Week 1: Proofs and Pancakes

The problems are designed to develop your skills with set cardinalities and how they relate to functions, provide some practice with inductive reasoning, and get you acquainted with your cohortmates.

Collaboration: You should work on the problems yourself, before discussing with others, and with your cohorts are your cohort meeting. By the Assessed Cohort Meeting, you and all of your cohortmates, should be prepared to present and discuss solutions to all of the assigned problems. In addition to discussing with your cohortmates, you may discuss the problems with any other current CS3102 students you want, and use any resources you want except for any materials from previous offerings of this course or complete solutions that might be available on the web, which are not permitted. **Unlike with most other weeks, you are welcome to collaborate with others on your writeup for this week.**

Problem 1: Cohortroductions

Everyone in your cohort should introduce themselves, provide your pronouns, teach all the other cohort members how to pronounce your name correctly, and share something interesting about yourself with the group. This can be anything you want, but a few suggestions:

1. What did you do over the summer break?
2. What most excites you about the return to in-person classes?
3. What is the most interesting thing about your hometown?
4. Who is your spirit animal?
5. If you could change anything about UVA, what would you change?

At the assessed cohort meeting, this will be treated like other questions: one of you will be selected, and will be expected to introduce yourself and the other members of your cohort, including pronouncing their names correctly and recounting something interesting you learned about each other cohort member.

Problem 2: Cohort Namesake

Each cohort has been named after someone who is (at least loosely) connected to theoretical computer science. You are not expected to do an extensive research project here, but find out something about the people your cohort is named after. At your cohort preparation meeting, everyone should share what they learned, and the group should settle a few interesting things that you'll share with your Cohort Leader (TA) at the Assessed Cohort Meeting. As an additional step for this problem, you should also agree on one interesting fact, and the best URL you found, about your cohort namesake, and one of your cohortees should post a message in #week1 on discord to share this with the rest of the class.

Problem 3: Induction Practice

Use induction to show that $n! < n^n$ for all $n \in \mathbb{N}$ where $n > 2$

Problem 4: Higher Induction Practice

Prove, using induction, that any binary tree of height h has at most 2^{h-1} leaves.

Note: We haven't defined a *binary tree* (and the book doesn't). An adequate answer to this question will use the informal understanding of a binary tree which we expect you have entering this class, but an excellent answer ($\star$) will include a formal definition of a binary tree and connect your proof to that definition.

Second Note: problems marked with a ($\star$) in this course are challenge problems. They are more challenging that other problems, but will be worth bonus points if solved well.

Problem 5: Busjections

The UTS buses have LED displays that give information about route, service, etc. These displays are 97 pixels wide and 17 pixels tall. Each pixel could have 1 of 2 colors: orange (on) or black (off). Answer these questions about the length of binary strings necessary to represent the bus display. Support your answer by describing a bijection between bus displays and sets of binary strings.

1. We will store the contents of display in binary. Assuming all configurations are represented with the same number of bits, what is the minimum number of bits required to do so? Justify your answer by demonstrating a bijection between all strings of the length you indicate, and all possible configurations of the display.
2. Suppose, to limit the amount of light pollution cause by buses, we require that no more than half of the pixels could be on at a time. Assuming all configurations are represented with the same number of bits, what is the minimum number of bits required to represent the contents of the display, now? **Hint**: begin with a bijection between configurations of the display which are majority orange vs. those that are majority black, noting that there are an odd number of total pixels.

Problem 6: Countable Programs

Prove that the set of all Python programs that you can execute on your laptop is countable.

Problem 7: Writeup

This semester you must use LaTeXto produce your writeup pdf. This week, your writeup is intended to get your started with LaTeX. In future weeks, your writeup will be one of your other problems for the week (as selected by your TA). Follow the instructions in the writeup section of the weekly guide.

There is nothing to do for this problem during your cohorts meetings.

Definition 1 *A rational number is a fraction* $\frac{a}{b}$ *where* a *and* b *are integers.*

Show $\sqrt{2}$ is irrational.

Proof.

□