Assignment 4

1) The random variable X has probability density function

$$f(x) = \begin{cases} ax + bx^2 & 0 < x < 1 \\ 0 & otherwise \end{cases}$$

If E[X]=0.6,
    a) Find a and b.
    b) Find P{X<1/2}
    c) Find Var(X).

2) The probability distribution of a random variable X is given by

$$P(x) = \begin{cases} 1/2 & x = 0 \\ 1/4 & x = 1 \\ 1/8 & x = 2 \\ 1/8 & x = 3 \\ 0 & otherwise \end{cases}$$

    a) Determine the mean and variance of X.
    b) If Y=(X-2)^2, find the mean and variance for Y

3) A large industrial firm purchases several new word processors at the end of each year, the exact number depending on the frequency of repairs in the previous year. Suppose that the number of word processors, X, that are purchased each year has the following probability distribution:

x	0	1	2	3
f(x)	1/10	3/10	2/5	1/5

If the cost of the desired model will remain fixed at $1200 throughout this year and a discount of 50 X^2 dollars is credited toward any purchase, how much can this firm expect to spend on new word processors at the end of this year?

4) In a gambling game a woman is paid $3 if she draws a jack or a queen and $5 if she draws a king or an ace from an ordinary deck of 52 playing cards. If she draws any other card, she loses. How much should she pay to play in the beginning of the game if the expected profit is "0"?

5) The length of time, in minutes, for an airplane to obtain clearance for take off at a certain airport is a random variable Y=3X-2, where X has the density function

$$f(x) = \begin{cases} \dfrac{1}{4} e^{-x/4} & x > 0 \\ 0 & otherwise \end{cases}$$

Find the mean and variance of the random variable Y.

6) An electrical firm manufactures a 100-watt light bulb, which, according to specifications written on the package, has a mean life of 900 hours with a standard deviation of 50 hours. At

most, what percentage of the bulbs fails to last even 700 hours? Assume that the distribution is symmetric about the mean.

7) A random variable X has a mean $\mu=12$, a variance $\sigma^2=9$, and an unknown probability distribution. Using Chebyshev's theorem, estimate

a) P(6<X<18);
b) P(3<X<21).

8) The postal service requires, on the average, 2 days to deliver a letter across town. The variance is estimated to be 0.4 $(day)^2$. If a business executive wants 99 percent of his letters delivered on time, how early should he mail them?