

Convolution Neural Networks

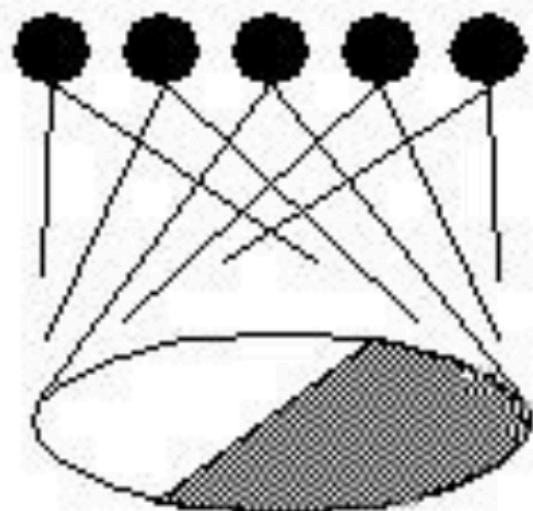
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Vision: Hierarchical Organization (Year: 1962)

Hubel & Weisel

topographical mapping

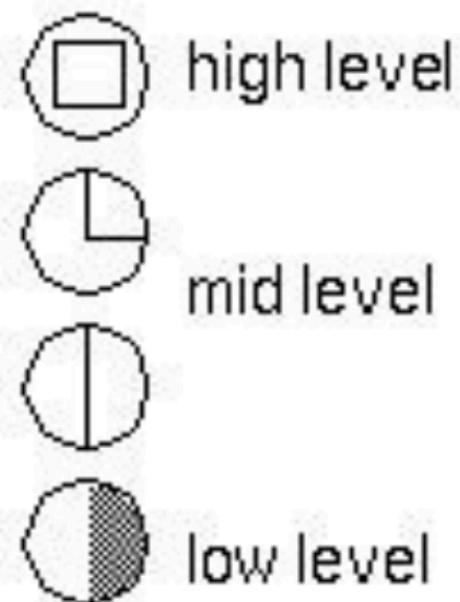
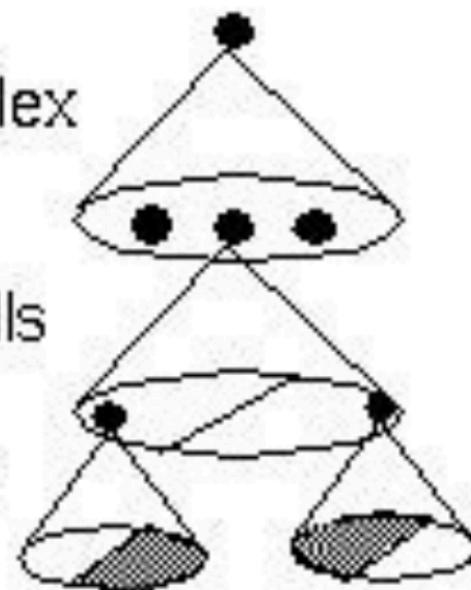


featural hierarchy

hyper-complex
cells

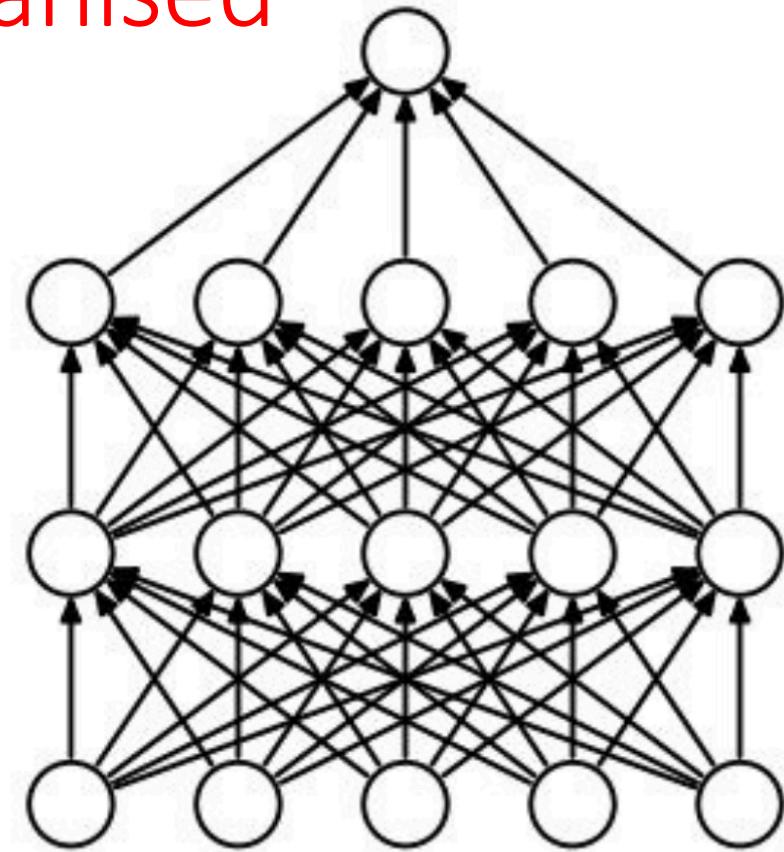
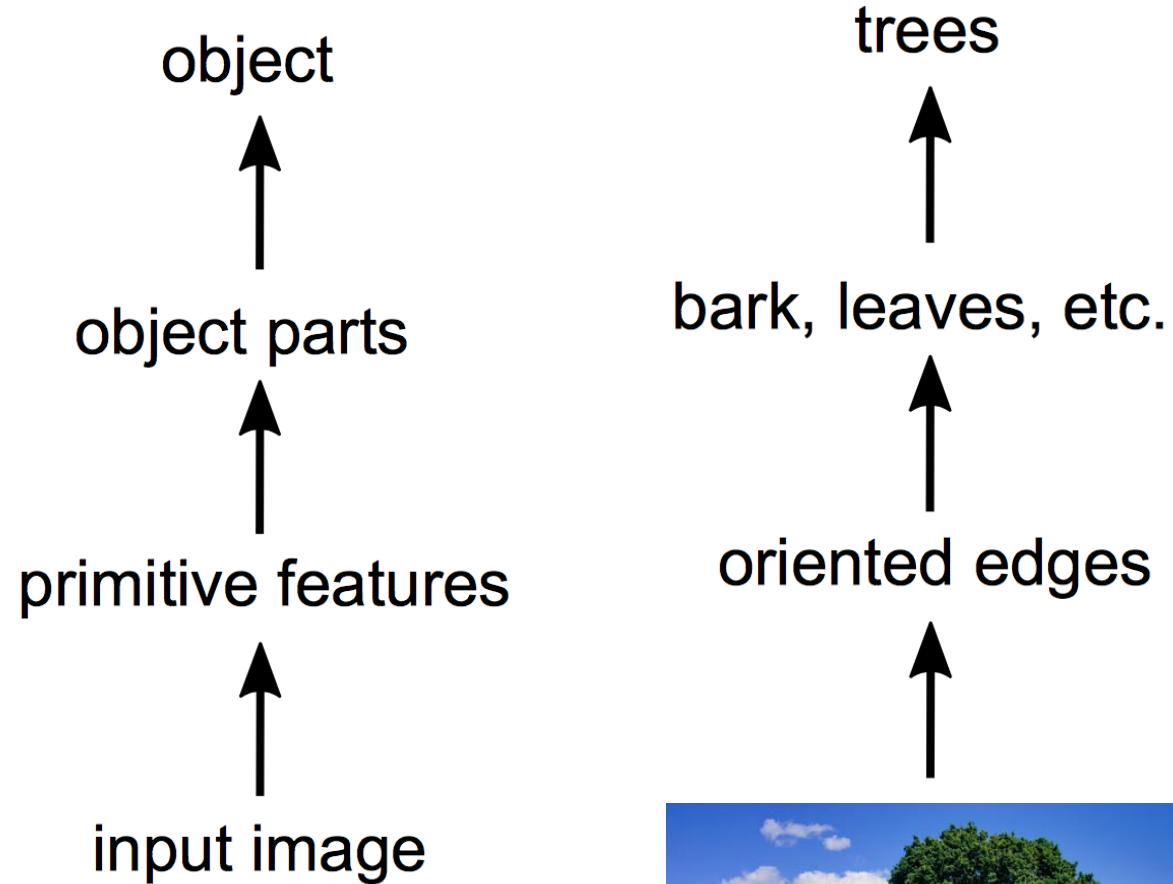
complex cells

simple cells



Why use hierarchical multi-layered models?

Biological vision is hierachically organised



What's wrong with standard neural networks?

Hard to Train

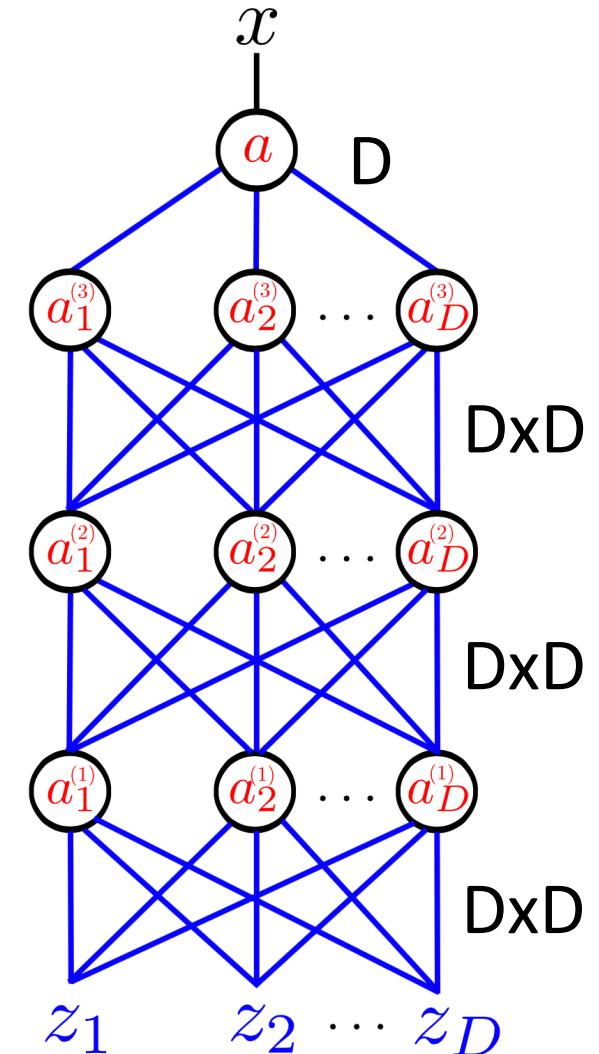
How many parameters does this network have?

Number of Parameters = $3 \times (D \times D) + D$

For a small $D = 32 \times 32 = 1024$ MNIST image:

Number of Parameters = $3 \times (1024 \times 1024) + 1024$

$\sim 3 \times 10^6$



Architecture of LeNet-5, Convolution Neural Network

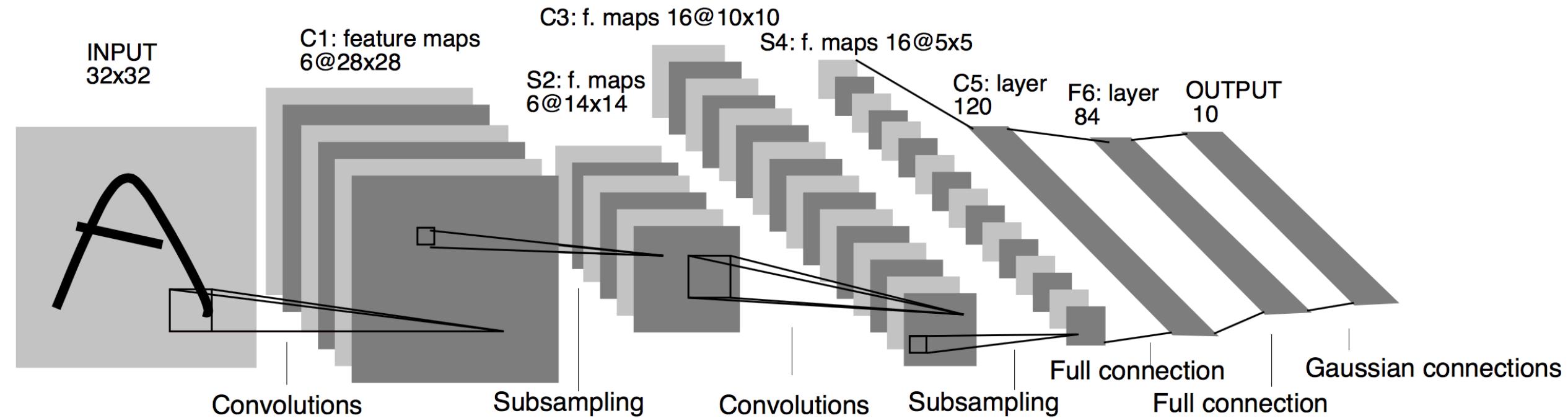
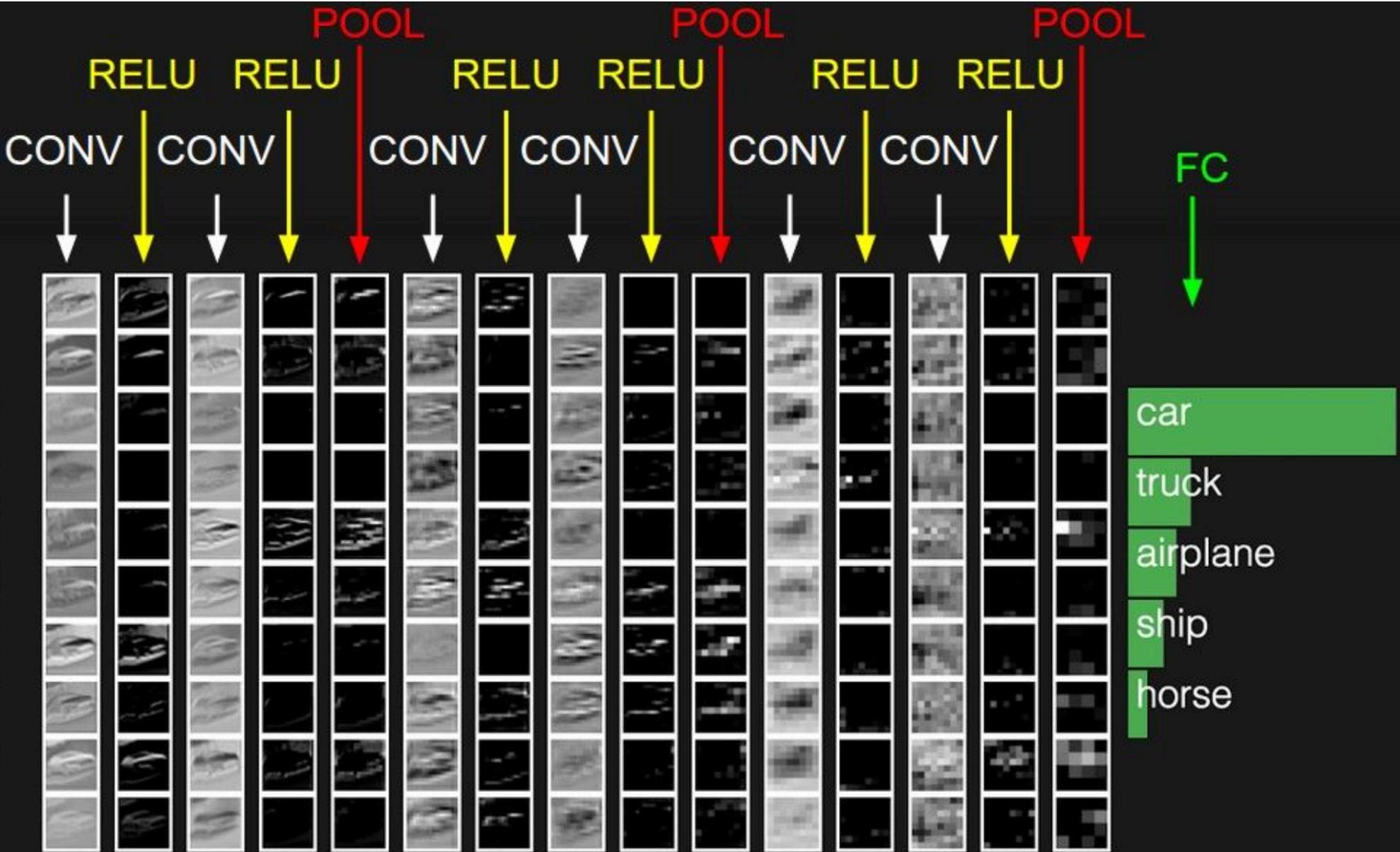


Fig. 2. Architecture of LeNet-5, a Convolutional Neural Network, here for digits recognition. Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

Proc. Of the IEEE, November 1998, "Gradient-Based Learning Applied to Document Recognition"



Review: What is convolution?

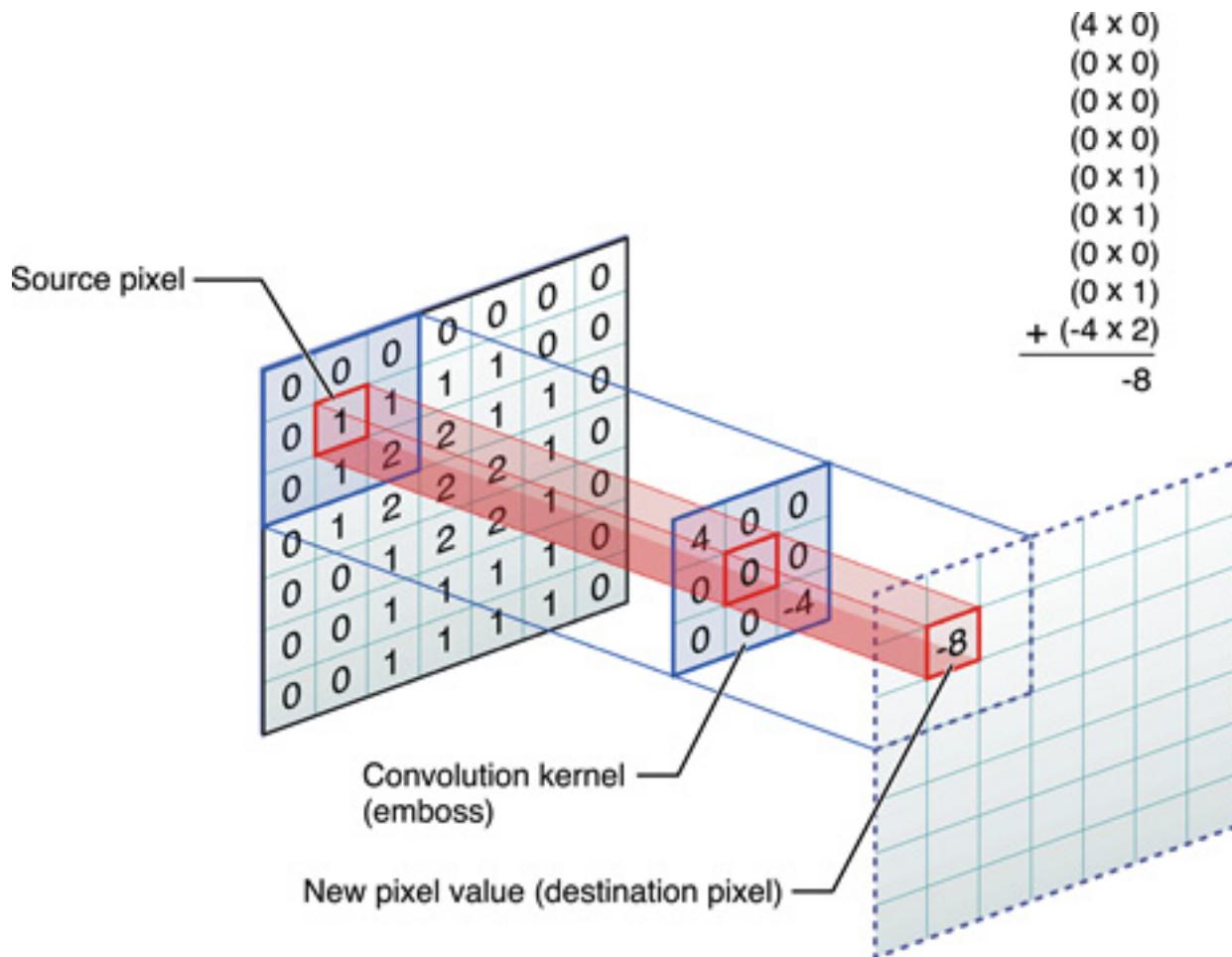
- Convolution is an important operation from signal processing
- A convolution is an integral that expresses the amount of overlap of one function as it is shifted over another function .

$$f * g = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau = \int_{-\infty}^{\infty} g(\tau)f(t-\tau)d\tau$$

- 2 Dimensional Discrete Function (Image)

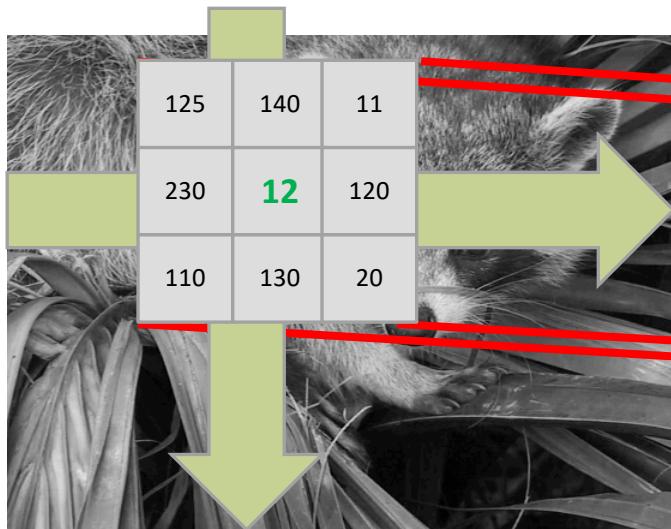
$$f[x,y] * g[x,y] = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} f[n_1,n_2] \cdot g[x-n_1, y-n_2]$$

2-Dimensional Convolution



Example: 2-Dimensional Convolution

A convolution is an integral (**discrete signals :Matrix Dot Product**) that expresses the amount of overlap of one function as it is shifted over another function



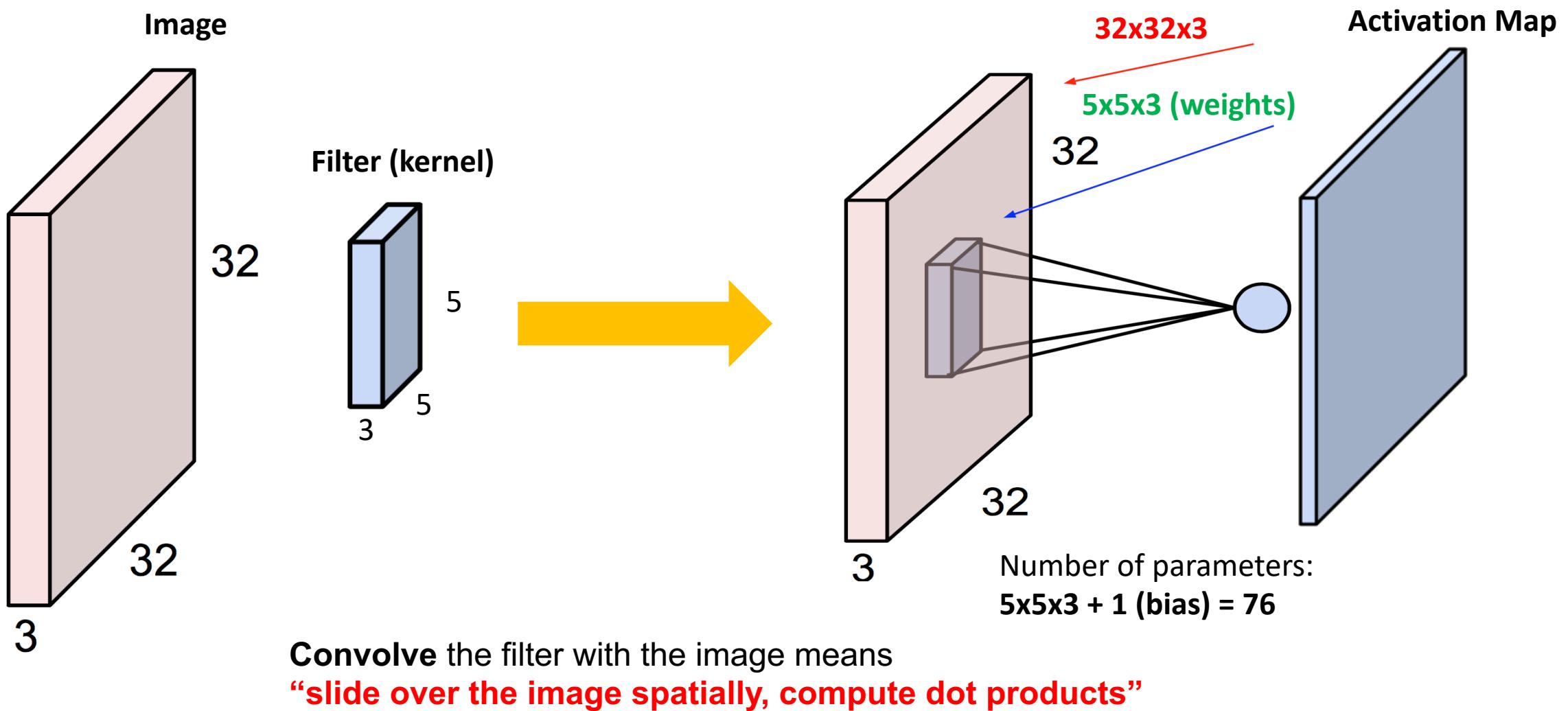
-1	-2	-1
0	0	0
1	2	1



-1	0	1
-2	0	2
-1	0	1



Convolution Layer



Input Volume (+pad 1) (7x7x3)

 $x[:, :, 0]$

0	0	0	0	0	0	0	0
0	2	0	1	1	0	0	0
0	2	1	2	2	1	0	0
0	2	0	0	1	2	0	0
0	1	1	2	2	1	0	0
0	0	1	0	2	2	0	0
0	0	0	0	0	0	0	0
$x[:, :, 1]$	0	0	0	0	0	0	0
0	1	2	1	1	2	0	0
0	1	2	1	2	0	0	0
0	2	0	1	2	2	0	0
0	2	2	2	1	0	0	0
0	0	1	0	2	2	0	0
0	0	0	0	0	0	0	0
$x[:, :, 2]$	0	0	0	0	0	0	0
0	0	0	2	0	0	0	0
0	1	1	1	0	2	0	0
0	2	1	1	2	1	0	0
0	0	2	1	1	0	0	0
0	0	0	2	1	2	0	0
0	0	0	0	0	0	0	0

Filter W0 (3x3x3)

 $w0[:, :, 0]$

1	1	1
1	1	1
0	-1	0

 $w0[:, :, 1]$

1	0	1
-1	1	0
-1	1	1

 $w0[:, :, 2]$

1	0	1
-1	1	0
-1	1	1

 $b0[:, :, 0]$

1

Filter W1 (3x3x3)

 $w1[:, :, 0]$

1	1	0
0	0	-1
0	0	1

 $w1[:, :, 1]$

-1	0	-1
-1	1	-1
-1	0	1

 $w1[:, :, 2]$

1	1	-1
-1	1	1
1	1	0

 $b1[:, :, 0]$

0

Output Volume

 $o[:, :, 0]$

1	1	0
0	0	-1
0	0	1

 $o[:, :, 1]$

-1	0	-1
-1	1	-1
-1	0	1

 $o[:, :, 2]$

1	1	-1
-1	1	1
1	1	0

 $b1[:, :, 0]$

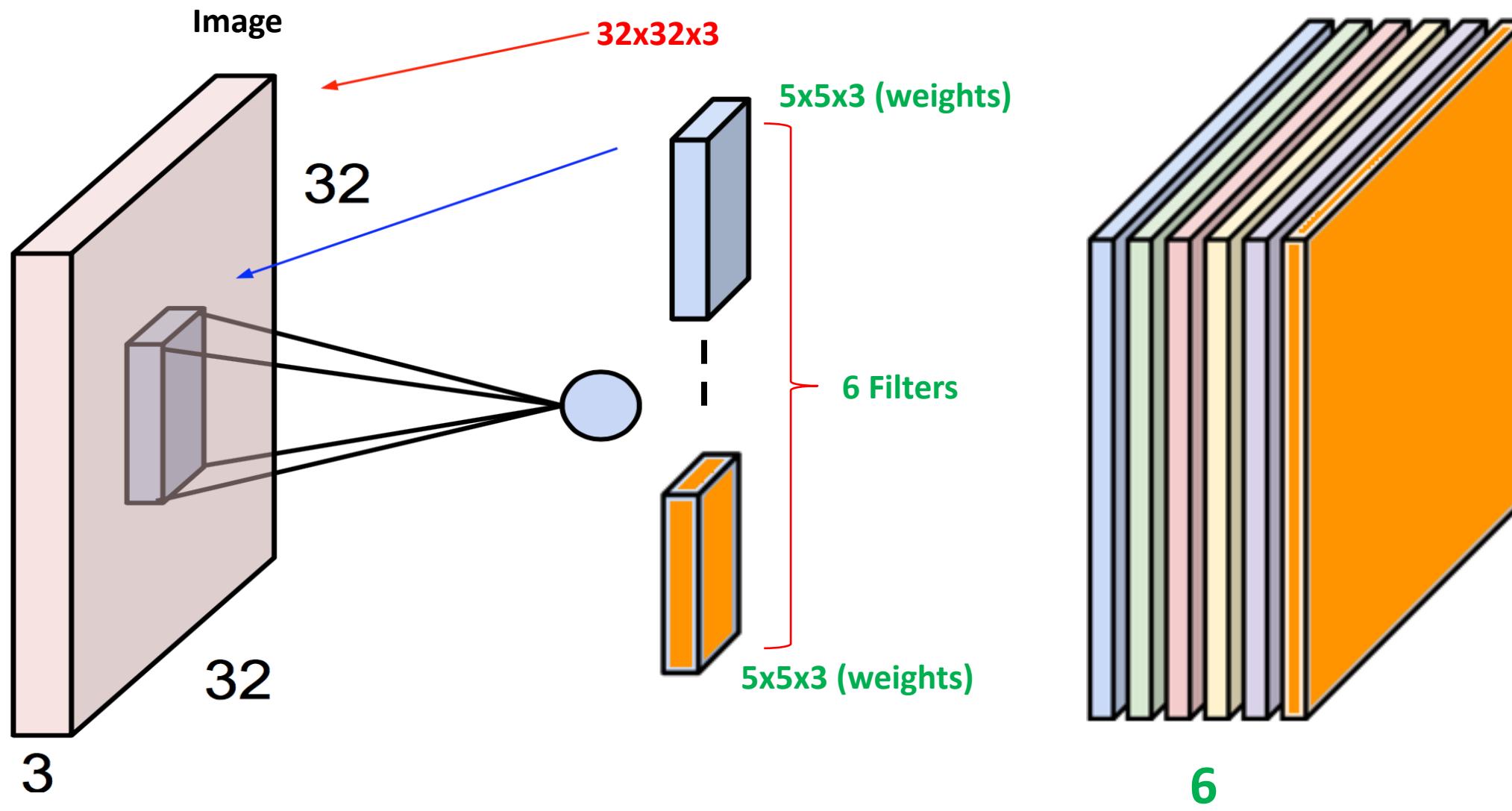
0

$$(2 \times 1) + (1 \times 1) + 0 + (1 \times 1) + (2 \times 1) + 0 + (2 \times 1) + (1 \times 1) + 0 = 9$$

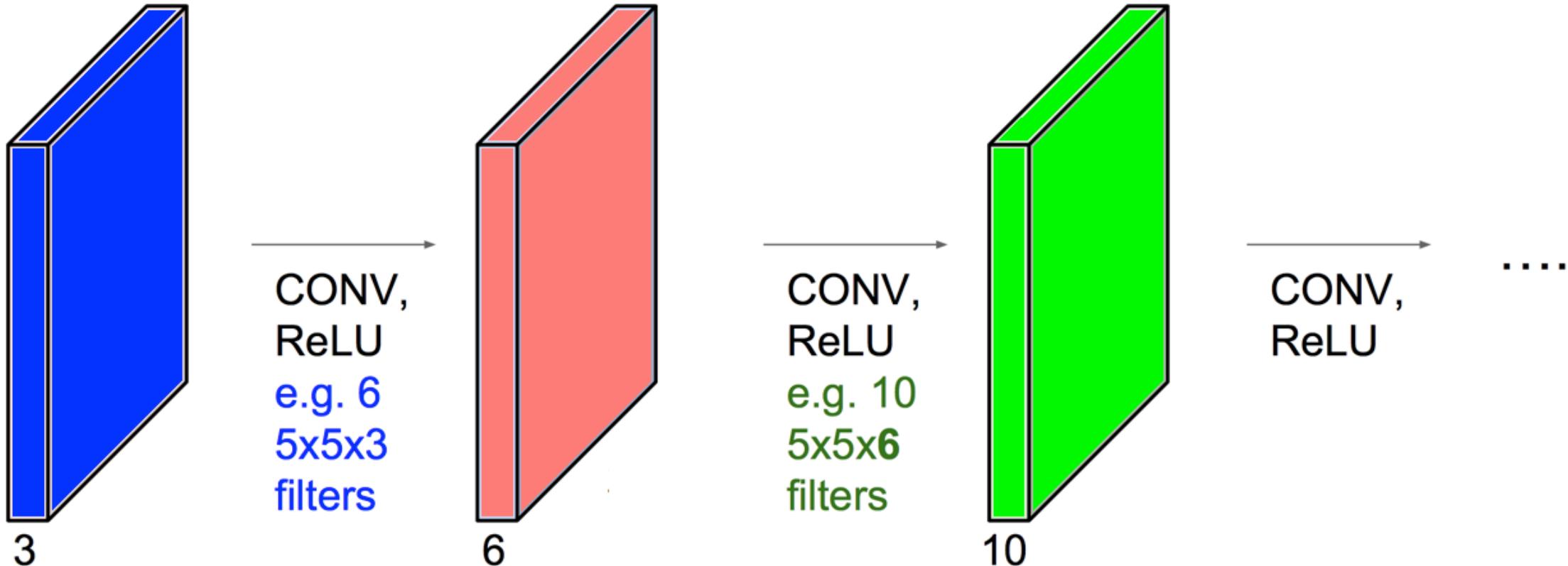
$$0 + 0 + 0 + (2 \times 1) + 0 + (1 \times -1) + 0 + 0 = 1$$

$$0 + 0 + 0 + (2 \times -1) + (1 \times 1) + 0 + (1 \times -1) + 0 + 0 = -2$$

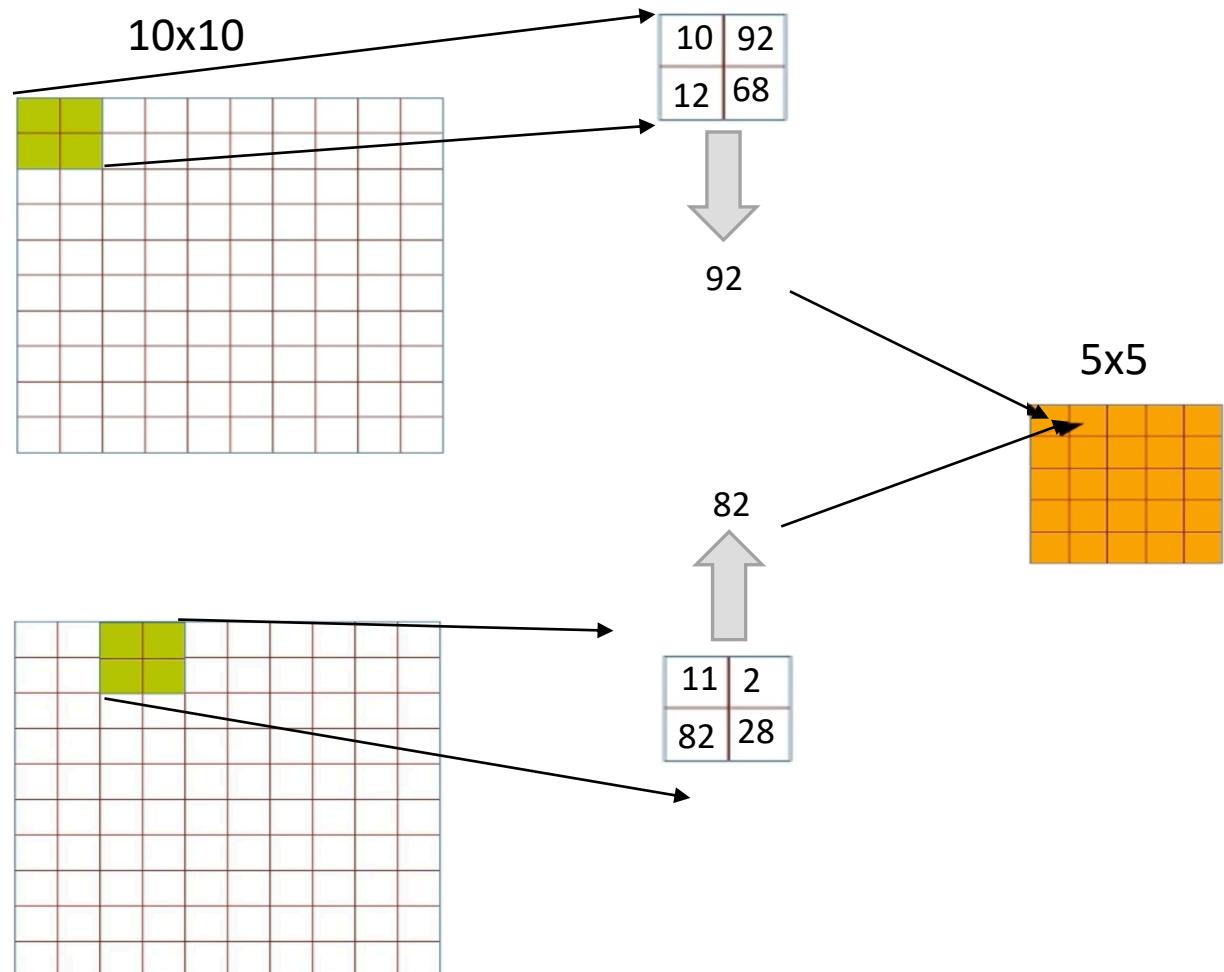
$$1 + 1 + (-2) + 9 = 9$$



Convolution Network



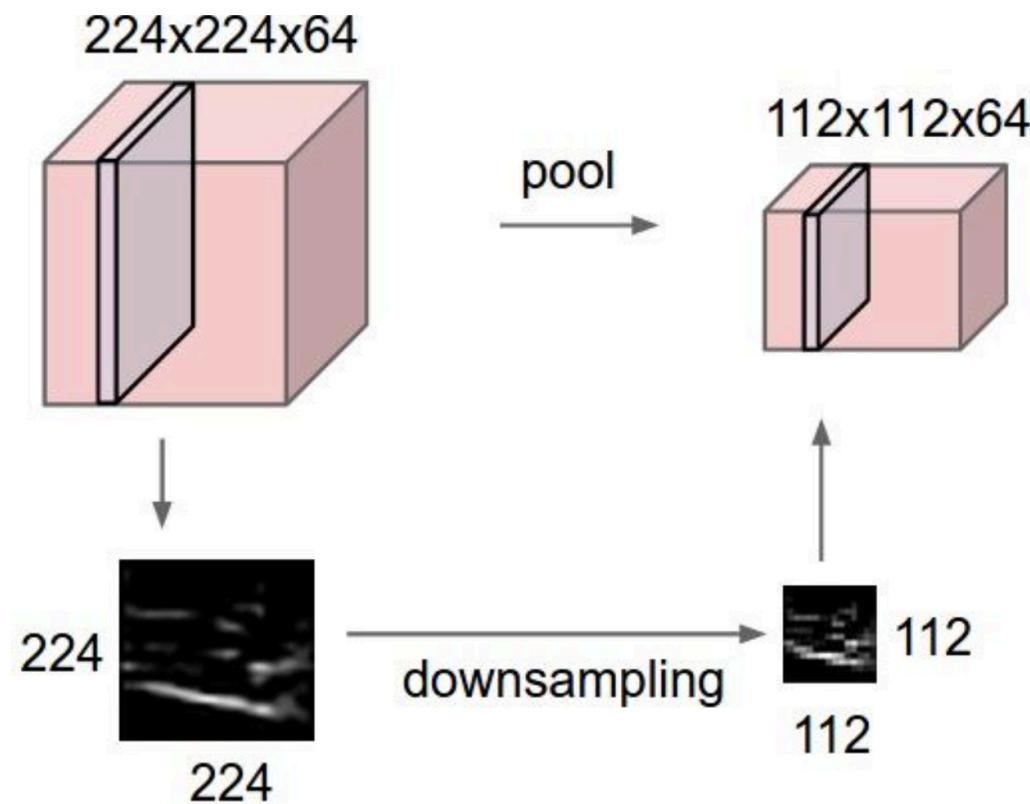
Downsampling = Max Pooling

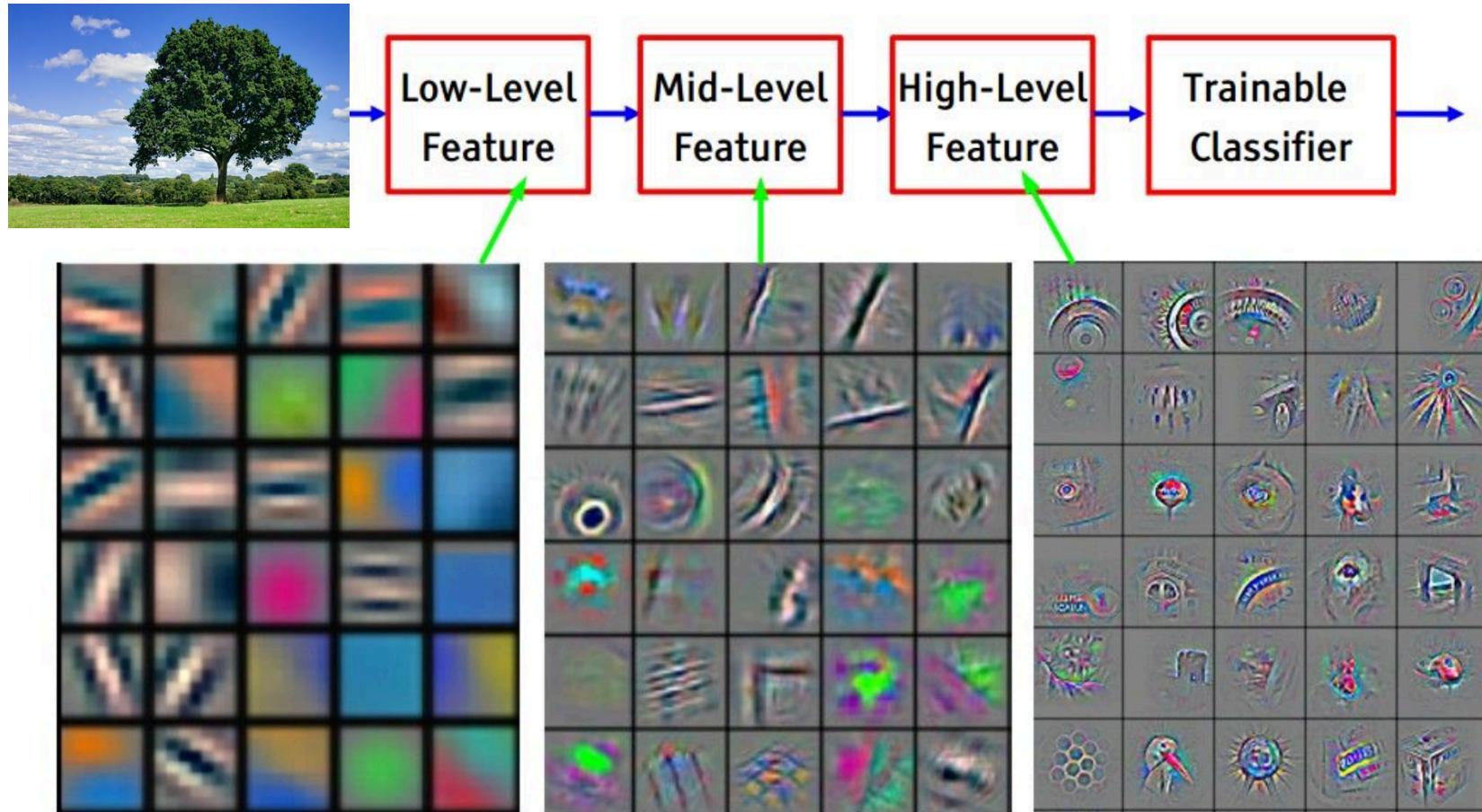


“Max Pooling” Layers to extract the “best” local feature

Pooling Layer

- Makes the representations smaller and more manageable
- Operates over each activation map independently





Feature visualization of convolution net trained on ImageNet from [Zeiler & Fergus 2013]