Ex :- $X \sim N(6,2)$

Compute $P[3 \le X \le 5] = P\left[\frac{3-6}{\sqrt{2}} \le \frac{X-6}{\sqrt{2}} \le \frac{5-6}{\sqrt{2}}\right]$

$= P\left[-\frac{3}{\sqrt{2}} \le Y \le -\frac{1}{\sqrt{2}}\right]$

$Y \to N(0,1)$

$= \Phi\left(-\frac{1}{\sqrt{2}}\right) - \Phi\left(-\frac{3}{\sqrt{2}}\right)$

$= \left\{1 - \Phi\left(\frac{1}{\sqrt{2}}\right)\right\} - \left\{1 - \Phi\left(+\frac{3}{\sqrt{2}}\right)\right\}$

$= \Phi\left(\frac{3}{\sqrt{2}}\right) - \Phi\left(\frac{1}{\sqrt{2}}\right)$.

* **Cauchy distⁿ :-**

X : Random variable

$$f_X(x) = \frac{1}{\pi\sigma\left\{1+\left(\frac{x-\mu}{\sigma}\right)^2\right\}}, \quad x \in \mathbb{R},\ \mu \in \mathbb{R},\ \sigma \in \mathbb{R}^+$$

It is called Cauchy distⁿ with location parameter $= \mu$ & scale parameter $= \sigma$.

If $\mu = 0$ & $\sigma = 1$, we call it as standard Cauchy dist^{n}.

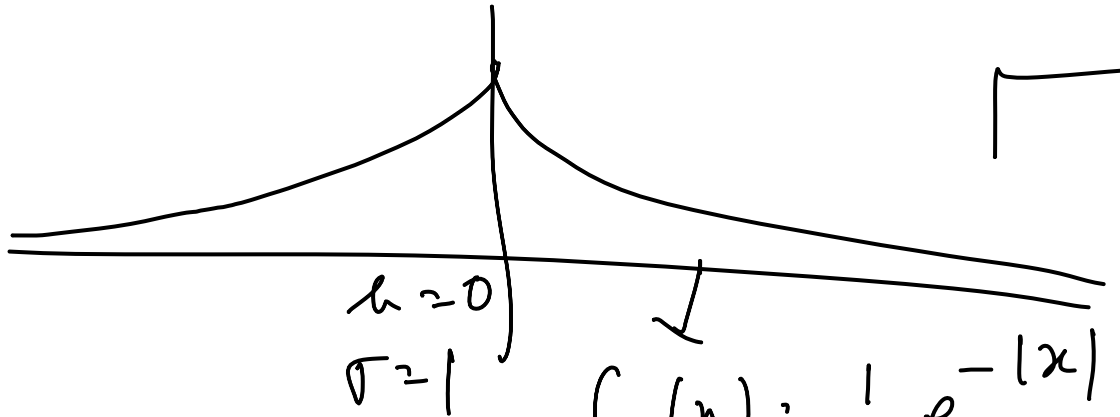
<u>Remark:-</u> For Cauchy dist^{n}, the moments of order ≥ 1 do NOT exist.

** <u>Laplace dist^{n}:-</u> (or double exponential).

X: Random variable.

$$f_X(x) = \frac{1}{2\sigma} e^{-\left|\frac{x-\mu}{\sigma}\right|}, \quad x \in \mathbb{R},\ \mu \in \mathbb{R},\ \sigma \in \mathbb{R}^+.$$

If $\mu = 0$ & $\sigma = 1$, it is called standard Laplace dist^{n}.



$f_X(x) = \frac{1}{2} e^{-|x|}$, $x \in \mathbb{R}$

→ This is just a mirror image of e^{-x} along the Y-axis.

→ Derive $M_X(t)$, $E(X)$ & $Var(X)$ (do it by yourself).

New Section:- (Function of Random variable):-

Application:- i) In Physics, $P = mf$, and X :- denotes f, and its distⁿ is F.

Want to know the distⁿ of P.

ii) Chirp signal model:-

$$Y_n = f(\cos\theta, \sin\theta) + \varepsilon_n$$

Y_n: n-th signal

ε_n: F (C.D.F)

Want to know the distⁿ of Y_n.

iii) Biological Science: Blood Pressure $= g$(Weight)

Weight: F (CDF)

Want to know the CDF of Blood Pressure.

A few Examples :—

Example I :— $P[X = x] = \left(\frac{1}{2}\right)^x , \quad x = 1, 2, \ldots$

$= 0 , \quad \text{o.w.}$

Define $Y = X - 1$ & want to derive the p.m.f of Y.

Range of Y $\leftarrow$ $\mathcal{Y} = \{0, 1, 2, \ldots \ldots\}$.

p.m.f. of Y $\leftarrow$ $P[Y = y] = P[X - 1 = y] = P[X = y+1] = \left(\frac{1}{2}\right)^{y+1}, \quad y = 0, \ldots$

$= 0 , \quad \text{o.w.}$

Example 2:- Suppose $\mathcal{X} = \{-1, 0, 1, 2\}$.

Given that $P[X = -1] = 0.2$, $P[X = 0] = 0.3$, $P[X = 1] = 0.4$,

$P[X = 2] = 0.1$.

Want to derive the p.m.f. of $Y = X^2$.

Range of Y $\mathcal{Y} = \{0, 1, 4\}$.

$P[Y = 0] = P[X = 0] = 0.3$.

$P[Y = 1] = P[X = +1] + P[X = -1] = 0.4 + 0.2$
$= 0.6$.

$P[Y = 4] = P[X = 2] = 0.1$.

Example 3:-

$X \sim$ Poisson (λ).

$$P[X = x] = \frac{e^{-\lambda}\lambda^x}{x!}, \quad \underbrace{x = 0, 1, 2, \ldots}_{\mathcal{X}}$$

Want to derive the p.m.f of $Y = X^2 + 3$.

Range of Y: $\mathcal{Y} = \{3, 4, 7, \ldots\}$.

pmf. of Y:

$$P[Y = y] = P[X^2 + 3 = y] = P[X^2 = y - 3]$$

$X \geq 0$ r.v.

$$= P[X = \sqrt{y-3}] = \frac{e^{-\lambda}\lambda^{\sqrt{y-3}}}{(\sqrt{y-3})!}, \quad y = 3, 4, 7, \ldots$$

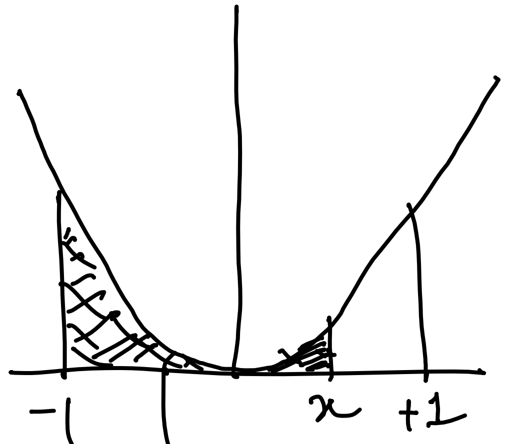
Tranformation (or function) of Continuous random variable :-

Suppose $X \sim F$ with p.d.f. f.

Want to know the p.d.f. of $Y = g(X)$, [suppose, g is any contn. fn.].

<u>Example</u> :- $X \sim$ PDF $f_X(x)$.

$$f_X(x) = \begin{cases} \dfrac{3x^2}{2} & \text{if } -1 \le x \le 1 \\ 0 & , \text{ o.w.} \end{cases}$$



$F_X(x) = P[X \le x] =$

The area of the shaded region will be $P[X \le x]$.

Suppose, we want to derive the CDF (or PDF) of $Y = X^2$.

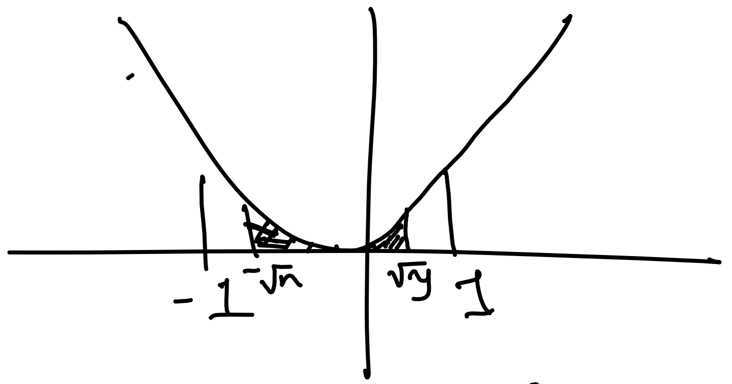
Method 1:- (Dist^n f^n approach) :- Try to derive the dist^n of Y, and from that try to derive the PDF of Y.

Recall the earlier example:-

$$1 \le X \le -1 \implies 0 \le Y \le 1 .$$

$$F_Y(y) = \begin{cases} 0 & \text{if } y < 0 \\ P[Y \le y] = P[X^2 \le y] & \text{if } 0 \le y \le 1 \\ 1 & \text{if } y > 1 \end{cases}$$

CDF of Y



Next, $P[X^2 \le y] = P[\omega : X^2(\omega) \le y] = P[-\sqrt{y} \le X \le \sqrt{y}]$

$$= \int_{-\sqrt{y}}^{\sqrt{y}} \frac{3x^2}{2}\,dx = 3\times 2 \int_0^{\sqrt{y}} \frac{3x^2}{2}\,dx = y^{3/2}$$

Therefore,

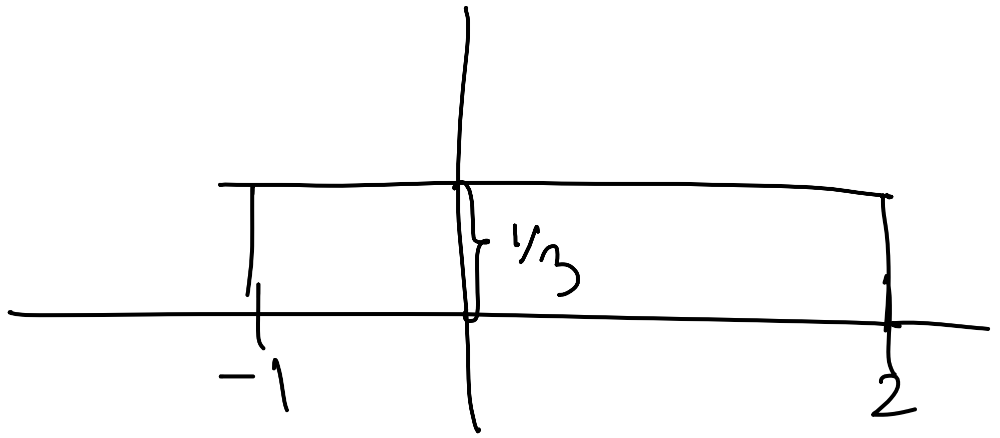
$$F_Y(y) = P[Y \le y] = \begin{cases} 0 & \text{if } \quad y < 0 \\ y^{3/2} & \text{if } \quad 0 \le y \le 1 \\ 1 & \text{if } \quad y > 1 \end{cases}.$$

Hence, the PDF of Y is $=$

$$\begin{cases} \dfrac{d}{dy}\left(y^{3/2}\right) = \dfrac{2}{3} y^{1/2} & \text{if } 0 \le y \le 1 \\ = 0 & , \text{ o.w.} \end{cases}$$

<u>Example 2:—</u>

$$f_X(x) = \frac{1}{3} \quad \text{if } -1 \le x \le 2$$
$$= 0 \quad , \text{ o.w.}$$



Want to derive the CDF or PDF of $Y = X^2$