## تمرین دوم طراحی الگوریتم ها

## 4-4 More recurrence examples

Give asymptotic upper and lower bounds for T(n) in each of the following recurrences. Assume that T(n) is constant for sufficiently small n. Make your bounds as tight as possible, and justify your answers.

a. 
$$T(n)=3T(n/2)+nlgn$$
.

b. 
$$T(n)=5T(n/5)+n/lgn$$
.

c. 
$$T(n)=4T(n/2)+n^{2\sqrt{n}}$$
.

d. 
$$T(n)=3T(n/3+5)+n/2$$
.

$$e$$
.  $T(n)=2T(n/2)+n/lgn$ .

f. 
$$T(n)=T(n/2)+T(n/4)+T(n/8)+n$$
.

$$g. T(n)=T(n-1)+1/n.$$

$$h. T(n)=T(n-1)+lgn.$$

$$i. \quad T(n)=T(n-2)+2lgn.$$

$$j$$
.  $T(n) = \sqrt{nT(\sqrt{n}) + n}$ .

## 6.4-2

Argue the correctness of HEAPSORT using the following loop invariant:

At the start of each iteration of the **for** loop of lines 2–5, the subarray A[1..i] is a max-heap containing the i smallest elements of A[1..n], and the subarray A[i + 1..n] contains the n - i largest elements of A[1..n], sorted.

## 6.4-5 \*

Show that when all elements are distinct, the best-case running time of heapsort is  $(n \lg n)$ .