③ a) $P(X<0.75) = \int_0^{0.75} 6x(1-x)\,dx = 6\left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^{3/4} = 6\left(\frac{9}{32} - \frac{9}{64}\right) = \overset{3}{\cancel{6}} \times \frac{9}{\underset{32}{\cancel{64}}} = \frac{27}{32}$

$= 0.84375$

b) $E(X) = \int_0^1 x\, 6x(1-x)\,dx = 6\int_0^1 (x^2 - x^3)\,dx = 6\left(\frac{x^3}{3} - \frac{x^4}{4}\right)\Big|_0^1$

$= 6\left(\frac{1}{3} - \frac{1}{4}\right) = 6 \times \frac{1}{12} = \frac{1}{2}$

c) $x \le 0 \Rightarrow F(x) = 0$

$0 < x < 1 \Rightarrow F(x) = \int_0^x 6t(1-t)\,dt = 6\int_0^x (t - t^2)\,dt = 6\left(\frac{t^2}{2} - \frac{t^3}{3}\right)\Big|_{t=0}^{t=x}$

$= 6\left(\frac{x^2}{2} - \frac{x^3}{3}\right) = 3x^2 - 2x^3$

$x \ge 1 \Rightarrow F(x) = 1$

$$F(x) = \begin{cases} 0 & , & x \le 0 \\ 3x^2 - 2x^3 & , & 0 < x < 1 \\ 1 & , & x \ge 1 \end{cases}$$

$P(0 < x \le 0.2) = P(X \le 0.2) - P(X \le 0) = F(0.2) - F(0)$

$= 3(0.2)^2 - 2(0.2)^3 - 0$

$= 0.104$

④ a) $f(x,y) \overset{?}{=} g(x)\cdot h(y) \xrightarrow{E} $ X ve Y ras. deg. leri bağımsızdır.

$\xrightarrow{H}$ X ve Y ras. deg. leri bağımlıdır.

$g(x) = \int_0^1 (x+y)\,dy = \left(xy + \frac{y^2}{2}\right)\Big|_0^1 = x + \frac{1}{2}, \quad 0 < x < 1$

$h(y) = \int_0^1 (x+y)\,dx = \left(\frac{x^2}{2} + xy\right)\Big|_0^1 = \frac{1}{2} + y, \quad 0 < y < 1$

$x+y \ne \left(x+\frac{1}{2}\right)\left(y+\frac{1}{2}\right)$ olduğundan X ve Y ras. deg. leri bağımlıdır.

b) $P(X < 1/2 \mid Y = 1/4) = \int_0^{1/2} f(x \mid y = 1/4)\,dx$

$f(x \mid y) = \frac{f(x,y)}{h(y)} = \frac{x+y}{y + \frac{1}{2}} \quad 0 < x < 1, \ 0 < y < 1$

$P(X < 1/2 \mid Y = 1/4) = \int_0^{1/2} \frac{x + 1/4}{\frac{1}{4} + \frac{1}{2}}\,dx = \int_0^{1/2} \left(\frac{4x+1}{4}\right)\frac{4}{3}\,dx = \frac{1}{3}(2x^2 + x)\Big|_0^{1/2}$

$= \frac{1}{3}\left(\frac{1}{2} + \frac{1}{2}\right) = \frac{1}{3}$

c) $\text{Cov}(X,Y) = E(XY) - E(X)E(Y)$

$E(XY) = \int_0^1\int_0^1 xy(x+y)\,dx\,dy = \int_0^1\int_0^1 (x^2y + xy^2)\,dx\,dy = \int_0^1 \left(\frac{x^3 y}{3} + \frac{x^2 y^2}{2}\right)\Big|_{x=0}^{x=1}dy$

$= \int_0^1 \left(\frac{y}{3} + \frac{y^2}{2}\right)dy = \left(\frac{y^2}{6} + \frac{y^3}{6}\right)\Big|_0^1 = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$