

## INTRODUCTION

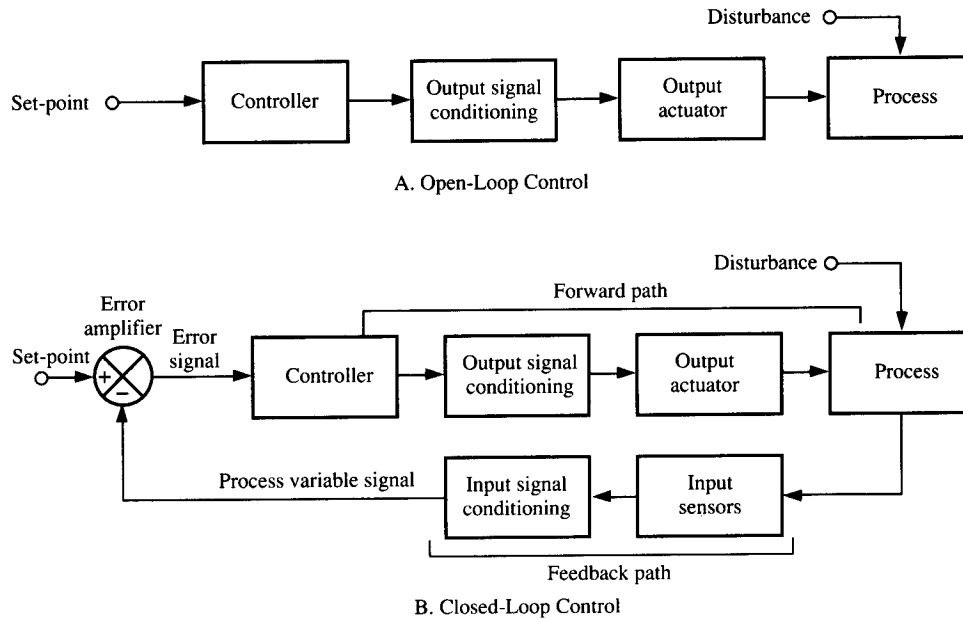
A control system can be described as a group of components arranged in such a way that the operational characteristics of a particular process are maintained at the desired level. Although control can be achieved mechanically, electrically, or through fluid power, this book is concerned with those systems that control the process by using electrical and electronic components.

To understand the need for control in even the most basic systems, consider the problem of regulating the temperature in an average household during the winter months. Technically, cycling the furnace ON and OFF at given intervals would keep the house at a relatively uniform temperature. Determining the proper duty cycle might require some experimentation, but after an appropriate cycle time had been determined, the temperature should remain constant. However, there are many variables, or system disturbances, that can change the duty cycle requirements. The outside temperature variations will change the rate at which heat is transferred to the outside. People entering and leaving the house will allow heat to escape. The number of people present in the house, along with their level of activity, will change the inside temperature. Obviously, constant monitoring and adjustment will be required if, in fact, the inside temperature is to remain constant. Rather than making these adjustments manually, it would be much more efficient (and less tiresome) to have the adjustments made automatically through the intervention of control devices.

In industrial processes the goal of control is usually much more critical than maintaining a constant household temperature. Automatic control systems are implemented in areas that would be hazardous or impossible for human operators to work in. They perform functions with greater speed, precision, and repeatability than are otherwise obtainable. The quality of the control systems in use can determine, for example, how well two machined parts will fit together or whether or not the welds on an auto chassis will break. That is, good or bad, identical items rolling out of a manufacturing facility should be just that—identical. Good electronic control allows products to be manufactured cheaper and faster and with a higher level of quality control.

Generally, a control system is classified according to the process that is being controlled. That is, temperature control systems control temperature, flow control systems control fluid flow, and level control systems control the height of material in holding bins or reservoirs. The nature of the controlling components also plays a part in system classification. In this respect, systems are usually classified as either analog or digital, depending on the electronic device technology employed. Control systems are also classified according to the presence or absence of feedback. Open-loop systems employ no feedback; closed-loop systems constantly monitor the process through a feedback loop.

Since all control systems share common structural characteristics, block diagrams are often used to describe them concisely. Depending on the particular textbook, application specification, or sales representative you are dealing with, the nomenclature associated with these block diagrams will vary. Each of the



**FIGURE 13-1**  
Typical System Control Diagrams

terms used in Figure 13-1B will now be explained, and some of the more common alternative names for them will be given. *Set-point*, *command*, and *reference* are all names used to describe the input that determines the desired or ideal operating point for the process. This is usually provided by a human operator, although it may also be supplied by another electronic circuit. The *process variable*, or *measured value*, or *controlled variable*, is the signal that contains information about the current process status. In other words, this is the feedback signal. Ideally, the process variable and the set-point should match (indicating that the process is operating exactly as desired). *Error amplifier*, *error detector*, *comparator*, and *summing point* are names given to the electronic components that determine whether the process operation matches the set-point. Usually, the device that makes the determination is nothing more than a differential amplifier. The output of the error amplifier is referred to as the *error signal* or the *system deviation signal*. The magnitude and the polarity of this signal will determine how the process will be brought back under control. The *controller* is the group of electronic components that, depending on the error signal, will produce the appropriate corrective signal. The *output actuator*, or *final correcting device*, is the component that directly affects a process change. Motors, heaters, fans, and solenoids are all examples of output actuators. For the controller output to drive the output actuator, the signal may have to be modified. For example, for an op amp to drive a 120 V AC motor, there must be some sort of power interface, as well as an

isolation circuit. Likewise, in the sensing of the process status to produce the process variable signal, some modification or conversion may be required. The components or circuits that provide this function are called *signal conditioners*.

As shown in Figure 13–1A, an open-loop control system consists of a controller, signal conditioning circuitry, an output actuator, and a process. Since the only input to the controller is the set-point, it is apparent that an open-loop system controls the process blindly. That is, the controller receives no information concerning the present status of the process, the need for any correction due to process disturbances, or the responses of the process to previous controller action. The previously described duty-cycle-controlled furnace controller operated in an open-loop mode. The set-point was established by a human operator, and the furnace responded to that set-point alone, ignoring completely any variation in room temperature due to external influences. Open-loop control systems are considerably cheaper and less complex than their closed-loop counterparts. The inevitable result of their use, however, is poor process control.

The presence of the feedback loop in Figure 13–1B classifies this system as closed-loop. Since the controllers in such systems receive information about the process status, they can compensate for external disturbances. In other words, they can automatically adjust process conditions to ensure that external events do not cause the process to stray from the set-point. To convert the duty-cycle-controlled furnace to a closed-loop system, only temperature-measuring devices and an error amplifier must be added. The error signal can then be used to control the length of time that the furnace stays on. Or the furnace can be allowed to run continuously, and the error signal can be used to control the amount of fuel burned. Closed-loop systems, although more expensive and complex than open-loop control, allow more flexibility and afford the operator tighter control over the process.

## THE ROLE OF THE CONTROLLER

At the heart of any electronic control system is the controller. This is the circuitry that accepts information from the input transducers and the signal conditioners and provides the signals that will ultimately be used to correct the error condition. The purpose of the controller is to maintain the process variable within an acceptable distance of the set-point. Under ideal circumstances, the process variable would never stray from the set-point. If conditions within the system attempted to change, the controller would sense this tendency and instantly produce an output that would prevent deviation from the set-point. The quality of a controller, therefore, depends on how closely the process variable tracks the set-point (how small an error can be maintained) and how quickly the controller can respond when the two do not match. The type of controller that is chosen for any particular application depends on the desired speed of response, the allowable system error, and the process dynamics.

### 13-3 ON/OFF CONTROLLERS

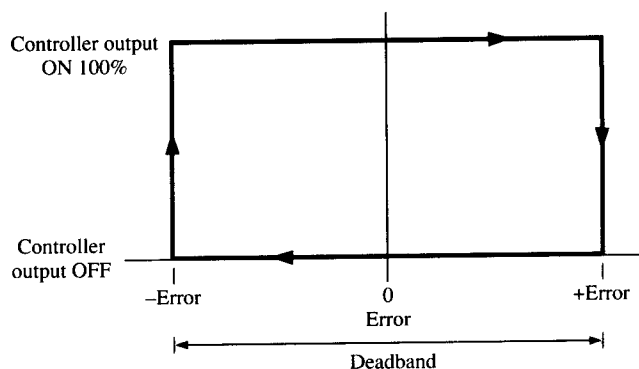
The *ON/OFF*, or *two-position*, controller attempts to control the process variable with an output that is either fully ON or fully OFF. If the sensed magnitude of the process variable is below a given threshold (the set-point), the controller output turns completely OFF. This action turns OFF the output actuator, and the process variable comes back into the vicinity of the set-point. If, on the other hand, the sensed magnitude of the process variable is above the set-point, the controller output turns completely ON, providing full power to the output actuator.

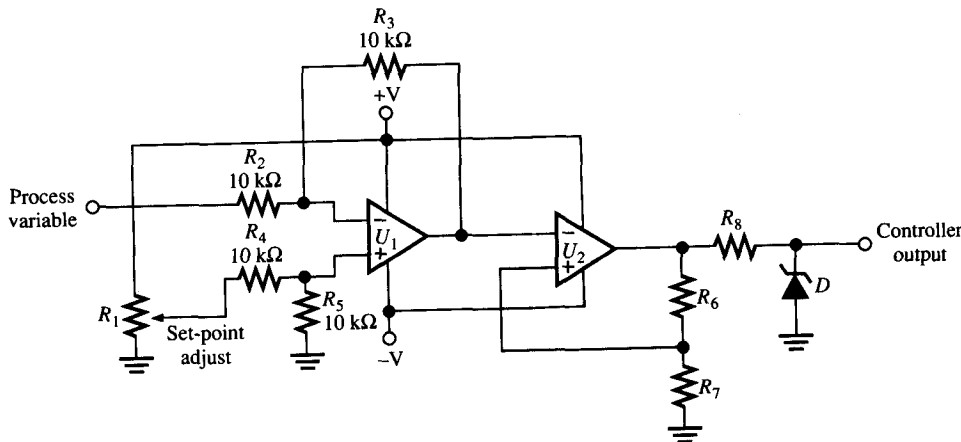
The preceding description is of a *direct-acting controller*—one in which the controller output and the process variable move in the same direction. An *inverse-acting controller* would produce a full-ON output if the process variable were below the set-point. Likewise, if the process variable were above the set-point, the controller output would be OFF. Changing a controller from direct- to inverse-acting (or vice versa) simply involves an inversion of the controller output.

For an ON/OFF controller to be practical, it must exhibit some degree of hysteresis. In other words, the magnitude of the process variable that forces the controller output ON must be different from the value that forces the output OFF. If no hysteresis exists, the output will oscillate, causing a loss of process control and possible damage to system components. Figure 13-2, the transfer curve for an ON/OFF controller, demonstrates this principle. For the controller output to be driven to full-ON, the error signal must decrease to the magnitude indicated by  $-Error$ . Once the output is ON, however, it will not turn OFF again until the error signal is above the magnitude indicated by  $+Error$ . A significant difference may exist between these two points.

In an inverse-acting temperature controller, for example, if the temperature rises above the set-point (creating a positive error) by a sufficient amount, the output of the controller will shut OFF the heater. This will cause the temperature to begin to decrease. A temperature decrease sufficient to cause the error signal to fall to zero will not turn the heater back ON. Instead, the temperature must decrease below the set-point (creating a negative error) by a certain amount before the controller output will again turn the heater ON.

**FIGURE 13-2**  
Hysteresis in an ON/OFF Controller





**FIGURE 13-3**  
Typical Op-Amp ON/OFF Controller

This error “window” is called the *deadband* for the controller. Basically, a deadband means that the error signal can change without affecting the output of the controller. Formally, the deadband is the difference between the error signal that turns the controller output fully ON and the error signal that turns the controller output fully OFF. That is,

$$\text{Deadband} = \text{Error}_{\text{ON}} - \text{Error}_{\text{OFF}} \quad (13.1)$$

The deadband may be expressed as a voltage or in terms of the process variable.

In some cases, the deadband is an inherent part of the system, caused by mechanical hysteresis, thermal lag, or some other characteristic. In many cases, however, a deadband must be implemented electronically. This implementation allows for adjustment and calibration of the system for individual applications. Remember that an ideal controller would reduce the error signal to zero. Because a deadband is required, an ON/OFF controller will always have a finite error. That is, the process variable will never be maintained at the desired set-point. The system will continually “hunt” for the proper value.

Figure 13-3 shows a typical ON/OFF controller. Op-amp  $U_1$ , a differential amplifier, serves as the error amplifier. Here it is set for unity gain, so its output (the error signal) is simply the difference between the set-point voltage and the voltage at the process variable input. This error voltage is then compared with a fraction of the output from  $U_2$ . The amount of feedback to the inverting input of  $U_2$  establishes the controller deadband. To drive the output of  $U_2$  to negative saturation, the error signal must increase to a magnitude greater than the voltage across  $R_7$  (the voltage across  $R_7$  is positive at this time, since the output of  $U_2$  is at  $+V_{\text{sat}}$ ). As soon as the output of  $U_2$  changes, however, the voltage across  $R_7$  will

be negative with respect to ground. Therefore, to drive the output back to positive saturation, the error signal will have to decrease to a magnitude that is more negative than the voltage across  $R_7$ . The span of error voltage (the deadband) that is required to drive the output of  $U_2$  from  $+V_{\text{sat}}$  to  $-V_{\text{sat}}$  (or from  $-V_{\text{sat}}$  to  $+V_{\text{sat}}$ ) is twice the voltage across  $R_7$ . The equation for this relationship is as follows:

$$\text{Deadband} = 2V_{\text{sat}} \left( \frac{R_7}{R_6 + R_7} \right) \quad (13.2)$$

In many cases, control will not be maintained if the output of the controller is allowed to swing from  $+V_{\text{sat}}$  to  $-V_{\text{sat}}$ . For example, if the controller is driving the coil of a DC relay, the contacts will be pulled in whether the controller provides an output of  $+V_{\text{sat}}$  or  $-V_{\text{sat}}$ . Therefore, the output must often be limited to values between 0 V and  $+V_{\text{sat}}$  (or between 0 V and  $-V_{\text{sat}}$ ). In Figure 13–3 resistor  $R_8$  and the zener diode serve to clamp the output of  $U_2$  to positive values. Note that when  $U_2$  is at  $-V_{\text{sat}}$ , the zener diode is forward-biased, and the resultant output is clamped to  $-0.7$  V. When  $U_2$  is at  $+V_{\text{sat}}$ , the output cannot increase beyond the reverse-avalanche zener voltage. If the zener chosen for this circuit avalanched at 4.5 V, the controller output would be limited to values between  $-0.7$  V and  $+4.5$  V, even though the supply voltage to  $U_2$  might be considerably higher. The value of  $R_8$  should be chosen so as to limit the forward and reverse current through the zener to a reasonable value.

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**EXAMPLE 13–1** Given the circuit of Figure 13–3, suppose that the op amps are powered with a supply of  $\pm 12$  V,  $R_6 = 100$  k $\Omega$ ,  $R_7 = 22$  k $\Omega$ , and  $D$  has an avalanche voltage of 6 V.

*Calculate the Circuit Deadband*

The deadband may be calculated from equation 13.2. Since  $U_2$  is powered with a supply of  $\pm 12$  V, it will saturate at about  $\pm 10.5$  V. Inserting this value and the values of  $R_6$  and  $R_7$  into the equation yields the following result:

$$\text{Deadband} = 2(10.5 \text{ V}) \left( \frac{22 \text{ k}\Omega}{122 \text{ k}\Omega} \right) = 3.8 \text{ V}$$

That is, if the error signal increases above zero by  $+1.9$  V, the output of  $U_2$  will be driven to negative saturation. To drive the output to positive saturation, the error signal will have to decrease to a value of  $-1.9$  V (a span of 3.8 V).

*Draw the Circuit Transfer Curve*

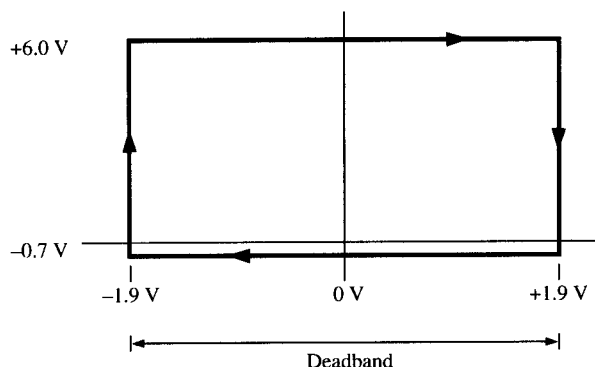
To draw the transfer curve, four distinct points are required: the deadband endpoints (+Error and –Error), the minimum controller output, and the maximum controller output. Since the deadband has already been calculated, only the output conditions must be determined. Note that even though the output of  $U_2$  may switch between  $+10.5$  V and  $-10.5$  V, the actual controller output is limited by  $R_8$  and the zener to either  $-0.7$  V or  $+6$  V.

With the error signal along the  $x$ -axis, and the controller output along the  $y$ -axis, the following points define the hysteresis curve (Figure 13–4) that is charac-

teristic of this circuit. It is important to include an arrow along the curve to indicate the direction of the process.

Error Signal (V)	Controller Output (V)
-1.9	-0.7
-1.9	+6
+1.9	-0.7
+1.9	+6

FIGURE 13-4



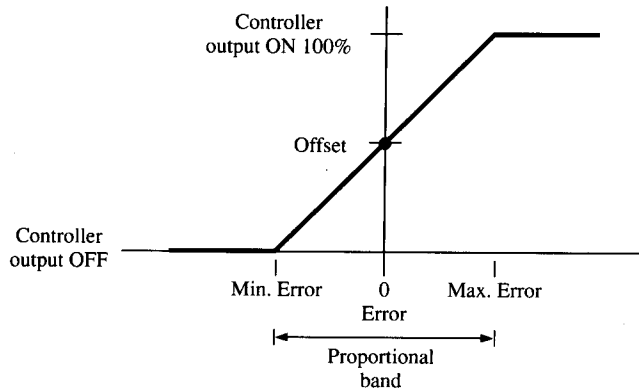
## 13-4 PROPORTIONAL CONTROLLERS

In many instances, the error window of an ON/OFF controller cannot be tolerated. A *proportional controller* allows tighter control of the process variable, because its output can take on any value between fully ON and fully OFF, depending on the magnitude of the error signal. As indicated by the name of this controller, the output changes proportionally with small changes in the error input. A proportional controller usually has a linear response (see Figure 13-5). That is, if the error input doubles, the controller output also doubles. Linearity, however, is not a requirement for this type of controller. For example, the output may be proportional to the square or the logarithm of the error signal. Such outputs, however, are the exception and not the rule.

Large error inputs can still drive the controller to positive or negative saturation. Proportional controllers, however, are operated in a region called the *proportional band*. Like the deadband of an ON/OFF controller, the proportional band is described as the change in error signal that will cause the controller output to swing from full-OFF to full-ON. Likewise, it may be expressed as an absolute voltage or in terms of the process variable. The proportional band for a linear controller can be calculated in terms of the controller output, as follows:

$$\text{Proportional band} = \frac{V_{\text{outmax}} - V_{\text{outmin}}}{A_V} \quad (13.3)$$

where  $A_V$  is the controller gain.



**FIGURE 13-5**  
Typical Proportional Controller Response Curve

There are three points of interest on the proportional-controller transfer curve in Figure 13-5. The first is the magnitude of error signal that drives the controller to full-ON. The second is the magnitude of error signal that drives the controller output to full-OFF. These two points need not be equally spaced on either side of the zero-error point. The factor that determines where the two points fall is called the *offset*. This is the point where the curve crosses the y-axis. More precisely, it is the output that the controller produces when the error signal is zero. Note that when the offset is exactly halfway between  $+V_{\text{sat}}$  and  $-V_{\text{sat}}$  (as it is in Figure 13-5),  $+ \text{Error}$  and  $- \text{Error}$  are spaced equally on either side of the zero error point. If, however, the offset is decreased, the effect is to move the entire curve to the right, without disturbing the slope of the line. Thus, the magnitude of the offset does not affect the magnitude of the proportional band.

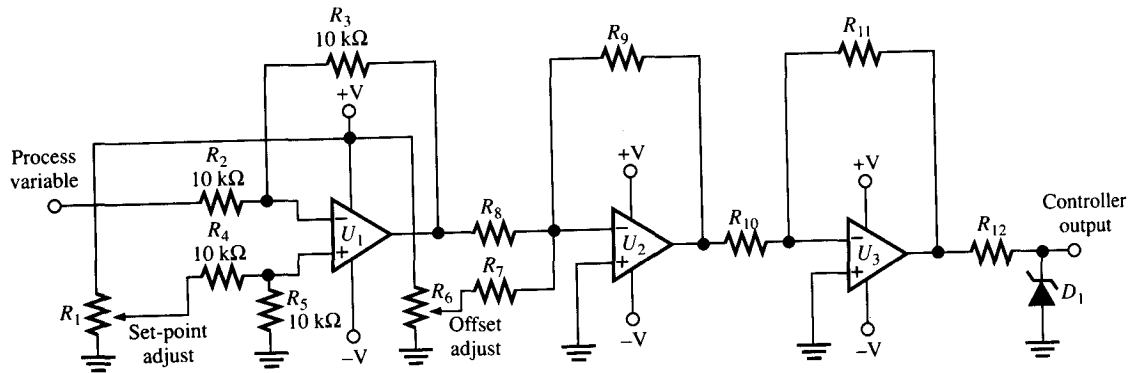
Since the proportional band says nothing about the actual magnitude of the error signals, it may be necessary to calculate them. Naturally, the minimum error signal will produce the minimum controller output, and the maximum error will produce the maximum controller output. Both of these values, however, are modified by the offset voltage. The minimum and maximum error signals can be calculated by the use of the following equations:

$$\text{Error}_{\text{min}} = \frac{V_{\text{out}_{\text{min}}}}{A_v} - V_{\text{offset}} \quad (13.4)$$

$$\text{Error}_{\text{max}} = \frac{V_{\text{out}_{\text{max}}}}{A_v} - V_{\text{offset}} \quad (13.5)$$

Shown in Figure 13-6 is a typical proportional controller. Differential amplifier  $U_1$  provides the error signal. As in the ON/OFF controller, it is set for unity gain, so the error voltage is the difference between the set-point voltage and the sensed process-variable voltage. Op-Amp  $U_2$  is an inverting summing amplifier. Thus, it can add an offset voltage to the error voltage before amplification. The





**FIGURE 13-6**  
Typical Op-Amp Proportional Controller

significance of the offset is that it allows the controller to maintain an output even when the error signal is near zero. If the controller output decreased to zero when there was no error (when the process variable and the set-point matched), the output actuator would turn OFF. This action would, in turn, cause the process variable to stray from the desired set-point. For this reason, proportional controllers, like ON/OFF controllers, are unable to maintain an error signal of zero. If the controller output is offset by a value that is exactly halfway between  $+V_{\text{sat}}$  and  $-V_{\text{sat}}$  for  $U_2$ , it is easier (and faster) for the controller to correct for changes in the process variable. When the error signal is zero, the controller output may be, for example,  $+5\text{ V}$ . This may not be the correct output required to maintain the error at zero. Perhaps an output of  $+7\text{ V}$  is required. With an offset of  $+5\text{ V}$ , the controller output needs to change by only  $2\text{ V}$  (as opposed to the entire  $7\text{ V}$ ). The offset will not allow the controller to maintain an error of zero; it simply facilitates correction of the process variable. The error can be reduced, however, by increasing the gain of  $U_2$ . This increase results in a narrower proportional band (a smaller error change will cause the output to swing full-scale). This action would be similar to reducing the deadband of an ON/OFF controller. There is a practical limit to the extent to which the error signal can be reduced by increasing the gain. The system will most likely oscillate long before the error can be totally reduced. Note that  $U_2$  controls two aspects of the transfer curve. The gain of  $U_2$  determines the slope of the line, and the magnitude of the offset positions the entire curve about the zero-error point.

**EXAMPLE 13-2** Given the circuit of Figure 13-6, suppose that the offset input is adjusted to  $+1\text{ V}$ . Assume that  $R_7$  and  $R_8$  are each  $10\text{ k}\Omega$  and that  $R_9$  is  $20\text{ k}\Omega$ . In addition, assume that  $U_3$  is operating with unity gain.

### Calculate the Circuit Proportional Band

Equation 13.3 can be used to calculate the proportional band. However, keep in mind that the output of the controller is clamped to  $-0.7\text{ V}$  and  $+6\text{ V}$ . Therefore, the minimum and maximum outputs are not  $\pm V_{\text{sat}}$  for  $U_2$ .

$$\text{Proportional band} = \frac{+6\text{ V} - (-0.7\text{ V})}{2} = 3.35\text{ V}$$

### Draw the Circuit Transfer Curve

The transfer curve is most easily drawn by calculating the minimum and maximum error inputs and simply plotting and connecting them. Since the gain of  $U_2$  is 2 and the offset is adjusted to  $+1\text{ V}$ ,  $U_2$  should produce an output of  $+2\text{ V}$  in response to an error of zero. Therefore, it is only necessary, as a quick check, to make sure that the curve crosses the y-axis at  $+2\text{ V}$ . Equations 13.4 and 13.5 give the following results:

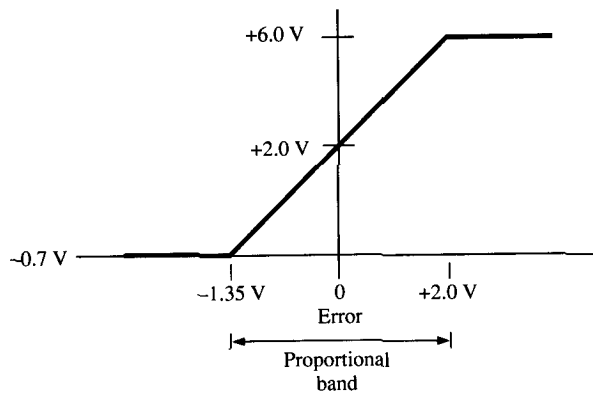
$$\text{Error}_{\min} = \frac{-0.7\text{ V}}{2} - 1\text{ V} = -1.35\text{ V}$$

$$\text{Error}_{\max} = \frac{+6\text{ V}}{2} - 1\text{ V} = 2\text{ V}$$

With the error signal along the x-axis and the controller output along the y-axis, the following coordinates define the endpoints of the curve (Figure 13-7):

Error Signal (V)	Controller Output (V)
-1.35	-0.7
+2	+6

**FIGURE 13-7**



## INTEGRAL CONTROLLERS

When implementing ON/OFF or proportional controllers in an electronic control system, the user must be able to tolerate a certain amount of error. In an ON/OFF

temperature-control system, for example, one must be content with the fact that the system will never quite stabilize at the desired temperature—it will fluctuate around the set-point. An *integral controller*, however, is capable of driving the error to zero and keeping it there. The main element of an integral controller is an integrator.

Remember from Chapter 6 that the magnitude of the output of an integrator is not proportional to the input. Rather, it is the rate of change of the output that is proportional to the input. The greater the input, the faster the output changes. If the input is increasing, the output changes at an increasing rate. One of the most important characteristics of an integrator, however, is its ability to maintain an output even after the input has decreased to zero. That is, when the input has fallen to zero, the output no longer changes; it simply maintains the output that was present when the input fell to zero.

Formally, the output of an integral controller may be expressed in terms of its error input as follows:

$$V_{\text{out}} = -\frac{1}{R_i C_f} \int_0^t V_{\text{error}} dt + V_0 \quad (13.6)$$

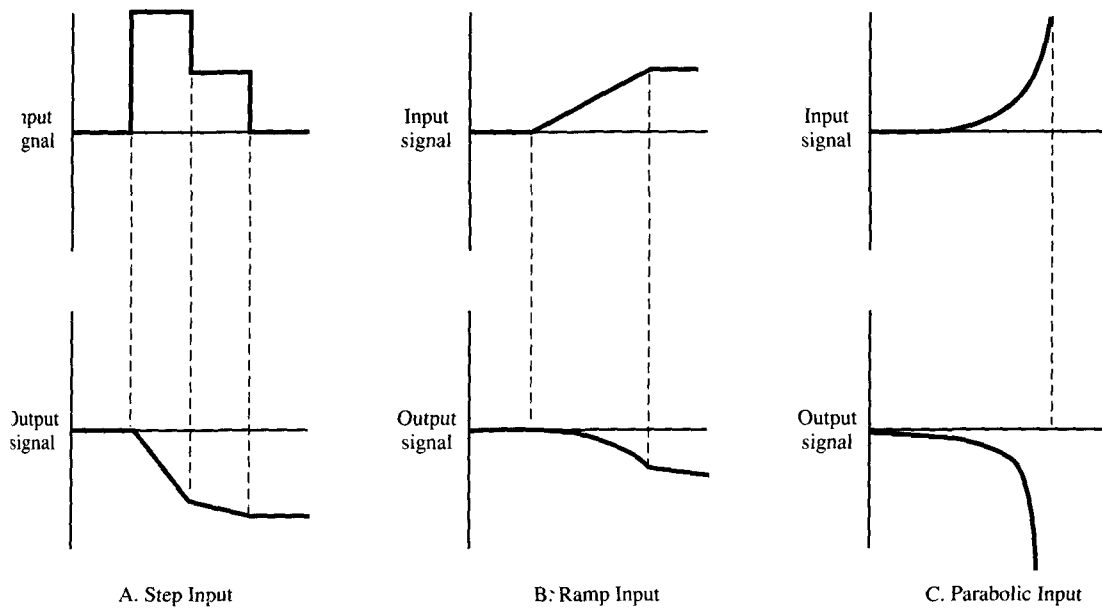
where  $V_0$  is the initial capacitor voltage,  $R_i$  is the op-amp input resistor, and  $C_f$  is the op-amp feedback capacitor. For the special case in which the error input is a steady DC, equation 13.6 simplifies to the following equation:

$$V_{\text{out}} = -\left(\frac{V_{\text{error}}}{R_i C_f}\right) t + V_0 \quad (13.7)$$

That is, when the error input is constant, the controller output changes linearly (*ramps*) as a function of time. Note that the actual output from an op-amp integrator is inverted (as indicated by the minus sign in equations 13.6 and 13.7). This inversion is of little consequence, however, since an extra gain or inversion stage can be easily added at the output.

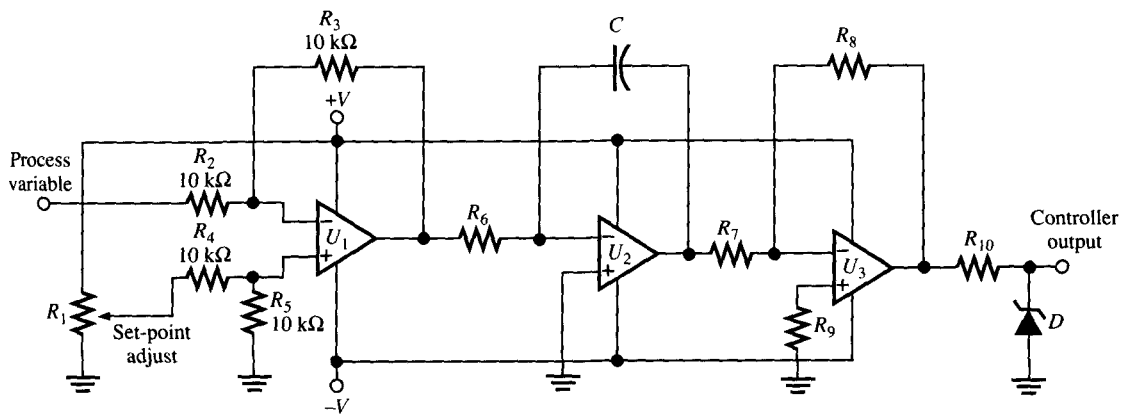
Three important assumptions are made for equation 13.6. First, it is assumed that the output of the integrator is ideal. Very few electronic circuits provide ideal performance. Second, it is assumed that the error signal (the input to the integrator) can be accurately represented by a mathematical equation. Depending on the nature of the controlled variable, it may be very difficult to describe the error signal mathematically. The function will be, at best, an approximation. Finally, it is assumed that the person calculating the output signal will have more than a cursory familiarity with the process of integration. The overall result is that unless the error input is a simple step function, the magnitude of the output of an integral controller may be very difficult to calculate. By analyzing some typical inputs, however, it is possible to predict the response of the output. Shown in Figure 13–8 are some common input functions, along with the ideal integrator outputs they would produce.

The integral controller of Figure 13–9 consists of three main blocks. As usual, the unity-gain differential amplifier,  $U_1$ , provides the error signal. If, in



**FIGURE 13-8**  
Typical Integrator Output Responses

fact, a higher gain is required, it is a simple matter to adjust the values of  $R_2$  through  $R_5$ . Op-amp  $U_2$  performs the actual integration. Remember from equation 13.6 that the values of  $R_6$  and  $C$  will determine the integration constant. As the error signal fluctuates,  $C$  will charge and discharge through  $R_6$ . Keeping in mind



**FIGURE 13-9**  
Op-Amp Integral Controller

that the integrator is to operate with its capacitor in the linear region of its charge curve, it is important to prevent the capacitor from fully charging. Whether the capacitor fully charges or not depends on three factors: the magnitude of the error signal, the length of time that the error signal is present, and the values chosen for  $R_6$  and  $C$ . Knowledge of the system characteristics is required to properly choose these values. That is, an integral controller must be precisely “tuned” to respond properly for a given application. Op-amp  $U_3$  has two main functions. It provides an inversion/gain stage for the output of the integrator. Moreover, it provides isolation of the integrator from the load. In this manner,  $U_2$  is able more easily to maintain the capacitor voltage. That is, the capacitor is not constantly being discharged directly through the load.

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**EXAMPLE 13-3** Given the integral controller of Figure 13-9, along with the error signal shown in Figure 13-10, determine the controller output. Assume that the values of  $R_6$  and  $C$  are  $10\text{ k}\Omega$  and  $0.05\text{ }\mu\text{F}$ , respectively, and that the initial charge on the capacitor is zero. Assume also that  $U_3$  is operating with unity gain.

Since the error signal is a step function, equation 13.7 can be used to calculate the controller output. The problem is best approached by dividing the input into five separate intervals and analyzing each interval individually.

*Interval 1 (0–0.5 ms)*

From 0 to 0.5 ms, the input is zero. Since the initial capacitor charge is zero, the output will remain at zero.

*Interval 2 (0.5–1.5 ms)*

From 0.5 ms to 1.5 ms, the error signal is a constant +1 V. At 0.5 ms, the output will begin to ramp (increase linearly), and 1 ms later (after a total elapsed time of 1.5 ms), the output will have attained a value of +2 V, found by using equation 13.7:

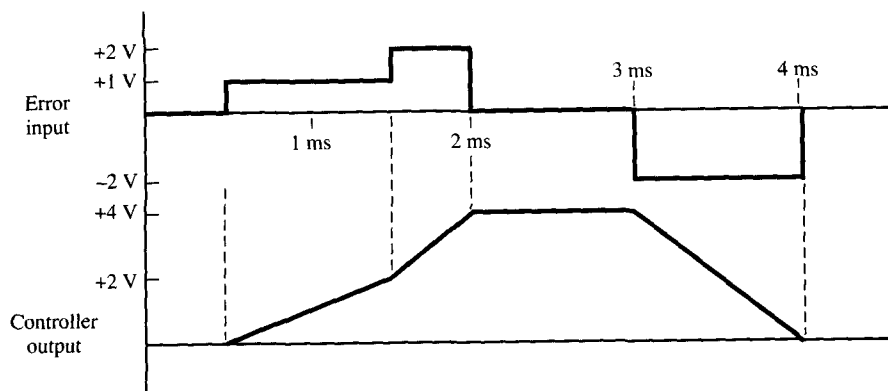
$$V_{\text{out}} = \left[ \frac{+1\text{ V}}{(0.05\text{ }\mu\text{F})(10\text{ k}\Omega)} \right] 1\text{ ms} = +2\text{ V}$$

*Interval 3 (1.5–2 ms)*

At 1.5 ms the output will have attained a value of +2 V. At this same instant, however, the error input steps up from +1 V to +2 V. The higher input voltage will cause the output voltage to ramp with a steeper slope. Throughout this 0.5 ms interval, the capacitor will accumulate enough charge to increase the output voltage by +2 V:

$$V_{\text{out}} = \left[ \frac{+2\text{ V}}{(0.05\text{ }\mu\text{F})(10\text{ k}\Omega)} \right] 0.5\text{ ms} + (+2\text{ V}) = +4\text{ V}$$

Since the initial charge was already +2 V, the resultant output at the end of 2 ms will be +4 V.



**FIGURE 13-10**

*Interval 4 (2–3 ms)*

Since the error input falls to zero during this interval, the rate of change of the output of the controller is zero. Therefore, the output will remain at +4 V.

*Interval 5 (3–4 ms)*

At the beginning of this interval, the error signal steps down from zero to –2 V, and it remains at that level for the remainder of the interval. Equation 13.7 is used to find that at the end of this 1 ms interval, the output has changed by –4 V:

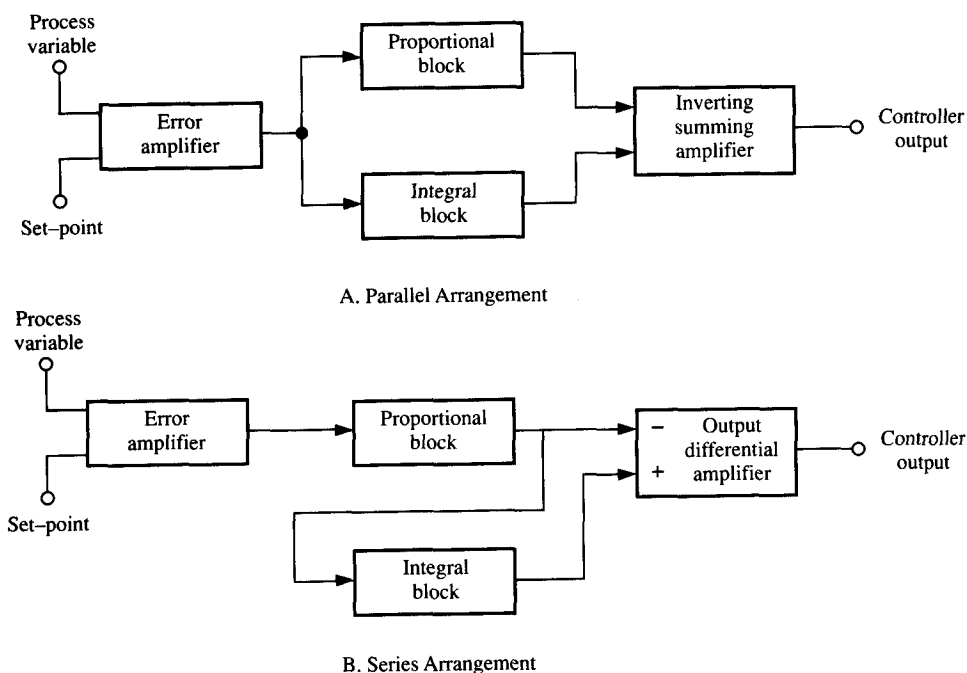
$$V_{\text{out}} = \left[ \frac{-2 \text{ V}}{(0.05 \text{ } \mu\text{F})(10 \text{ k}\Omega)} \right] 1 \text{ ms} + (+4 \text{ V}) = 0 \text{ V}$$

Since the initial capacitor voltage was +4 V, the resultant output will be zero.

## 13-6 PROPORTIONAL-INTEGRAL CONTROLLERS

Integral controllers almost never exist as isolated controllers. They are usually used in conjunction with some other type of controller. The reason is that although they are capable of driving the error signal to zero, they have very poor transient response. That is, the output responds slowly to rapid changes in error signal. Examining Figure 13-10 should make this quite obvious. Notice that as the error signal steps up to a higher value, the output of the controller only begins to ramp. Depending upon the value of  $R$  and  $C$ , it may be a considerable amount of time before the output reaches the proper value. So, even though the error signal will be reduced to zero in a system that incorporates integral control, the system will react sluggishly to dynamic system disturbances.

By combining proportional and integral principles in the same controller, the advantages of each can be realized. The proportional controller reacts quickly but cannot reduce the error to zero. The integral controller reacts slowly but, over a

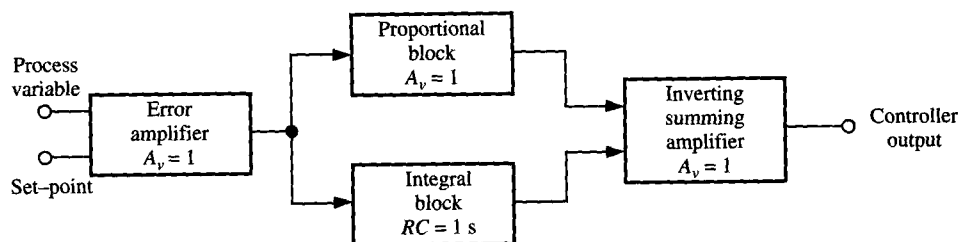


**FIGURE 13-11**  
Typical Proportional-Integral Controller Arrangements

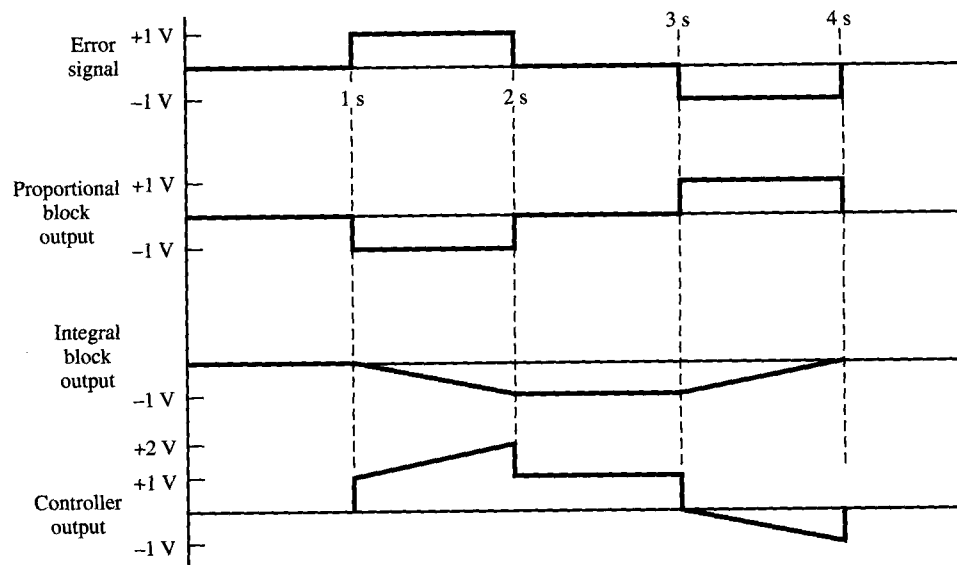
period of time, can eliminate the error. The proportional-integral controller exploits the fact that these two controllers naturally complement each other.

Generally, the proportional block and the integral block may be combined in one of two ways. As shown in Figure 13-11, the two blocks may be arranged in series or in parallel. In either case, the input to the controller is the familiar differential error amplifier, and the output is a summed combination of the outputs of the integral and proportional controller sections. The main difference between the two arrangements is the speed of response. Since the integral block in a series arrangement can receive an amplified error signal, the output in a series arrangement will change at a greater rate than the corresponding output in a parallel arrangement. This action will tend more rapidly to force the error to zero.

To understand the operation of a typical proportional-integral controller, consider the circuit of Figure 13-12. Since the characteristics of proportional and integral controllers have been discussed, the controllers are, for simplicity, represented as blocks. For this analysis, assume that the error amplifier, the proportional block, and the output summing amplifier are all operating with unity gain. In addition, assume that the product  $RC$  for the integral block is 1 s (the product of resistance and capacitance is in units of time). Since the proportional block has a gain of 1, its output will simply be an inverted image of the error signal. The



A. Parallel Arrangement



B. Circuit Response

**FIGURE 13-12**

Response of Parallel Proportional-Integral Controller to a Step Input

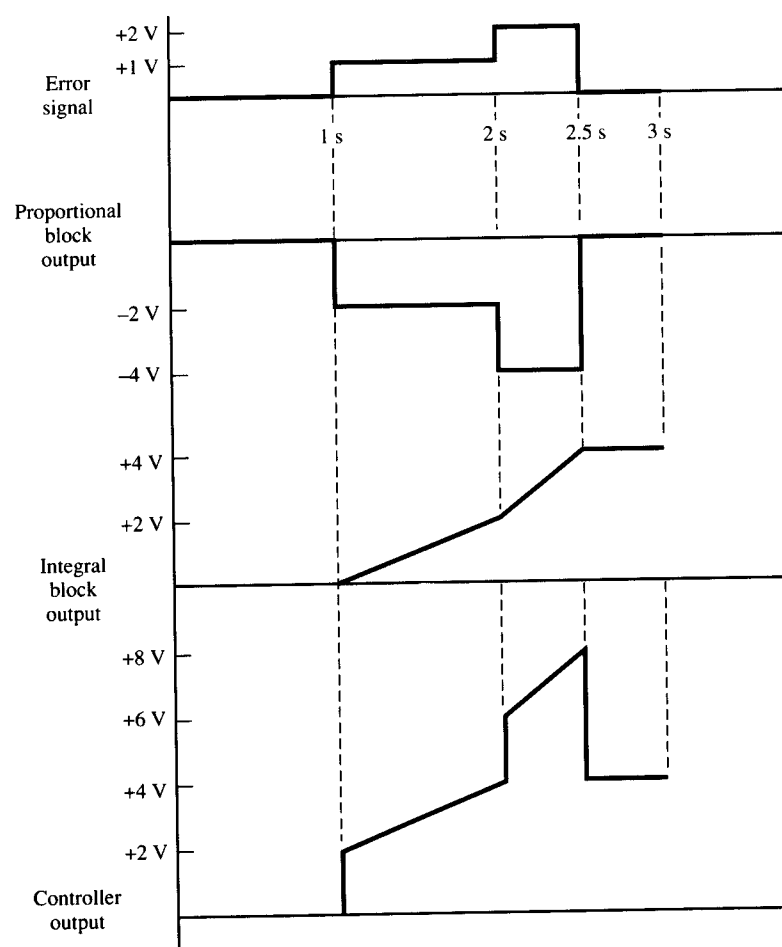
integral block receives the same error input, and after 1 s (when the error signal steps up to +1 V), the output begins to ramp. According to equation 13.7, after an interval of 1 s (a total elapsed time of 2 s) with a constant input of +1 V, the integral block output will be -1 V (remember that the integrator is still an inverting amplifier). At this time, the error signal falls to zero. This input causes the output of the integral block to remain constant at the -1 V level. After a total elapsed time of 3 s, the error signal steps down to a value of -1 V for a duration of 1 s. Again the integral block output begins to ramp. After 1 s the output due to this error step will be +1 V, but since the initial capacitor voltage was -1 V, the net output will be 0 V. Since the inverting summing amplifier has a gain of 1, the



outputs from the integral and proportional blocks are algebraically added (and inverted) to obtain the overall controller output. The result is shown at the bottom of Figure 13–12.

**EXAMPLE 13–4** Given the series proportional-integral controller of Figure 13–11*B*, along with the error signal shown in Figure 13–13, accurately sketch the controller output. Assume that the error amplifier and the differential amplifier are operating with unity gain. Assume also that the proportional block is operating with a gain of 2, and that the  $RC$  product for the integral block is 1 s.

As with the integral controller example, it is best to segment the error signal into separate intervals and analyze each individually. It can be seen that four



**FIGURE 13–13**

separate intervals exist. Remember that it is important to analyze the intervals where the error signal is zero, because the integral block will maintain an output during those times.

*Interval 1 (0–1 s)*

If the error input to the proportional block is zero, the output must also be zero. Since the input to the integral block is provided by the proportional block, the integral block output will not change. We assume that the initial capacitor voltage in the integral block is zero, so the output of this block will remain at zero. Since the final controller output is the algebraic difference between the integral and proportional block outputs, the controller yields 0 V for this entire interval.

*Interval 2 (1–2 s)*

After 1 s the error signal steps up to 1 V, and it remains at this level for a duration of 1 s. In response, the proportional block output steps down to a value of  $-2$  V for the same duration (remember that the proportional block has a gain of 2). This  $-2$  V is applied to the input of the integral block. Equation 13.7 can be used to determine the resultant output at the end of the interval:

$$V_{\text{out}} = - \left( \frac{-2 \text{ V}}{1 \text{ s}} \right) 1 \text{ s} + 0 \text{ V} = +2 \text{ V}$$

The output of the integral block, therefore, will begin to ramp linearly in the positive direction as soon as the error signal steps. At the end of a 1 s interval, the output will have achieved a value of  $+2$  V. The controller output can be described by the following equation:

$$\text{Controller output} = V_{\text{out integral}} - V_{\text{out proportional}}$$

Since both inputs to the differential amplifier are linear, it is necessary to calculate the final output only at the beginning and the end of the interval and to connect those points with a straight line. For this interval the endpoint calculations are as follows:

$$\text{Endpoint 1} = 0 \text{ V} - (-2 \text{ V}) = +2 \text{ V}$$

$$\text{Endpoint 2} = +2 \text{ V} - (-2 \text{ V}) = +4 \text{ V}$$

Plotting and connecting these points will yield the controller's overall response for this interval.

*Interval 3 (2–2.5 s)*

For this interval the error input steps up an additional 1 V. The input to the proportional block is now  $+2$  V. The output of this block, therefore, will be driven to a potential of  $-4$  V for a duration of 0.5 s. This potential is applied to the input of the integral block for the same duration. The output of the integral block will begin to ramp linearly in the positive direction. At the end of the 0.5 s interval, the output will be

$$V_{\text{out}} = - \left( \frac{-4 \text{ V}}{1 \text{ s}} \right) 0.5 \text{ s} + 2 \text{ V} = +4 \text{ V}$$

At the beginning of this interval, the integral output starts at +2 V, and it linearly ramps positive to a final value of +4 V.

The controller output throughout this interval can again be determined by calculating the endpoint voltages:

$$\text{Endpoint 1} = +2 \text{ V} - (-4 \text{ V}) = +6 \text{ V}$$

$$\text{Endpoint 2} = +4 \text{ V} - (-4 \text{ V}) = +8 \text{ V}$$

Plotting and connecting these points will yield the overall controller response throughout this interval.

*Interval 4 (2.5–3 s)*

For the duration of this interval, the error signal has fallen to zero. This will cause the proportional block output to fall to zero, and the integral block output to hold its current output constant at +4 V. Since neither output is changing, there is only one calculation to be made to determine the overall controller response:

$$\text{Controller output} = +4 \text{ V} - 0 \text{ V} = +4 \text{ V}$$


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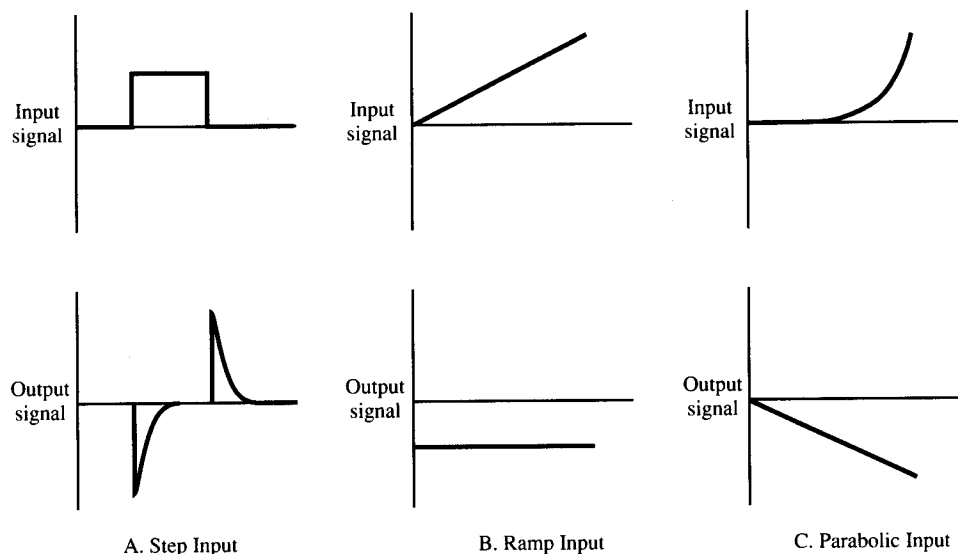
## 13-7 DERIVATIVE CONTROLLERS

In many cases a process may possess an inherent inertia or hysteresis. For example, in the heating of water, it takes a significant addition of energy to raise the temperature. Therefore, there will be a lag from the time heat is applied to the time the temperature actually rises. The significance of this phenomenon is that a system disturbance will not immediately result in a deviation from the set-point. More important, however, once the error has been detected, the system responds just as slowly to the corrective action. To overcome this sluggish response, an exaggerated correction is required. In other words, if the controller produces a large corrective signal in response to a minute error, the system will be brought back under control more quickly (even though it possesses a large inertia). There is a flaw with this theory, however, for if the corrective signal remains large, the controller will overcompensate for the error and possibly break into oscillation. What is really required is a corrective action that is initially large but tapers off as time goes on. Such an output is characteristic of a *derivative controller*.

The derivative controller has as its basic element a differentiator. Remember from Chapter 6 that the output of a differentiator is proportional to the rate of change of its input. Mathematically, the output may be expressed in terms of the input as follows:

$$V_{\text{out}} = -RC \left( \frac{dV_{\text{in}}}{dt} \right) \quad (13.8)$$

In this equation  $dV/dt$  (the derivative of  $V$  with respect to  $t$ ) is simply the time rate of change of the input voltage. If the input is zero or a constant DC, the output of the differentiator will be zero. If the input is a step function, the steep slope will surely send the differentiator into saturation. If, however, the input changes at a



**FIGURE 13-14**  
Ideal Differentiator Output Responses

linear rate (a ramp), the output will maintain a constant value that is equal to the slope of the function (expressed in volts/second) multiplied by the product  $RC$ . More complex input functions naturally produce more complex outputs. Calculation of these requires a familiarity with calculus.

Equation 13.8 assumes ideal circuit operation. It also assumes that the input function is known and can be accurately described by a mathematical equation. Remember that no circuit will produce ideal results. In addition, very often the input function is nothing better than an approximation. By analyzing some typical inputs, however, the response of the output can be predicted. Shown in Figure 13-14 are some common input functions, along with the ideal differentiator outputs that would be produced.

Examination of equation 13.8 shows that as the rate of change of the input signal increases, the output of the differentiator increases proportionally. High-frequency transients (noise) that are induced in the circuit will produce substantial outputs, at times even saturating the amplifier. This tendency can be reduced by inserting a resistor in series with the input capacitor. The high-frequency gain is then limited to the ratio of the feedback resistor to the input resistor.

A more significant drawback is that a derivative controller responds only to *changes* in the error signal. That is, if the system has a steady-state error, a derivative controller will take no corrective action. Unlike ON/OFF, proportional, and integral controllers, a derivative controller is never used alone. Instead, it is implemented in the proportional-integral-derivative (PID) controllers that have become the industry standard.

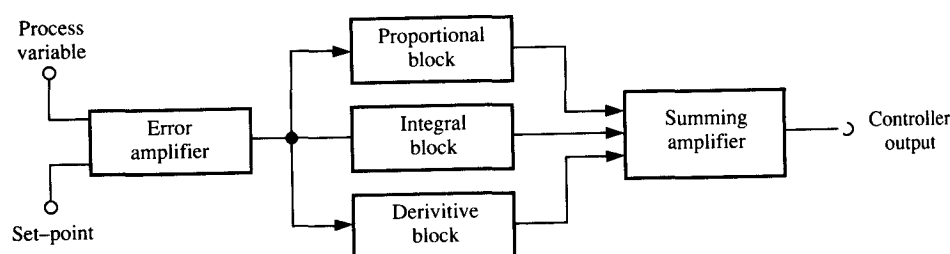
## 13-8 PROPORTIONAL-INTEGRAL-DERIVATIVE (PID) CONTROLLERS

As may be inferred from the name, proportional-integral-derivative (PID) controllers make use of the attractive attributes of all three controllers. The proportional block provides fast response to system disturbances, the derivative portion ensures that sudden disturbances will be met with an aggressive attempt to correct the error, and the integral section provides a means of eventually eliminating the error altogether.

Although many PID variations are possible, a common parallel configuration, shown in Figure 13-15, will be examined here. Each element receives the same error signal, and the outputs of all the elements are added through a summing amplifier. Since the response characteristics of all three blocks have been discussed, it should be a simple task to predict the output response to a change in error signal, as long as the error is represented by a relatively simple function.

The process of adjusting each of the three blocks in a PID controller is called *tuning*. The manner in which a PID controller must be tuned depends on the configuration of the controller, the characteristics of the process being controlled, and the desired controller performance. That is, if the same controller configuration was applied to two different processes, each would require a different tuning procedure. The procedure of tuning is by no means a simple task. Literature published by the manufacturer of the controller is often used as a guideline, although computer simulation programs have become popular because results can be observed quickly, without the necessity of starting up the process. Like any software simulation, however, the accuracy of the results depends on how well the system response can be modeled.

Two precautions must be observed when implementing PID control. Both stem from the fact that the action of the integral or derivative block can mask the effects of the other blocks in the controller. For example, if there is a sudden change (step) in error, the derivative block will most likely saturate, causing a corresponding saturation to occur in the summing amplifier. This sudden error change could be caused by a disturbance in the process or by a change in the set-



**FIGURE 13-15**  
Block Diagram of a Typical Parallel PID Controller

point. The result may be an overcompensation, causing the process to oscillate. As another example, if a large error is present in the system for a substantial period of time, the output of the integral block may be forced into saturation. Even if the error is driven back to zero, the integral output will remain at saturation. This output, too, will cause the process to overshoot, until the resultant negative error brings the integral block out of saturation.

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**EXAMPLE 13-5** Given the circuit of Figure 13-15, along with the error signal shown in Figure 13-16, sketch the resultant controller response. Assume that the error amplifier, the proportional block, and the summing amplifier are all operating with unity gain. In addition, assume that the  $RC$  products for the integral and derivative blocks are 1 s and 0.2 s, respectively.

Examination of the given error signal reveals that there are three distinct time intervals to be analyzed. For each interval, the output signal from each of the three controller blocks must be calculated and sketched. When all three responses have been obtained, it will be a simple matter to add them algebraically to obtain the overall controller response.

*Interval 1 (0–1 s)*

For this first interval, the error remains at zero. Here it will be assumed that each of the outputs was previously at zero. Since the error is not changing, the outputs of the integral and derivative blocks will remain at zero. Because the error is at a level of 0 V, the proportional block will likewise output 0 V for this interval.

*Interval 2 (1–3 s)*

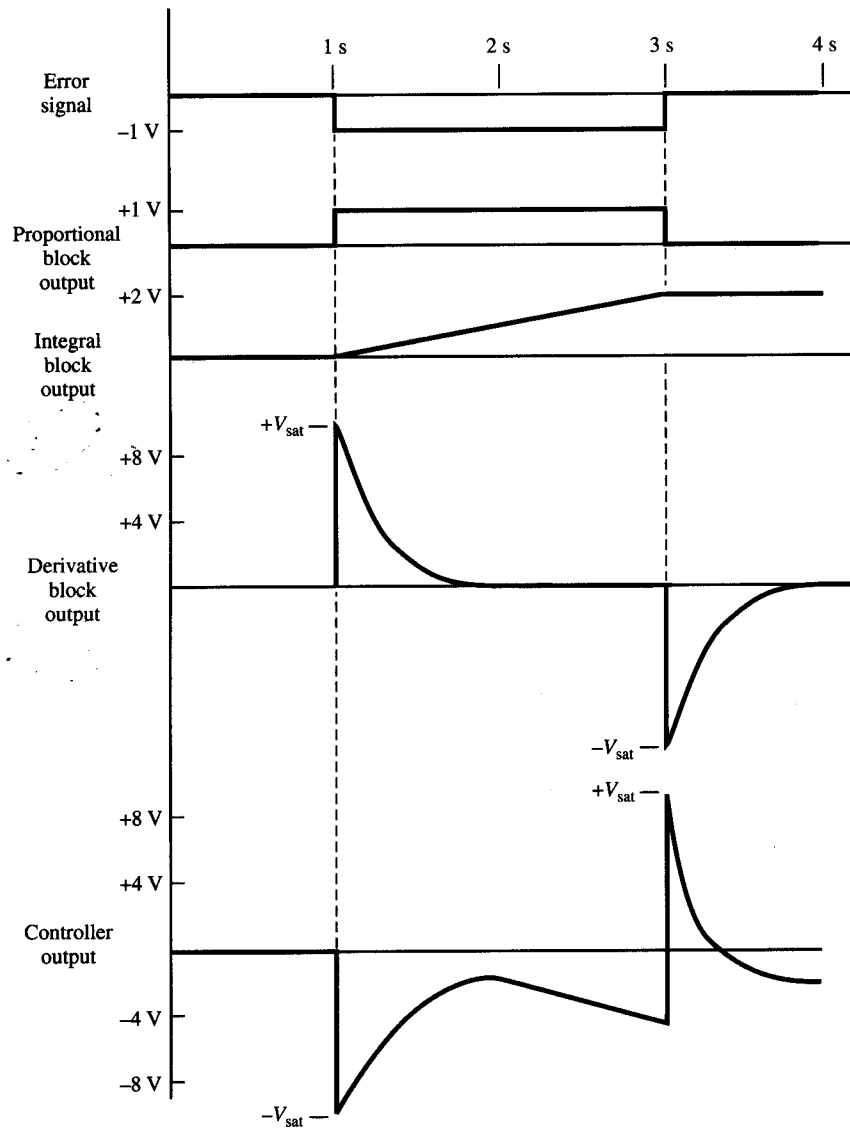
The error signal steps down to a level of  $-1$  V at the beginning of this interval, and it remains at that level for a duration of 2 s. In response, the proportional block, having a gain of 1, will step up to  $+1$  V for the same duration. The integral block output will begin to ramp linearly, and at the end of the interval, it will have attained a value calculated as follows:

$$V_{\text{out}} = - \left( \frac{-1 \text{ V}}{1 \text{ s}} \right) 2 \text{ s} + 0 \text{ V} = +2 \text{ V}$$

Therefore, the integral block output will ramp linearly from 0 V at 1 s to  $+2$  V at 3 s. The differentiator, on the other hand, will be driven to positive saturation immediately in response to the negative step of the error signal at 1 s. As the differentiator capacitor charges, the derivative block output will fall. Due to the 0.2 s time constant of the resistor and the capacitor, the capacitor should be fully charged after 1 s (5 time constants). The output, therefore, should follow the characteristic exponential decay curve from  $+V_{\text{sat}}$  at 1 s to 0 V at 2 s.

*Interval 3 (3–4 s)*

At the end of the preceding interval, when the error stepped back up to 0 V, the proportional block output likewise fell to 0 V. The integral block will maintain its  $+2$  V output, since its input has fallen to zero. The derivative block, however,



**FIGURE 13-16**

exhibits much more dynamic behavior. The tremendous positive slope of the error signal step will drive the output of the derivative block into negative saturation. Because of the same capacitor change characteristic described for interval 2, the output will return to zero by the exponential decay curve. After a period of 1 s, the output of the derivative block will again be zero.

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## SUMMARY

- Electronic control systems are used to compensate automatically for external disturbances to processes. Control systems are used to carry out operations in hazardous locations, to reduce variations in product quality, and to reduce production costs.
- The electronic “brain” behind the control system is a group of components that are collectively called the controller. The controller accepts the system error signal and produces an output that is used to correct the error condition.
- An ON/OFF controller attempts to maintain control of the process by switching the output actuator ON or OFF according to the magnitude of the error signal. An ON/OFF controller cannot reduce the system error to zero. The range of system error that will cause the controller output to cycle from ON to OFF is referred to as the deadband.
- A proportional controller maintains control by generating an output proportional to the system error signal. Although a proportional controller usually results in a residual system error lower than that of an ON/OFF controller, it still cannot reduce the error to zero. The range of system error for which the controller will produce a proportional output is referred to as the proportional band.
- An integral controller produces an output whose rate of change is proportional to the magnitude of the system error signal. Since the controller can maintain an output even when its input is zero, this controller can eliminate system error altogether.
  
- Proportional-integral controllers combine the swift response of proportional controllers with the error elimination capability of integral controllers. Since integration is a function of time, these controllers must be tuned to ensure that the output can respond to variations in the process being controlled.
- Proportional-integral-derivative (PID) controllers produce outputs that depend on the magnitude, duration, and rate of change of the system error signal. Sudden system disturbances are met with an aggressive attempt to correct the condition. A PID controller can reduce the system error to zero faster than any other controller. Because it has an integrator and a differentiator, however, this controller must be custom-tuned to each process being controlled.



## EXERCISES

1. Give two examples of open-loop control systems. Identify the controller, the output actuator, and the process for each.
2. Give two examples of closed-loop control systems. Identify the error amplifier, the controller, the output actuator, and the process for each.
3. State one common disadvantage that ON/OFF and proportional controllers share.
4. For the circuit of Figure 13–3, what circuit change or modification would increase the circuit deadband?
5. How would an ON/OFF controller respond if the deadband were too narrow?
6. List two ways to decrease the proportional band of the controller in Figure 13–6.
7. If a home heating system is controlled by an ON/OFF controller, what will be the effect of widening the deadband?
8. If more precise control is desired from a proportional controller, should the proportional band be increased or decreased?
9. In Example 13–2, if the full-scale error is 8.5 V, what is the proportional band, expressed as a percentage?
10. Which of the following output actuators is likely to be driven by an ON/OFF controller?
  - a. DC motor
  - b. power amplifier
  - c. solid-state relay
  - d. solenoid valve
11. Which of the following output actuators is likely to be driven by a proportional controller?
  - a. DC motor
  - b. power amplifier
  - c. solid-state relay
  - d. solenoid valve
12. Give an example of a process for which it would be advantageous to use PID control instead of proportional-integral control.
13. When a proportional-integral or PID controller is tuned, what controller characteristic is actually being adjusted?

14. Given the circuit of Figure 13–3, assume that  $R_6 = R_7$ . Draw the response curve for the circuit.
15. For Exercise 14, if the full-scale error is 10 V, what is the circuit deadband, expressed as a percentage?
16. In Figure 13–6, suppose that  $R_7 = R_8 = 1 \text{ k}\Omega$ . Suppose also that  $R_9 = 3.3 \text{ k}\Omega$ . If  $U_3$  is operating with unity gain and the offset is adjusted to 0.5 V, draw the circuit response curve, and calculate the controller proportional band.
17. Given the information in Exercise 16, rework the problem for the situation in which  $R_{10}$  and  $D$  have been removed. (Assume that  $U_2$  and  $U_3$  saturate at  $\pm 10 \text{ V}$ .)
18. For Example 13–3, redraw the output response for  $R_6 = 1 \text{ k}\Omega$ . Use the given error signal.
19. For Figure 13–12, redraw the output for the condition where the proportional block has a gain of 2. Use the given error signal.
20. For Example 13–4, redraw the output response for the condition where the proportional block has a gain of 1 and the  $RC$  product for the integral block is 0.5 s. Use the given error signal.
21. For Example 13–5, show how the output response would change if the  $RC$  time constants for both the integral and derivative blocks were changed to 0.5 s. Use the given error signal.