Week 1: Proofs and Pancakes

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Problem 7: Writeup

Definition 1 *A rational number is a fraction $\frac{a}{b}$ where a and b are integers.*

Show $\sqrt{2}$ is irrational.

Proof.

For a rational number $\frac{a}{b}$, without loss of generality we may suppose that a and b are integers which share no common factors, as otherwise we could remove any common factors (i.e. suppose $\frac{a}{b}$ is in simplest terms). To say $\sqrt{2}$ is irrational is equivalent to stating that 2 cannot be expressed in the form $(\frac{a}{b})^2$. Equivalently, this says that there are no integer values for a and b satisfying

$$a^2 = 2b^2 \tag{1}$$

We argue by *reductio ad absurdum* (proof by contradiction). Assume toward reaching a contradiction that Equation 1 holds for a and b being integers without any common factor between them. It must be that a^2 is even, since $2b^2$ is divisible by 2, therefore a is even. If a is even, then for some integer c

$$\begin{aligned} a &= 2c \\ a^2 &= (2c)^2 \\ 2b^2 &= 4c^2 \\ b^2 &= 2c^2 \end{aligned}$$

therefore, b is even. This implies that a and b are both even, and thus share a common factor of 2. This contradicts our hypothesis, therefore our hypothesis is false. □