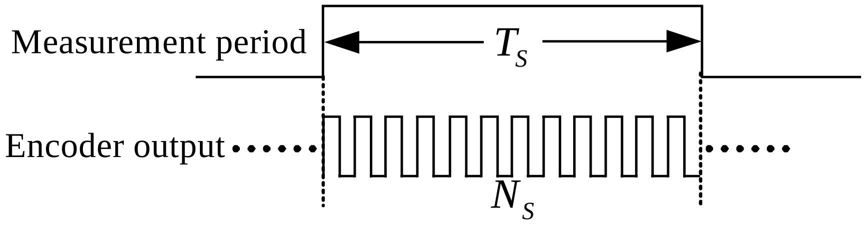
5. Speed Measurement

5.1 Speed Sensor Measurement

(1) The principle of measurement

(1.a) Pulse Number Measurement



T_s : the measurement period,

N_s : Output pulse number of the encoder in the measurement period

The frequency of output pulse of the encoder is:

$$f_e = \frac{N_s}{T_s} \qquad \text{(pulse/s)} \qquad (1)$$

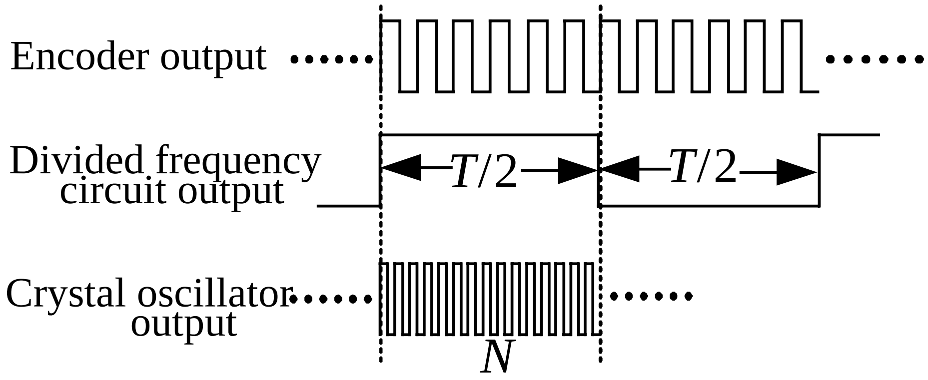
Assume that the output pulse number one cycle of the encoder is K (pulse/cycle), the motor speed can be expressed as

$$n_1 = \frac{f_e}{K} \times 60 \qquad \text{(cycle/m)} \qquad (2)$$

Substitute (1) into (2) to obtain

$$n_1 = \frac{N_s}{T_s K} \times 60 \qquad \text{(cycle/m)} \qquad (3)$$

(1.b) Pulse Width Measurement



The frequency of output pulse of the encoder is f_e.

The frequency of output pulse of the divided-M frequency circuit is

$$f_M = \frac{f_e}{M} \tag{4}$$

Assume that the high oscillation frequency of a crystal oscillator is f_i, then

$$\frac{f_M}{f_i} = \frac{1}{2N} \tag{5}$$

Substitute (4) into (5) to get

$$\frac{f_e/M}{f_i} = \frac{1}{2N} \tag{6}$$

Equation (6) becomes

$$f_e = \frac{Mf_i}{2N} \qquad \text{(pulse/s)} \tag{7}$$

Assume that the output pulse number one cycle of the encoder is K (pulse/cycle)，the motor speed can be expressed as

$$n_2 = \frac{f_e}{K} \times 60 \qquad \text{(cycle/m)} \tag{8}$$

Substituting (7) into (8) to obtain

$$n_2 = \frac{Mf_i}{2NK} \times 60 \qquad \text{(cycle/m)} \tag{9}$$

(2) Speed Measurement Error

(2.a) Pulse Number Measurement

Assume that the measured pulse number of the encoder in the measurement period T_S is $N_s - 1$, the motor speed can be expressed as

$$n_1' = \frac{N_s - 1}{T_s K} \times 60 \qquad (10)$$

The speed measurement error is expressed as:

$$\Delta n_1 = \left| \frac{n_1 - n_1'}{n_1} \right| \times 100\% \qquad (11)$$

Substituting (3) and (10) into (11) yields

$$\Delta n_1 = \frac{1}{N_s} \times 100\% \qquad (12)$$

Substituting (3) into (12) to obtain

$$\Delta n_1 = \frac{60}{n_1 T_s K} \times 100\% \qquad (13)$$

From (13), for a fixed T_s and K, the speed n_1 decreases, the speed measurement error Δn_1 increases.

(2.b) Pulse Width Measurement

Assume that the measured pulse number of the encoder in the measurement period $T/2$ is $N-1$, the motor speed can be expressed as

$$n_2' = \frac{M f_i}{2(N-1)K} \times 60 \qquad (14)$$

The speed measurement error is expressed as:

$$\Delta n_2 = \left| \frac{n_2 - n_2'}{n_2} \right| \times 100\% \qquad (15)$$

Substituting (9) and (14) into (15) yields

$$\Delta n_2 = \frac{1}{N-1} \times 100\% \tag{16}$$

Substituting (9) into (16) to get

$$\Delta n_2 = \frac{2n_2 K}{60Mf_i - 2n_2 K} \times 100\% \tag{17}$$

According to (17), for a fixed f_i, M and K, the speed n_2 increases, the speed measurement error Δn_2 increases. However, for a large $f_i = 2$ MHz, the speed measurement error Δn_2 is small.

(3) Maximum and minimum motor speed analysis

(3.a) Pulse Number Measurement

Assume that the encoder K =500 (pulse/cycle), the maximum and minimum numbers of N_s are 2^{20} and 1, $T_S = 0.01s$.

According to (3), the maximum and minimum motor speed are

$$n_{1\max} = \frac{2^{20}}{0.01 \times 500} \times 60 = 12582912 \text{ rpm}$$

and

$$n_{1\min} = \frac{1}{0.01 \times 500} \times 60 = 12 \text{rpm}$$

(3.b) Pulse Width Measurement

Assume that the encoder K =500 (pulse/cycle), the maximum and minimum numbers of N are 2^{20} and 1, $f_i = 2 \times 10^6$ Hz, M=8.

According to (9), the maximum and minimum motor speed are

$$n_{2\max} = \frac{8 \times 2 \times 10^6}{2 \times 1 \times 500} \times 60 = 9600000 \text{ rpm}$$

and

$$n_{2\min} = \frac{8 \times 2 \times 10^6}{2 \times 2^{20} \times 500} \times 60 = 0.92\text{rpm}$$

Table 1 Speed measurement error, encoder K =500 pulse/cycle

Speed(rpm)	Speed measurement error of the pulse number measurement (%)	Speed measurement error of the pulse width measurement (%)
1700	0.7	0.1774
1200	1	0.1252
600	2	0.0625
120	10	0.0125
100	12	0.0104
80	15	0.0083
60	20	0.0063
40	30	0.0042
20	60	0.0021
15	80	0.0016
12	100	0.0013
10	Cann’t measure	0.0010
1	Cann’t measure	0.0001

Table 2 Speed measurement error, encoder K =1000 pulse/cycle

Speed(rpm)	Speed measurement error of the pulse number measurement (%)	Speed measurement error of the pulse width measurement (%)
1700	0.35	0.3554
1200	0.5	0.2506
600	1	0.1252
120	5	0.0250
100	6	0.0208
80	7.5	0.0167
60	10	0.0125
40	15	0.0083
20	30	0.0042
15	40	0.0031
12	50	0.0025
10	100	0.0021
1	Cann’t measure	0.0002

Table 3 Speed measurement error, encoder K =2000 pulse/cycle

Speed(rpm)	Speed measurement error of the pulse number measurement (%)	Speed measurement error of the pulse width measurement (%)
1700	0.175	0.7134
1200	0.25	0.5025
600	0.5	0.2506
120	2.5	0.0500
100	3	0.0416
80	3.75	0.0333
60	5	0.0250
40	7.5	0.0167
20	15	0.0083
15	20	0.0062
12	25	0.0050
10	50	0.0042
1	100	0.0004

5.2 Speed Sensorless Measurement

(1) Speed Observer (IEEE Trans. on Industry Application, Vol. 30. No. 5, 1994, pp.1219-1224)

The IM can be described by the following state equation in the stationary reference frame:

$$\frac{d}{dt}\begin{bmatrix} i_s \\ \psi_r \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} i_s \\ \psi_r \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} v_s$$
$$\Rightarrow \frac{d}{dt}x = Ax + Bv_s \tag{1}$$

$$i_s = Cx \tag{2}$$

$i_s = [i_{ds} \quad i_{qs}]^T$: Stator current, $\psi_r = [\psi_{dr} \quad \psi_{qr}]^T$: Rotor flux

$v_s = [v_{ds} \quad \mathrm{v}_{qs}]^T$: Stator voltage

$A_{11} = -\{R_s/(\sigma L_s) + (1-\sigma)/(\sigma\tau_r)\}I = a_{r11}I$

$A_{12} = L_m/(\sigma L_s L_r)\{(1/\tau_r)I - \omega_r J\} = a_{r12}I + a_{i12}J$

$A_{21} = (L_m/\tau_r)I = a_{r21}I$

$A_{22} = -(1/\tau_r)I + \omega_r J = a_{r22}I + a_{i22}J$

$B_1 = 1/(\sigma L_s)I = b_1 I$

$C = [I \quad 0], \qquad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

R_s, R_r : Stator and rotor resistance, L_s, L_r : Stator and rotor self - inductance

L_m : Mutual inductance, σ : Leakage coefficient, $\sigma = 1 - L_m^2/(L_s L_r)$

$\tau_\mathrm{r} = L_r/R_r$: Rotor time constant, ω_r : Motor angular velocity

The state observer

$$\frac{d}{dt}\hat{x} = \hat{A}\hat{x} + Bv_s + G(\hat{i}_s - i_s)$$
$$\hat{i}_s = C\hat{x} \tag{3}$$

From (1) and (3)

$$\frac{d}{dt}(x-\hat{x}) = Ax - \hat{A}\hat{x} - G(C\hat{x} - Cx)$$
$$= Ax - A\hat{x} + A\hat{x} - \hat{A}\hat{x} + GC(x-\hat{x})$$
$$= A(x-\hat{x}) + (A-\hat{A})\hat{x} + GC(x-\hat{x})$$

$$\Rightarrow \quad \frac{d}{dt}e = (A+GC)e - \Delta A\hat{x} \tag{4}$$

where $e = x - \hat{x}$, $\Delta A = \hat{A} - A$.

$$\Delta A = \hat{A} - A = \begin{bmatrix} \hat{A}_{11} - A_{11} & \hat{A}_{12} - A_{12} \\ \hat{A}_{21} - A_{21} & \hat{A}_{22} - A_{22} \end{bmatrix}$$
$$= \begin{bmatrix} 0 & L_m/(\sigma L_s L_r)\{-(\hat{\omega}_r - \omega_r)J\} \\ 0 & (\hat{\omega}_r - \omega_r)J \end{bmatrix} = \begin{bmatrix} 0 & -\Delta\omega_r J/c \\ 0 & \Delta\omega_r J \end{bmatrix}$$

where $c = \sigma L_s L_r / L_m$, $\Delta\omega_r = \hat{\omega}_r - \omega_r$.

Define Lyapunov function candidate

$$V = e^T e + (\hat{\omega}_r - \omega_r)^2/\lambda \tag{5}$$

where λ is a positive constant.

The time derivitive of V becomes

$$\frac{d}{dt}V = e^T\frac{de}{dt} + \frac{de^T}{dt}e + 2\Delta\omega_r\frac{d}{dt}\hat{\omega}_r/\lambda$$
$$= e^T[(A+GC)e - \Delta A\hat{x}] + [(A+GC)e - \Delta A\hat{x}]^T e + 2\Delta\omega_r\frac{d}{dt}\hat{\omega}_r/\lambda$$
$$= e^T[(A+GC) + (A+GC)^T]e - e^T\Delta A\hat{x} - \hat{x}^T\Delta A^T e + 2\Delta\omega_r\frac{d}{dt}\hat{\omega}_r/\lambda$$

Since

$$-e^T\Delta A\hat{x} - \hat{x}^T\Delta A^T e = -2e^T\Delta A\hat{x}$$

$$= -2[e_{ids} \quad e_{iqs} \quad e_{\phi dr} \quad e_{\phi qr}]\begin{bmatrix} 0 & 0 & 0 & \Delta\omega_r / c \\ 0 & 0 & -\Delta\omega_r / c & 0 \\ 0 & 0 & 0 & \Delta\omega_r \\ 0 & 0 & -\Delta\omega_r & 0 \end{bmatrix}\begin{bmatrix} \hat{i}_{ds} \\ \hat{i}_{qs} \\ \hat{\phi}_{dr} \\ \hat{\phi}_{qr} \end{bmatrix}$$

$$= -2(e_{ids}\Delta\omega_r\hat{\phi}_{qr} / c - e_{iqs}\Delta\omega_r\hat{\phi}_{dr} / c + e_{\phi dr}\Delta\omega_r\hat{\phi}_{qr} - e_{\phi qr}\Delta\omega_r\hat{\phi}_{dr})$$

$$\cong -2\Delta\omega(e_{ids\,r}\hat{\phi}_{qr} - e_{iqs}\hat{\phi}_{dr}) / c$$

then

$$\frac{d}{dt}V = e^T\{(A+GC)^T + (A+GC)\}e - 2\Delta\omega_r(e_{ids}\hat{\phi}_{qr} - e_{iqs}\hat{\phi}_{dr})/c + 2\Delta\omega_r\frac{d}{dt}\hat{\omega}_r / \lambda \quad (6)$$

where $e_{ids} = i_{ds} - \hat{i}_{ds}$, $e_{iqs} = i_{qs} - \hat{i}_{qs}$.

$$\frac{d}{dt}\hat{\omega}_r = \lambda(e_{ids}\hat{\phi}_{qr} - e_{iqs}\hat{\phi}_{dr}) / c \quad (7)$$

$$\hat{\omega}_r = K_P(e_{ids}\hat{\phi}_{qr} - e_{iqs}\hat{\phi}_{dr}) + K_I\int(e_{ids}\hat{\phi}_{qr} - e_{iqs}\hat{\phi}_{dr})dt \quad (8)$$

(2) Reduced Extended Kalman Filter (REKF)

$$\frac{d}{dt}i_{ds}^{s}=-(\frac{R_s}{\sigma L_s}+\frac{1-\sigma}{\sigma\tau_r})i_{ds}^{s}+\frac{L_m}{\sigma L_s L_r\tau_r}\phi_{dr}^{s}+\frac{L_m\omega_r}{\sigma L_s L_r}\phi_{qr}^{s}+\frac{1}{\sigma L_s}v_{ds}^{s} \quad (1.a)$$

$$\frac{d}{dt}i_{qs}^{s}=-(\frac{R_s}{\sigma L_s}+\frac{1-\sigma}{\sigma\tau_r})i_{qs}^{s}-\frac{L_m\omega_r}{\sigma L_s L_r}\phi_{dr}^{s}+\frac{L_m}{\sigma L_s L_r\tau_r}\phi_{qr}^{s}+\frac{1}{\sigma L_s}v_{qs}^{s} \quad (1.b)$$

$$\frac{d}{dt}\phi_{dr}^{s}=\frac{L_m}{\tau_r}i_{ds}^{s}-\frac{1}{\tau_r}\phi_{dr}^{s}-\omega_r\phi_{qr}^{s} \quad (1.c)$$

$$\frac{d}{dt}\phi_{qr}^{s}=\frac{L_m}{\tau_r}i_{qs}^{s}+\omega_r\phi_{dr}^{s}-\frac{1}{\tau_r}\phi_{qr}^{s} \quad (1.d)$$

$$T_e=\frac{3PL_m}{4L_r}(\phi_{dr}^{s}i_{qs}^{s}-\phi_{qr}^{s}i_{ds}^{s})=J\frac{d\omega_r}{dt}+B\omega_r+T_L \quad (2)$$

Equation (2) can be rewritten as:

$$\frac{d\omega_r}{dt}=\frac{3PL_m}{4JL_r}(i_{qs}^{s}\phi_{dr}^{s}-i_{ds}^{s}\phi_{qr}^{s})-\frac{B}{J}\omega_r-\frac{1}{J}T_L \quad (3)$$

$$\frac{d}{dt}T_L=0 \quad (4)$$

$$\frac{d}{dt}(\frac{1}{\tau_r})=0 \quad (5)$$

We define the state variables as follows:

$$\begin{aligned}\underline{x}(t)&=\begin{bmatrix}x_1(t) & x_2(t) & x_3(t) & x_4(t) & x_5(t)\end{bmatrix}^{\mathrm{T}}\\ &=\begin{bmatrix}\phi_{dr}^{s} & \phi_{qr}^{s} & \omega_r & T_L & \tau_r^{-1}\end{bmatrix}^{\mathrm{T}}\end{aligned} \quad (6)$$

According to Eqs. (1.c), (1.d), (3), (4) and (5), the dynamic behavior of an induction motor is modeled as

$$\dot{\underline{x}}(t) = f[\underline{x}(t), \underline{u}(t), t] + \underline{w}(t) \tag{7}$$

where

$$f(\underline{x}, \underline{u}, t) = \begin{bmatrix} L_m i_{ds}^s x_5 - x_1 x_5 - x_2 x_3 \\ L_m i_{qs}^s x_5 + x_1 x_3 - x_2 x_5 \\ \dfrac{3PL_m}{4JL_r}(i_{qs}^s x_1 - i_{ds}^s x_2) - \dfrac{B}{J} x_3 - \dfrac{1}{J} x_4 \\ 0 \\ 0 \end{bmatrix} \tag{8}$$

According to Eqs. (1.a) and (1.b), the discrete-time measurement model can be written as

$$\underline{y}(t_i) = h[\underline{x}(t_i), t_i] + \underline{v}(t_i) \tag{9}$$

where

$$\underline{y}(t_i) = \begin{bmatrix} \dot{i}_{ds}^s + (\dfrac{R_s}{\sigma L_s} + \dfrac{1-\sigma}{\sigma \tau_r}) i_{ds}^s - \dfrac{1}{\sigma L_s} v_{ds} \\ \dot{i}_{qs}^s + (\dfrac{R_s}{\sigma L_s} + \dfrac{1-\sigma}{\sigma \tau_r}) i_{qs}^s - \dfrac{1}{\sigma L_s} v_{qs} \end{bmatrix} \tag{10a}$$

$$h(\underline{x}, t) = \frac{L_m}{\sigma L_s L_r} \begin{bmatrix} x_1 x_5 + x_2 x_3 \\ -x_1 x_3 + x_2 x_5 \end{bmatrix} \tag{10b}$$

$w(t)$:process noise , $v(t)$: measurement noise .

The REKF Algorithm

1) Prediction of state:

$$\underline{\hat{x}}(k+1|k) = \underline{\hat{x}}(k|k) + \int_{t_{k-1}}^{t_k} f(\underline{\hat{x}}(t|t_k), \underline{u}(t), t)dt \tag{11}$$

2) Estimation of error covariance matrix:

$$P_m(k+1|k) = \Phi(k+1|k)P_m(k|k)\Phi^T(k+1|k) + Q_d \tag{12}$$

where

$$\Phi(k+1|k) = \exp(F(k)t_s) \approx I_n + t_s F(k) \tag{13}$$

$$F[k] = \frac{\partial f[\underline{x}(t), \underline{u}(t), t]}{\partial \underline{x}} \bigg|_{\underline{x} = \underline{\hat{x}}(k|k)}$$

$$= \begin{bmatrix} -\hat{x}_5 & -\hat{x}_3 & -\hat{x}_2 & 0 & L_m i_{ds}^s \\ \hat{x}_3 & -\hat{x}_5 & \hat{x}_1 & 0 & L_m i_{qs}^s - \hat{x}_2 \\ \frac{-3PL_m i_{qs}^s}{4JL_r} & \frac{-3PL_m i_{ds}^s}{4JL_r} & \frac{-B}{J} & \frac{-1}{J} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{14}$$

$$Q_d = \int_{t_k}^{t_{k+1}} \Phi(t_{k+1}|\tau)Q(\tau)\Phi^T(t_{k+1}|\tau)d\tau \tag{15}$$

3) Computation of Kalman filter gain:

$$K_m(k+1) = P_m(k+1|k)H^T(k+1)[H(k+1)P_m(k+1|k)H^T(k+1) + R(k+1)]^{-1} \tag{16}$$

where

$$H = \frac{\partial h(\underline{x}, t)}{\partial \underline{x}} \bigg|_{\underline{x} = \underline{\hat{x}}} = \frac{L_m}{\sigma L_s L_r} \begin{bmatrix} \hat{x}_5 & \hat{x}_3 & \hat{x}_2 & 0 & \hat{x}_1 \\ -\hat{x}_3 & \hat{x}_5 & -\hat{x}_1 & 0 & \hat{x}_2 \end{bmatrix} \tag{17}$$

4) The update of the error covariance matrix is:

$$P_m(k+1|k+1) = [I_n - K_m(k+1)H(k+1)]P_m(k+1|k) \tag{18}$$

5) State estimation:

$$\underline{\hat{x}}(k+1|k+1) = \underline{\hat{x}}(k+1|k) + K_m(k+1)\underline{e}(k+1) \qquad (19)$$

where

$$\underline{e}(k+1) = \underline{y}(k+1) - h[\underline{\hat{x}}(k+1|k), k+1] \qquad (20)$$