Problem Set 1

Due: **9:59pm, Monday, 30 January**

This assignment is designed to develop your skills with formal definitions and to provide some practice with inductive reasoning. This problem set has two parts: one should be completed by writing your answers in the ps1.tex LaTeX template, and the other in the ps1.ipynb Jupyter notebook. See https://uvatoc.github.io/ps1/ for directions for the Jupyter part. You will submit your solutions to both parts in GradeScope separately — the first as a PDF file with your answers to the questions in this template, and the second by submitting your Jupyter notebook.

Collaboration Policy: You may discuss the problems with anyone you want. You are permitted to use any resources you find for this assignment **other than solutions from previous cs3102/cs3120 courses**. You should write up your own solutions and understand everything in them, and submit only your own work. You should note in the *Collaborators and Resources* box below the people you collaborated with and any external resources you used (you do not need to list resources you used for help with LaTeX or Jupyter/Python).

Collaborators and Resources: TODO: replace this with your collaborators and resources (if you did not have any, replace this with *None*)

To do the LaTeX part of this assignment:

1. Visit https://www.overleaf.com and register for an Overleaf account (if you don't already have one). UVA has a site license to Overleaf, so if you register with your @virginia.edu email address you will have full access to all the Overleaf features for free.
2. After opening this read-only Overleaf project, while you are logged in, select the "Menu" button at the top-left of Overleaf, and select "Copy Project". You will have an opportunity to rename the project, and then Overleaf will create a new copy of the project which you can edit.
3. Open your copy of the project and in the left side of the browser, you should see a file directory containing ps1.tex. Click on ps1.tex to see the LaTeX source for this file, and enter your solutions in the marked places. (You will also see the uvatoc.sty file, a "style" file that defines useful macros for cs3120. You are welcome to look at this file but should not need to modify it.)
4. The first thing you should do in ps1.tex is set up your name as the author of the submission by replacing the line, `\submitter{TODO: your name}`, with your name and UVA id, e.g., `\submitter{Grace Hopper (gmh1a)}`.
5. Write insightful and clear answers to all of the questions in the marked spaces provided.
6. Before submitting your ps1.pdf file, also remember to:
   - List your collaborators and resources, replacing the TODO in `\collaborators{TODO: replace ...}` with your collaborators and resources. (Remember to update this before submitting if you work with more people.)
   - Replace the second line in ps1.tex, `\usepackage{uvatoc}` with `\usepackage[response]{uvatoc}` so the directions do not appear in your final PDF.

Problem 1 *Induction Practice:* Prove that for any natural number $n \geq 2$, $n! < n^n$.

Note: For this problem (and any other problems where we don't explicitly state that you should use a particular definition), you can use the intuitive informal definition of natural numbers, and assume all of the familiar operations are defined and behave as expected.

Answer:

Problem 2 *Higher Induction Practice:* Prove that any binary tree of height h has at most 2^{h-1} leaves.

Note: We haven't defined a *binary tree* (and the book doesn't). An adequate answer to this question will use the informal understanding of a binary tree which we expect you have entering this class, but an excellent answer will include a definition of a binary tree and connect your proof to that definition.

Answer:

Problem 3 *Addition is Commutative:* For this problem, we will use the successor definition of Natural Numbers (from Class 2) as follows.

Definition 1 (Natural Numbers) We define the *Natural Numbers* as:

1. **0** is a Natural Number.
2. If n is a Natural Number, $\mathbf{S}(n)$ is a Natural Number.

We will use this definition of addition:

Definition 2 (Sum) The *sum* of two Natural Numbers a and b (denoted as $a + b$) is defined as:

1. If a is **0**, then $a + b$ is b.
2. Otherwise, a is $\mathbf{S}(p)$ for some Natural Number p, and $a + b$ is $\mathbf{S}(p + b)$.

Prove that addition as defined above is *commutative*: for all Natural Numbers a and b, $a + b$ is equal to $b + a$. (we defined equality for our Natural Number representation in Class 2).

Answer:

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Problem 4 *Busjections*

The UTS buses have LED displays that give information about route, service, etc. These displays are 97 pixels wide and 17 pixels tall. Each pixel could have one of two possible colors: orange (on) or black (off). Answer these questions about the length of binary strings necessary to represent the bus display. Support your answer by describing a *bijection* between bus displays and sets of binary strings.

1. We will store the contents of display in binary. Assuming all configurations are represented with the same number of bits, what is the minimum number of bits required to do so? Justify your answer by demonstrating a bijection between all strings of the length you indicate, and all possible configurations of the display.
2. Suppose, to limit the amount of light pollution cause by buses, we require that no more than half of the pixels could be on at a time. Assuming all configurations are represented with the same number of bits, what is the minimum number of bits required to represent the contents of the display given this restriction? **Hint**: begin with a bijection between configurations of the display which are majority orange vs. those that are majority black, noting that there are an odd number of total pixels.

Answer:

Problem 5 *Countable Programs:* Prove that the set of all Python programs that you can execute on your laptop is *countable*.

Answer:

This is the end of the LaTeX problems for PS1. Remember to follow the last step in the directions on the first page to prepare your PDF for submission, and to also complete the problems in the Jupyter notebook, see https://uvatoc.github.io/ps1/.

