# ENM 207

Lecture 7

Distribution Function

Cumulative Distribution Function

## Distribution Function / Cumulative Distribution Function

- Let X be a random variable (discrete or continuous)
- The cumulative distribution function of X is the function that represented by F(x) for a random variable X is equal to the probability

$$F(x) = P(X \leq x)$$

- This function is called the distribution function or cumulative distribution function.
- Contain complete information about the probability model of the random variable

PMF $\Longleftrightarrow$ CDF

# Cumulative Distribution Function

**Theorem**: For a discrete random variable X with
$R$x={x1,x2,…} & $\mathbf{x_1 \le x_2 \le \ldots}$

1) $F(-\infty) = 0$ and $F(\infty) = 1$ ⟶ From 0 to 1

2) $\forall x' \ge x,\ F(x') \ge F(x)$ ⟶ Monotonic Increase

3) For $x_i \in Rx$ and $\varepsilon$ = +small number
$F(x_i) - F(x_i - \varepsilon) = p(x_i)$ ⟶ Discontinuity = p(x)

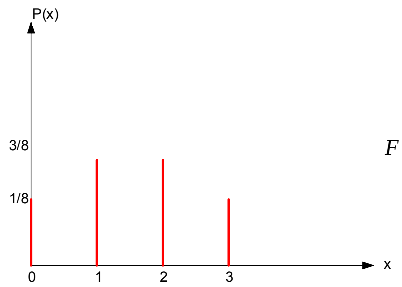
4) $F(x) = F(x_i)\ \ \forall x,\ x_i \le x < x_i+1$ ⟶ Horizon line

## F(x) for Discrete Random Variable

**Example**

i)

| X | P(x) |
|---|---|
| 0 | 1/8 |
| 1 | 3/8 |
| 2 | 3/8 |
| 3 | 1/8 |



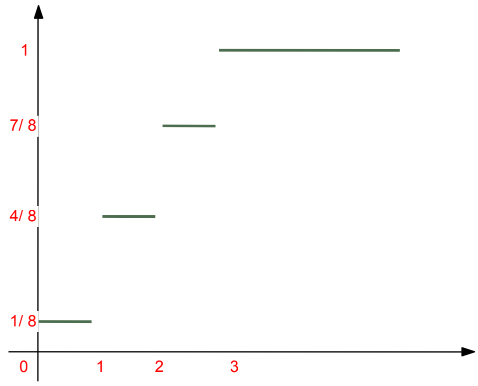

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{8} & 0 \le x < 1 \\ \frac{4}{8} & 1 \le x < 2 \\ \frac{7}{8} & 2 \le x < 3 \\ 1 & x \ge 3 \end{cases}$$

Find these probabilities:

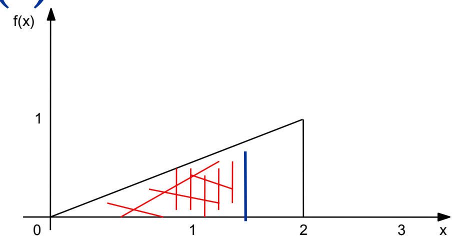
P(x=1)=? = (4/8) –(1/8) = 3/8

P(1≤ x <2) = P(x =1) =3/8

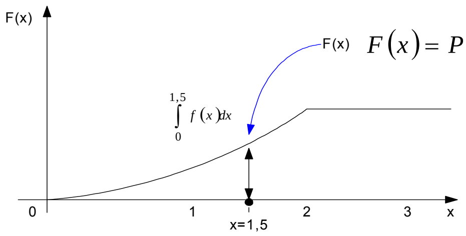
## Graphical Representation of F(x)




## F(x) for Continuous Random Variable




The value of the height for x=1.5 i. e., F(x =1.5)=$P(X \leq 1.5)$ in the figure below is equal to the value of red shaded area in the figure above.

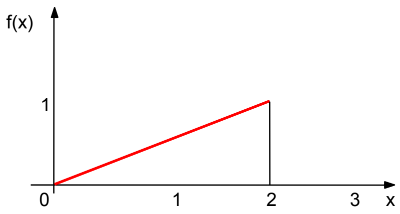



$$F(x) = P(X \leq x) = \int_0^x \frac{1}{2} x dx = \frac{1}{2} \frac{x^2}{2}\Bigg|_0^x = \frac{x^2}{4}$$

# Example

Let X be a continuous random variable with the following distribution

$$f(x) = \begin{cases} \frac{1}{2}x & 0 \le x \le 2 \\ 0 & o.w \end{cases}$$




$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{4}x^2 & 0 \le x \le 2 \\ 1 & x > 2 \end{cases}$$

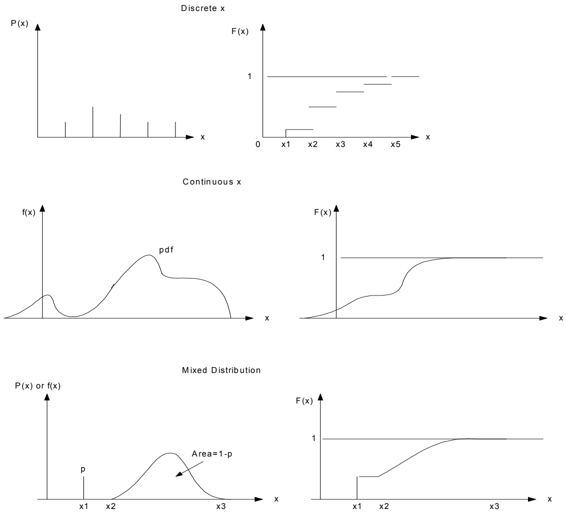
# F(x) for Continuous Random Variable

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)\,dt$$

If F(x) has a first derivative ,then

$$f(x) = \frac{dF(x)}{dx}$$

## F(x) for Discrete, Continuous and Mixed R.V.




## Example

$$f(x) = \begin{cases} cx & 0 \le x \le 1 \\ 0 & \text{dd} \end{cases}$$

$a)$ c = ?

b)P($0.2 < y < 0.5$) = ?

## Example

$$f(x) = \begin{cases} cy^2 & 0 \le x \le 1 \\ 0 & \text{dd} \end{cases}$$

$a)$ c = ?

b) $F$ (y) = ?

$c)F(1) = ?$

$d)F(0.5) = ?$

$e)P(1 \le y \le 1.5) = ?$

## Example

$$f(y) = \begin{cases} ce^{-y} & y > 0 \\ 0 & o.w. \end{cases}$$

$a)$ Find the value of c ?

$b)$ Find the cdf

$c)$ Compute $F(2.6)$

$d)$ Show that F(0) = 0 and $F(\infty) = 1$

$e)$ Compute $P(1 \le y \le 5)$

# References


- Walpole, Myers, Myers, Ye, (2002),
  - *Probability & Statistics for Engineers & Scientists*
- Dengiz, B., (2004),
  - *Lecture Notes on Probability,* http://w3.gazi.edu.tr/web/bdengiz
- Hines, Montgomery, (1990),
  - *Probability & Statistics in Engineering* & Management *Science*