Final Exam (Preview)

> Read this page and fill in your name, pledge, and email ID now.
> **Do not open past this page until you are told to begin.**

Name: _____

UVA Email ID: _____

For this exam, you must **work alone**. You are not permitted to obtain help from people other than asking clarifying questions of the course staff. You are not permitted to provide help to others taking the exam. **You may not use any resources other than your brain and body, the one page of notes you prepared, and a simple writing implement like a pen or pencil.**

Sign below to indicate that you understand these expectations and can be trusted to behave honorably:

Signed: _____

> Our goal is to design exams that do not incentivize the intellectual dishonesty that is typically incentivized by school assignments and that you are all experts at, as demonstrated by your ability to achieve the level of success needed in high school to be admitted to the University. Hence, please keep in mind that the exam will be graded in a way that does not reward intentionally obfuscated or deceptive answers — if you do not know how to solve a problem, or get stuck at a step in a proof, it is much better to state that clearly and explain what you know that might be relevant or useful towards solving the problem, than to fabricate an answer that you know is wrong.
>
> Although fairly generous partial credit will be awarded for answers that state that you do not know how to solve the asked problem and either solve an easier one or show something you can do that is related to the given problem, answers that we believe are deliberately deceptive will receive negative scores (worse than the 0 a blank answer would receive for any question).

The exam has 9 questions, each of which awards a good answer with 10 points (you can also get up to **10 points for filling in the three blanks** on this cover page well enough so we can read your name and id), so an exam with all good answers would be worth 100 points. Answers that are considerably better than "good" can be worth additional points.

For each question, we either identify the length a good answer should have, or else provide ample space to hold a full-credit answer. If you need more space, you can use the backs of pages, but include clear markings and arrows to indicate the answer that should be graded. We will assume anything not inside an answer box or clearly marked from one, is your scratch work that should not be considered in scoring your answers. Questions are organized around topics, not by difficulty. Within a question, sub-parts are usually, but not necessarily, ordered by difficulty.

Useful Definitions

Recall these definitions from class. We provide them here, and you can (carefully) rip this page out of the exam if it is helpful to you. These are all definitions you have seen before, and nothing should be surprising in them. We use $\mathbb{R}_+$ to denote the non-negative reals, and $\mathbb{R}^+$ to denote the positive reals (so $\mathbb{R}_+ = \mathbb{R}^+ \cup \{0\}$). We use $\mathcal{M}(w)$ to denote the Turing Machine described by the bitstring w.

Definition 1 (O) A function $f : \mathbb{N} \to \mathbb{R}_+$ is in the set $O(g(n))$, defined for any function $g : \mathbb{N} \to \mathbb{R}_+$ iff there exist two constants $c \in \mathbb{R}^+, n_0 \in \mathbb{N}$ such that: $\forall n > n_0.f(n) \leq cg(n)$.

Definition 2 (Ω) A function $f : \mathbb{N} \to \mathbb{R}_+$ is in the set $\Omega(g(n))$, defined for any function $g : \mathbb{N} \to \mathbb{R}_+$ iff there exist two constants $c \in \mathbb{R}^+, n_0 \in \mathbb{N}$ such that: $\forall n > n_0.f(n) \geq cg(n)$.

Definition 3 (Θ) A function $f : \mathbb{N} \to \mathbb{R}_+$ is in the set $\Omega(g(n))$, defined for any function $g : \mathbb{N} \to \mathbb{R}_+$ iff $f(n) \in O(g(n))$ and $f(n) \in \Omega(g(n))$

Definition 4 (Accepts) The language, A_{TM}, is defined as:

$$A_{TM} = \{(w, x) \mid \mathcal{M}(w) \text{ accepts } x\}$$

The corresponding function, *Accepts*, is defined as:

$$Accepts(w, x) = \mathbf{1} \text{ if } \mathcal{M}(w) \text{ accepts } x, \text{ otherwise } \mathbf{0}$$

Definition 5 (Halts) The language, $Halts_{TM}$, is defined as:

$$Halts_{TM} = \{w \mid \mathcal{M}(w) \text{ terminates when run on a blank input }\}$$

True, False, or Unknown

1. For each of the following, circle one of the choices to indicate whether the statement is known to be *True,* is known to be *False,* or *Unknown* if its validity depends on something that is either currently unknown or not specified in the question.

For full credit, your answer must include a short but clear and convincing justification to support your answer. When your answer is *Unknown,* your answer should make it clear what unknown the validity of the statement depends on (for example, that it is equivalent to a statement whose truth is currently unknown to anyone).

. . .

2. Automata and Regular Expressions

(a) Draw a non-deterministic finite state machine (NFA) for . . .

(b) Give a regular expression for the language: . . .

Always, Sometimes, Never

3. For a function $f : \{0, 1\}^* \rightarrow \{0, 1\}$ that can be implemented by . . .

Proving Uncomputability

4. In this question, your goal is to show that the function $FLIPS_0$ defined below is uncomputable.

. . .

Complexity Classes

5. Suppose that we have problems A and C where A reduces to C in polynomial time. . . .

Modeling Computing

6. Computing Models

(a) Explain the important difference between . . .

Asymptotics

7. Let $f(n) = 2^n n^2$ and . . .

Induction

8. . . .

Countable, Uncountable, Unknown

9. For each set described below, indicate whether its cardinality is *Countable, Uncountable,* or *Unknown* (not determined by the question if it is countable or uncountable). Circle one option and give a proof of your answer.

. . .

(a) . . .

(b) . . .

A function f is *monotonically increasing* if for two any inputs x, y, if $x \geq y$ then $f(x) \geq f(y)$. Note that . . .

(c) . . .

(d) . . .

(e) ($\star$ **Bonus!**) . . .

Oral Exam Request

If you are worried that your performance on this exam does not reflect well how much you have actually learned in the class, you may check fill in the minimum grade expectation box below to request an independently scheduled oral exam:

If my course grade would be lower than _____, I would like to schedule an oral exam.

We will decide on (preliminary) final grades before looking at this. If you fill in a grade here, and your final grade would be lower than the grade you entered here, we will contact you with an opportunity to schedule an oral exam (which will be either in person or on zoom). At the exam, you will be expected to demonstrate your knowledge and understanding by explaining a few key definitions from the class and answering questions (including doing a proof). It is your choice to do the oral exam, and your final grade may go either up or down as a result of the oral exam depending on your performance.

Optional Feedback

(Optional) This question is optional and will not affect your grade.

Anything else you want us to know? Feel free to explain why you think your performance on this exam might not fairly reflect your understanding of the course materials, or any other comments you want on the exam, the course, or just draw a picture or your favorite complexity class.)