

Use Lagrange interpolation to find a polynomial that passes through the points:

$$(-1, 0), (2, 1), (3, 1), (5, 2)$$

$$L_1(x) = \frac{(x-2)(x-3)(x-5)}{(-1-2)(-1-3)(-1-5)} = \frac{(x-2)(x-3)(x-5)}{-72}$$

$$L_2(x) = \frac{(x+1)(x-3)(x-5)}{(2+1)(2-3)(2-5)} = \frac{(x+1)(x-3)(x-5)}{9}$$

$$L_3(x) = \frac{(x+1)(x-2)(x-5)}{(3+1)(3-2)(3-5)} = \frac{(x+1)(x-2)(x-5)}{-8}$$

$$L_4(x) = \frac{(x+1)(x-2)(x-3)}{(5+1)(5-2)(5-3)} = \frac{(x+1)(x-2)(x-3)}{36}$$

$$P_3(x) = 0 \cdot \frac{(x-2)(x-3)(x-5)}{-72} + 1 \cdot \frac{(x+1)(x-3)(x-5)}{9} +$$

$$1 \cdot \frac{(x+1)(x-2)(x-5)}{-8} + 2 \cdot \frac{(x+1)(x-2)(x-3)}{36}$$

$$= \frac{(x+1)(x-3)(x-5)}{9} - \frac{(x+1)(x-2)(x-5)}{8} + \frac{(x+1)(x-2)(x-3)}{18}$$

$$= \frac{1}{24}x^3 - \frac{1}{4}x^2 + \frac{11}{24}x + 3$$