KISA SINAV II Cevap Anahtarı

(1) $3y[n]-4y[n-1]+y[n-2]=x[n]$; $x[n]=(\frac{1}{2})^n$ $y[-1]=1$ $y[-2]=2$

$y[n]=?$

$$3Y_I(z)-4\{z^{-1}Y_I(z)+y[-1]\}+\{z^{-2}Y_I(z)+z^{-1}y[-1]+y[-2]\}=X_I(z)$$

$$Y_I(z)\cdot(3-4z^{-1}+z^{-2})=2-z^{-1}+\frac{z}{z-\frac{1}{2}}$$

$$Y_I(z)\ \frac{3(z-1)(z-\frac{1}{3})}{z^2}=\frac{3z^2-2z+\frac{1}{2}}{z(z-\frac{1}{2})}$$

$$Y_I(z)=\frac{z(3z^2-2z+\frac{1}{2})}{3(z-1)(z-\frac{1}{2})(z-\frac{1}{3})}$$

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$$=\frac{3}{2}\frac{1}{1-z^{-1}}-\frac{1}{1-z^{-1}(\frac{1}{2})}+\frac{1}{2}\frac{1}{1-\frac{1}{3}z^{-1}}$$

$$y[n]=\left(\frac{3}{2}-\left(\frac{1}{2}\right)^n+\frac{1}{2}\left(\frac{1}{3}\right)^n\right)u[n]$$

(2) $x(t)=\delta(t)+3e^{-2t}u(t)-8e^{-3t}u(t)$

$$X(s)=1+\frac{3}{s+2}-\frac{8}{s+3}=\frac{(s+2)(s+3)+3(s+3)-8(s+2)}{(s+2)(s+3)}$$

$$=\frac{s^2+5s+6+3s+9-8s-16}{(s+2)(s+3)}=\frac{(s+1)(s-1)}{(s+2)(s+3)}$$

$$H(s)=\frac{Y(s)}{X(s)}=\frac{\frac{(s-1)(s+2)}{(s+4)(s+3)}}{\frac{(s+1)(s-1)}{(s+2)(s+3)}}=\frac{(s+2)^2}{(s+1)(s+4)}$$

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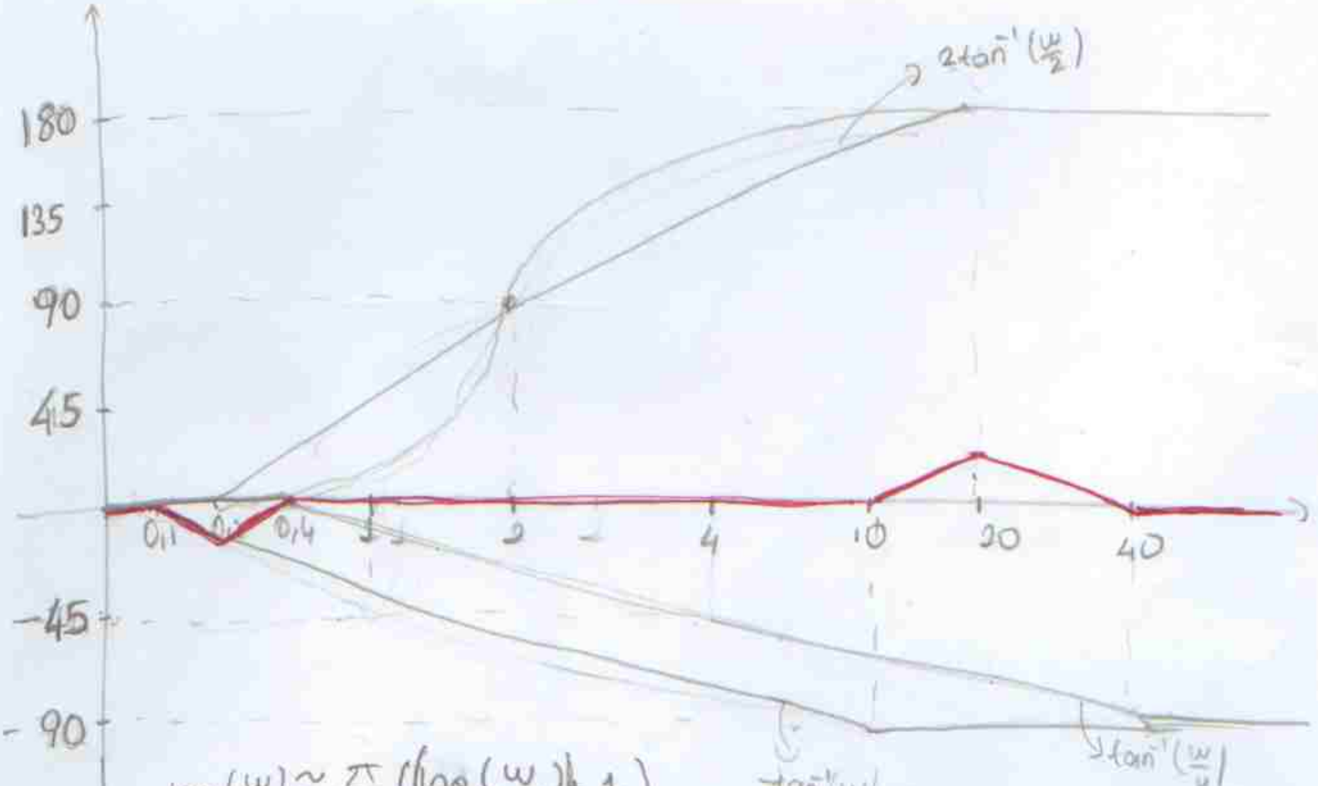
$$H(jw)=\frac{(jw+2)^2}{(jw+1)(jw+4)}=\frac{\frac{1}{4}(jw+2)^2}{\frac{1}{4}(jw+1)(jw+4)}=\frac{(\frac{jw}{2}+1)^2}{(\frac{jw}{4}+1)(jw+1)}$$

$$20\log|H(jw)|=40\log|1+\tfrac{jw}{2}|-20\log|1+jw|-20\log|1+\tfrac{jw}{4}|$$

kırılma frekans. $w_1=2$ $w_2=1$ $w_3=4$



$\tan(\frac{w}{2_1}) \cong \frac{\pi}{4}(\log(\frac{w}{2_1})+1)$

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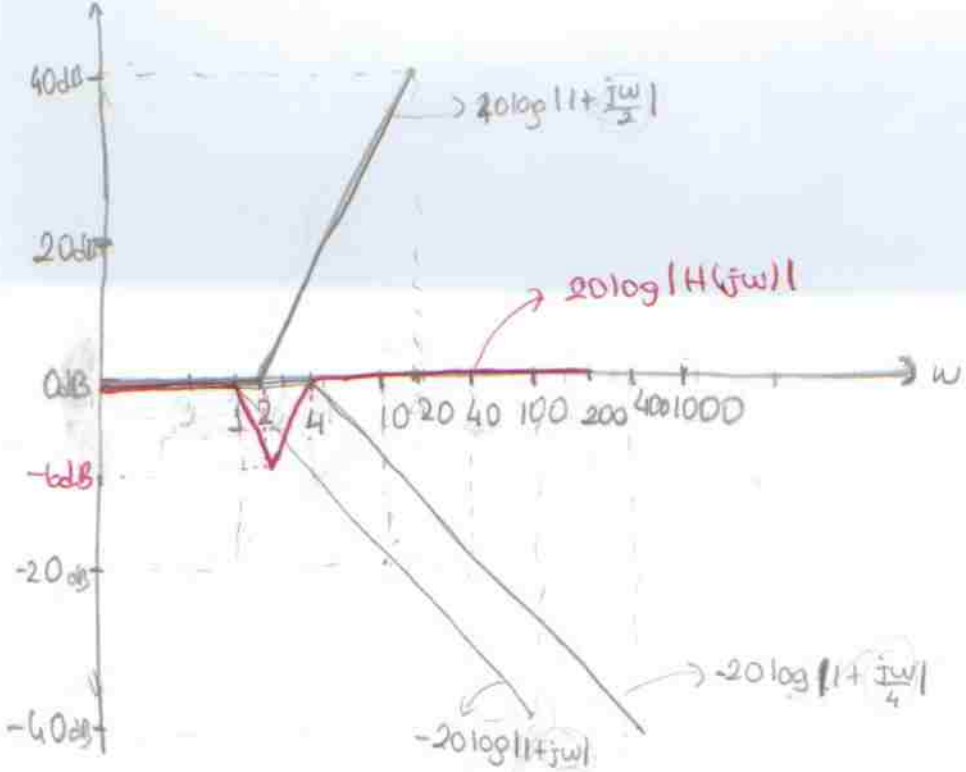
		$2\tan^{-1}(\frac{w}{2})$	$-\tan^{-1}(\frac{w}{4})$	$-\tan^{-1}(w)$	
		$\frac{\pi}{2}(\log(\frac{w}{2})+1)$	$-\frac{\pi}{4}(\log(\frac{w}{4})+1)$	$-\frac{\pi}{4}(\log(w)+1)$	
$w=0{,}1$	$\angle H(jw) =$	0	0°	0	0°
$w=0{,}2$	$\angle H(jw) =$	0	0°	-13,5°	-13,5°
$w=0{,}4$	$\angle H(jw) =$	27°	0	-27°	0°
$w=1$	$\angle H(jw) =$	63°	-18°	-45°	0°
$w=2$	$\angle H(jw) =$	90°	-31°	-59°	0°
$w=4$	$\angle H(jw) =$	117°	-45	-72°	0°
$w=10$	$\angle H(jw) =$	153°	-63°	-90°	0°
$w=20$	$\angle H(jw) =$	180°	-76°	-90°	14°
$w=40$	$\angle H(jw) =$	180°	-90°	-90°	0°



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$w=1 \quad 20\log|H(jw)| = 0 - 0 - 0 = 0\,dB$

$w=2 \quad 20\log|H(jw)| = 0 - 6 - 0 = -6\,dB$

$w=4 \quad 20\log|H(jw)| = 12{,}04 - 12{,}04 - 0 = 0\,dB$

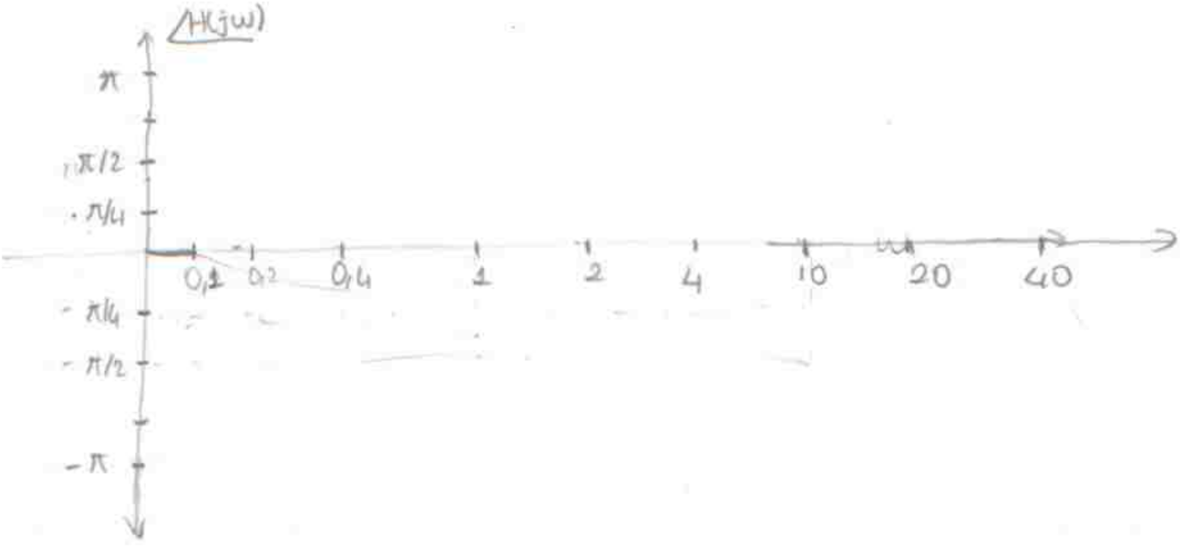
$w=10 \quad 20\log|H(jw)| = 27{,}95 - 20 - 7{,}95 = 0\,dB$

$w=20 \quad 20\log|H(jw)| = 40 - 26 - 14 = 0\,dB$

$w=40 \quad 20\log|H(jw)| = 52 - 32 - 20 = 0\,dB$

$\angle H(jw) = 2 \cdot \tan^{-1}\left(\frac{w}{2}\right) - \tan^{-1}\left(\frac{w}{4}\right) - \tan^{-1}(w)$

$w_1 = 2$	0,2	20
$w_2 = 1$	0,1	10
$w_3 = 4$	0,4	40



(3) $H(j\omega) = \prod_{n=1}^{N} \left(\frac{1-nj\omega}{1+nj\omega}\right)$

$$|H(j\omega)| = \prod_{n=1}^{N} \frac{|1-nj\omega|}{|1+nj\omega|} = \prod_{n=1}^{N} \frac{\sqrt{1+n^2\omega^2}}{\sqrt{1+n^2\omega^2}} = \prod_{n=1}^{N} 1 = 1 \quad \text{(15)}$$

$$\angle H(j\omega) = \sum_{n=1}^{N} \tan^{-1}(n\omega) - \tan^{-1}(-n\omega) \qquad \text{(30)}$$

$$= \sum_{n=1}^{N} 2\tan^{-1}(n\omega)$$

$$\tau(\omega) = -\frac{d\angle H(j\omega)}{d\omega} = -2\sum_{n=1}^{N} \frac{d(\tan^{-1}(n\omega))}{d\omega} = -2\sum_{n=1}^{N} \frac{n}{(n\omega)^2+1}$$

(15)