$* \; P(0<X<0.5,\ Y>0.25) = \int_{0.25}^{0.5}\int_{0.25}^{x} 3x\,dy\,dx = \int_{0.25}^{0.5} 3xy\Big|_{y=0.25}^{y=x} dx$ (3)

$$= \int_{0.25}^{0.5} 3(x^2-0.25x)\,dx = 3\left(\frac{x^3}{3} - \frac{0.25x^2}{2}\right)\Big|_{0.25}^{0.5}$$

$$= 3\left(\frac{0.125}{3} - \frac{0.0625}{2} - \frac{0.015625}{3} + \frac{0.015625}{2}\right)$$

$$= 0.0390625 = \frac{5}{128}$$

(3)

$$* \; P(Y>0.25) = \int_{0.25}^{1} \frac{3}{2}(1-y^2)\,dy = \frac{3}{2}\left(y - \frac{y^3}{3}\right)\Big|_{0.25}^{1}$$

$$= \frac{1}{2}(3y-y^3)\Big|_{0.25}^{1} = \frac{1}{2}(3-1-0.75+0.015625) = 0.6328125 = \frac{81}{128}$$

(2) $P(0<X<0.5 \mid Y>0.25) = \dfrac{0.0390625}{0.6328125} = 0.061728 = \dfrac{5}{81}$

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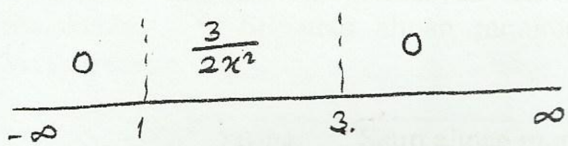
4 a) 8



$x<1 \Rightarrow F(x)=0$ (2)

$1 \le x \le 3 \Rightarrow F(x) = \int_1^x \frac{3}{2t^2}\,dt = -\frac{3}{2t}\Big|_1^x = -\frac{3}{2x} + \frac{3}{2} = \frac{3}{2}\left(1-\frac{1}{x}\right)$ (4)

$x>3 \Rightarrow F(x)=1$ (2)

b) $P(1.5<X<4) = P(X<4) - P(X<1.5)$ (1)

4

$= F(4) - F(1.5)$ (2)

$= 1 - \frac{3}{2}\left(1-\frac{2}{3}\right) = 1 - \frac{1}{2} = \frac{1}{2}$ (1)

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$$P(1.5<X<4) = \int_{1.5}^{3} \frac{3}{2x^2}\,dx = \frac{1}{2}$$

c) $Var(X) = E(X^2) - \mu^2$ (1)

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(2) $E(X) = \mu = \int_1^3 x \cdot \frac{3}{2x^2}\,dx = \frac{3}{2}\int_1^3 \frac{1}{x}\,dx = \frac{3}{2}(\ln x)\Big|_1^3 = \frac{3}{2}(\ln 3 - \underbrace{\ln 1}_{0}) = \frac{3}{2}\ln 3 \cong 1.6479$

(2) $E(X^2) = \int_1^3 x^2 \cdot \frac{3}{2x^2}\,dx = \int_1^3 \frac{3}{2}\,dx = \frac{3}{2}x\Big|_1^3 = \frac{3}{2}(3-1) = 3$

(1) $Var(X) = 3 - (1.6479)^2 = 0.2844 \Rightarrow$ st. sapma $= \sqrt{0.2844} = 0.5333$ (2)