EE 455 Solution to Homework # 6 Georghiades

Given: November 26, 2003
Due: December 9, 2003.

1. Solve problem 8.10 in the textbook.

Solution

a) Each segment of the wire-line can be considered as a bandpass filter with bandwidth $W = 1200$ Hz. Thus, the highest bit rate that can be transmitted without ISI by means of binary PAM is

$$R = 2W = 2400 \text{ bps}$$

b) The probability of error for binary PAM transmission is

$$P_2 = Q\left[\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right]$$

Hence, using mathematical tables for the function $Q[\cdot]$, we find that $P_2 = 10^{-7}$ is obtained for

$$\sqrt{\frac{2\mathcal{E}_b}{N_0}} = 5.2 \Longrightarrow \frac{\mathcal{E}_b}{N_0} = 13.52 = 11.30 \text{ dB}$$

c) The received power P_R is related to the desired SNR per bit through the relation

$$\frac{P_R}{N_0} = R\frac{\mathcal{E}_b}{N_0}$$

Hence, with $N_0 = 4.1 \times 10^{-21}$ we obtain

$$P_R = 4.1 \times 10^{-21} \times 1200 \times 13.52 = 6.6518 \times 10^{-17} = -161.77 \text{ dBW}$$

Since the power loss of each segment is

$$L_s = 50 \text{ Km } \times 1 \text{ dB/Km } = 50 \text{ dB}$$

the transmitted power at each repeater should be

$$P_T = P_R + L_s = -161.77 + 50 = -111.77 \text{ dBW}$$

2. Solve problem 8.14 in the textbook.

Solution

The bandwidth of the channel is

$$W = 3000 - 300 = 2700 \text{ Hz}$$

Since the minimum transmission bandwidth required for bandpass signaling is R, where R is the rate of transmission, we conclude that the maximum value of the symbol rate for the given channel is $R_{\max} = 2700$. If an M-ary PAM modulation is used for transmission, then in order to achieve a bit-rate of 9600 bps, with maximum rate of $R_{\max}$, the minimum size of the constellation is $M = 2^k = 16$. In this case, the symbol rate is

$$R = \frac{9600}{k} = 2400 \text{ symbols/sec}$$

and the symbol interval $T = \frac{1}{R} = \frac{1}{2400}$ sec. The roll-off factor α of the raised cosine pulse used for transmission is determined by noting that $1200(1+\alpha) = 1350$, and hence, $\alpha = 0.125$. Therefore, the squared root raised cosine pulse can have a roll-off of $\alpha = 0.125$.

3. Consider the following block code: $C = \{(000000),\ (010101),\ (101010),\ (111111)\}$.

   (a) Is the code linear? Explain.

   (b) What is the minimum Hamming distance for this code?

   (c) For a BSC channel with uncoded error-rate $p = 10^{-3}$, what is the codeword-error probability for this code?

Solution

(a) The code is linear since if $\mathbf{v}_1 \in C$ and $\mathbf{v}_2 \in C$ then $\mathbf{v}_1 + \mathbf{v}_2 \in C$.

(b) The minimum distance equals the smallest weight (excluding the all-zero codeword), i.e. $d_{\min} = 3$.

(c) The code can correct $t = 1$ errors. Thus (to a good approximation)

$$P(e) = \sum_{k=2}^{6} \binom{6}{k} p^k (1-p)^{6-k} = 1.496 \times 10^{-5}.$$

4. Consider a block code described by the following generator matrix:

$$\mathbf{G} = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}.$$

   (a) Construct the code (i.e. find all codewords and their bit assignments) and find its minimum Hamming distance.

   (b) If the code is used on a BSC with $p = 10^{-4}$, what is the resulting codeword-error probability?

Solution

(a) There are 8 messages, 3 bits each.

000	000000
001	010011
010	001110
011	011101
100	110110
101	100101
110	111000
111	101011

The minimum distance is 3, i.e. it can correct one error.

(b)

$$P(e) = \sum_{k=2}^{6} p^k(1-p)^{6-k} = 1.50 \times 10^{-7}.$$

5. For the above code, suppose the message $\mathbf{m} = (110)$ is at the input of the encoder.

   (a) What is the output of the encoder?

   (b) If the received vector corresponding to the message sent is $\mathbf{r} = (101000)$, find the decision of the decoder.

Solution

   (a) When the input is (110), the output is (111000).

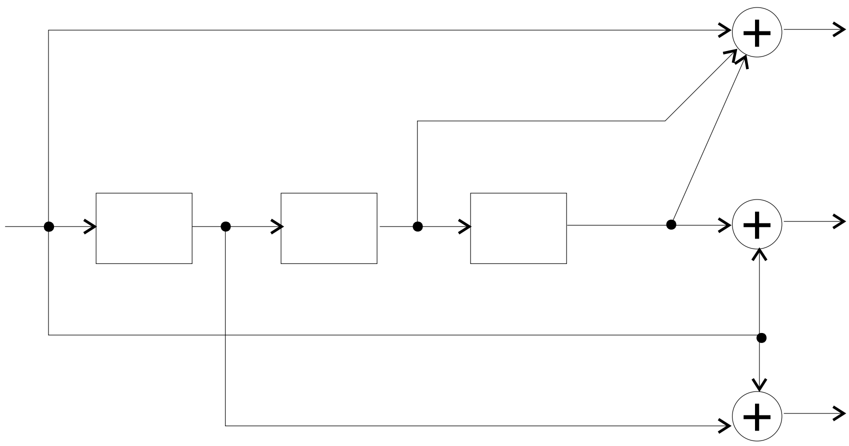
   (b) The decoder finds the codeword that is closest to $\mathbf{r}$ in Hamming distance, i.e. $\mathbf{v} = (111000)$. The decoded message is thus (110).

6. Draw a circuit diagram similar to the one given in class (with the shift-registers) for a convolutional encoder with $k = 1$ described by

$$\mathbf{g}_1 = (1011) \quad \mathbf{g}_2 = (1100) \quad \mathbf{g}_3 = (1001)$$

   What is the constraint length and the rate of this code?

Solution



The constraint length of the code is $K = 4$ and the rate is $R = 1/3$.

7. If the input to the convolutional encoder above is $\mathbf{x} = (1101001)$, what is the corresponding output vector $\mathbf{y}$? Assume that the shift-registers are reset to zero before the first input bit enters the encoder.

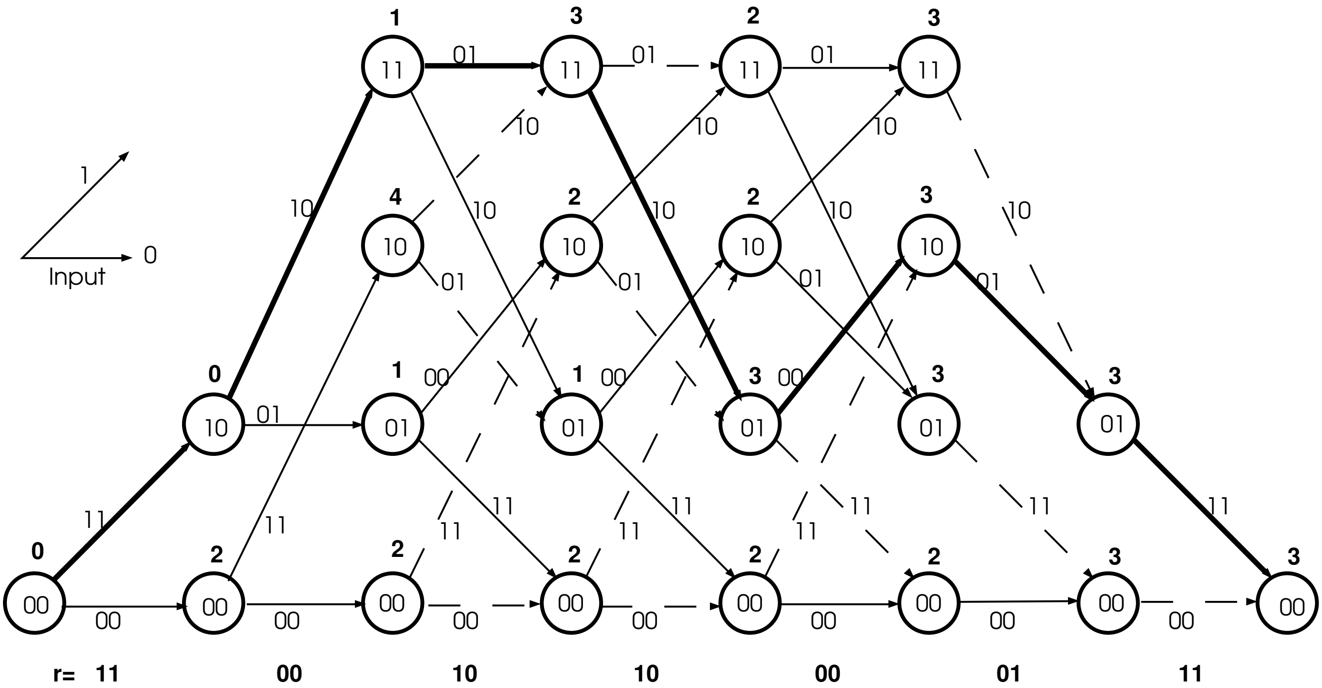
Solution

The output is:

$$\mathbf{y} = (111, 101, 110, 110, 111, 100, 010\cdot)$$

8. Consider the (2,1,2) convolutional code given in class (whose trellis diagram is attached). Use the Viterbi algorithm to decode the following received vector: $\mathbf{r} = (11, 00, 10, 10, 00, 01, 11)$. The input to the encoder is a 5-bit information sequence followed by two zero bits to reset the state.

Solution



Decoded sequence: 1 1 1 0 1 0 0