ENM 207

Lecture 2

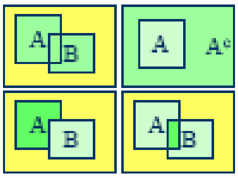
Probability

What is Probability

- The term *probability* has come to widely used in everyday life to quantify the degree of belief in an event of interest.
- For example;
  - There is a 0.2 probability of rain showers
  - The probability that brand X personal computer will survive 100000 hours of operation without repair is 0.75
- Probability is the study of randomness and uncertainty
- Probability gives the likelihood of an event or a set of events

Probability Mathematics

- Set Theory
  - Set operation
  - Set properties



- Set is a collection of objects.
- Sets are generally represented by capital letters A, B, C etc...

Set Theory

- There are three different representations for sets
  - All members of A are listed.

    A={1, 2, 3, 4}
  - The set A is described in words.

    A consists of all real numbers between "0" and "1" , inclusive.
  - We can simply write to describe the above set

    $A=\{X \mid 0 \leq x \leq 1\}$

    (Set A contains X such that X is between 0 and 1)

Set Theory

a $\in$ A ("a" is element of A)
a $\notin$ A (a is not a member of A)

Universal Set : The set of all objects under consideration, it is represented by **S** or **U**.

Empty or Null Set : If there is no any object in the set, then it is called empty or null set $\varnothing$

Subset : If there is a relation between A and B , then A is a subset of B
A $\subset$ B

Equal Sets : If A $\subset$ B and B $\subset$ A then A and B are equal sets A= B

Set Theory

- For every set A ,
  - $\varnothing \subset$ A
  - A $\subset$ S (A is a subset of S)

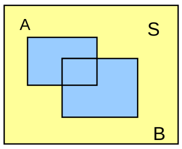
For example;
S : all real numbers

$$A = \{x \mid x^2 + 2x - 3 = 0\}, \qquad \text{i.e. } A = \{-3,1\}$$
$$B = \{x \mid (x-2)(x^2 + 2x - 3) = 0\}, \qquad \text{i.e. } B = \{-3, 2, 1\}$$
$$C = \{x \mid x =, -3, 2, 1\}$$

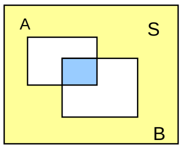
$$then \ \ A \subset B \quad \text{and} \quad B = C$$

Set Operations

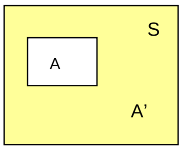
- A∪B: The union of A and B is the set of members that belong to at least one of the sets A and B.



- A∩B: The intersection of A and B is the set of members that belong to both A and B.



- The complement of a set A, denoted by A', is the set of all members in S that are not contained in A.



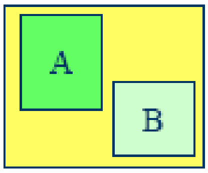
Important Set Properties

1. Mutually Exclusive

$A_i \cap A_j = \phi \quad \text{for } i \neq j$

$A \cap B = \phi \rightarrow$ called

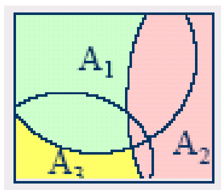
Disjoint for only 2 sets



2. Collectively Exhaustive

$$A_1 \cup A_2 \cup ... \cup A_n = S$$

$$\bigcup_{i=1}^{n} A_i = S$$



Example

- Roll a die.
- Define events A, B, and C as
  - A = rolling a 6, i.e. A={6}
  - B = rolling even, i.e. B={2,4,6}
  - C = rolling odd, i.e. C= {1,3,5}
- A and C are mutually exclusive
- A and B are not mutually exclusive
- B and C are mutually exclusive

Important Set Properties

properties

a) $A \cup B = B \cup A$ "commutative laws"

b) $A \cup (B \cup C) = (A \cup B) \cup C$ "associative laws"

c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ "distributed property"

d) $A \cup \varnothing = A$

e) $(A \cup B)' = A' \cap B'$ "de morgon"

a) $A \cap B = B \cap A$

b) $A \cap (B \cap C) = (A \cap B) \cap C$

c) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

d) $A \cap \varnothing = \varnothing$

e) $(A \cap B)' = A' \cup B'$ "de morgon"

Example

If $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and
$A = \{0, 2, 4, 6, 8\}$,
$B = \{1, 3, 5, 7, 9\}$
$C = \{2, 3, 4, 5\}$,
$D = \{1, 6, 7\}$,

list the elements of the sets corresponding to the following events:

a) $A \cup C = \{0, 2, 3, 4, 5, 6, 8\}$
b) $(C' \cap D) \cup B = \{1, 3, 5, 6, 7, 9\}$
c) $(S \cap C)' = ?$

Example (Walpole, ex. 10, p 30)

An engineer firm is hired to determine if certain waterways in Virginia are safe for fishing. Samples are taken from three rivers

a) List the elements of a sample space *S*, using the letters *F* for "safe to fish" and *N* for "not safe to fish"?

b) List the elements of *S* corresponding to event *E* that at least two of the rivers are safe for fishing?

c) Define an event that has as its elements the points

{FFF, NFF, FFN, NFN}

References

- Walpole, Myers, Myers, Ye, (2002),
  - *Probability & Statistics for Engineers & Scientists*
- Dengiz, B., (2004),
  - *Lecture Notes on Probability,* http://w3.gazi.edu.tr/web/bdengiz
- Hines, Montgomery, (1990),
  - *Probability & Statistics in Engineering* & Management *Science*