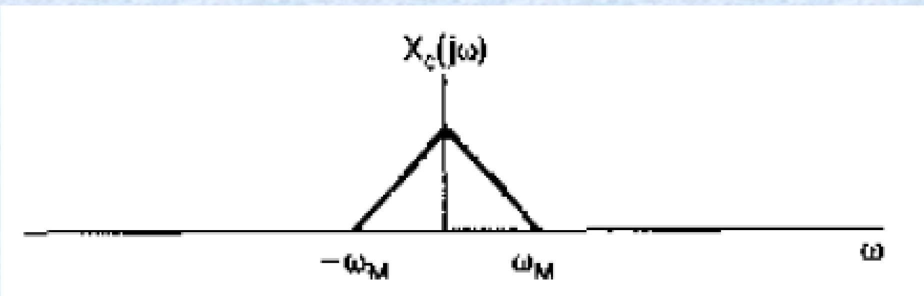
$X_p(e^{j\omega})$
$\frac{1}{N}$
ω_M
ω_M
π
2π
ω

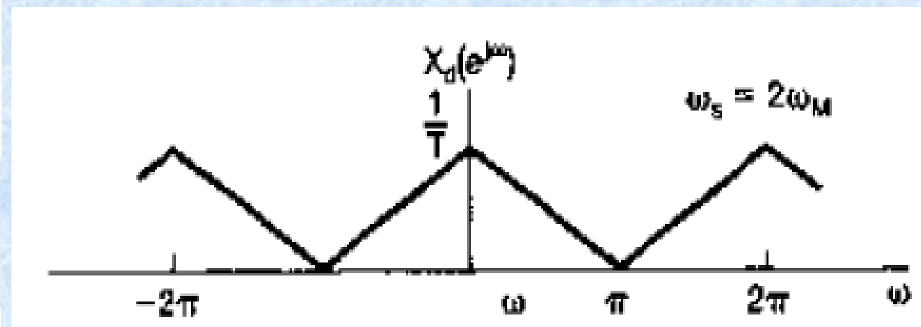


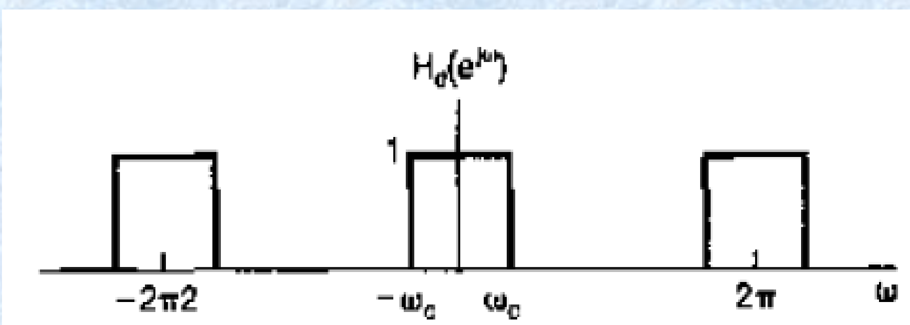
$X_d(e^{j\omega})$
$\frac{1}{N}$
$-N\omega_M$
$N\omega_M$
π
2π
ω

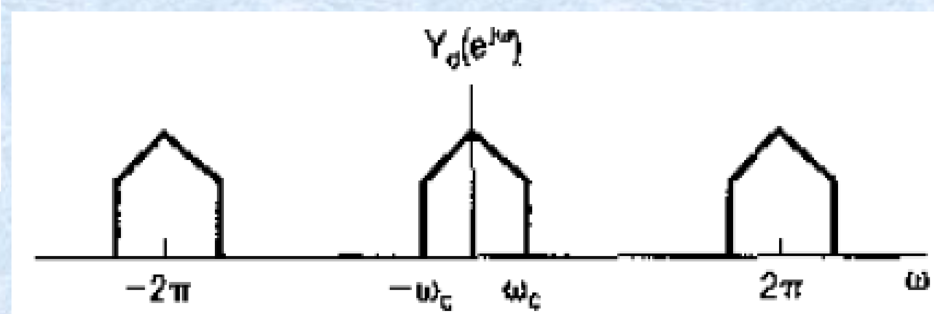
Ayrık Zamanlı Ondalıklama ve Ara Değerleme



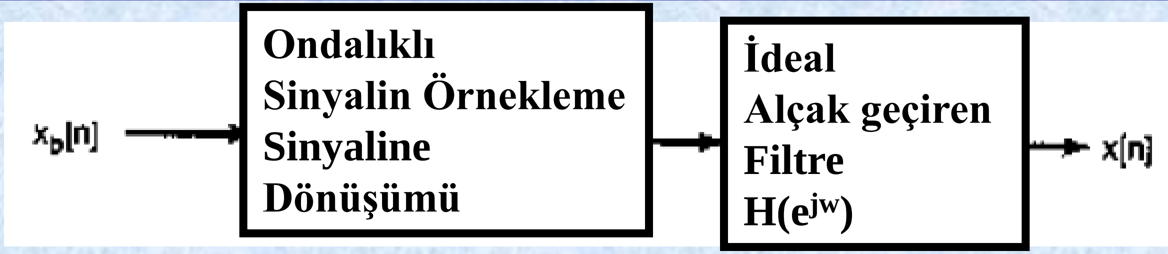


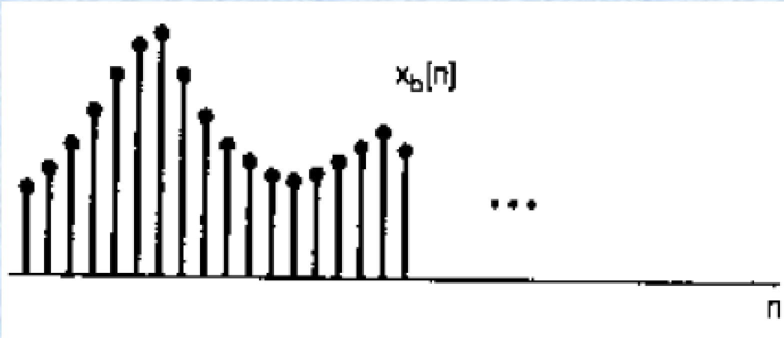


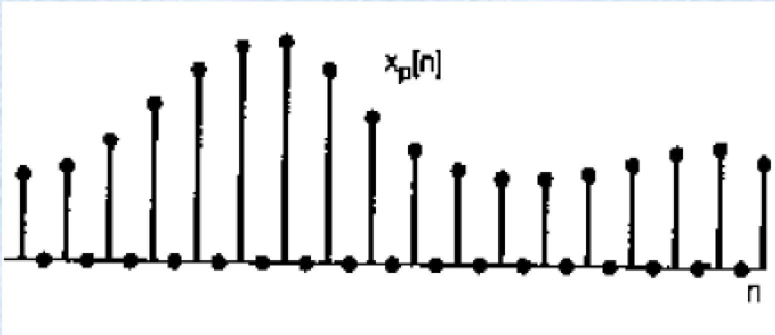


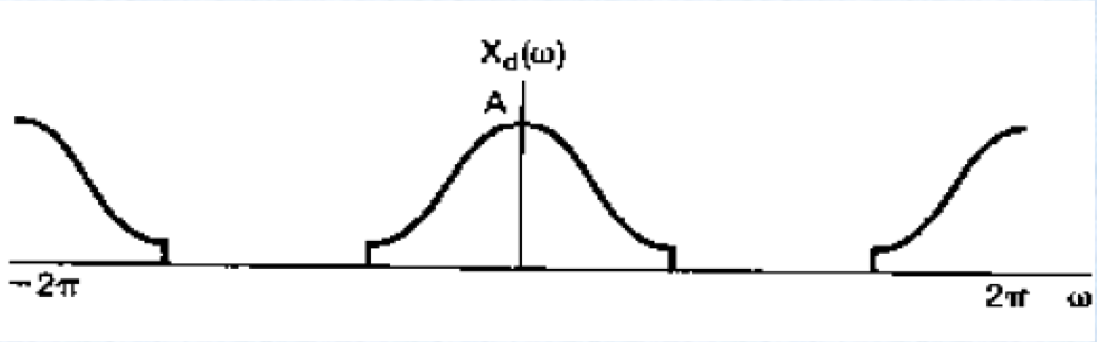


Ayrık Zamanlı Ondalıklama ve Ara Değerleme

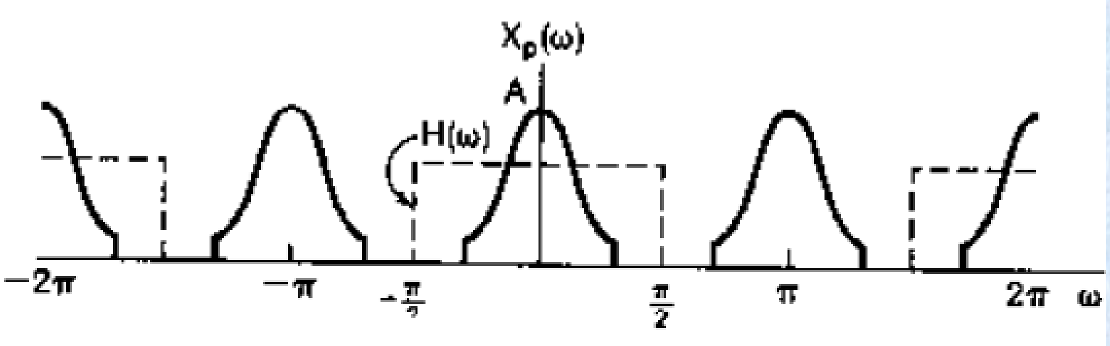




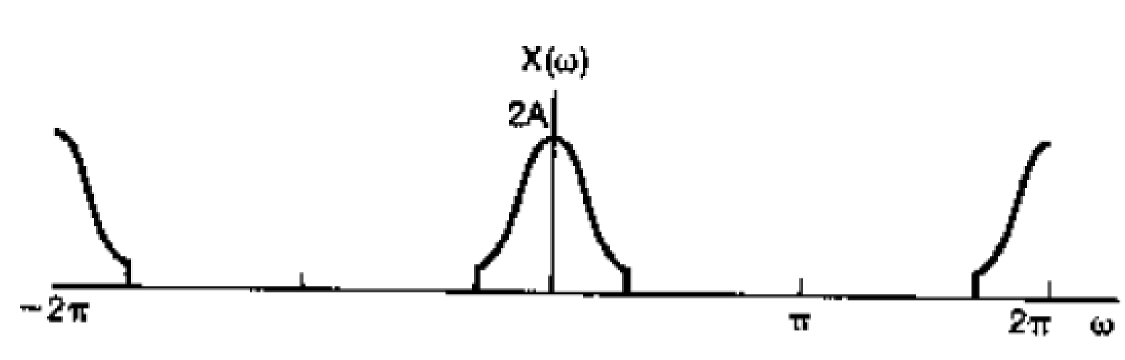




$X_d(\omega)$
A
-2π
2π
ω



$X_p(\omega)$
A
$H(\omega)$
-2π
$-\pi$
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
π
2π
ω



$X(\omega)$
2A
-2π
π
2π
ω

Ayrık Zamanlı Sinyallerinin Örneklenmesi

Örnek :