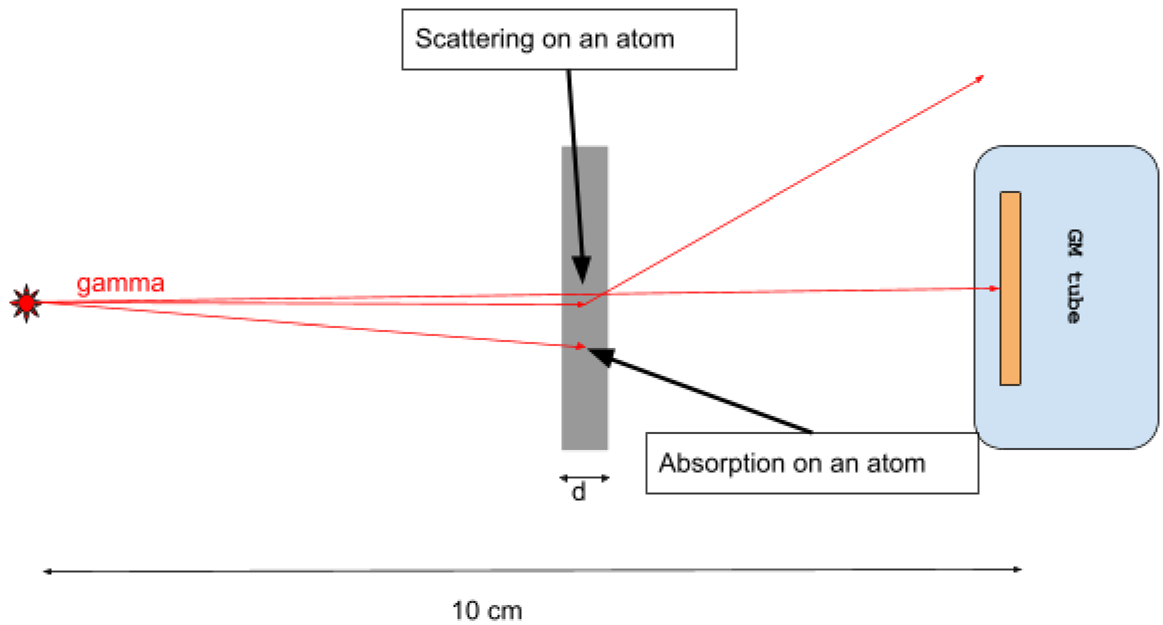
Part1: Experiments on Attenuation

What happens to gammas when they interact with the surrounding materials? At the ~MeV energy scale there are three primary processes: photoelectric effect; Compton scattering; pair production (>1.022 MeV).

Cross section and attenuation of gammas

In this experiment we will measure the transmission of gammas through different materials, such as lead and plastic.



Before we start remember the formula that we derived in class for the effects of materials on gamma transmission:

$t = \frac{N}{N_0} = \exp(-\ \mu\rho d),$

Where N is the transmitted count, N_0 is the incident count (before the material), d is the material thickness (in cm), ρ is density in g/cm^3, and μ is the mass attenuation coefficient in units of cm^2/g. Note that μ strongly depends on the material *and* the energy of the gamma. You can look up values of μ on the [XCOM database] at the National Institute of Standards and Technology (NIST). Also note that in this equation ρd always come as a product – this product is often referred to as the *areal density* of the material (in units of g/cm^2). We often describe the absorbing material in areal density, rather than in d and rho separately.
The goal of this experiment is to make measurements of transmission=N/N_0 and compare to the prediction as estimated by the XCOM database.

However measuring N and N_0 is not so trivial because there is background! When you measure either of these counts there is always some background mixed in. Thus you need to measure the background separately (with no source present!) and then subtract it from both N and N_0.

Step 1: background

Run your counter for 5 min and measure the background counts. Note: the source, any source, needs to be put far away from your detector. Rather than working in absolute counts, we will work in **counts per minute** (CPM), which we will denote as r (for **rate**).
Q1: what are your counts in 5 min?

$c_{bg} =$

Q2: what is the error/uncertainty on our 5 min count? Remember, Poisson statistics…Units: 1

$\sigma \; =$

Q3: now compute your background rate in counts-per-minute (CPM) and the error. Remember the formula from statistics for error propagation for $f = const. \times x$! Units: 1

$r_{bg} =$

$\sigma_{bg} =$

Step 2: open beam

You have now nailed down the background – great! Now let's get a hold of N_0. Place the ^{60}Co source about 10 cm away from your detector and measure for 5 min. Record your counts, calculate the error on them, and then determine the CPM rate and the error on it, similar to how you did it in Step 1. Units: 1

$r_0 =$

$\sigma_0 =$

Step 3: attenuation for lead

Now repeat Step 2 with TWO Pb absorbers in the way. Place it right up against the detector. If you are using the Pb absorber marked as S then it has an areal density of $\rho d = 2 \cdot 3.63\ g/cm^2$.
Like before estimate the rate in CPM and the error: Units: 1

r_{Pb} =

σ_{Pb} =

Step 4: calculate transmission

The final step! Your transmission is going to be

$t = \frac{r-r_{bg}}{r_0-r_{bg}}$

Calculate t based on the values from Steps 1-3. Units: 1

t_{Pb} =

Now you need to determine the error! Use $\sigma^2 = \sum(\sigma_i \partial f/\partial x_i)^2$ and the formula above for t to calculate the formula for σ_t: Units: 4

Calculate the numerical value of σ_t = Units: 1

Step 5: compare to theory

Use XCOM tables to determine the transmission for 60Co gammas (~1.1 MeV) and $\rho d = 2 \cdot 3.63\ g/cm^2$ of lead. Units: 1

t_{XCOM} =

How does your result compare to XCOM? Calculate Units: 1+2

Z =

And the associated probability p =

Attenuation for plastic and for 137Cs.

Repeat the steps from the lead experiment here for 3cm of plastic and a 137Cs source. Note that you do NOT need to take a new background (no source) measurement, you can just use the old one.

Determine the CPM rate for open beam with 137Cs. Note that the 137Cs needs to be shielded by absorber R (we consider it as part of the source): Units: 1

$r_0 =$

$\sigma_0 =$

Attenuated rate for plastic: Units: 1

$r_{plastic} =$

$\sigma_{plastic} =$

Calculate transmission: Units: 1

$t_{plastic} =$

$\sigma_t =$

Calculate the theoretical value for transmission from XCOM, using 0.662 MeV for 137Cs, plastic thickness of 3cm, density of 1g/cc, and plastic's formula of CH2 (note: you need to use materials input on XCOM, not elements!) Units: 1

$t_{XCOM} =$

From here determine your Z and the associated probability Units: 1+2

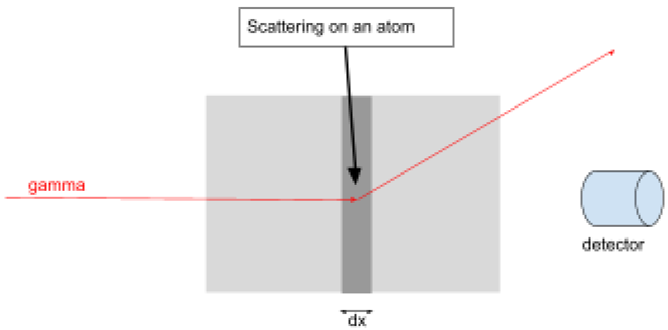
$Z_{plastic} =$

$p =$

Appendix

Cross section and mass attenuation coefficient

Let's assume that the combined cross section for all these interactions is σ (note, this is different from st. deviation!). Remember, cross section is in the units of area, and it denotes the "effective area" of the scatterer. It typically has the unit of 1 barn = 10^{-24} cm^2. This may sound like a tiny number, however given how many atoms there are, 1 barn ends up being a very large number.



Let's consider the thin sliver of material, dx, as depicted in the image above, where the scattering happens. You want to know the total area covered by all the atoms in the sliver, if one atom has an effective area of σ. If the material density is ρ, and the atomic weight of the material is A, then the total area covered by all the atoms is the total number of atoms X σ. The total number of atoms is volume of the sliver times density times Avogadro's number divided by atomic weight. So the total area is

$Atomic\ area\ =\ \sigma \cdot YZ \cdot dx \cdot \rho N_{Av}/A$

Where Y and Z are the height and the width of your material. And the probability that a random particle streaming through the material will interact with one atom is just the ratio of this "Atomic area" and the total area, hence:

$dP\ =\ [Atomic\ area]/YZ\ =\ dx\, \sigma\rho N_{Av}/A$

We next want to ask ourselves: if N number of particles are shot at the material, how many of them will interact in the thin sliver dx?

$\#\ of\ Interacted\ particles\ =\ NdP\ =\ Ndx\, \sigma\rho N_{Av}/A$

But remember, the change in N, i.e. dN, is exactly equal to the # of interacted particles, with a minus sign (# of particles is positive, while the change in N is negative). So we get the following first order homogeneous differential equation that governs the number N:

$-\ dN\ =\ N\, dx\, \sigma\rho N_{Av}/A$

Here we denote $\mu\ \equiv\ \sigma N_{Av}/A$, and we call it the mass attenuation coefficient. The new equation takes the following form:

$-\ dN\ =\ N\, dx\, \mu\rho$

By moving some variables around we get $\frac{dN}{N} = -dx\,\mu\rho$, which we integrate for $0 \le x' \le x$ and $N_0 \ge N' \ge N$, and it gives us

$\frac{N}{N_0} = \exp(-\mu\rho x)$

Note that μ strongly depends on the material *and* the energy of the gamma. You can look up values of μ on the [XCOM database] at the National Institute of Standards and Technology (NIST).

Waiting times

SOLUTION to Problem S.3:

If you had a count, what's the probability of having no counts for time t, and then a count in the tiny window dt? This may be confusing, because with the formula above we are used to thinking of x as a discrete variable (one count, two counts, but no 1.324 counts…)...however the time t is continuous. So when it comes to the distribution of waiting time we are working not with probabilities, but rather with *probability density functions*. Let's denote this probability density as $p(t)$, which tells us the probability density of having a 2nd count at time t. The actual probability is then $p(t)dt$. A 2nd count time t away requires two things: no counts for time t, and then a count in the time window dt right after that. The total probability should be the product of these two.

Thus we can write

$p(t)dt = P(x = 0) \cdot rdt$

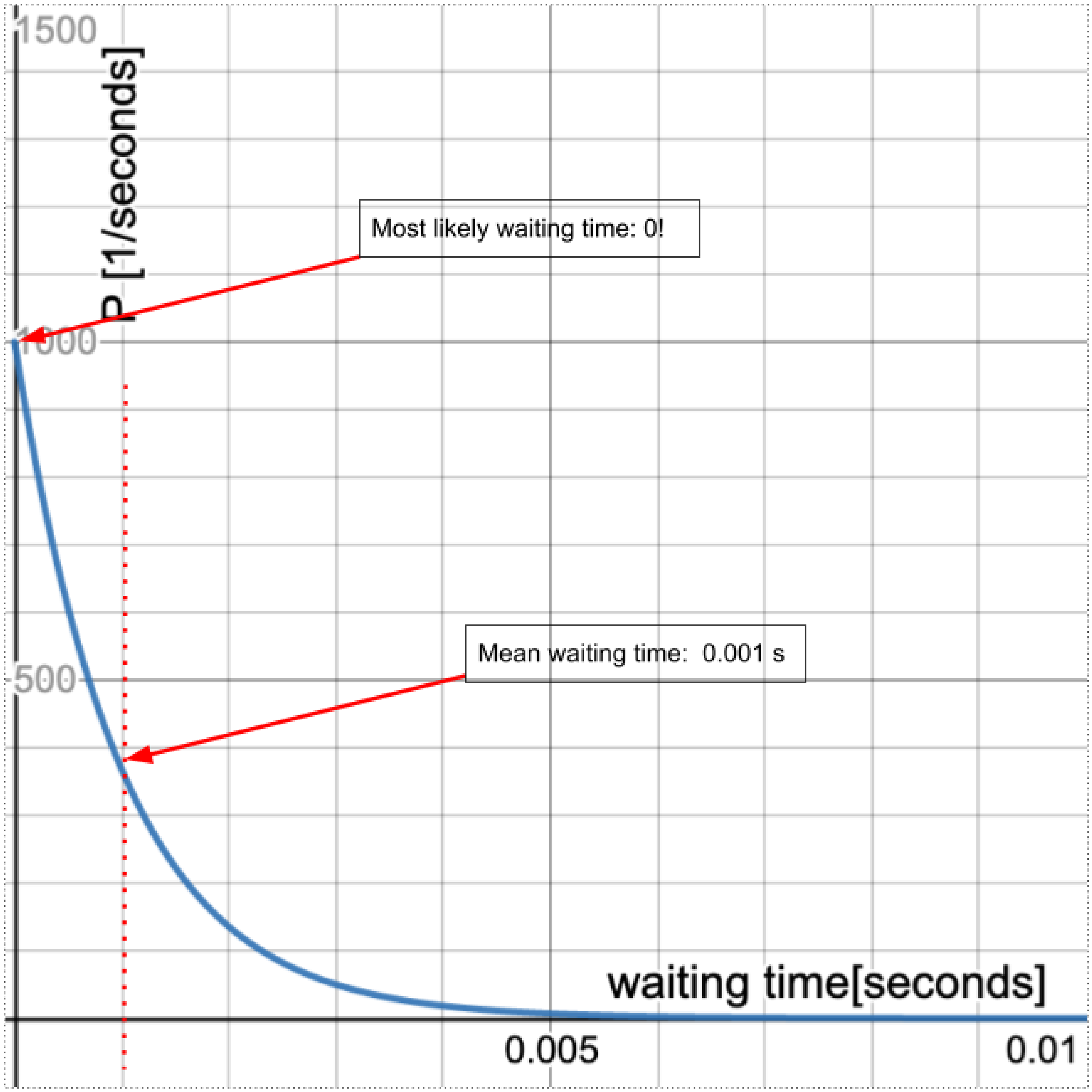
where r is the rate of counts per second. To be clear, if you know your rate then the number of expected counts within a time window dt is rdt. Similarly, the expectation $\bar{x}$ of counts in the time window t preceding that is $\bar{x} = rt$. Let's plug these, and $x = 0$ (no counts for time t) into the Poisson's equation $P(x) = \bar{x}^{x} \exp(-\bar{x})/x!$. This gives us

$p(t)dt = P(x = 0) \cdot rdt = ((rt)^0 \exp(-rt)/0!) \cdot rdt = r\exp(-rt)dt$.

The dt's cancel, and we get

$p(t) = r\exp(-rt)$

Below is a plot of the distribution for the value of r=1000 counts/second.



1500
P [1/seconds]
Most likely waiting time: 0!
1000
Mean waiting time: 0.001 s
500
waiting time[seconds]
0.005
0.01