ENM 207

Lecture 11

Some Useful Continuous Distributions

Continuous Uniform Distribution

Suppose that X is a continuous random variable assuming all values equally likely in the interval [a,b] , where a and b are finite.

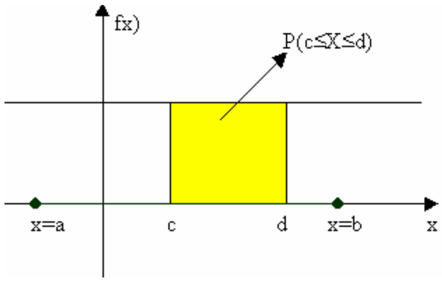
The density function of the continuous uniform random variable *X* on the interval [a, b] is

$$f(x) = \begin{cases} \dfrac{1}{b-a} & a \le x \le b \\ 0 & o.w. \end{cases}$$

It is said that *X* is uniformly distributed over interval [a,b].

Continuous Uniform Distribution

The graph of probability density function (pdf) of *X* is



Continuous Uniform Distribution

f(x) is a probability density function , since

1) $f(x) \geq 0\ ,\ \begin{cases} b>0 & a<0 \quad b-a>0 \\ b<0 & a<0 \quad b-a>0 \end{cases}$

2) $\int_{-\infty}^{\infty} f(x)dx = \int_{a}^{b} f(x)dx = \int_{a}^{b} \frac{1}{b-a} dx = \frac{1}{(b-a)}.(b-a) = 1$

Continuous Uniform Distribution

The mean of the uniformly distributed random variable X is

$$\mu = E(x) = \int_a^b x \frac{1}{b-a} dx = \frac{1}{b-a} \left. \frac{x^2}{2} \right|_a^b = \frac{b^2 - a^2}{2(b-a)} = \frac{b+a}{2}$$

Mean of random variable *X* represents the midpoint of the interval [a,b] , as we would expect intuitively.

Continuous Uniform Distribution

The variance of *X* is

$$\sigma^2 = V(x) = E(x^2) - [E(x)]^2 = \int_a^b x^2 \frac{1}{b-a} dx - \left[\frac{b+a}{2}\right]^2$$

$$= \frac{1}{b-a} \left. \frac{x^3}{3} \right|_a^b - \frac{(b+a)^2}{4} = \frac{b^3 - a^3}{3(b-a)} - -\frac{(b+a)^2}{4}$$

$$= \frac{b^2 + ab + a^2}{3} - \frac{(b^2 + 2ab + a^2)}{4}$$

$$= \frac{4b^2 + 4ab + 4a^2 - 3b^2 - 6ab - 3a^2}{12} = \frac{b^2 - 2ab + a^2}{12}$$

$$= \frac{(b-a)^2}{12}$$

Continuous Uniform Distribution

Example

A point is chosen at random on the line segment [0,2]. What is the probability that the chosen point lies between 1 and 3/2 ?

Letting *x* represent the coordinate of the chosen point we have that the pdf of *X* is given by

$$f(x) = \begin{cases} \frac{1}{2} & 0 < x < 2 \\ 0 & \text{o.w.} \end{cases}$$

And hence

$$p\left(1 \le x \le \frac{3}{2}\right) = \int_1^{\frac{3}{2}} \frac{1}{2}\,dx = \frac{x}{2}\Big|_1^{\frac{3}{2}} = \frac{3}{4} - \frac{1}{2} = \frac{1}{4}$$

Continuous Uniform Distribution

Example

The hardness , say that the number of a speciment of steel is assumed to be a continuous random variable uniformly distributed over [50,70] on the B scale. Hence

$$f(h) = \begin{cases} \frac{1}{20} & 50 < h < 70 \\ 0 & o.w. \end{cases}$$

Compute the probability that the hardness of a randomly selected steel specimen is less than 65.

Continuous Uniform Distribution

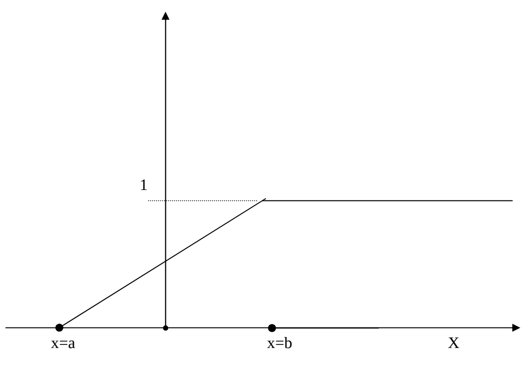
Cumulative distribution function (cdf) of a uniform distribution is

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(s)ds = \int_{a}^{x} \frac{1}{b-a} ds = \frac{x-a}{b-a}$$

$$F(x) = \begin{cases} 0 & x < a \\ \dfrac{x-a}{b-a} & a \le x < b \\ 1 & x \ge b \end{cases}$$

Continuous Uniform Distribution

The graph of cdf of uniform distribution is



References

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