A Hybrid Image Compression Technique using Quadtree Decomposition and Parametric Line Fitting for Synthetic Images

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Abstract

This paper presents a new hybrid scheme for image data compression using quadtree decomposition and parametric line fitting. In the first phase of encoding, the input image is partitioned into quadrants using quadtree decomposition. To prevent from very small quadrants, a constraint of minimal block size is imposed during quadtree decomposition. Homogeneous quadrants are separated from non-homogeneous quadrants. In the second phase of encoding, the non-homogeneous quadrants are scanned row-wise. Luminance variation of each scanned row is fitted using parametric line at specified level of tolerance. The output data is entropy coded. Experimental results show that the proposed scheme performs better than well-known lossless image compression techniques for several types of synthetic images, e.g., clip-art, cartoon, animation, scientific plot, medical image etc.

keywords: *image, compression, quadtree, parametric line, fitting.*

1 Introduction

The two broad classes of images are natural and synthetic. Natural images occur in nature and are captured by digital devices like camera. The pixel intensities of natural images vary smoothly and there is correlation in the neighboring pixel values. Synthetic images are computer generated, and include animation, clip-art, cartoons, medical images, text images, maps etc. The pixel intensities of synthetic images do not vary smoothly but contain a small discrete set of values. There are large areas (quadrants) of a uniform color and there are sharp changes in color.

The objective of image compression is to reduce redundancy of the image data in order to be able to store or transmit data in an efficient form. Due to extensive research on lossy compression of natural images, several techniques based on discrete cosine transform (JPEG) , wavelet transform (JPEG2000), and fractal coding have been developed. The advancement and availability of new software tools to create and generate synthetic images has raise the interest in research community to investigate and devise better techniques for compression of synthetic images. Synthetic images do not compress well using lossy compression techniques [4]. The two most widely used formats for lossless compression are GIF and PNG. In this paper we presented a new technique for synthetic image compression using quadtree decomposition and parametric line fitting. Since our method is based on **Q**uadtree and **P**arametric **L**ine, therefore we would call our method (QPL). We compared the performance of our technique with well known lossless image compression techniques, GIF and PNG.

Organization of the rest of the paper is as follows: Related work is discussed in section 2. Framework is described in section 3. Section 4 is the most important section and it describes the encoding and decoding algorithms. Experiments and results are presented in Sect. 5. Section 6 gives insight view of the proposed method. Final concluding remarks are in Sect. 7.

2 Related Work

GIF and PNG are the most prevalent lossless image compression techniques. A concise overview of GIF and PNG image compression techniques is described in following sections 2.1 and 2.2 respectively.

2.1 Graphics Interchange Format (GIF)

The Graphics Interchange Format (GIF) is a lossless 8-bit-per-pixel bitmap image format that was introduced by CompuServe in 1987 [13, 4]. GIF images are compressed using the Lempel-Ziv-Welch (LZW) coding [12], a dictionary-based technique to exploit redundancy. The initial size of dictionary is 2^9, while this

fills up, the dictionary size is doubled, until the maximum dictionary size of 4096 is reached. Afterwards the compression algorithm behaves like a static dictionary algorithm. A more detailed description can be found in [5, 3]. GIF is widely used for lossless compression of both natural and synthetic images. While GIF works well with synthetic images, and pseudo color or color-mapped images, it is generally not the most efficient way to compress natural images, photographs, satellite images [10]. LZW coding, used in GIF, scans pixels from left to right, top to bottom. Therefore, horizontal patterns are effectively compressed but vertical patterns are not [11].

2.2 Portable Network Graphics (PNG)

Portable Network Graphics (PNG) is a bitmap image format that employs lossless data compression. PNG was created to improve upon and replace the GIF format, as an image-file format not requiring a patent license [14]. The compression algorithm used in PNG is based on LZ77 [15], a dictionary-based compression technique. PNG uses *deflate* [8] implementation of LZ77. At each step the encoder examines three bytes. If it cannot find a match of three bytes, it abandons the first byte and examines the next three bytes. So, at each step it either abandons the value of a single byte, or a literal, or a pair $\langle$*match length, offset*$\rangle$. The alphabets of the *literal* and *match length* are combined to form an alphabet of size 286 (indexed between 0-285). The indices 0-255 represent literal bytes and the index 256 is an end-of-block symbol. The remaining 29 indices represent the codes for ranges of lengths between 3 and 258. A more detailed description and standard tables for representation of *match length* and Huffman codes can be found in [5, 7]. For most images, PNG can achieve greater compression than GIF.

3 Framework

The subsequent two sections, 3.1 and 3.2 describe the framework of our system that is based on quadtree and parametric line.

3.1 Quadtree

Quadtree is a data structure that is widely used for image storage, representation and processing [2, 9]. Quadtree is most often used to partition a 2-D space by recursively subdividing it into four quadrants or blocks until each quadrant contains only pixels of one color or luminance. Recursively subdividing may result a quadrant that contains only single pixel. This conventional quadtree decomposition has following drawbacks: (1) The overhead of representing a single pixel by quadtree is not desirable for image compression. It may take more space to represent a single pixel by quadtree than without using it. (2) Due to subdividing criteria, even if a single pixel in a quadrant is of different color or luminance then quadtree decomposition would divide that quadrant into four quadrants. As a consequence of this, there may be three quadrants with same luminance value. In other words, the boundaries between quadrants does not necessary represent quadrant of different luminance. To overcome the first drawback; in our method we imposed a constraint of minimum block size on quadtree decomposition. It means that a quadrant would not be further divided into four quadrants if its size is equal to the predefined minimum block size. The constraint of minimum block size safeguards our method from the overhead of representing very small quadrants (e.g., quadrants of size less than 4×4) by a quadtree. The constraint based quadtree decomposition results in two types of quadrants, (a) homogeneous quadrants, i.e., quadrants that contain only pixels of one color or luminance, (b) non-homogeneous quadrants, i.e., quadrants that contain pixels of more than one color or luminance. We represented only homogeneous quadrants using quadtree. Non-homogeneous quadrants are represented by parametric line as described in next section 3.2.

3.2 Parametric Line

Parametric line is essentially a straight line obtained by linear interpolation between two points (control points). To generate a parametric line that interpolates $k+1$ points, k line segments are used. Equation of j^{th} segment between points p_j and p_{j+1} can be written as follows:

$$q_j(t) = (1-t)\,p_j + t\,p_{j+1},\ t \in [0,1],\ 1 \leq j \leq k, \quad (1)$$

where $q_j(t)$ is an interpolated point between control points p_j and p_{j+1} at parameter value t. To generate n points between p_j and p_{j+1} inclusive, the parameter t is divided into $n-1$ intervals between 0 and 1 inclusive such that $q_j(0) = p_j$ and $q_j(1) = p_{j+1}$.

In order to represent the non-homogeneous quadrants, we scanned the image data row wise and fitted the parametric line to pixels of non-homogeneous quadrants. Parametric line fitting helps to further reduce the data size in two ways. First, the parametric line fitting helps to represent the pixels of one color/luminance

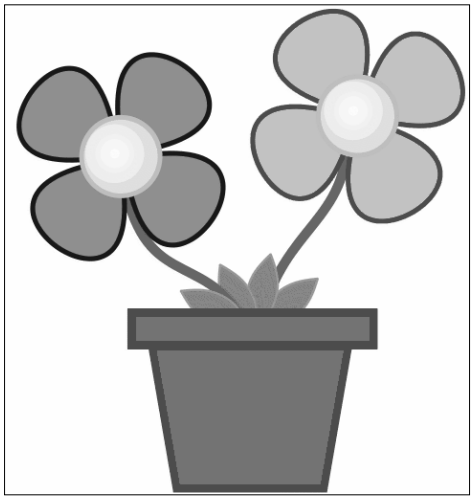


Figure 1: Original image of size 509×486. The image size is not a power of 2.

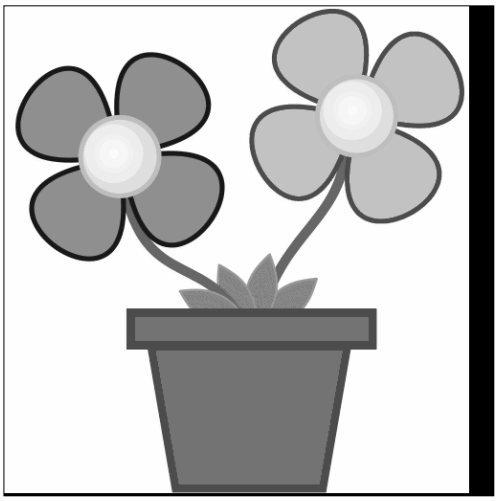


Figure 2: The image of Fig. 1 is padded. Size of the padded image is 512×512. Padded area on right and bottom is shown in black color.

with smaller data set. Second, the parametric line fitting merges the data of a row, belong to more than one non-homogeneous quadrant, as a single data set. This single merged row removes the artificial boundaries between quadrants that have been imposed by quadtree decomposition. It is very likely that at the boundaries of two adjacent non-homogeneous quadrants, pixels have same luminance. By merging quadrants, large number of pixels can be represented by small output data obtained from parametric line fitting. This also solves the second drawback of conventional quadtree representation of image, as described in the previous section 3.1.

4 Algorithm

This section presents the details of our algorithm for image compression. The encoding part of the algorithm consists of two main phases. The first phase consists of quadtree decomposition and the second phase consists of parametric line fitting. The decoding is relatively a simple process and we described it in a single phase.

Let we have a gray-scale image I of size $w \times h$, as shown in Fig. 1. Let $p_{i,j}$ is luminance of a pixel at spatial location (i, j), where $0 \leq p_{i,j} \leq 255$, $1 \leq i \leq h$ and $1 \leq j \leq w$.

4.1 Encoding

4.1.1 Phase 1: Quadtree Encoding

1. If size of image I is not a power of 2 then pad the right and bottom borders of I with -1's. The Fig 2 shows padded image, where padded area is shown in black color (internally image matrix has -1 value for padded area). Let J is the padded image of size, $w' \times h'$. For example, if size of I is 509×486 then after padding, the size of J would be 512×512.
2. Specify threshold of minimum block size B^{lmt} for quadtree decomposition. For example when $B^{lmt} = 4$, the minimum block size would be 4×4.
3. Apply the quadtree decomposition to image J. This yields homogeneous quadrants and non-homogeneous quadrants. Figure 3 shows homogeneous and non-homogeneous quadrants of a quadtree decomposed image, minimum block size is 4×4. Non-homogeneous quadrants have pixels of more than one luminance value; therefore it is not worth to save each pixel (total 16 pixels in a minimum size quadrant) of non-homogeneous quadrants individually.
4. Entropy encode the data of homogeneous quadrants (blocks). Each homogeneous quadrants is identified by, (i) size of the quadrant, because width and height of a quadrant are always equal, therefore only single value is required to save, (ii) (x, y) coordinates of upper left corner of the quadrant, (iii) luminance of the quadrant. There are multiple quadrants of same size, therefore for efficient storage, we created a list of sizes and each element of list has reference towards (ii) and (iii) of all quadrants whose size is equal to

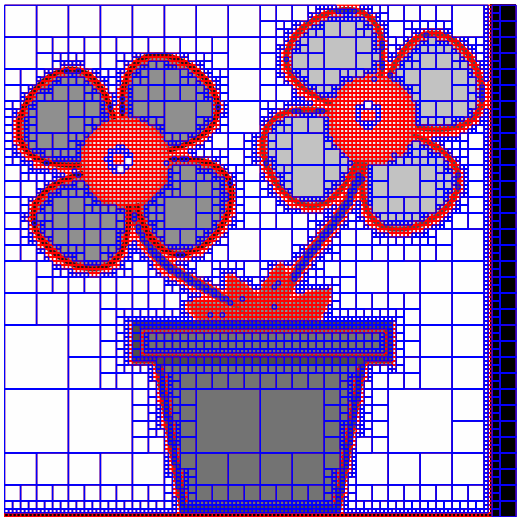


Figure 3: A quadtree decomposed image; minimum block size is 4×4. Boundaries of homogeneous and non-homogeneous quadrants are shown with blue and red colors respectively.

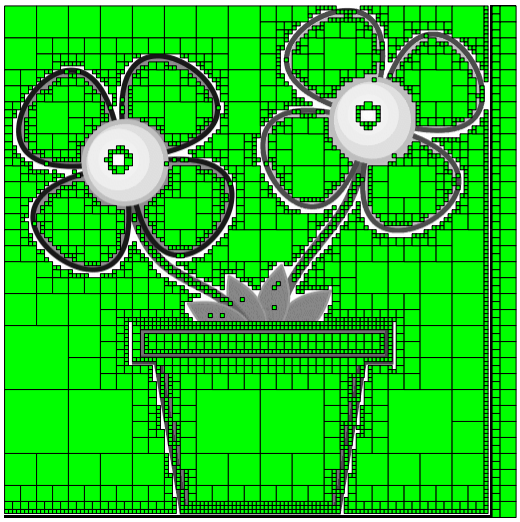


Figure 4: A quadtree decomposed image, minimum block size is 4×4. Homogeneous quadrants are filled with -1, shown in green color. Non-homogeneous quadrants are shown with their actual luminance values.

corresponding element in the list. There is no need to save the data of any quadrant in J which is completely outside of original image I. It can easily be determined by comparing the upper left corner coordinates of a quadrant with the value of w and h.

5. Replace the luminance values of pixels of homogeneous quadrants in J with -1. In Fig. 4 we showed homogeneous quadrants with green color, while non-homogeneous quadrants are shown with their actual luminance value. Let the K is the image we obtained after replacing pixels of homogeneous quadrants in J with -1.

4.1.2 Phase 2: Parametric Line Encoding

1. Specify threshold of fitting λ^{lmt}. The value of λ^{lmt} is 0 for lossless fitting (compression), while $\lambda^{lmt} > 0$ for lossy compression.
2. Scan the pixels of image K row by row. During scanning skip the pixels of values -1. In other words, we are skipping the pixels of homogeneous quadrants, because they are already represented by quadtree. Let R_i is the set of pixels in the i^{th} row of image K. Let R'_i is the set of pixels in the i^{th} row of image K, excluding pixels of value -1, $|R'_i| \leq |R_i|$ [1].
3. Apply the fitting process to pixel values of each row of image K separately as follows:
   (a) $R'_i = \{p_{i,1}, p_{i,2}, \ldots, p_{i,n}\}$ consists of luminance values of all pixels in i^{th} row (excluding pixels of homogeneous quadrants). Each element in R'_i can be considered as a point in 1-dimensional Euclidean space. We call the set points in R'_i as *original data*.
   (b) Take the first and the last pixels of R'_i as breakpoints, i.e., $BP = \{p_{(i,1)}, p_{(i,n)}\}$. For example, $BP = \{p_{(i,1)}, p_{(i,105)}\}$, assuming there are 105 pixels in the i^{th} row, excluding pixels of -1 value.
   (c) Divide the R'_i into segments. A segment is a set of all points (pixels) between two adjacent breakpoints. There is only one segment in the first iteration between $p_{(i,1)}$ and $p_{(i,n)}$, but when new breakpoints would be added in the following iterations then the number of segments would be increased. For example, suppose there are four breakpoints, i.e., $BP = \{p_{(i,1)}, p_{(i,55)}, p_{(i,75)}, p_{(i,105)}\}$. Then there would be three segments, $S_1 =$

[1] $|A|$ denotes cardinality of set A

$\{p_{(i,1)},\ldots,p_{(i,55)}\}$, $S_2 = \{p_{(i,55)},\ldots,p_{(i,75)}\}$, and $S_3 = \{p_{(i,75)},\ldots,p_{(i,105)}\}$.

(d) Apply the fitting process to each segment by taking each pair of adjacent breakpoints as the first and the last point of parametric line and obtain the *fitted data* by Eq. 1. Number of segments and number of points in the fitted data are equal to number of segments and number of points in the original data respectively. Let Q_i is the set of points in the fitted data, then $Q_i = \{q_{i,1}, q_{i,2}, \ldots, q_{i,n}\}$.

(e) Compute the squared distance for each point between R'_i and Q_i i.e., $d_j^2 = \left\| p_{(i,j)} - q_{(i,j)} \right\|^2$, $1 \leq j \leq n$.

(f) Find $\lambda^{max} = Max\left(d_1^2, d_2^2, \ldots, d_n^2\right)$, $\lambda^{max} \in k^{th}$ segment.

(g) If $\lambda^{max} > \lambda^{lmt}$ then replace the k^{th} segment by two new segments at the point where the squared distance is maximum. For example, if $\lambda^{max} = d_{37}^2$, it means that squared distance between $p_{(i,37)}$ and $q_{(i,37)}$ is maximum. Since $p_{(i,37)}$ is in segment $S_1 = \{p_{(i,1)},\ldots,p_{(i,55)},\}$, therefore split S_1 at $p_{(i,37)}$ and replace S_1 by $S_{1a} = \{p_{(i,1)},\ldots,p_{(i,37)}\}$ and $S_{1b} = \{p_{(i,37)},\ldots,p_{(i,55)}\}$.

(h) Add a new breakpoint bp_{new} in the set of breakpoints, i.e., $BP = \{BP\} \bigcup \{bp_{new}\}$. For example, if before spitting, $BP = \{p_{(i,1)}, p_{(i,55)}, p_{(i,75)}, p_{(i,105)}\}$ and $bp_{new} = p_{(i,37)}$ then after splitting, $BP = \{p_{(i,1)}, p_{(i,37)}, p_{(i,55)}, p_{(i,75)}, p_{(i,105)}\}$.

(i) Go to step 3c and repeat the fitting process with the new set of breakpoints, until the λ^{max} between any two points of original and fitted data is less than or equal to λ^{lmt}. Figures 5 - 11 show how the fitting process works for a row of image data. Pixels belong to third row of a image are scanned, while pixels belong to homogeneous quadrants are skipped (because they are already represented by quadtree). Then parametric line is fitted to pixel values. Initially first and last pixels are taken as breakpoints. Due to break-and-fit strategy other pixels are added as new breakpoints during fitting process. Eventually all data is fitted with 0 error (lossless coding) with very few breakpoints. It is also worth to note that by using only 3 breakpoints, as shown in Figure 6, sufficient fitting accuracy is achieved. This implies that the method is also suitable for lossy coding and yields higher compression (less number of breakpoints gives higher compression) with very little distortion.

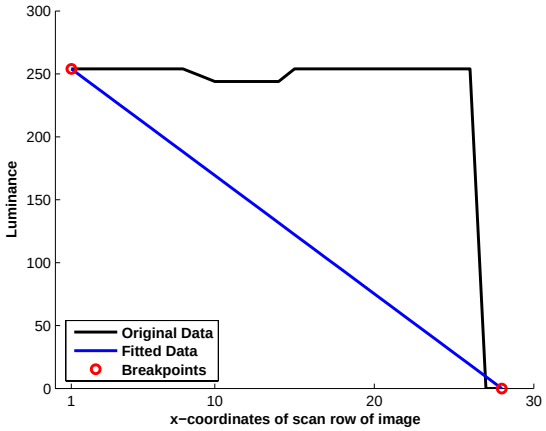


Figure 5: Fitting with 2 breakpoints.

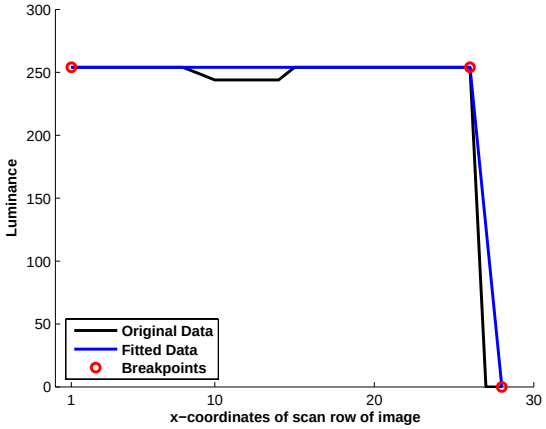


Figure 6: Fitting with 3 breakpoints.

(j) Entropy encode the final set of breakpoints (BP) and count of interpolation points (C) between breakpoints. This is required to decode the spline fitted data. For example, if the final set of breakpoints is, $BP = \{p_{(i,1)}, p_{(i,37)}, p_{(i,55)}, p_{(i,75)}, p_{(i,105)}\}$, then count of interpolating points would be: $C = \{37-1+1, 55-37+1, 75-55+1, 105-75+1\}$, i.e., $C = \{37, 19, 21, 31\}$.

4.2 Decoding

1. Create an empty image (all pixels of 0 values) L of size $w' \times h'$. The size of L is equal to the padded

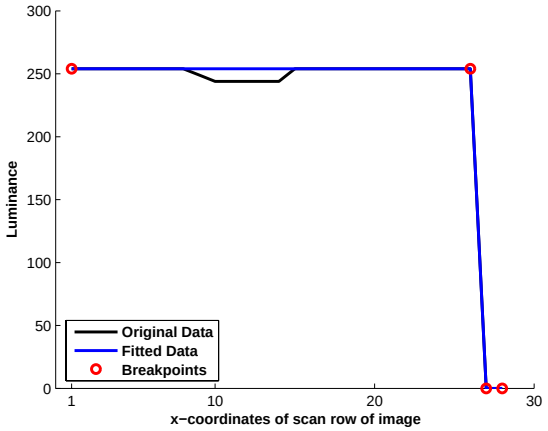


Figure 7: Fitting with 4 breakpoints.

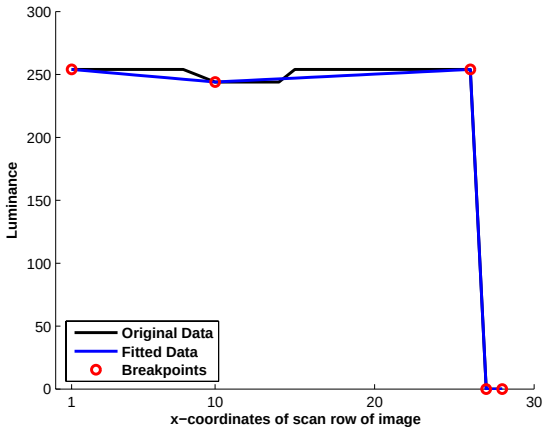


Figure 8: Fitting with 5 breakpoints.

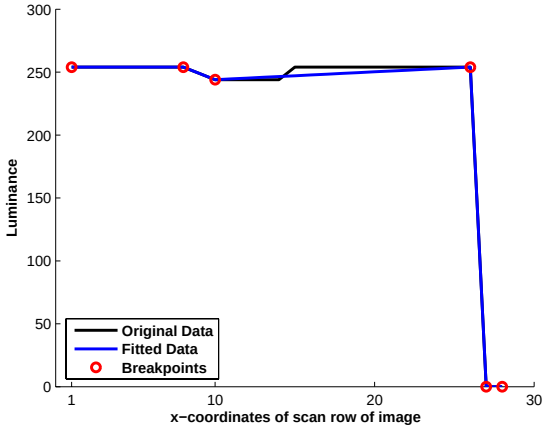


Figure 9: Fitting with 6 breakpoints.

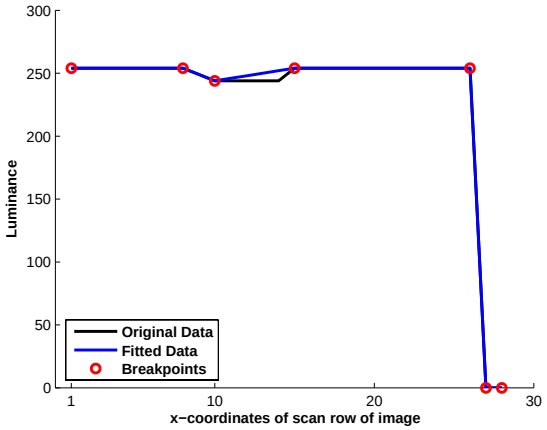


Figure 10: Fitting with 7 breakpoints.

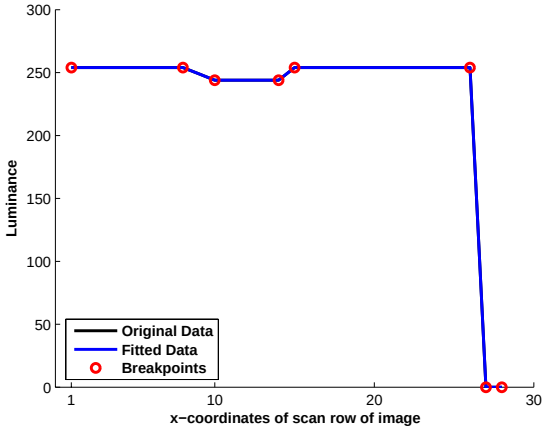


Figure 11: Fitting with 8 breakpoints.

image J.

2. Assign -1 value to each pixel of homogeneous quadrants of L. We know the coordinates of homogeneous quadrants, because we saved the size and the coordinates of homogeneous quadrants during step 4 of quadtree encoding process.
3. Using BP and C, obtained in the step 3j of parametric line encoding process, perform the parametric line interpolation. This yields the parametric line data Q_{pl}.
4. Fill L with Q_{pl} with skipping pixels of -1 value. In other words we are skipping homogeneous quadrants. This yields portion of image decoded by parametric line.
5. Fill the homogeneous quadrants of L using actual values of homogeneous quadrants. We saved the val-

ues of each block of homogeneous quadrants in
4 of encoding process, besides its size and co
nates.

6. Trim L to size $w \times h$, i.e,. equal to the size o
original image I. This is the decoded image.

5 Experiments and Results

We tested our method on various types of syntheti
ages such as clip-art, animation, cartoon, medica
age, scientific plot, wavelet transformed, etc. Ta
gives the details of input images. All input image
bitmap images, with 256 possible gray-levels (0-
stored at 8 bit per pixel. Figure 1, and Figures
17 show input images. Images are scaled to fit o
per. In figures with white background, rectangle b
are drawn around images to visualize boundaries of images. Table 2 shows the bit-rate performance of GIF, PNG and our method (QPL). From Table 2 it is evident that the QPL preformed better than GIF and PNG for all images except for *Text* image, where PNG performs slightly better. The good performance of QPL is due to the fact that in the first phase, it exploits both horizontal and vertical redundancy using quadtree structure. GIF and PNG do not exploit horizontal and vertical redundancy simultaneously. In the second phase, QPL further reduces the redundancy of those pixels showing constant or linear luminance variation by parametric line fitting. By imposing minimum block size constraint our quadtree decomposition does not fall in the trap of very small blocks.

Table 1: Details of input images used in the simulation.

Image Name	Type	$w \times h$
Planter	Clip-art	509×486
Play	Cartoon	420×315
Text	Text	704×395
Style	Computer Animation	250×314
Bone	Medical (X-ray)	560×420
Wavelet	Wavelet transform	400×352
Meshgrid	Scientific plot	560×420

6 Discussion

***Encoding/Decoding of RGB image*:** We described the algorithm for gray-scale image. To apply the algorithm on true color or RGB or HSV image, each dimension (channel) is processed separately.

***Lossless and Lossy Compression*:** The proposed

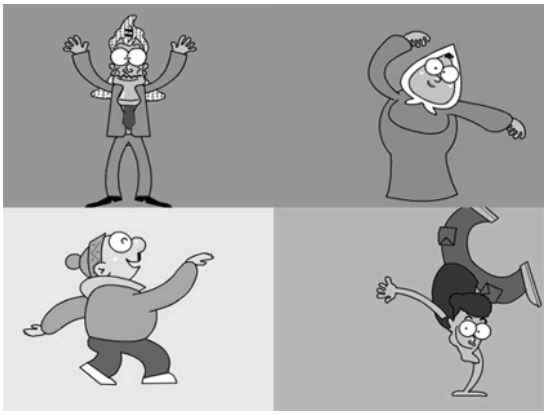
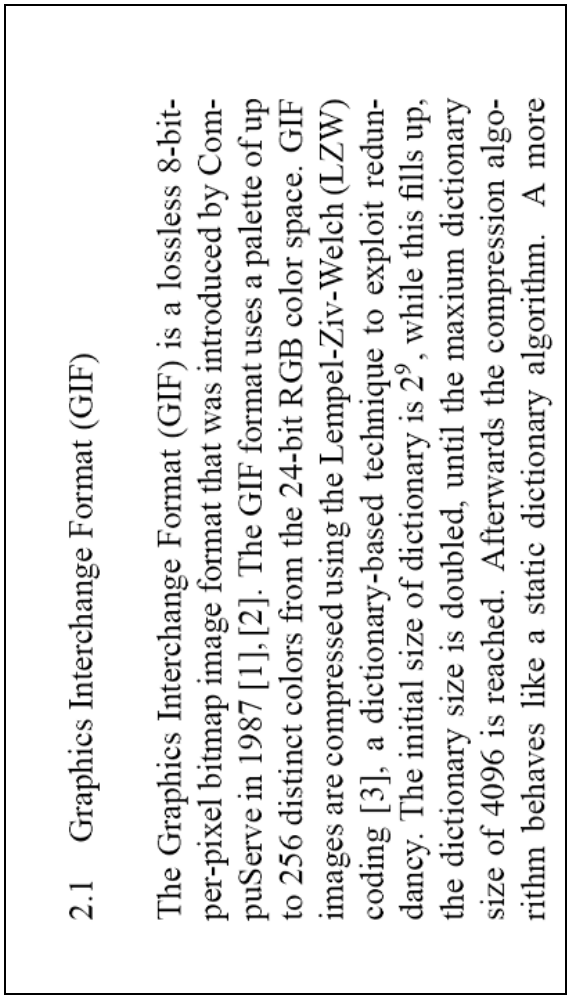


Figure 12: *Play*, a cartoon image.



2.1 Graphics Interchange Format (GIF)

The Graphics Interchange Format (GIF) is a lossless 8-bit-per-pixel bitmap image format that was introduced by CompuServe in 1987 [1], [2]. The GIF format uses a palette of up to 256 distinct colors from the 24-bit RGB color space. GIF images are compressed using the Lempel-Ziv-Welch (LZW) coding [3], a dictionary-based technique to exploit redundancy. The initial size of dictionary is 2^9, while this fills up, the dictionary size is doubled, until the maxium dictionary size of 4096 is reached. Afterwards the compression algorithm behaves like a static dictionary algorithm. A more

Figure 13: *Text*, a text image. Original image is rotated 90 degree counter-clockwise.

Table 2: Bitrate (bit per pixel, bpp) performance of GIF, PNG and QPL for lossless compression. For QPL minimum block size is 4×4.

Image Name	bpp (GIF)	bpp (PNG)	bpp (QPL)
Planter	0.6633	0.7965	0.6180
Play	1.5689	1.5618	1.3497
Text	1.7398	1.5150	1.5220
Style	0.8728	1.0517	0.8204
Bone	0.5452	0.5113	0.4389
Wavelet	0.3891	0.4497	0.3587
Meshgrid	0.3866	0.4215	0.3349

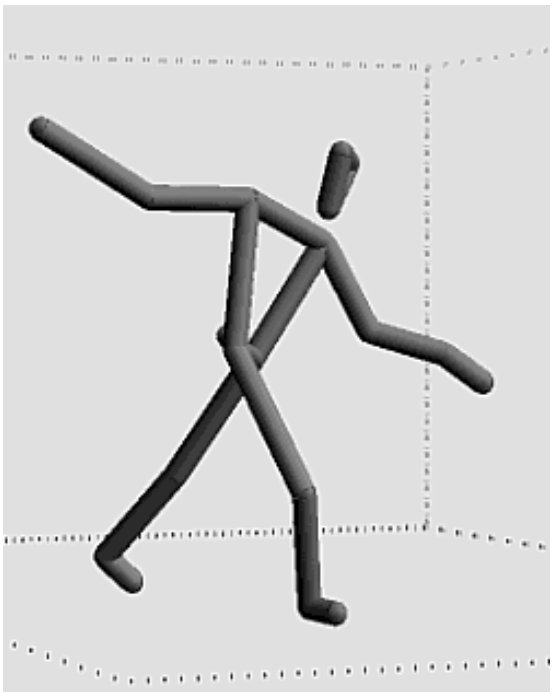


Figure 14: *Style*, an animation image.

method can be used for both lossless and lossy compression, while both GIF and PNG are inherently lossless compression schemes and cannot be used for lossy compression. In our method, the only difference between lossless and lossy compression is the value of parameter λ^{lmt}, i.e., threshold of fitting. The value of λ^{lmt} is zero for lossless compression, while λ^{lmt} is greater than zero for lossy compression.

***Using higher degree Bézier curves*:** Parametric line is

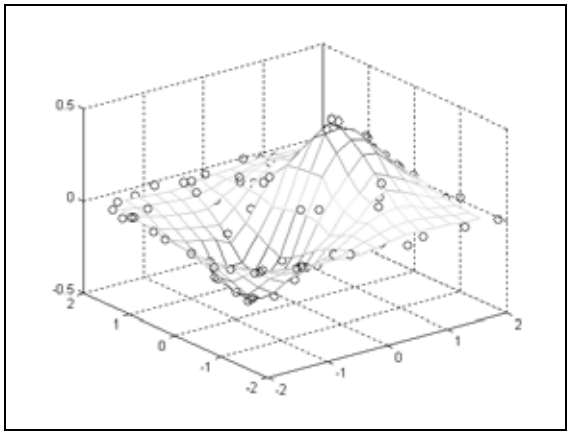


Figure 15: *Meshgrid*, a scientific plot.

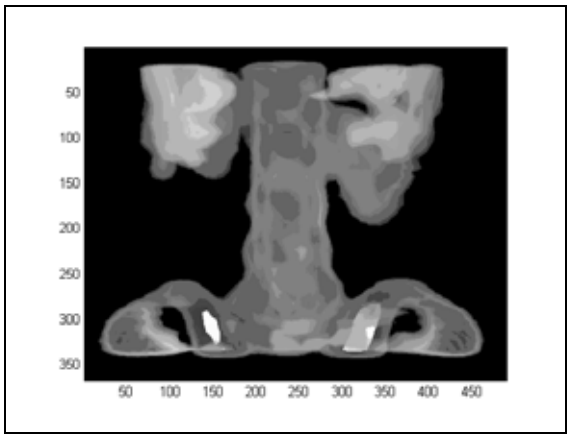


Figure 16: *Bone*, a medical (x-ray) image.

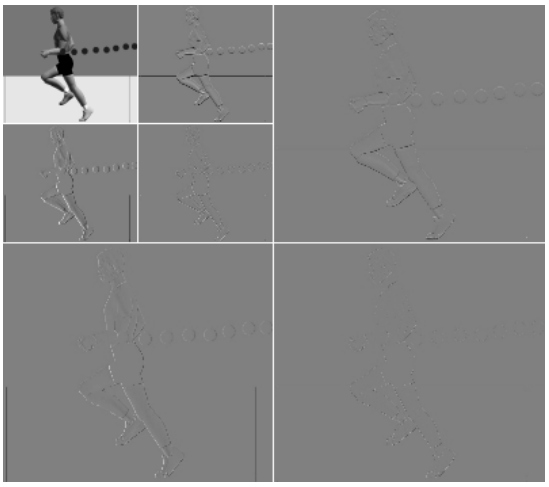


Figure 17: *Wavelet*, a wavelet transform image.

a linear Bézier curve. Mathematical model of parametric line (linear Bézier) is analogous to higher degree Bézier curves such as quadratic or cubic Bézier curves. After experiments, we found that parametric line is more suitable than higher degree Bézier curves; both from compression and computational perspectives. Fitting by any form of straight line e.g., polyline [1, 6] would yield the same results as parametric line.

7 Conclusion

We presented a new hybrid scheme for image data compression using quadtree decomposition and parametric line fitting. We described the encoding and decoding steps of algorithm. The encoding composed of two main phases. In the first phase, the method finds the homogeneous and non-homogeneous quadrants using quadtree decomposition with constraint of minimum block size. Homogeneous quadrants are represented by quadtree. In the second phase, the method fits the parametric line to non-homogeneous pixels of each row. Experimental results show that the proposed scheme performs better than well-known lossless image compression techniques for several types of synthetic images.

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