# D_worksheet

March 13, 2023

## 1 Worksheet D

***variationalform*** https://variationalform.github.io/

***Just Enough: progress at pace*** https://variationalform.github.io/

https://github.com/variationalform

Simon Shaw https://www.brunel.ac.uk/people/simon-shaw.



This document uses python

and also makes use of LaTeX

in Markdown

### 1.1 What this is about:

This worksheet is based on the material in the notebook

- regress: polynomial and logistic regression.

Note that while the 'lecture' notebooks are prefixed with `1_`, `2_` and so on, to indicate the order in which they should be studied, the worksheets are prefixed with `A_`, `B_`, …

[1]:
```
# useful imports
import numpy as np
import seaborn as sns
import matplotlib.pyplot as plt
from sklearn import linear_model
```

#### 1.1.1 Exercise 1

A straight line has gradient $m = 2$ and $y$-intercept $c = 4$. Sketch it, and determine the value of $x$ for which $y = 8$.

[2]:
```
# Answer here - create more cells as necessary
```

### 1.1.2 Exercise 2

A straight line with gradient $m$ passes though the point $(x_0, y_0)$ then show that $y - y_0 = m(x - x_0)$. This is called the *point slope* form.

[3]:
```
# Answer here - create more cells as necessary
```

### 1.1.3 Exercise 3

A line with gradient $m = 5$ passes through $(x, y) = (-1, 2)$. Find the equation of the line in the form $y = mx + c$.

[4]:
```
# Answer here - create more cells as necessary
```

### 1.1.4 Exercise 4

Recall Anscombe's data set. We used the following codse to split it into its four sub-sets, and we also produced scateterplots, as shown below for the first data subset.

```
dfa = sns.load_dataset('anscombe')
print("The size of Anscombe's data set is:", dfa.shape)
dfa.dataset.unique()
dfa1 = dfa.loc[dfa['dataset'] == 'I']
dfa2 = dfa.loc[dfa['dataset'] == 'II']
dfa3 = dfa.loc[dfa['dataset'] == 'III']
dfa4 = dfa.loc[dfa['dataset'] == 'IV']
sns.scatterplot(data=dfa1, x="x", y="y")
dfa1.describe()
```

Implement linear regression for this fist dataset `dfa1`. Then implement ridge and LASSO regression. Plot your regreesion lines on the same plot and include the underlying data.

You might find the following useful:

```
dfreg = dfa1.sort_values('x', ascending = True).reset_index(drop=True)
```

After this you can either reassign `dfa1 = dfreg` or work directly with `dfreg`.

[5]:
```
# Answer here - create more cells as necessary
```

### 1.1.5 Exercise 5

Repeat Exercise 4 but with `dfa2`.

[6]:
```
# Answer here - create more cells as necessary
```

### 1.1.6 Exercise 6

Repeat Exercise 5 but with `dfa3`.

[7]:
```
# Answer here - create more cells as necessary
```

### 1.1.7 Exercise 7

Repeat Exercise 6 but with `dfa4`.

[8]:
```
# Answer here - create more cells as necessary
```

## 2 Outline Suggested Solutions

The following are suggestions for solutions of the above problems. Please have a go first though before looking at these.

### 2.0.1 Solution 1

The line passes through the vertical axis at $y = 4$ and climbs a vertical distance of 2 for every unit of horizontal distance. When $y = 8$, we have from $y = mx + c$ that $x = (y - c)/m = 2$.

### 2.0.2 Solution 2

At another (arbitrary but distinct) point $(x, y)$ the gradient is $m = (y - y_0)/(x - x_0)$. The *point slope* form follows.

### 2.0.3 Solution 3

$y - y_0 = m(x - x_0)$ with $(x_0, y_0) = (-1, 2)$ and $m = 5$. Therefore

$$y = mx - mx_0 + y_0 = 5x - (5 \times -1 - 2) = 5x + 7.$$

### 2.0.4 Solution 4

An outline solution to Exercise 4 follows.

[9]:
```
dfa = sns.load_dataset('anscombe')
print("The size of Anscombe's data set is:", dfa.shape)
```

```
The size of Anscombe's data set is: (44, 3)
```

[10]:
```
dfa.dataset.unique()
```

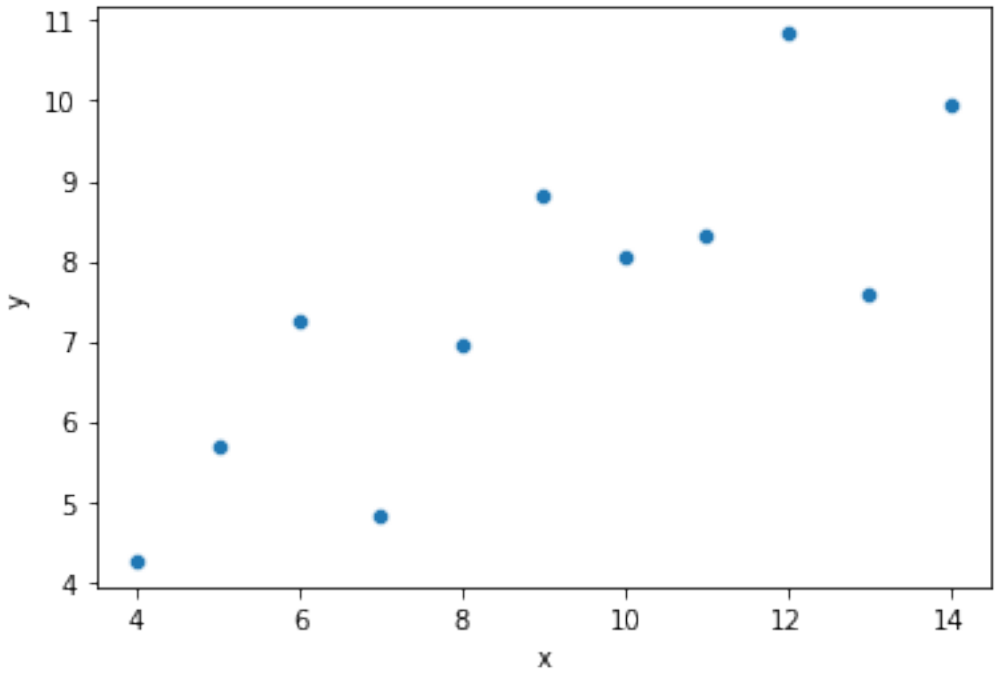
[10]:
```
array(['I', 'II', 'III', 'IV'], dtype=object)
```

[11]:
```
dfa1 = dfa.loc[dfa['dataset'] == 'I']
dfa2 = dfa.loc[dfa['dataset'] == 'II']
dfa3 = dfa.loc[dfa['dataset'] == 'III']
dfa4 = dfa.loc[dfa['dataset'] == 'IV']
```

[12]:
```
sns.scatterplot(data=dfa1, x="x", y="y")
dfa1.describe()
```

[12]:

| | x | y |
|---|---|---|
| count | 11.000000 | 11.000000 |
| mean | 9.000000 | 7.500909 |
| std | 3.316625 | 2.031568 |
| min | 4.000000 | 4.260000 |
| 25% | 6.500000 | 6.315000 |
| 50% | 9.000000 | 7.580000 |
| 75% | 11.500000 | 8.570000 |
| max | 14.000000 | 10.840000 |




[13]:
```
dfa1.head()
```

[13]:

| | dataset | x | y |
|---|---|---|---|
| 0 | I | 10.0 | 8.04 |
| 1 | I | 8.0 | 6.95 |
| 2 | I | 13.0 | 7.58 |
| 3 | I | 9.0 | 8.81 |
| 4 | I | 11.0 | 8.33 |

[14]:
```
dfreg = dfa1.sort_values('x', ascending = True).reset_index(drop=True)
```

[15]:
```
dfreg.head()
```

[15]:

| | dataset | x | y |
|---|---|---|---|
| 0 | I | 4.0 | 4.26 |
| 1 | I | 5.0 | 5.68 |
| 2 | I | 6.0 | 7.24 |
| 3 | I | 7.0 | 4.82 |
| 4 | I | 8.0 | 6.95 |

[16]:
```
X_vals = dfreg.iloc[:,1].values.reshape(-1,1)
y_vals = dfreg.iloc[:,2].values.reshape(-1,1)
#print(X_vals,'\n', y_vals)
```

[17]:
```
# standard (usual) regression
reg_usual = linear_model.LinearRegression()
reg_usual.fit(X_vals, y_vals)
# Make predictions
y_pred_usual = reg_usual.predict(X_vals)
print('reg_usual_coef_ = ', reg_usual.coef_)
print('reg_usual_intercept_ = ', reg_usual.intercept_)
```

```
reg_usual_coef_ =  [[0.50009091]]
reg_usual_intercept_ =  [3.00009091]
```

[18]:
```
# ridge regression
reg_ridge = linear_model.Ridge(alpha=0.5)
reg_ridge.fit(X_vals, y_vals)
# Make predictions
y_pred_ridge = reg_ridge.predict(X_vals)
print('reg_ridge_coef_ = ', reg_ridge.coef_)
print('reg_ridge_intercept_ = ', reg_ridge.intercept_)
```

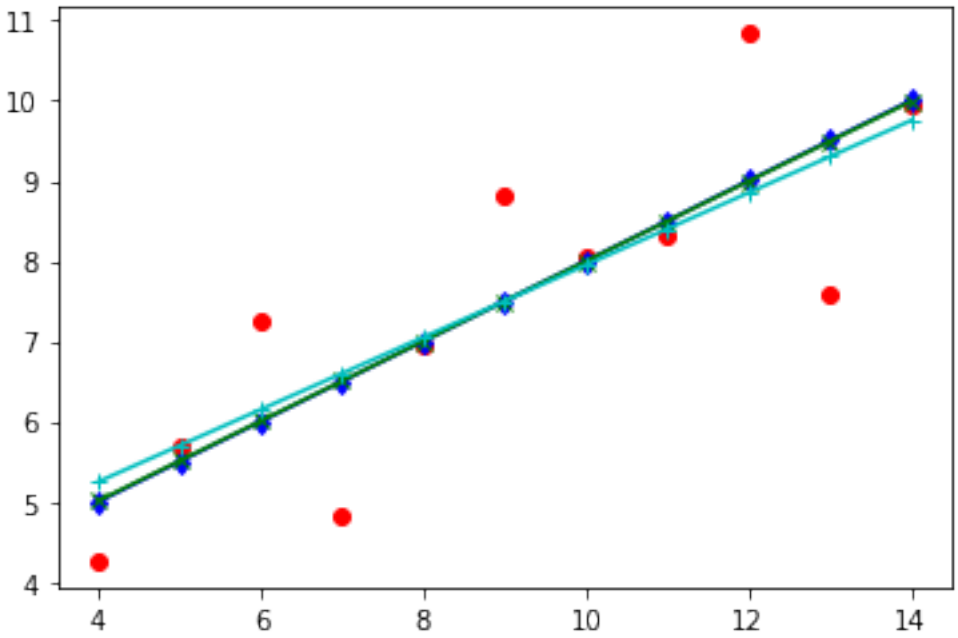
```
reg_ridge_coef_ =  [[0.49782805]]
reg_ridge_intercept_ =  [3.0204566]
```

[19]:
```
# LASSO regression
reg_lasso = linear_model.Lasso(alpha=0.5)
reg_lasso.fit(X_vals, y_vals)
# Make predictions
y_pred_lasso = reg_lasso.predict(X_vals)
print('reg_lasso_coef_ = ', reg_lasso.coef_)
print('reg_lasso_intercept_ = ', reg_lasso.intercept_)
```

```
reg_lasso_coef_ =  [0.45009091]
reg_lasso_intercept_ =  [3.45009091]
```

[20]:
```
plt.plot(X_vals,y_vals,'.r',marker='o')
plt.plot(X_vals,y_pred_usual,'b',marker='d')
plt.plot(X_vals,y_pred_ridge,'g',marker='x')
plt.plot(X_vals,y_pred_lasso,'c',marker='+')
```

[20]: [<matplotlib.lines.Line2D at 0x7fbb08b44470>]




### 2.0.5 Solution 5

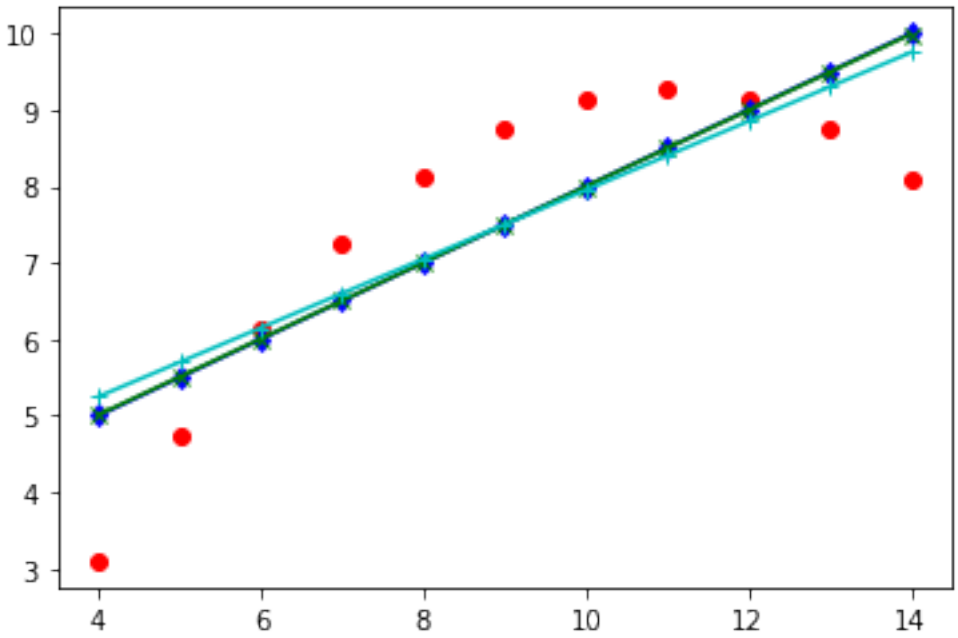
An outline solution to Exercise 5 follows.

[21]:
```
dfreg = dfa2.sort_values('x', ascending = True).reset_index(drop=True)
X_vals = dfreg.iloc[:,1].values.reshape(-1,1)
y_vals = dfreg.iloc[:,2].values.reshape(-1,1)

# standard (usual) regression
reg_usual = linear_model.LinearRegression()
reg_usual.fit(X_vals, y_vals)
# Make predictions
y_pred_usual = reg_usual.predict(X_vals)
print('reg_usual_coef_ = ', reg_usual.coef_)
print('reg_usual_intercept_ = ', reg_usual.intercept_)

# ridge regression
reg_ridge = linear_model.Ridge(alpha=0.5)
reg_ridge.fit(X_vals, y_vals)
# Make predictions
y_pred_ridge = reg_ridge.predict(X_vals)
print('reg_ridge_coef_ = ', reg_ridge.coef_)
print('reg_ridge_intercept_ = ', reg_ridge.intercept_)
```

```
# LASSO regression
reg_lasso = linear_model.Lasso(alpha=0.5)
reg_lasso.fit(X_vals, y_vals)
# Make predictions
y_pred_lasso = reg_lasso.predict(X_vals)
print('reg_lasso_coef_ = ', reg_lasso.coef_)
print('reg_lasso_intercept_ = ', reg_lasso.intercept_)

plt.plot(X_vals,y_vals,'.r',marker='o')
plt.plot(X_vals,y_pred_usual,'b',marker='d')
plt.plot(X_vals,y_pred_ridge,'g',marker='x')
plt.plot(X_vals,y_pred_lasso,'c',marker='+')
```

```
reg_usual_coef_ =  [[0.5]]
reg_usual_intercept_ =  [3.00090909]
reg_ridge_coef_ =  [[0.49773756]]
reg_ridge_intercept_ =  [3.02127108]
reg_lasso_coef_ =  [0.45]
reg_lasso_intercept_ =  [3.45090909]
```

[21]: [<matplotlib.lines.Line2D at 0x7fbaf9454780>]



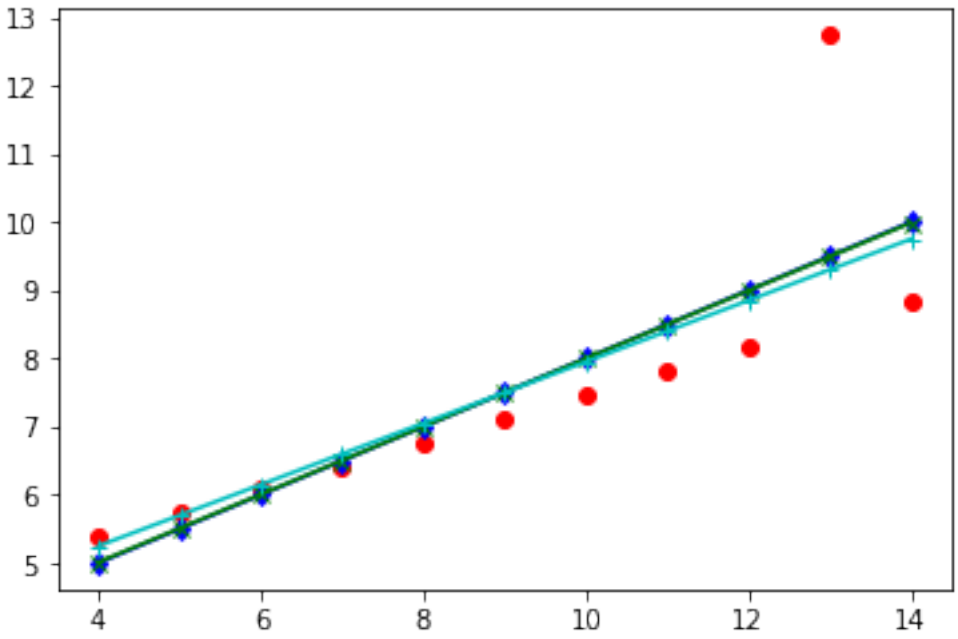

### 2.0.6 Solution 6

An outline solution to Exercise 6 follows.

[22]:
```
dfreg = dfa3.sort_values('x', ascending = True).reset_index(drop=True)
X_vals = dfreg.iloc[:,1].values.reshape(-1,1)
y_vals = dfreg.iloc[:,2].values.reshape(-1,1)

# standard (usual) regression
reg_usual = linear_model.LinearRegression()
reg_usual.fit(X_vals, y_vals)
# Make predictions
y_pred_usual = reg_usual.predict(X_vals)
print('reg_usual_coef_ = ', reg_usual.coef_)
print('reg_usual_intercept_ = ', reg_usual.intercept_)

# ridge regression
reg_ridge = linear_model.Ridge(alpha=0.5)
reg_ridge.fit(X_vals, y_vals)
# Make predictions
y_pred_ridge = reg_ridge.predict(X_vals)
print('reg_ridge_coef_ = ', reg_ridge.coef_)
print('reg_ridge_intercept_ = ', reg_ridge.intercept_)

# LASSO regression
reg_lasso = linear_model.Lasso(alpha=0.5)
reg_lasso.fit(X_vals, y_vals)
# Make predictions
y_pred_lasso = reg_lasso.predict(X_vals)
print('reg_lasso_coef_ = ', reg_lasso.coef_)
print('reg_lasso_intercept_ = ', reg_lasso.intercept_)

plt.plot(X_vals,y_vals,'.r',marker='o')
plt.plot(X_vals,y_pred_usual,'b',marker='d')
plt.plot(X_vals,y_pred_ridge,'g',marker='x')
plt.plot(X_vals,y_pred_lasso,'c',marker='+')
```

```
reg_usual_coef_ =  [[0.49972727]]
reg_usual_intercept_ =  [3.00245455]
reg_ridge_coef_ =  [[0.49746606]]
reg_ridge_intercept_ =  [3.02280543]
reg_lasso_coef_ =  [0.44972727]
reg_lasso_intercept_ =  [3.45245455]
```

[22]:
```
[<matplotlib.lines.Line2D at 0x7fbae8b86c50>]
```




### 2.0.7 Solution 7

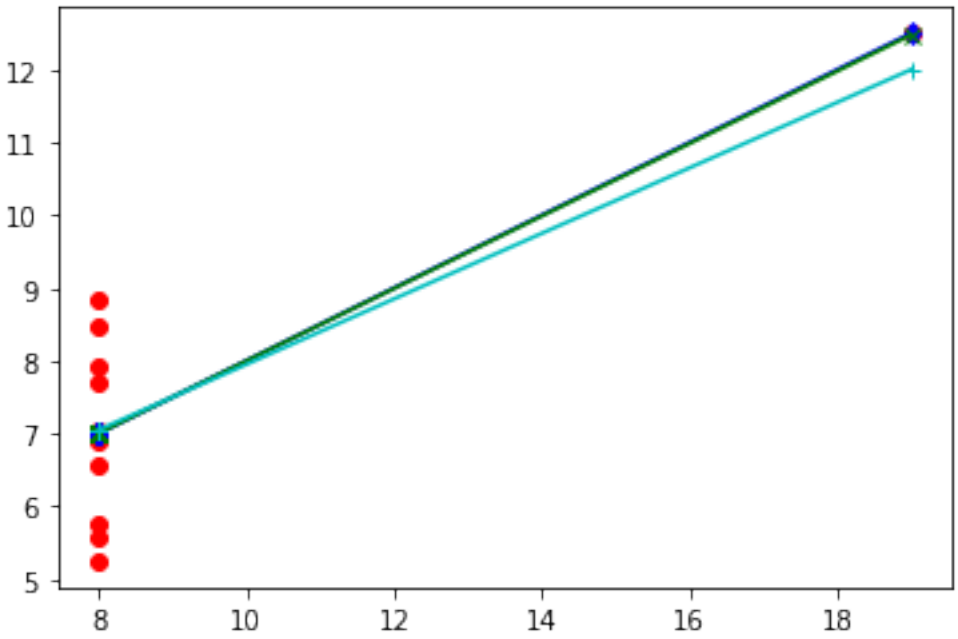
An outline solution to Exercise 7 follows.

[23]:
```
dfreg = dfa4.sort_values('x', ascending = True).reset_index(drop=True)
X_vals = dfreg.iloc[:,1].values.reshape(-1,1)
y_vals = dfreg.iloc[:,2].values.reshape(-1,1)

# standard (usual) regression
reg_usual = linear_model.LinearRegression()
reg_usual.fit(X_vals, y_vals)
# Make predictions
y_pred_usual = reg_usual.predict(X_vals)
print('reg_usual_coef_ = ', reg_usual.coef_)
print('reg_usual_intercept_ = ', reg_usual.intercept_)

# ridge regression
reg_ridge = linear_model.Ridge(alpha=0.5)
reg_ridge.fit(X_vals, y_vals)
# Make predictions
y_pred_ridge = reg_ridge.predict(X_vals)
print('reg_ridge_coef_ = ', reg_ridge.coef_)
print('reg_ridge_intercept_ = ', reg_ridge.intercept_)

# LASSO regression
reg_lasso = linear_model.Lasso(alpha=0.5)
```

```
reg_lasso.fit(X_vals, y_vals)
# Make predictions
y_pred_lasso = reg_lasso.predict(X_vals)
print('reg_lasso_coef_ = ', reg_lasso.coef_)
print('reg_lasso_intercept_ = ', reg_lasso.intercept_)

plt.plot(X_vals,y_vals,'.r',marker='o')
plt.plot(X_vals,y_pred_usual,'b',marker='d')
plt.plot(X_vals,y_pred_ridge,'g',marker='x')
plt.plot(X_vals,y_pred_lasso,'c',marker='+')
```

```
reg_usual_coef_ =  [[0.49990909]]
reg_usual_intercept_ =  [3.00172727]
reg_ridge_coef_ =  [[0.49764706]]
reg_ridge_intercept_ =  [3.02208556]
reg_lasso_coef_ =  [0.44990909]
reg_lasso_intercept_ =  [3.45172727]
```

[23]: `[<matplotlib.lines.Line2D at 0x7fbad8a63da0>]`




## 2.1 Technical Notes, Production and Archiving

Ignore the material below. What follows is not relevant to the material being taught.

### Production Workflow

- Finalise the notebook material above

- Clear and fresh run of entire notebook
- Create html slide show:
  - `jupyter nbconvert --to slides D_worksheet.ipynb`
- Set `OUTPUTTING=1` below
- Comment out the display of web-sourced diagrams
- Clear and fresh run of entire notebook
- Comment back in the display of web-sourced diagrams
- Clear all cell output
- Set `OUTPUTTING=0` below
- Save
- git add, commit and push to FML
- copy PDF, HTML etc to web site
  - git add, commit and push
- rebuild binder

Some of this originated from https://stackoverflow.com/questions/38540326/save-html-of-a-jupyter-notebook-from-within-the-notebook These lines create a back up of the notebook. They can be ignored. At some point this is better as a bash script outside of the notebook

[24]:
```
%%bash
NBROOTNAME='D_worksheet'
OUTPUTTING=1

if [ $OUTPUTTING -eq 1 ]; then
  jupyter nbconvert --to html $NBROOTNAME.ipynb
  cp $NBROOTNAME.html ../backups/$(date +"%m_%d_%Y-%H%M%S")_$NBROOTNAME.html
  mv -f $NBROOTNAME.html ./formats/html/

  jupyter nbconvert --to pdf $NBROOTNAME.ipynb
  cp $NBROOTNAME.pdf ../backups/$(date +"%m_%d_%Y-%H%M%S")_$NBROOTNAME.pdf
  mv -f $NBROOTNAME.pdf ./formats/pdf/

  jupyter nbconvert --to script $NBROOTNAME.ipynb
  cp $NBROOTNAME.py ../backups/$(date +"%m_%d_%Y-%H%M%S")_$NBROOTNAME.py
  mv -f $NBROOTNAME.py ./formats/py/
else
  echo 'Not Generating html, pdf and py output versions'
fi
```

```
Not Generating html, pdf and py output versions
```