1) $\left.\begin{matrix} Z = X+Y \\ W = Y \end{matrix}\right\}$ $\begin{matrix} X = Z - W = w_1(x,y) \\ Y = W = w_2(x,y) \end{matrix}$ $\begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = 1 = J$

$g(z,w) = 24.(z-w)(w).|1|$

$$g(z,w) = \begin{cases} 24(zw-w^2) & , 0<z<1, 0<w<z \\ 0 & , d.d. \end{cases}$$

a) $h(z) = \int_0^z 24(zw-w^2)\,dw = 24\int_0^z (zw-w^2)\,dw$

$= 24\left[\frac{zw^2}{2} - \frac{w^3}{3}\right]\Big|_0^z = 24\left[\frac{z^3}{2} - \frac{z^3}{3}\right] = 24.\frac{z^3}{6} = 4z^3$

$$h(z) = \begin{cases} 4z^3 & , 0<z<1 \\ 0 & , d.d. \end{cases}$$

b) $\int_{1/2}^{3/4} 4z^3\,dz = z^4\Big|_{1/2}^{3/4} = \frac{81}{256} - \frac{1}{16} = \frac{65}{256} = 0.254$

2) Y: başlangıç miktarı, X: satılan miktar

$Z = Y - X$: kalan miktar $\Rightarrow h(z) = ?$

$\left.\begin{matrix} Z = Y-X \\ W = Y \end{matrix}\right\}$ $\begin{matrix} X = w_1(z,w) = w - z \\ Y = w_2(z,w) = w \end{matrix}$ $\begin{matrix} 0<z<w \\ 0<w<1 \end{matrix}$

$g(z,w) = f(w-z), w).|J|$

$\begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix} = -1 = J$ $|J| = 1$ $g(z,w) = \begin{cases} 2 & , \begin{matrix} 0<z<w \\ 0<w<1 \end{matrix} \\ 0, & d.d. \end{cases}$

$\Rightarrow h(z) = \int_0^1 2\,dw = 2 \Rightarrow h(z) = \begin{cases} 2, & 0<z<w \\ 0, & d.d. \end{cases}$

3)

$$f(x_1) = \begin{cases} e^{-x_1} & , x_1 > 0 \\ 0 & , d.d. \end{cases} \quad , f(x_2) = \begin{cases} e^{-x_2} & , x_2 > 0 \\ 0 & , d.d. \end{cases}$$

x_1 ve $x_2 \rightarrow$ bağımsız $\Rightarrow f(x_1, x_2) = f(x_1) \cdot f(x_2)$

$$f(x_1, x_2) = \begin{cases} e^{-(x_1+x_2)} & , \begin{array}{l} x_1 > 0 \\ x_2 > 0 \end{array} \\ 0 & , d.d. \end{cases}$$

$Y_1 = X_1 + X_2 \qquad X_1 = Y_1 Y_2$

$Y_2 = \dfrac{X_1}{X_1 + X_2} \qquad X_2 = Y_1(1 - Y_2) \quad \underbrace{}_{Y_1 - (Y_1 Y_2)}$

$\Rightarrow J = \begin{vmatrix} Y_2 & Y_1 \\ 1 - Y_2 & -Y_1 \end{vmatrix} = \dots = -Y_1$

$g(Y_1, Y_2) = f(Y_1 Y_2 , Y_1 - (Y_1 Y_2)) , |-Y_1|$

$$g(y_1, y_2) = \begin{cases} y_1 e^{-y_1} & , y_1 > 0 , y_2 > 0 \\ 0 & , d.d. \end{cases}$$

$h(y_1) = \int_0^1 y_1 e^{-y_1} dy_2 = y_1 e^{-y_1}$

$h(y_2) = \int_0^\infty y_1 e^{-y_1} dy_1 \qquad y_1 = u , e^{-y_1} dy_1 = dv , du = dy_1 , v = -e^{-y_1}$ //

$h(y_2) = -y_1 e^{-y_1} \Big|_0^\infty - \int_0^\infty -e^{-y_1} dy_1 = 0 + (-e^{-y_1}) \Big|_0^\infty$

$= 0 + \dfrac{1}{e^{y_1}} \Big|_\infty^0 = 0 + 1 = 1$

$g(y_1, y_2) = h(y_1) \cdot h(y_2)$ olduğuna göre Y_1 ve Y_2 r. değişkenleri bağımsızdır.

(4) $X_1 = -\sqrt{Y}$, $-1 < x < 0$ $J_1 = \frac{-1}{2\sqrt{Y}}$

$X_2 = \sqrt{Y}$, $0 < x < 1$ $J_2 = \frac{1}{2\sqrt{Y}}$

$$g(y) = f(-\sqrt{y}) \cdot |J_1| + f(\sqrt{y})|J_2|$$

$$= \frac{1-\sqrt{y}}{2} \cdot \frac{1}{2\sqrt{y}} + \frac{1+\sqrt{y}}{2} \cdot \frac{1}{2\sqrt{y}} = \frac{1}{2\sqrt{y}}$$

$$g(y) = \begin{cases} \frac{1}{2\sqrt{y}} & , 0<y<1 \\ 0 & , \text{d.d.} \end{cases}$$

(5) $$f(x) = \begin{cases} pq^{x-1} & , x=1,2,3,\ldots \\ 0 & , \text{d.d.} \end{cases}$$ $\sum_{x=0}^{\infty} \alpha^n = \frac{1}{1-\alpha}$

$$M_x(t) = E[e^{tx}] = \sum_{x=1}^{\infty} e^{tx} pq^{x-1} = p\sum_{x=1}^{\infty} e^{tx} \cdot q^{x-1}$$

$$= p\sum_{x=1}^{\infty} (e^t)^x \frac{q^x}{q} = \frac{p}{q}\sum_{x=1}^{\infty} (e^t \cdot q)^x = \frac{p}{q} \cdot \frac{1}{1-e^t q} - \frac{p}{q}$$

$$= \frac{p}{q}\left(\frac{1}{1-e^t q} - 1\right) = \frac{p}{q}\left(\frac{1-1+e^t q}{1-e^t q}\right) =$$

$$= \frac{p}{q}\left(\frac{qe^t}{1-e^t q}\right) = \frac{pe^t}{1-e^t q}$$

$$\left.\frac{\partial M_x(t)}{\partial t}\right|_{x=0} = \left.\frac{pe^t(1-qe^t) - pe^t(-qe^t)}{(1-qe^t)^2}\right|_{t=0}$$

$$= \left.\frac{pe^t(1-qe^t+qe^t)}{(1-qe^t)^2}\right|_{t=0} = \frac{p}{(1-q)^2} = \frac{1}{p}$$

$$\left.\frac{\partial^2 M_x(t)}{\partial t^2}\right|_{t=0} = \left.\frac{pe^t(1-qe^t)^2 - p.e^t\,2(1-qe^t)(-qe^t)}{(1-qe^t)^4}\right|_{t=0}$$

$$= \left.\frac{pe^t\,(1-\cancel{qe^t})\,[\,(1-qe^t)+2qe^t\,]}{(1-qe^t)^{\cancel{4}\,3}}\right|_{t=0}$$

$$= \frac{\cancel{p}[\,p+2q\,]}{p^{\cancel{3}\,2}} = \frac{1+q}{p^2}$$

$$\mathrm{Var}(x) = \frac{1+q}{p^2} - \left(\frac{1}{p}\right)^2 = \frac{q}{p^2}\,//$$