3D Reconstruction of the Density Field: Using Redshift Information in Weak Lensing

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ABSTRACT: We present a new method for constructing three-dimensional mass maps from gravitational lensing shear data. We solve the lensing inversion problem using truncation of singular values (within the context of generalized least squares estimation) without a priori assumptions about the statistical nature of the signal. Our method yields near-optimal angular resolution of the lensing reconstruction and allows cluster sized halos to be de-blended robustly. It allows for mass reconstructions which are 2-3 orders-of-magnitude faster than the Wiener filter approach, which will become increasingly important for future large-scale lensing surveys, such as LSST and DES. Using the SVD framework, we discuss the limitations in attainable redshift resolution for nonparametric tomographic mass reconstructions.

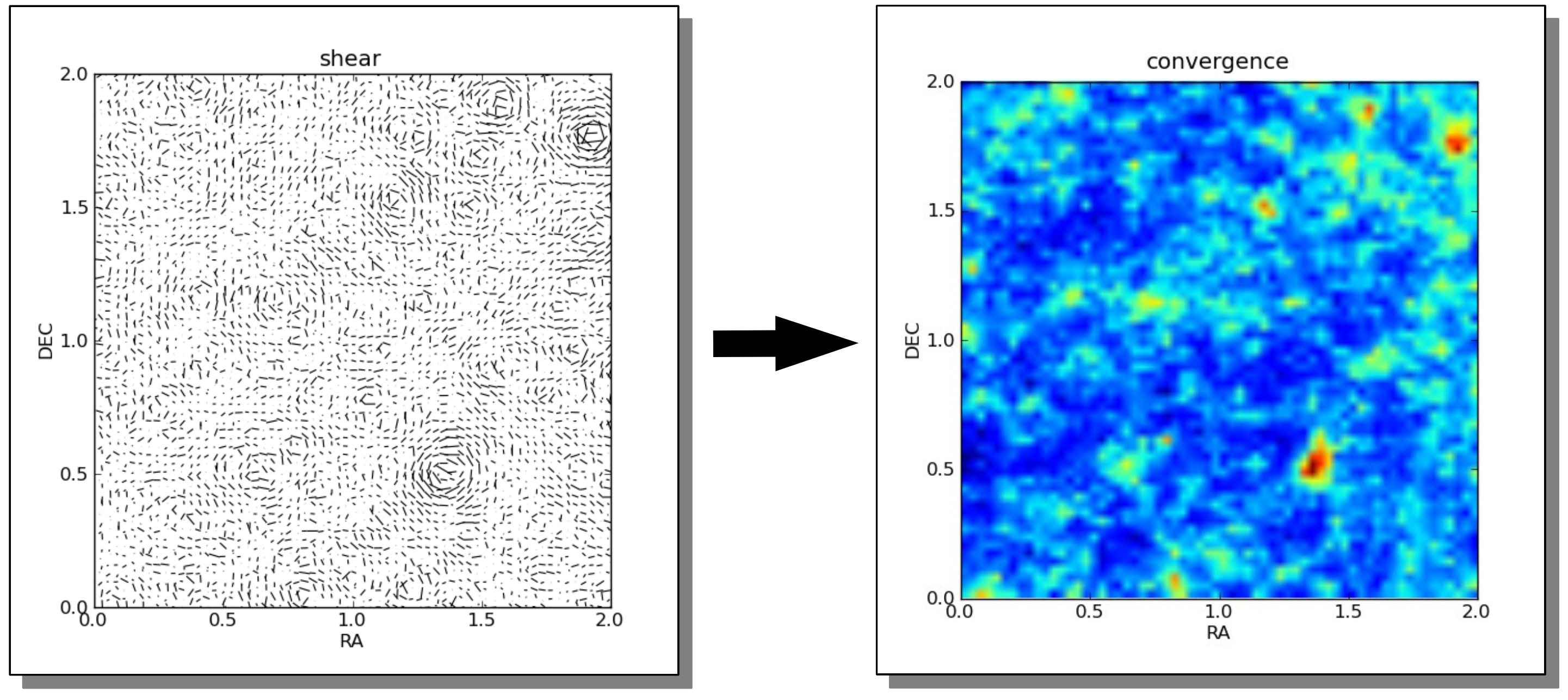
KEY QUESTIONS:

To what extent can a *linear* and *non-parametric* weak lensing analysis recover line-of-sight information about structure?

What level of photometric redshift accuracy is needed to optimally recover 3D structure?

Can a reconstruction be performed quickly enough to be useful in large datasets like LSST & DES?

Basic (2D) Weak Lensing Approach

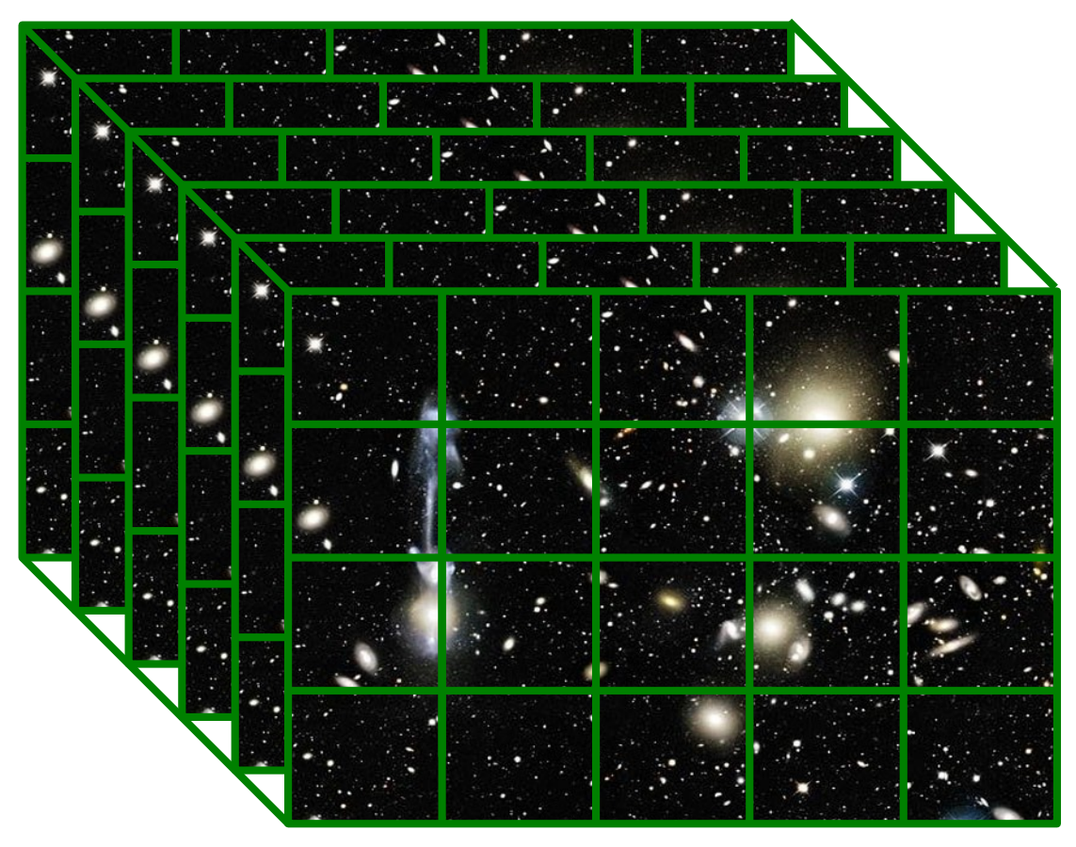


Shear is measured from the ellipticity of distant galaxies

Convergence is related to the line-of-sight integral of mass

Extension to three dimensions

We divide source galaxies into planes in redshift.



Use 2D approach in each redshift plane, and invert the line-of-sight integrals to obtain the 3D distribution of gravitational mass.

This can be expressed as a linear inversion:

$$\gamma = M_{\gamma\delta}\boldsymbol{\delta} + \boldsymbol{n_\gamma}$$

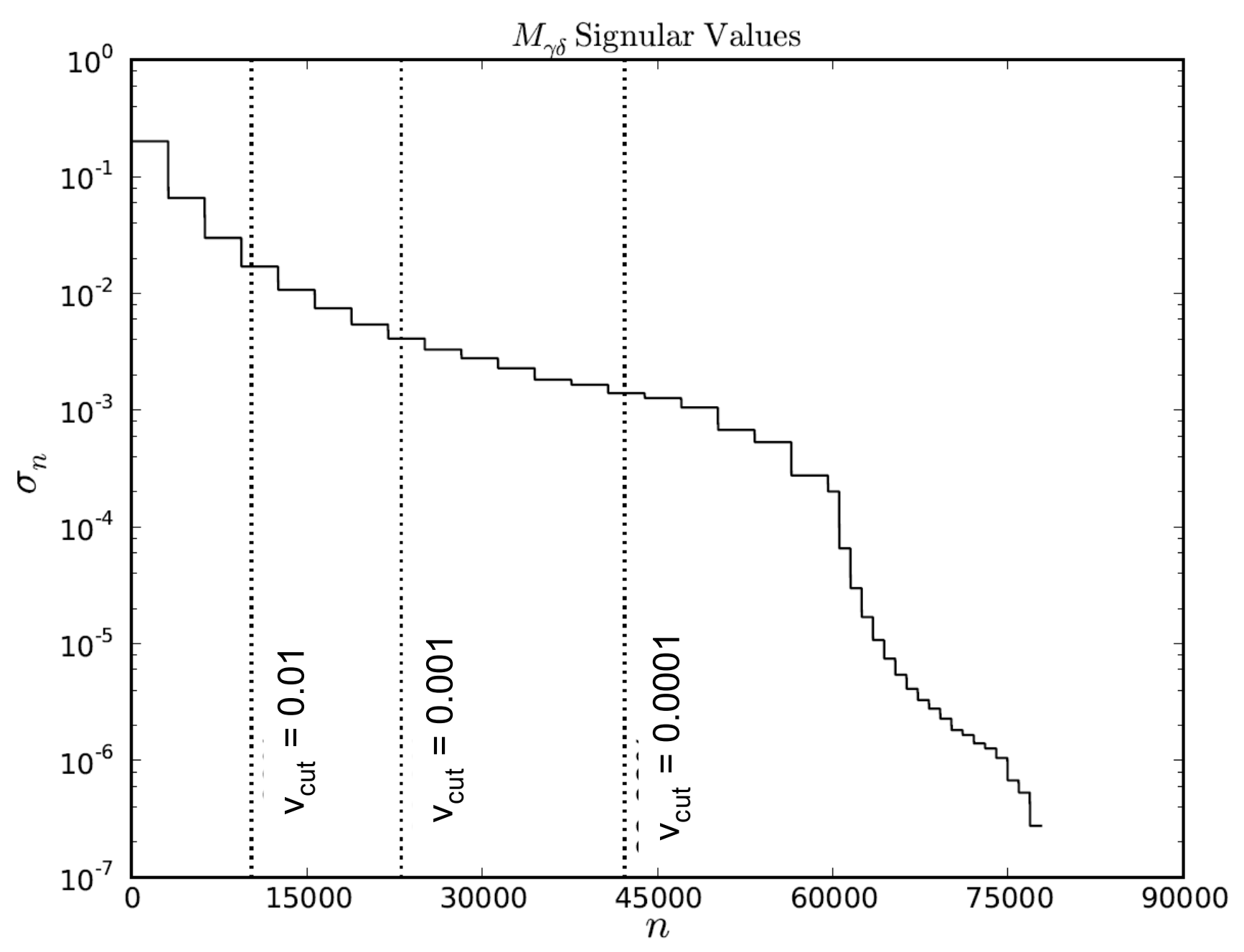
shape noise
3D density distribution
transformation matrix
measured shear

We express the transformation matrix as a Singular Value Decomposition:

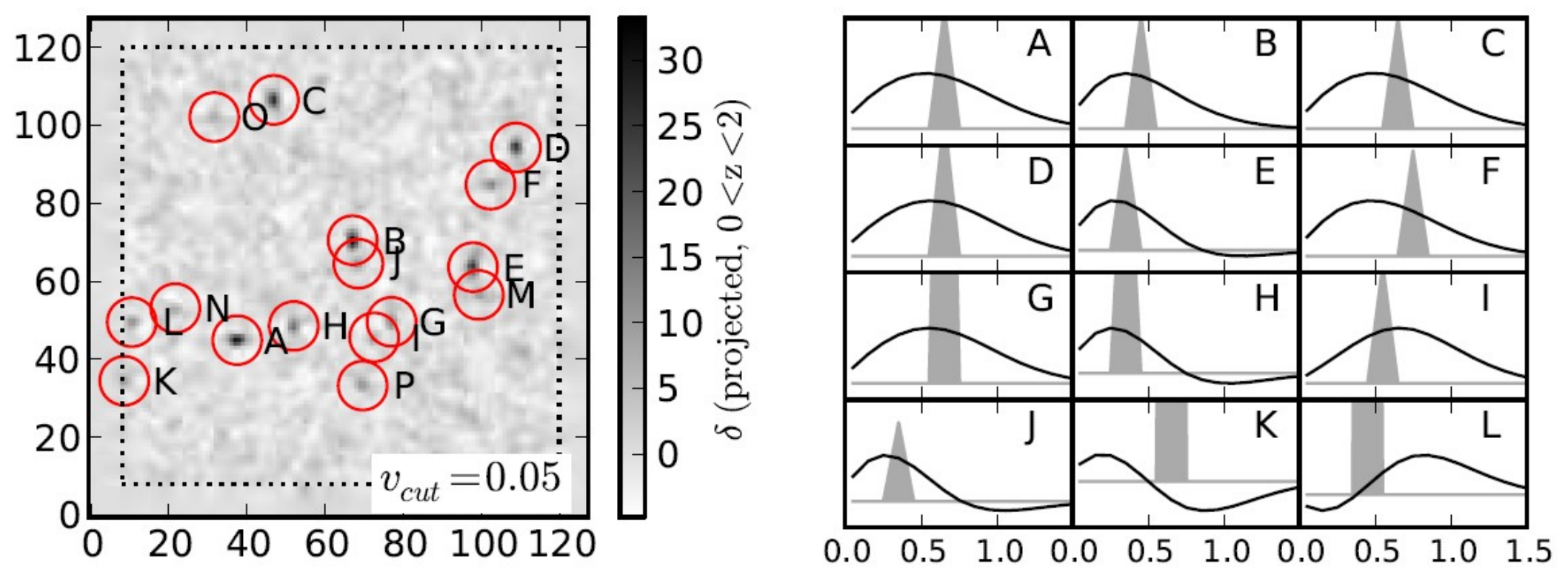
$$M_{\gamma\delta} = U\Sigma V^\dagger$$

Small singular values lead to large errors in inversion

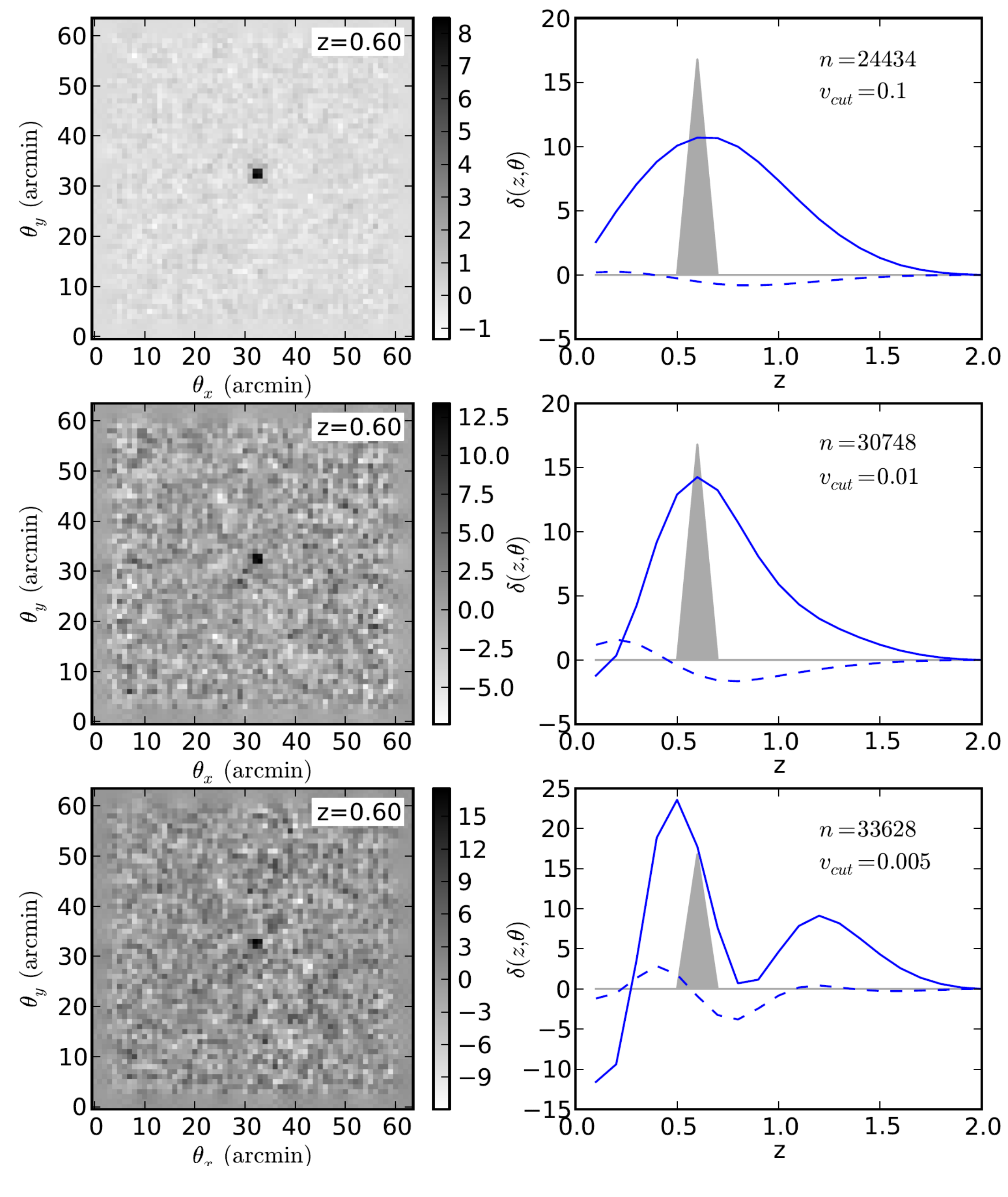
Singular Values drop off steeply: the many small values lead to large errors in the reconstruction



Solution: remove the singular vectors associated with small singular values. v_{cut} refers to the fraction of the of variance removed from the reconstruction (see singular value plot, left)



Left: projected density. *Right:* line-of-sight reconstruction at each halo. Gray region is input, black line is the reconstruction. Halos are robustly de-blended in angular space, but redshift resolution is lacking. Can this be addressed by varying the singular value cutoff?

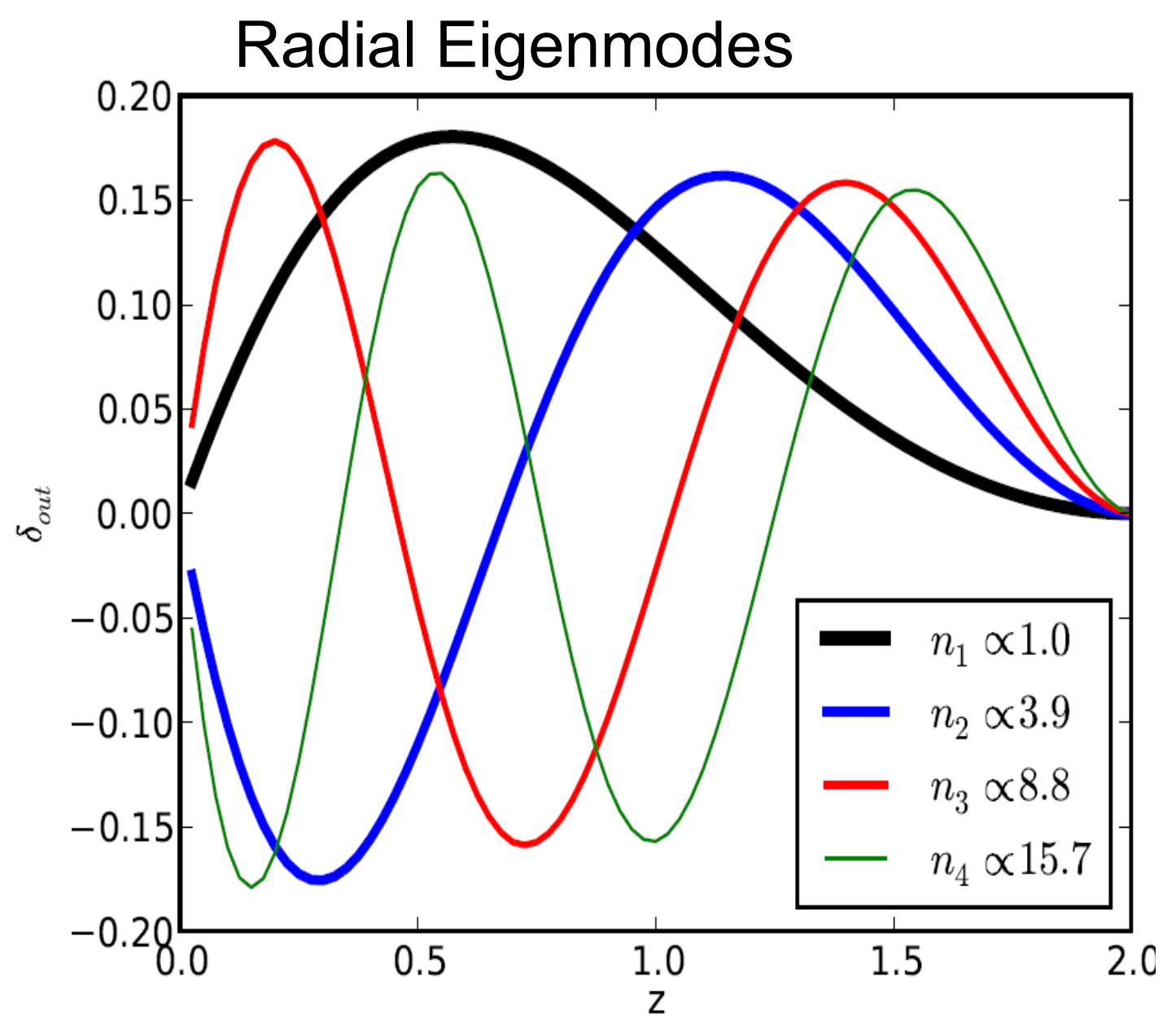


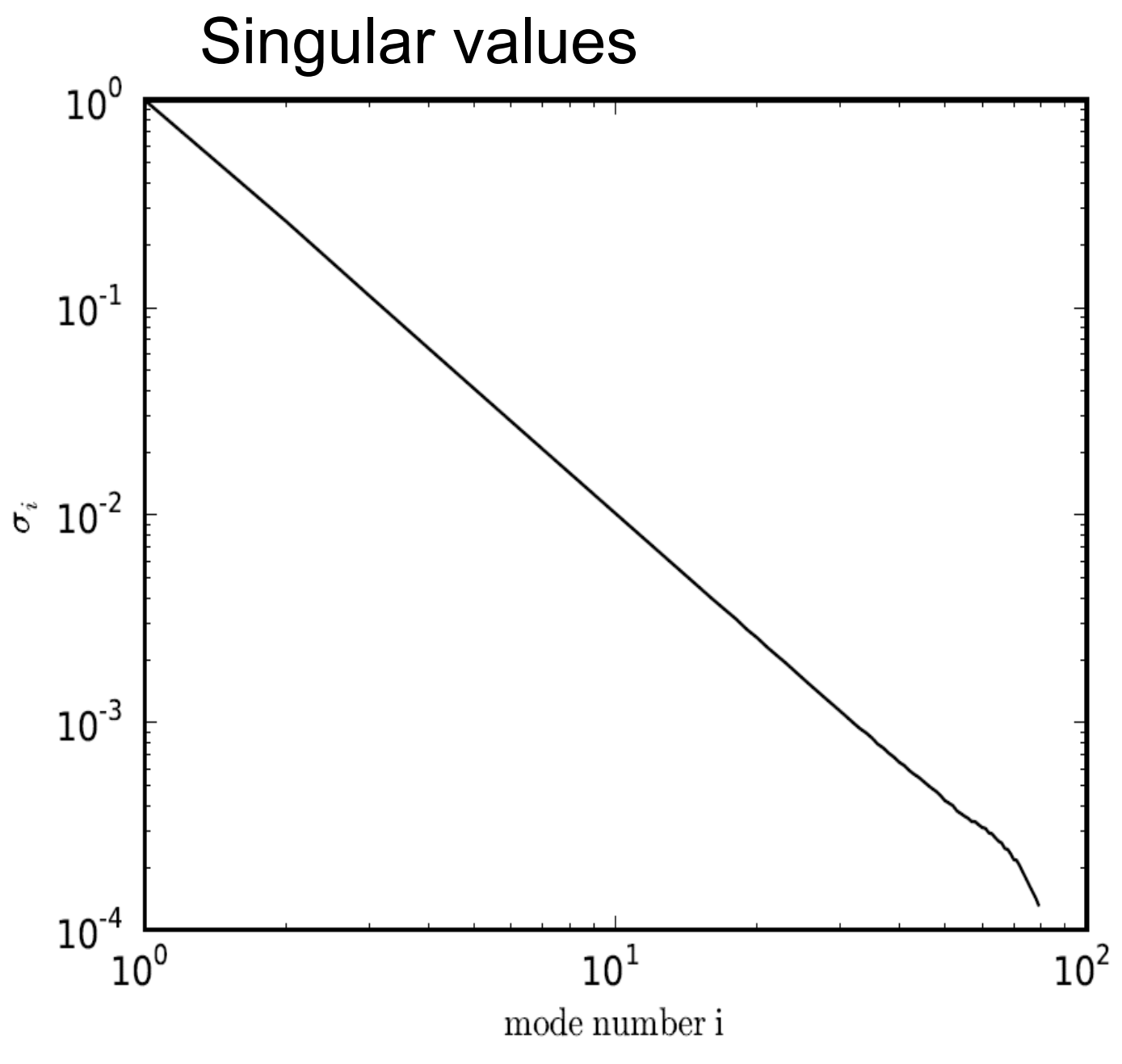
Here we reconstruct a single NFW halo with decreasing singular value cutoffs. We clearly see the trade-off between bias for high filtering levels (top row: note the spreading of the peak in redshift space) and noise contamination for low filtering levels (bottom row: note the spurious peaks along the line-of-sight).
The dashed line in each plot shows the B-mode contribution.

The singular vectors of the transformation matrix are orthonormal modes in delta which diagonalize the noise in the reconstruction. The noise level is proportional to the inverse of the square of the associated singular value.

This shows us the extent to which we can expect tomographic weak lensing to recover the radial density profile. In a few words: not so well. Any reconstruction will necessarily have poor constraints in the z direction.

Eigenmodes are labeled with their noise level relative to that of the fundamental mode.





Computation time:

Using a few reasonable approximations and some linear-algebraic tricks, we are able to compute the above SVD very fast – for a 20,000 square degree survey with arcminute pixels, a single workstation with sufficient memory could perform the full computation in a timescale of a few hours. This is much faster than previously explored techniques, e.g. Wiener filtering approaches.
(see VanderPlas *et al.* 2011 for details)

RESULTS:

- *Linear* and *non-parametric* weak lensing methods are severely limited in their ability to recover accurate halo redshifts
- For nonparametric tomographic mass mapping, accurate photometric redshifts are not the limiting factor. Rather, shape noise is the limiting factor.
- Tomographic mapping of DES and LSST-scale surveys is practical using this method

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