ENM 207

Lecture 3
Counting Method

Counting Sample Points

- In many cases, we shall be able to solve a probability problem by counting the number of points in the sample space without actually listing each element.
- The fundamental principle of counting is *multiplication rule*.

If experiment A has **n** possible outcomes,
and experiment B has **k** possible outcomes,

→Then there are **nk** possible outcomes
when you perform both experiments

Counting Sample Points

Example 1:

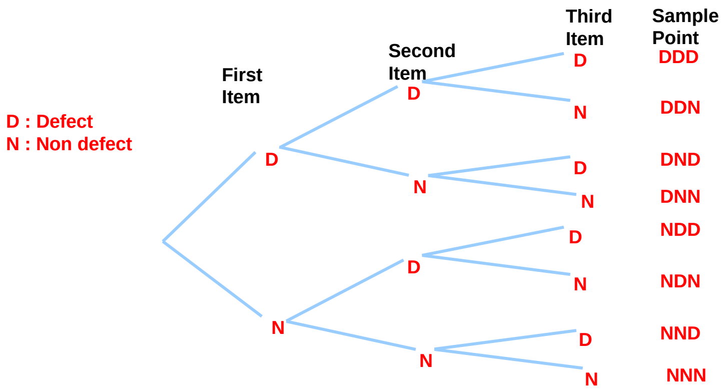
How many sample points are in the sample space when a pair of dice is thrown once?

Example 2:

8 possible colors to paint living room, 5 possible colors to paint the kitchen , how many ways to select and purchase two cans of paint?

Tree Diagram

- A tree diagram lists the elements of the sample space systematically.



Three items are selected at random from a manufacturing process. Each item is Classified defective or non defective.

FACTORIAL NOTATION

The product of positive integers from 1 to n is denoted by the special symbol n! and read “n factorial”.

$$n! = 1.2.3....(n-2).(n-1).n$$

$$5! = 1.2.3.4.5 = 120$$

Permutations

- We are interested in a sample space that contains as elements all possible orders or arrangements of a group of objects.
- For example:
  - We may want to know how many different arrangements are possible for sitting 6 people around a table
  - We may ask how many different orders are possible for drawing 2 lottery tickets from a total of 20.

A permutation is an arrangement of all or part of a set of objects

The number of permutations of *n* distinct objects is *n* !

The number of permutations of size *r* in *n* distinct objects is

$$P_n^r = \frac{n!}{(n-r)!}$$

Permutations

The number of permutations of size k is obtained from the general multiplication rule as follows:

- The first element can be chosen in **n** ways,
- The second element can be chosen in **n-1** ways,
- The third element can be chosen in **n-2** ways,
- and so on ;
- Finally for each way of choosing the first **k-1** elements, the **k^th^** element can be chosen in **n-(k-1) = n-k+1** ways, thus

The number of permutations of size **k** in **n** distinct objects is denoted by

$$P_n^k = n.(n-1)(n-2)(n-3)......(n-k+2)(n-k+1) = \frac{n!}{(n-k)!}$$

Permutations

Example: Three awards (research, teaching and service) will be given one year for a class of 25 graduate students in a statistics department. If each student can receive at most one award, how many possible selections are there?

Permutations

Example: A president and a treasure are to be chosen from a student club consisting of 50 people. How many different choices of officers are possible if

a. There are no restrictions,
b. A will serve only if he is president,
c. B and C will serve together or not at all,
d. D and E will not serve together?

Permutations

- Permutations that occur by arranging objects in a circle are called as circular permutations.

The number of permutations of *n* distinct objects in a circle is (*n*-1)!

- Example : n people can be sit around a table in (n-1)! different form.

The number of distinct permutations of *n* things of which n_1 are of one kind, n_2 of a second kind, …., n_k of a *k*th kind is

$$\frac{n!}{n_1!n_2!\dots n_k!}$$

Example: How many distinct permutations can be formed from all the letters of each word:

i) them ii) unusual iii) sociological

Permutations

- Sometimes, we are concerned with the number of ways of partitioning a set of *n* objects into *r* subsets called cells.
- A partition has been achieved
  - if the intersection of every possible pair of *r* subsets is the empty set ∅, and
  - if the union of all subsets gives the original set.
- The order of the elements within a cell is no importance.
- Example: {a, e, i, o, u}

  The possible partitions into cells in which the first cell contains 4 elements and the second cell 1 element are 5.

Permutations

The number of ways of partitioning a set of *n* objects into *r* cells with

n_1 elements in the first cell, n_2 elements in the second, and so forth, is

$$\binom{n}{n_1, n_2, \ldots, n_r} = \frac{n!}{n_1! n_2! \ldots n_k!}$$

where $n_1 + n_2 + \ldots + n_r = n$

Example: In how many ways can 7 scientists be assigned to one triple and two double hotel rooms?

Combinations

- In many problems, we are interested in the number of ways of *selecting* *r* objects from *n* without regard to order.
- These selections are called combinations.
- A combination is actually a partition with two cells, the one cell containing the *r* objects selected and other cell containing the (*n*-*r*) objects that are left.

The number of combinations of size *r* at *n* distinct objects

$$C_n^r = \binom{n}{r} = \frac{n!}{r_1!(n-r)!}$$

Combinations

Example: consider the set {A,B,C,D,E} consisting of 5 elements.

We know that there are

5!/(5-3)!=60 permutations of size 3 and
5!/ 3!(5-3)!= 10 combinations of size 3

Example: find the number of permutations of size 3 consisting of the elements of A,B,C.

3! = 3 x 2 x 1 = 6
(A,B,C) (A,C,B) (B,A,C) (B,C,A) (C,A,B) and (C,B,A)

Some special combinations

$$i)C_n^0 = \frac{n!}{0!(n-0)!} = 1$$

$$ii)C_n^1 = \frac{n!}{1!(n-1)!} = n$$

$$iii)C_n^n = \frac{n!}{n!(n-n)!} = 1$$

References

- Walpole, Myers, Myers, Ye, (2002),
  - *Probability & Statistics for Engineers & Scientists*
- Dengiz, B., (2004),
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  - *Probability & Statistics in Engineering* & Management *Science*