

(a) $\Omega = \{0, 1, 2, \dots\}$

$$P(A) = \sum_{x \in A} \frac{e^{-\lambda} \lambda^x}{x!}, \lambda > 0$$

$$P(A) \geq 0 \text{ by (i)}$$

$$P(\Omega) = \sum_{x \in \Omega} \frac{e^{-\lambda} \lambda^x}{x!} = e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{x!} = 1$$

$$\text{and } A_1, A_2, \dots, A_i \cap A_j = \emptyset \text{ for } i \neq j$$

Define $A_i = \{x \in \Omega : i-1 < x < i+1\} = \{i\}$

$$P(\bigcup_{i=1}^{\infty} A_i) = P\{\{1, 2, \dots\}\} = 1 - P(x=0)$$

$$\sum_{i=1}^{\infty} P(A_i) = P(A_1) + \dots = 1 - P(x=0) = \sum_{x \in \Omega \setminus A_i} P(x)$$

$$P(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i) \quad - (\text{iii}) \Rightarrow P \text{ is prob measure}$$

(b) Similar to (a)

(c) $P(A) \geq 0$

$$P(\Omega) = 0 \quad (\because \Omega \text{ is infinite}).$$

P is not prob measure.

Note: Also taking $A_i = \{i\} \quad i = 1, 2, \dots$ Then $\Omega \setminus A_i = \{1, 2, \dots\} - \text{infinite # of elements}$

$$P(\bigcup_{i=1}^{\infty} A_i) = 0 \neq \sum_{i=1}^{\infty} P(A_i) = \sum_{i=1}^{\infty} 1$$

2nd part

(a) $P(E) = \sum_{j=3}^{\infty} \frac{e^{-\lambda} \lambda^j}{j!} = 1 - e^{-\lambda} \left(1 + \lambda + \frac{\lambda^2}{2!}\right)$

$$P(F) = \sum_{j=1}^2 \frac{e^{-\lambda} \lambda^j}{j!} = e^{-\lambda} \left(\lambda + \frac{\lambda^2}{2!}\right)$$

$$P(E \cup F) = \sum_{j=1}^2 \frac{e^{-\lambda} \lambda^j}{j!} = 1 - e^{-\lambda} \frac{\lambda^0}{0!} = 1 - e^{-\lambda}$$

S by others.

(2) $\Omega = \mathbb{R}$

$$(a) P(I) = \int_I \frac{1}{2} e^{-|x|} dx \geq 0 \quad + I$$

$$P(\Omega) = \frac{1}{2} \int_{-\infty}^{\infty} e^{-|x|} dx = \frac{1}{2} \left(\int_{-\infty}^0 e^x dx + \int_0^{\infty} e^{-x} dx \right) = 1$$

$$I_1 \cap I_2 = \emptyset; P(I_1 \cup I_2) = \frac{1}{2} \left[\int_{I_1} e^{-|x|} dx + \int_{I_2} e^{-|x|} dx \right] = P(I_1) + P(I_2)$$

extend P to \mathcal{A} (sigma algebra) P is prob measure

(b) Similar to (a)

(c) $P(\Omega) = P(\mathbb{R}) = 0 \neq 1$, P is not prob measure

(3) $P(\text{exactly one of } A \text{ or } B)$

$$= P((A \cap B^c) \cup (A^c \cap B))$$

$$= P(A) + P(B) - 2 P(A \cap B) \quad - \text{on simplification.}$$

$$P(A \cap B) = P(A) P(B) = P(A) P(B^c) = P(A \cap B^c), \quad - \text{1st eqn.}$$

$$[\text{using } P(A) = P(A \cap B) + P(A \cap B^c).]$$

2nd & 3rd eqns can be proved in a similar way.

(5) For $n=2$

$$P(A_1 \cup A_2) = P(A_1 \cup (A_1^c \cap A_2))$$

$$= P(A_1) + P(A_1^c \cap A_2)$$

$$= P(A_1) + [P(A_2) - P(A_1 \cap A_2)] \quad - \text{true for } n=2$$

Proof by induction

Assume that it is true for $n=m$, then

$$P(\bigcup_{k=1}^{m+1} A_k) = P\left(\left(\bigcup_{k=1}^m A_k\right) \cup A_{m+1}\right)$$

$$= P\left(\bigcup_{k=1}^m A_k\right) + P(A_{m+1}) - P\left(\left(\bigcup_{k=1}^m A_k\right) \cap A_{m+1}\right)$$

$$\text{L.H.S} = \left[\sum_{k=1}^n P(A_k) - \sum_{k_1 < k_2} P(A_{k_1} \cap A_{k_2}) + \sum_{k_1 < k_2 < k_3} P(A_{k_1} \cap A_{k_2} \cap A_{k_3}) - \dots + (-1)^{m-1} P\left(\bigcap_{k=1}^m A_k\right) \right]$$

$$+ P(A_{m+1}) - P\left(\bigcup_{k=1}^m (A_k \cap A_{m+1})\right) \quad \dots \quad (1)$$

$$P\left(\bigcup_{k=1}^m (A_k \cap A_{m+1})\right) = \sum_{k=1}^m P(A_k \cap A_{m+1}) - \sum_{k_1 < k_2} P(A_{k_1} \cap A_{m+1}) \cap (A_{k_2} \cap A_{m+1})$$

$$+ \sum_{k_1 < k_2 < k_3} P(A_{k_1} \cap A_{m+1} \cap A_{k_2} \cap A_{m+1} \cap A_{k_3} \cap A_{m+1}) - \dots$$

$$+ (-1)^{m-1} P\left(\bigcap_{k=1}^m A_k \cap A_{m+1}\right) \quad \dots \quad (2)$$

Using (2) in (1) gives

$$P\left(\bigcup_{k=1}^{m+1} A_k\right) = \sum_{k=1}^{m+1} P(A_k) - \sum_{k_1 < k_2}^{m+1} P(A_{k_1} \cap A_{k_2}) + \sum_{k_1 < k_2 < k_3} P(A_{k_1} \cap A_{k_2} \cap A_{k_3}) - \dots + (-1)^m P\left(\bigcap_{k=1}^{m+1} A_k\right).$$

$$(6) \quad \Omega = \{0, 1, 2, \dots\}. \quad P(\{j\}) = c \frac{2^j}{j!}; \quad j=0, 1, 2, \dots$$

$$(a) \quad 1 = P(\Omega) = \sum_{j=0}^{\infty} P(\{j\}) = \sum_{j=0}^{\infty} c \frac{2^j}{j!} = c e^2$$

$$\Rightarrow c = e^{-2}$$

$$(b) \quad P(A) = \sum_{j=2}^{\infty} e^{-2} \frac{2^j}{j!}; \quad P(B) = \sum_{j=3}^{\infty} e^{-2} \frac{2^j}{j!}$$

$$P(C) = \sum_{j=0}^{\infty} e^{-2} \frac{2^{2j+1}}{(2j+1)!}$$

$$P(B \cap C) = P(\{3\}) + P(\{5\}) + \dots = \dots$$

Other probs can be computed in a similar manner

(7) Favorite models of dinosaurs numbered 1, 2, 3 (say)

Define events

A_i = model #i not found in 6 packets.
 $i=1, 2, 3$

$$\begin{aligned} \text{req'd prob} &= P(A_1^c \cap A_2^c \cap A_3^c) \\ &= 1 - P(A_1^c A_2^c A_3^c)^c \\ &= 1 - P(A_1 \cup A_2 \cup A_3) \\ &= 1 - [P(A_1) + P(A_2) + P(A_3) - P(A_1 A_2) - P(A_1 A_3) \\ &\quad - P(A_2 A_3) + P(A_1 A_2 A_3)] - (1) \end{aligned}$$

Note that

$$\left. \begin{aligned} P(A_i) &= \left(\frac{4}{5}\right)^6 & \forall i \\ P(A_i A_j) &= \left(\frac{3}{5}\right)^6 & \forall i \neq j \\ P(A_1 A_2 A_3) &= \left(\frac{2}{5}\right)^6 \end{aligned} \right\} - (2)$$

use (2) in (1) to get the desired prob.

(8) A_i : match at position i

P(at least one match)

$$\begin{aligned} &= P(A_1 \cup A_2 \cup \dots \cup A_m) \\ &= \sum_{i=1}^n P(A_i) - \sum_{i < j} P(A_i A_j) + \dots + (-1)^{n-1} P(\bigcap_{i=1}^n A_i) \end{aligned}$$

with
 $P(A_{i_1} \cap A_{i_2} \cap \dots \cap A_{i_r}) = \frac{(n-r)!}{n!}; \quad 1 \leq i_1 < i_2 < \dots < i_r \leq n$
 $r=1(1)n$

$$\Rightarrow \text{req'd prob} = 1 - \frac{1}{2!} + \frac{1}{3!} + \dots + (-1)^{n-1} \frac{1}{n!}$$

(9)

(a) A_i : event that i^{th} bill goes to i^{th} envelope
 $(i=1 \dots n)$

Reqd prob

$$\begin{aligned} P\left(\bigcap_{i=1}^n A_i^c\right) &= P(\cup A_i)^c \\ &= 1 - P(\cup A_i) \\ &= 1 - \left[\sum P(A_i) - \sum_{i,j} P(A_i A_j) + \dots + (-1)^{n-1} P(A_1 \dots A_n) \right] \\ &= 1 - \underbrace{\sum_{i=1}^n P(A_i)}_{Q_1} + \underbrace{\sum_{\substack{i,j \\ i \neq j}} P(A_i A_j)}_{Q_2} + \dots + (-1)^n P(A_1 \dots A_n) \end{aligned}$$

In $Q_i \rightarrow \binom{n}{i}$ terms each equal to $\frac{(n-i)!}{n!} \left(= \frac{1}{(n-i)_i} \right)$

$$\begin{aligned} \Rightarrow P\left(\bigcap A_i^c\right) &= 1 - \binom{n}{1} \frac{1}{(n)_1} + \binom{n}{2} \frac{1}{(n)_2} - \dots + (-1)^n \frac{1}{(n)_n} \\ &= 1 - \frac{n!}{1!(n-1)!} \cdot \frac{(n-1)!}{n!} + \frac{n!}{2!(n-2)!} \cdot \frac{(n-2)!}{n!} - \dots + (-1)^n \frac{1}{n!} \\ &= 1 - \frac{1}{1!} + \frac{1}{2!} - \dots + (-1)^n \frac{1}{n!} = \sum_{i=0}^n (-1)^i \frac{1}{i!} \end{aligned}$$

(b) B_i : i^{th} envelope get i^{th} bill & i^{th} cheque

reqd prob $P\left(\bigcap B_i^c\right) = 1 - P(\cup B_i)$

$$= 1 - \sum P(B_i) + \sum_{\substack{i,j \\ i \neq j}} P(B_i B_j) - \dots + (-1)^n P(B_1 \dots B_n)$$

\longleftrightarrow

R_1 $\underbrace{R_2}_{R_n}$

In $R_i \rightarrow \binom{n}{i}$ term each equal to $\left(\frac{(n-i)!}{n!} \cdot \frac{(n-i)!}{n!} \right)$.

$$\Rightarrow P(\bigcap_{i=1}^n B_i^c)$$

$$= 1 - P\left(\bigcup_{i=1}^n B_i\right) = 1 - \sum_{i=1}^n P(B_i) + \sum_{1 \leq i < j} P(B_i B_j) - \dots + (-1)^n P(B_1 B_2 \dots B_n)$$

$$= 1 - R_1 + R_2 - R_3 + \dots + (-1)^n R_n$$

$$\begin{aligned} R_i &= \binom{n}{i} \frac{(n-i)!}{n!} \times \frac{(n-i)!}{n!} \\ &= \frac{n!}{i! (n-i)!} \cdot \frac{(n-i)!}{n!} \cdot \frac{(n-i)!}{n!} \\ &= \frac{1}{i!} \cdot \frac{1}{(n)_i} \quad | \quad (n)_i = \frac{n!}{(n-i)!} \end{aligned}$$

$$\Rightarrow P\left(\bigcap_{i=1}^n B_i^c\right) = \sum_{i=0}^n (-1)^i \frac{1}{i! (n)_i}$$

(10)

$$(i) P(A \cup B | C) = \frac{P((A \cup B) \cap C)}{P(C)} = \frac{P(Ac \cap Bc)}{P(C)}$$

$$= P(A|C) + P(B|C) - P(AB|C)$$

$$(ii) P(A^c | C) = \frac{P(A^c C)}{P(C)} = \frac{P(C) - P(Ac)}{P(C)} = 1 - P(A|C)$$

(11)

$$(a) \text{ true } - P(A|B) + P(A^c | B) = \frac{P(AB)}{P(B)} + \frac{P(A^c B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

$$(b) P(A|B) = \frac{P(AB)}{P(B)} ; P(A|B^c) = \frac{P(AB^c)}{P(B^c)} = \frac{P(A) - P(AB)}{1 - P(B)}$$

Take $A \subset B$, $P(A) > 0$, $P(B-A) > 0$

$$P(A|B) + P(A|B^c) = \frac{P(AB)}{P(B)} + \frac{P(AB^c)}{P(B^c)}$$

$$= \frac{P(A)}{P(B)} < 1 \quad \text{False}$$

(c)

Take $A \subset B$, i.e. $B^c \subset A^c$

$$P(A|B) + P(A^c | B^c)$$

$$= \frac{P(AB)}{P(B)} + \frac{P(A^c B^c)}{P(B^c)}$$

$$= \frac{P(A) > 0}{P(B)} + \frac{P(B^c)}{P(B^c)} > 1 \quad \text{false}$$

(12)

$$(a). P(A|B) = \frac{1}{4} \Rightarrow P(AB) \neq 0 \quad \text{(i.e. } AB \neq \emptyset\text{)} \quad \text{False}$$

$$(b). A \subset B \Rightarrow P(AB) = P(A) \quad \text{False}$$

$$\text{Given } P(B|A) = \frac{P(AB)}{P(A)} = \frac{1}{2}$$

$$\Rightarrow P(AB) \neq P(A) \quad \text{False}$$

$$(c) P(A^c | B^c) = \frac{P(A^c | B^c)}{P(B^c)} = \frac{1 - P(A) - P(B) + P(AB)}{1 - P(B)} \quad - (*)$$

$$P(B|A) = k$$

$$\Delta P(A) = \frac{1}{4}, \Rightarrow P(AB) = \frac{1}{8}$$

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{\frac{1}{8}}{\frac{1}{2}} = \frac{1}{4} \Rightarrow P(B) = \frac{1}{2}$$

$$(*) \Rightarrow P(A^c | B^c) = \frac{1 - \frac{1}{4} - \frac{1}{2} + \frac{1}{8}}{\frac{1}{2}} = \frac{3}{4}$$

(a). $P(\text{exactly 3 white balls, out of 4})$.

$$= \binom{4}{3} \left(\frac{1}{2}\right)^3 \cdot \frac{1}{2} = \dots$$

(b) A : first ball placed is white

$$P(A) = \frac{1}{2}$$

B : urn contains exactly 3 white balls

$$P(B|A) = \frac{P(AB)}{P(A)} = \frac{\frac{1}{2} \cdot \binom{3}{2} \left(\frac{1}{2}\right)^3}{\frac{1}{2}} = \frac{3}{8}$$

D : A occurs before B

$$P(D) = p_A + p_A p_C + p_A p_C^2 + \dots$$

$$= \frac{p_A}{1 - p_C} = \frac{p_A}{p_A + p_B}$$

A_i : component i functions

$$P(\text{System functions}) = 1 - P(\bigcap A_i^c)$$

$$= 1 - (1-p)^n$$

(16) Ans : Question i is among the 90 questions that the student can answer correctly

$$\text{reqd prob } P(A_1 A_2 A_3) = P(A_1) P(A_2 | A_1) P(A_3 | A_1 A_2)$$

$$= \frac{90}{100} \cdot \frac{89}{99} \cdot \frac{88}{98}$$

(17)

App Bayes Thm

$$\text{reqd prob} = \frac{\frac{1}{2} \times \frac{2}{3}}{\frac{1}{2} \times \frac{2}{3} + \frac{2}{3} \times \frac{1}{3}} =$$

(18) A^+, B^+, C^+, D^+ - events that A, B, C, D passes the paper

Bayes Thm $\Rightarrow P(D^+ | A^+) = \frac{P(A^+) P(D^+ | A^+)}{P(D^+)}$

$$\text{reqd prob} = P(A^+ | D^+) = \frac{P(A^+) P(D^+ | A^+)}{P(D^+)} =$$

$$P(D^+ | A^+) = \left(\frac{1}{3}\right)^3 + \left(\frac{3}{2}\right)\left(\frac{2}{3}\right)^2 \cdot \frac{1}{3} = \frac{13}{27}$$

Also

$$P(D^+) = P(D^+ | A^+) P(A^+) + P(D^+ | A_+^c) P(A_+^c)$$

$$P(D^+ | A_+^c) = P(D \text{ passes with } + | A \text{ passes } -) = \left(\frac{3}{1}\right) \frac{2}{3} \left(\frac{1}{3}\right)^2 + \left(\frac{3}{1}\right) \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^0 = \frac{14}{27}$$

$$P(D^+) = \frac{13}{27} \cdot \frac{1}{3} + \frac{14}{27} \cdot \frac{2}{3} = \frac{41}{81}$$

$$\Rightarrow P(A^+ | D^+) = \frac{\frac{13}{27} \cdot \frac{1}{3}}{\frac{41}{81}} = \frac{13}{41}$$

(19) silver coin in other

$P(\text{gold coin in one drawer})$. Bayes' Thm

$$= \frac{\frac{1}{3} \times \frac{1}{2}}{\frac{1}{3} \times 1 + \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times 0} = \frac{1}{3}.$$

(20)

(a) $P(c_1 c_2 c_3 c_4) = \prod_i P(c_i) = -$

(b) $P(c_1^c \cap c_2^c \cap c_3^c \cap c_4^c) = \prod_i P(c_i^c)$ [explain why?]
indep

(c) $P(c_1 c_2^c c_3^c c_4^c) + P(c_1^c c_2 c_3^c c_4^c) \Rightarrow c_1^c, c_2^c, c_3^c, c_4^c$ are
also indep.
 $+ P(c_1^c c_2^c c_3 c_4^c) + P(c_1^c c_2^c c_3^c c_4)$
 $= P(c_1) \prod_{i=2}^4 P(c_i^c) + - - - .$

(d). $P(\text{at least one hit}) = 1 - P(\text{no one hit})$

$$= 1 - P(c_1^c c_2^c c_3^c c_4^c).$$

(21)

$$P(\bigcap_i A_i^c) = \prod_i P(A_i^c)$$

$$= \prod_i (1 - P(A_i)) \leq \prod_i \exp(-P(A_i))$$

$$\left[0 < x < 1 \right. \\ \left. 1 - x < e^{-x} \right]$$

$$\text{i.e. } P(\bigcap_i A_i^c) \leq \exp\left(-\sum_i P(A_i)\right).$$

(22) $\Omega = \{1, 2, 3, 4\}$ \mathcal{F} : power set

$$P(\{i\}) = \frac{1}{4} \quad i = 1, 2, 3, 4$$

$$A = \{1, 4\}, B = \{2, 4\}, C = \{3, 4\}.$$

$$P(A) = P(B) = P(C) = \frac{1}{2}$$

$$P(AB) = P(AC) = P(BC) = \frac{1}{4}; P(ABC) = \frac{1}{4}$$

$$\Rightarrow P(AB) = P(A)P(B), P(AC) = P(A)P(C)$$

$$\& P(BC) = P(B)P(C).$$

i.e. A, B, C are pairwise indep

but $P(ABC) = \frac{1}{4} \neq P(A)P(B)P(C) = \frac{1}{8}$

$\Rightarrow A, B, C$ not mutually indep.

(23) Counter example

In prr prob setup take

~~A~~ $A = \{1, 2\}, B = \{3, 4\}, C = \{1\}$.

$$P(A|B) < P(A)$$

$$P(B|C) < P(B) \text{ but } P(A|C) > P(A).$$

$\Rightarrow C$ does not carry negative information about A .

(24) A_i : i girls are in the list
 $i = 0, 1, 2, 3$

B : 1st student is girl

C : 2nd student is boy - to obtain $P(C|B)$

$$P(B) = P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + P(A_3)P(B|A_3)$$

$$= \frac{\binom{3}{1}\binom{5}{3}}{\binom{8}{4}} \times \frac{1}{4} + \frac{\binom{3}{2}\binom{5}{2}}{\binom{8}{4}} \times \frac{2}{4} + \frac{\binom{3}{3}\binom{5}{1}}{\binom{8}{4}} \times \frac{3}{4}$$

$$P(B) = \frac{105}{4 \times \binom{8}{4}}$$

$$\begin{aligned}
 P(c|B) &= \frac{P(cB)}{P(B)} = \frac{P(c \cap (\bigcup_i A_i B))}{P(B)} = \frac{P(\bigcup_i c A_i B)}{P(B)} \\
 &= \sum_{i=1}^3 \frac{P(c A_i B)}{P(B)} = \sum_{i=1}^3 P(c|A_i B) \frac{P(A_i B)}{P(B)} \\
 &= \sum_{i=1}^3 P(c|A_i B) P(A_i|B) \\
 &= 1 \times P(A_1|B) + \frac{2}{3} \times P(A_2|B) + \frac{1}{3} \times P(A_3|B)
 \end{aligned}$$

$$\begin{aligned} P(A_1|B) &= \frac{P(A_1) P(B|A_1)}{P(B)} = \frac{2}{7} \\ P(A_2|B) &= \frac{P(A_2) P(B|A_2)}{P(B)} = \frac{4}{7} \quad \& P(A_3|B) = \frac{1}{7} \end{aligned}$$

$$\Rightarrow P(C|B) = 1 \times \frac{2}{7} + \frac{2}{3} \times \frac{4}{7} + \frac{1}{3} \times \frac{1}{7}$$

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(25) A & B are in series

C & D in parallel

(a) $P(\text{the system works})$

$$= P(A \cap B \cap (C \cup D))$$

$$= P(A \cap B) P(C \cup D)$$

$$= P(A) P(B) (P(c) + P(D) - P(c) P(D))$$

$$= 0.9 \times 0.9 (0.8 + 0.8 - 0.8 \times 0.8)$$

$$= - -$$

(b) $P(c \text{ is not working} \mid \text{system is working})$

$$= \frac{P(c \text{ is not working} \cap \text{system is working})}{P(\text{system is working})}$$

$$= \frac{P(A \cap B \cap c^c \cap D)}{P(\text{system is working})}$$

$P(\text{system is working}) \leftarrow \text{from (a)}$

$$= \frac{P(A) P(B) P(c^c) P(D)}{P(\text{system is working})}$$

(26)

A_i : event that a fly survives i th application.

$$i = 1, 2, 3, 4. \quad | \text{ Prob }$$

Note that $A_4 \subset A_3 \subset A_2 \subset A_1$

$$\Rightarrow A_4 = A_1 \cap A_2 \cap A_3 \cap A_4$$

(a) req prob = $P(\text{fly survives 4 applications})$

$$= P(A_1, A_2, A_3, A_4)$$

$$= P(A_4)$$

$$= P(A_1) P(A_2 | A_1) P(A_3 | A_1, A_2) P(A_4 | A_1, A_2, A_3)$$

$$= (1 - 0.8) \downarrow (1 - 0.4) \downarrow (1 - 0.2) \downarrow (1 - 0.1)$$

(from the given conditions)

$$= 0.2 \times 0.6 \times 0.8 \times 0.9$$

$$(b) P(A_4 | A_1) = \frac{P(A_4 \cap A_1)}{P(A_1)} = \frac{P(A_4)}{P(A_1)}$$

$$= 0.6 \times 0.8 \times 0.9.$$

(27) B_i : event that i of the paintings are forgeries
 $i = 0(1)5$

$$P(B_0) = 0.76, P(B_1) = 0.09, P(B_2) = 0.02, P(B_3) = 0.01 \\ P(B_4) = 0.02 \text{ & } P(B_5) = 0.1 \text{ (given cond's)}$$

A : event that the painting sent for authentication turns out to be a forgery.

$$\text{reqd prob} = P(B_5 | A) = \frac{P(B_5) P(A|B_5)}{\sum_{i=0}^5 P(B_i) P(A|B_i)}$$

↑
Bayes rule

$$\text{reqd } P(A) = \sum_{i=0}^5 P(B_i) P(A|B_i) \\ = 0.76 \times 0 + 0.09 \times \frac{1}{5} + 0.02 \times \frac{2}{5} \\ + 0.01 \times \frac{3}{5} + 0.02 \times \frac{4}{5} + 0.10 \times 1 \\ = \frac{0.10 \times 1}{P(A)}$$