# **ENM 207**

### Lecture 8

The Expected Value and Variance of a Random Variable

### The Expected Value for a Random Variable

- Since there is a range of possible values of a random variable, we would be interested in some central value such as the average.
- If X is a discrete random variable with probability function p(x), its weighted average value, denoted by E(x) or μ,is

$$\mu = E(x) = \sum_{all \ x_i} x_i p(x_i)$$

### The Expected Value for a Random Variable

Similarly, for a continuous random variable X with p.d.f. f(x), the mean value is

$$E(x) = \int_{-\infty}^{\infty} x \ f(x) dx$$

The expected value of a function g(x):

$$E[g(x)] = \begin{cases} \sum_{\substack{all \ x_i \\ \infty}} g(x_i) p(x_i) & \text{, if } x \text{ discrete} \end{cases}$$

$$\int_{-\infty}^{\infty} g(x) f(x) dx & \text{, if } x \text{ continuous} \end{cases}$$

## Example

Xi	0	1	2	3	
p(x <sub>i</sub> )	1/27	10/27	8/27	8/27	

$$E(x) = \sum_{\forall i} x_i p(x_i) = 0 \frac{1}{27} + 1 \frac{10}{27} + 2 \frac{8}{27} + 3 \frac{8}{27} = \frac{50}{27} = 1.85$$

A lot containing 7 components is sampled by a quality inspector; the lot contains 4 good components and 3 defective components. A sample of 3 is taken by the inspector. Find the expected value of the number of good components in this sample

# Example

Let *X* be the random variable that denotes the life in hours of a certain electronic device. The probability density function is

$$f(x) = \begin{cases} \frac{20000}{x^3} & x > 100\\ 0 & \text{o.w.} \end{cases}$$

$$\mu = E(X) = \int_{100}^{\infty} x \frac{20000}{x^3} dx = \int_{100}^{\infty} \frac{20000}{x^2} dx = 200 \text{ hours}$$

### SOME USEFUL EXPECTATION THEOREMS

#### **Theorem**

Let y be a discrete random variable with probability distribution p(y) and let c be a constant. Then the expected value or mean of c is

$$E(c) = c$$

Then the expected value of cy is

$$E(cy) = cE(y)$$

### SOME USEFUL EXPECTATION THEOREMS

#### **Theorem**

Let y be a discrete random variable with probability distribution p(y) and let  $g_1(y), g_2(y), ..., g_k(y)$  be functions of y. Then

$$E[g_1(y) + g_2(y) + \dots + g_k(y)] = E[g_1(y)] + E[g_2(y)] + \dots + E[g_k(y)]$$

## Example 4.5. p 92/ Walpole

Let X be a random variable with density function

$$f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2\\ 0 & \text{o.w.} \end{cases}$$

Find the expected value of g(X) = 4X + 3

$$E(4X+3) = \int_{-1}^{2} \frac{(4x+3)x^2}{3} dx = \frac{1}{3} \int_{-1}^{2} (4x^3 + 3x^2) dx = 8$$

### VARIANCE OF RANDOM VARIABLE

 Let Y be d.r.v. with probability distribution p(y) then the variance of y is

$$\sigma^2 = E[(x-\mu)^2] = E(x^2) - \mu^2 = E(x^2) - [E(x)]^2$$

$$\sigma^{2} = E(x^{2} - 2\mu x + \mu^{2}) = E(x^{2}) - 2\mu E(x) + \mu^{2}$$
$$= E(x^{2}) - 2\mu^{2} + \mu^{2} = E(x^{2}) - \mu^{2}$$

The standard deviation of y is the positive square root of the variance of y:

 $\sigma = \sqrt{\sigma^2}$ 

Let Hi & Ti denote the observation of a head & a tail on the ith toss, for i=1,2.

Simple event	Description	P(Ei)	# of heads
E1	H1H2	1/4	2
E2	H1T2	1/4	1
E3	T1H2	1/4	1
E4	T1T2	1/4	0

# Example

For the random variable

- Obtain probability mass function
- Calculate
  - Expected value of r.v.
  - Variance of r.v.

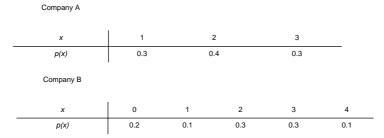
The mathematical model for d.r.v. y is given below:

yi	7	8	9	10	11	12	13
p(y <sub>i</sub> )	0.10	0.10	0.15	0.20	0.20	0.15	0.10

- a) Show that it is a probability mass function
- b) P(y=9)=?
- c) P(y<12)=?
- d) Calculate E[X]
- e) Calculate Var[x]

## Example

Let the random variable X represents the number of automobiles that are used for official business purposes on any given workday. The probability distributions for company A and B are:



Show that the variance of the probability distribution of company B is greater than that for company A.

y= is the # of firing pins tested in a sample of five selected from a large lot.

Suppose the cost of inspecting of a single pin is \$300 if the pin is defective and \$100 if not. Then the total cost c (in dollars) of the inspection is given by the equation c=200+100y.

Find the mean and the variance of c.

y	P(y)
1	0.6
2	0.3
3	0.1

## Example

 The hospital period, in days, for patients following treatment for a certain type of kidney disorder is a random variable Y = X + 4, where X has the density function

$$f(x) = \begin{cases} \frac{32}{(x+4)^3}, & x > 0\\ 0, & \text{elsewhere.} \end{cases}$$

 Find the average number of days that a person is hospitalized following treatment for this disorder.

# References

- Walpole, Myers, Myers, Ye, (2002),
  - Probability & Statistics for Engineers & Scientists
- Dengiz, B., (2004),
  - □ Lecture Notes on Probability, http://w3.gazi.edu.tr/web/bdengiz
- Hines, Montgomery, (1990),
  - □ Probability & Statistics in Engineering & Management Science