

0-1 Sequences

COUNTING INVERSIONS is the topic of the problem ¹ in this note.

¹ Tung Kam Chuen. 0-1 sequences. 2016. URL <https://open.kattis.com/problems/sequences>

Problem

You are given a sequence, in the form of a string with characters '0', '1', and '?' only. Suppose there are k '?'s. Then there are 2^k ways to replace each '?' by a '0' or a '1', giving 2^k different 0-1 sequences (0-1 sequences are sequences with only zeroes and ones).

For each 0-1 sequence, define its number of inversions as the minimum number of adjacent swaps required to sort the sequence in non-decreasing order. In this problem, the sequence is sorted in non-decreasing order precisely when all the zeroes occur before all the ones. For example, the sequence 11010 has 5 inversions. We can sort it by the following moves: 11010 \rightarrow 11001 \rightarrow 10101 \rightarrow 01101 \rightarrow 01011 \rightarrow 00111.

Find the sum of the number of inversions of the 2^k sequences, modulo $10^9 + 7$.

Bibliography

Tung Kam Chuen. 0-1 sequences. 2016. URL <https://open.kattis.com/problems/sequences>.