# DIP Project

## Poisson Image Editing

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# UNDERSTANDING

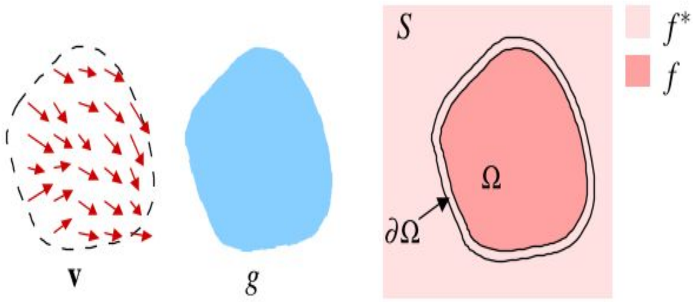
- We aim to seamlessly blend source images into target images
- In the region of interest, we try to make the boundary values of the source image equal to the target image’s boundary values, and the gradients inside the region equal to the gradients of the source interior
- The gradients of the source act as a guidance field
- We thus set up a Poisson equation to perform interpolation in the region of interest
- In this way, the reconstructed function in the region of interest interpolates the boundary conditions inwards, while following the spatial variations of the guidance field as closely as possible
- This leads to seamless cloning

# THEORY

**Guided Interpolation**

We detail image interpolation using a guidance vector field(in this case the gradient of the source). As it is enough to solve the interpolation problem for each color component separately, we consider only scalar image functions. Figure on the right illustrates the notations: let **S**, a closed subset of $\mathbf{R}^2$ , be the image definition domain, and let **Ω** be a closed subset of **S** with boundary ∂ **Ω.** Let **f*** be a known scalar function defined over **S** minus the interior of **Ω** and let **f** be an unknown scalar function defined over the interior of **Ω**. Finally, let **v** be a vector field defined over **Ω**.The simplest interpolant **f** of **f*** over **Ω** is the membrane interpolant defined as the solution of the minimization problem:

$$\min_f \iint_\Omega |\nabla f|^2 \text{ with } f|_{\partial\Omega} = f^*|_{\partial\Omega}, \qquad (1)$$




where $\nabla . = \left[\frac{\partial .}{\partial x}, \frac{\partial .}{\partial y}\right]$ is the gradient operator

# THEORY

The solution to this equation is the following:

$$\Delta f = 0 \text{ over } \Omega \text{ with } f|_{\partial\Omega} = f^*|_{\partial\Omega}, \qquad (2)$$

Where $\Delta$ is the Laplacian operator. In our case we approximate the Laplacian as the 3*3 matrix below and solve the equation in a discrete sense over each pixel.

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | -4 | 0 |
| 1 | 0 | 1 |

# THEORY

A guidance field is a vector field **v** used in an extended version of the minimization problem (1) :

$$\min_{f} \iint_{\Omega} |\nabla f - \mathbf{v}|^2 \text{ with } f|_{\partial\Omega} = f^*|_{\partial\Omega}, \quad (3)$$

whose solution is the unique solution of the following Poisson equation with Dirichlet boundary conditions :

$$\Delta f = \operatorname{div}\mathbf{v} \text{ over } \Omega, \text{ with } f|_{\partial\Omega} = f^*|_{\partial\Omega}, \quad (4)$$

where $\operatorname{div}\mathbf{v} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$ is the divergence of **v** = (u,v). Thus, 3 such equations are solved separately for each of the 3 channels.

# SEAMLESS CLONING

In case of seamless cloning the basic choice for the guidance field **v** is a gradient field taken directly from a source image. Denoting by **g** this source image, the interpolation is performed under the guidance of $\mathbf{v} = \nabla g,$

Using equation **2** this reduces to

$$\Delta f = \Delta g \text{ over } \Omega, \text{ with } f|_{\partial\Omega} = f^*|_{\partial\Omega}.$$

# MIXING GRADIENTS

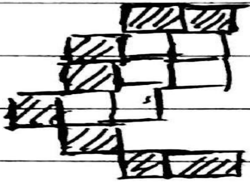
With the tool described in the previous section, no trace of the destination image **f***  is kept inside **Ω.** However,there are situations where it is desirable to combine properties of **f*** with those of **g**, for example to add objects with holes, or partially transparent ones, on top of a textured or cluttered background.

So instead of using the guidance vector field as the gradient of the source only,we retain the stronger of the two (the source and the target) on each point of **Ω** as the value of the guidance vector field at that point. Mathematically :

$$\text{for all } \mathbf{x} \in \Omega,\ \mathbf{v}(\mathbf{x}) = \begin{cases} \nabla f^*(\mathbf{x}) & \text{if } |\nabla f^*(\mathbf{x})| > |\nabla g(\mathbf{x})|, \\ \nabla g(\mathbf{x}) & \text{otherwise.} \end{cases}$$

# IMPLEMENTATION DETAILS

Let $I$ be the composite image and $S$ be the source image and $T$ be the target image.



□ - Inside the Boundary.

- Boundary Pixel

1) For a pixel $P$ inside like □

$$\Delta^2 I(x,y) = \Delta^2 S(x,y)$$

$$\Rightarrow I(x+1,y) + I(x-1,y) + I(x,y-1) + I(x,y+1) - 4I(x,y)$$

$$= S(x+1,y) + S(x-1,y) + S(x,y-1) + S(x,y+1) - 4S(x,y)$$

2) For a pixel $P$ on the boundary

$$I(x,y) = T(x,y)$$

To implement this we build a sparse matrix $A$ containing the coefficients and two column matrices for the laplacian of the source and the composite.

all pixels in the region

$$\begin{bmatrix} & \\ 0 & -1 & 4 & 0 & -1 & 0 & 0 & 0 & \cdots & \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ & \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$$

$S(x+1,y)+S(x,y+1)+S(x-1,y)+S(x,y-1)-4S(x,y)$

$T(x,y)$

For pixel corresponding to target pixels ~~&~~ only one coefficient will be 1 in the coefficient matrix and others will be 0. The corresponding value at RHS in the same row will be $T(x,y)$

For pixels inside, corresponding entries will be $-1$, and 4 and RHS will be $+[S(x+1,y)+S(x,y+1)+S(x-1,y)+S(x,y-1)]-4S(x,y)$

While implementation we number the corresponding pixels with numbers from 1 to $n$ for ease of building the matrix. Numbering is done so that while building the row for $I(x,y)$ values at $(x-1,y)$, $(x,y-1)$, $(x,y+1)$ and $(x+1,y)$ can be accessed.

# APPLICATION SCENARIOS

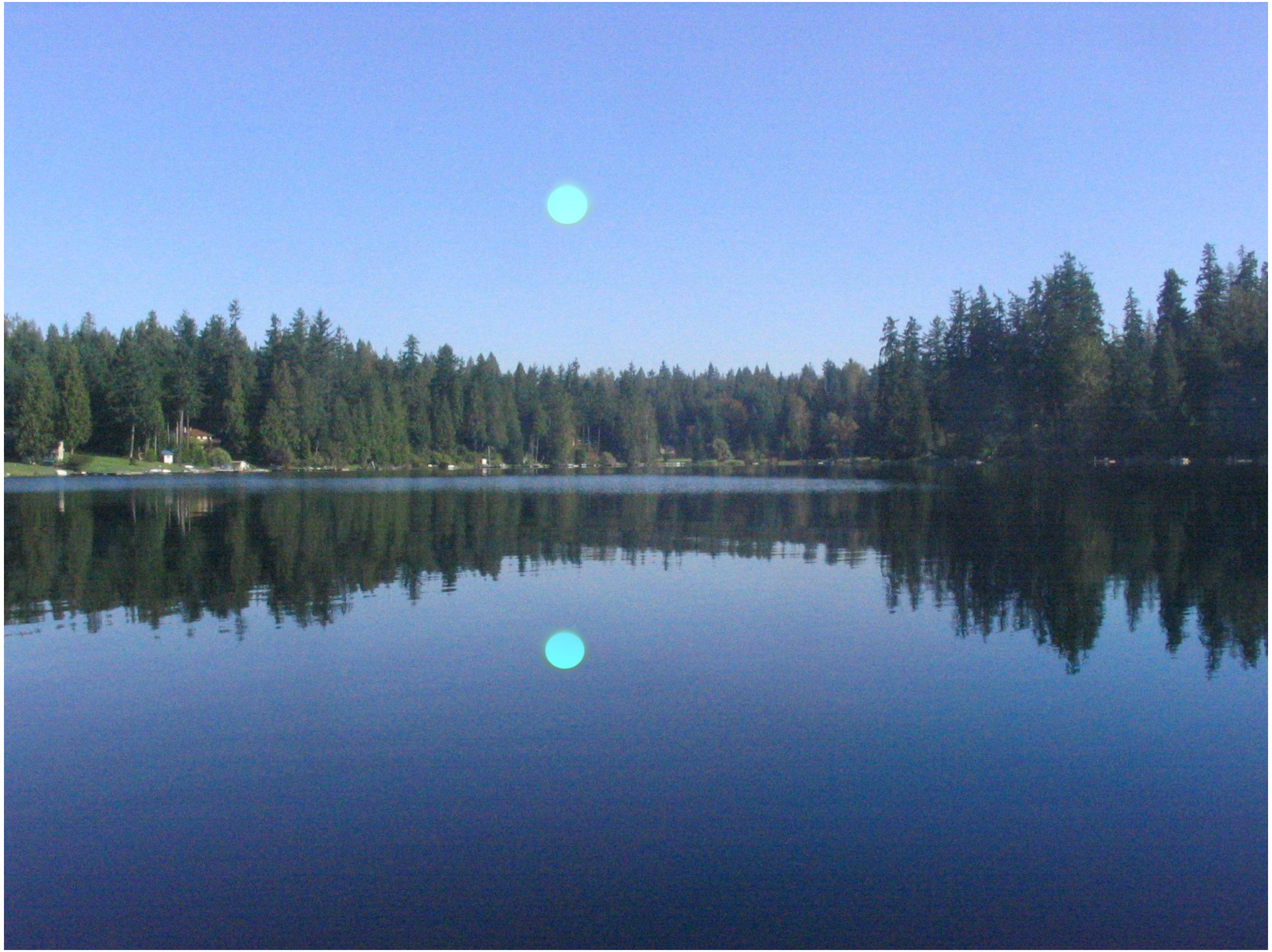
# SEAMLESS CLONING

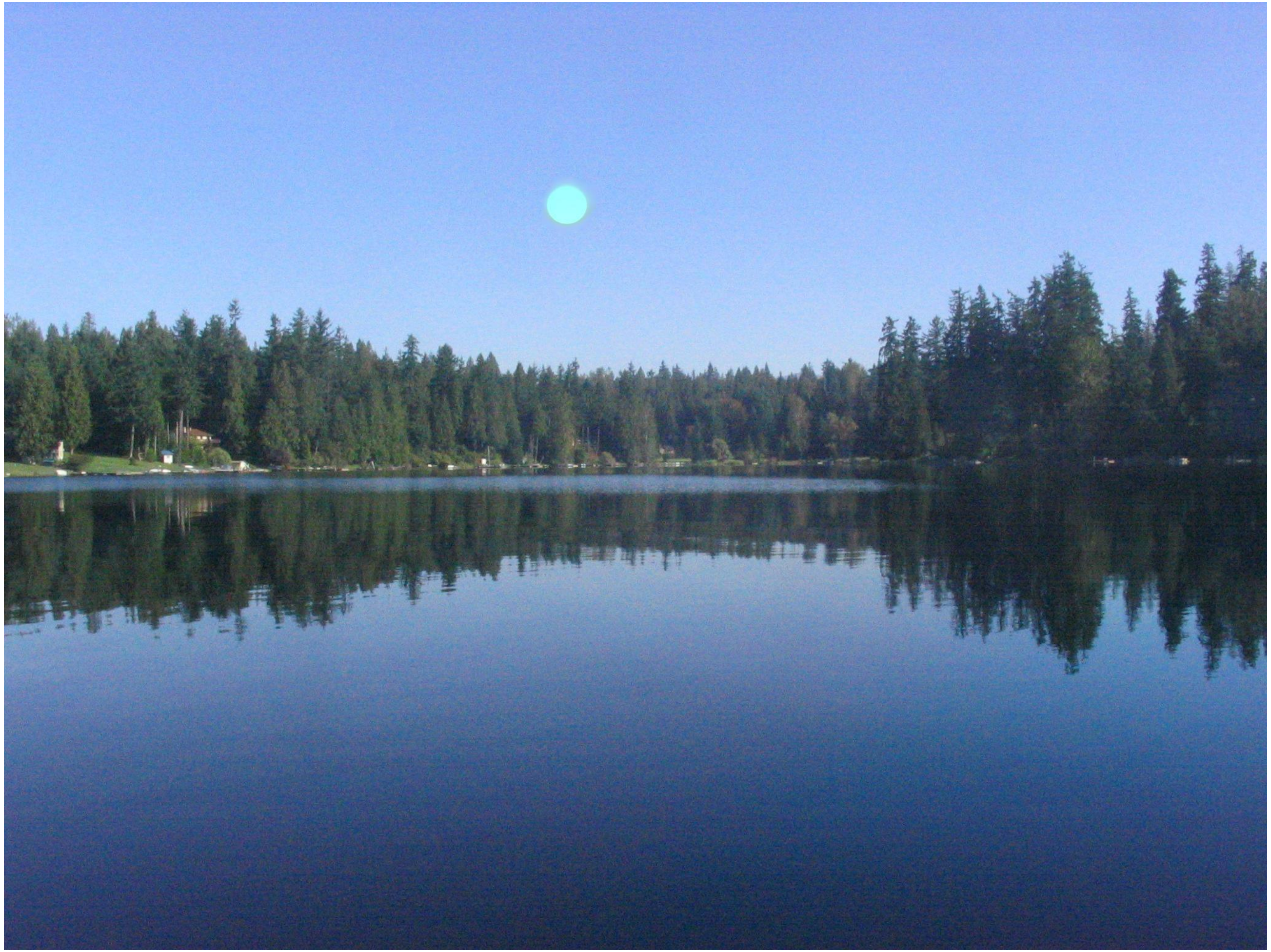


JPN 0 - 0 GRE
51:59
Continental
BAND-AID
2014 FIFA WORLD CUP BRAZIL

AUS 1 - 1 NED
25:32
CIBC
SONY
4K
SEE MORE DETAIL
BE MOVED
PORTO

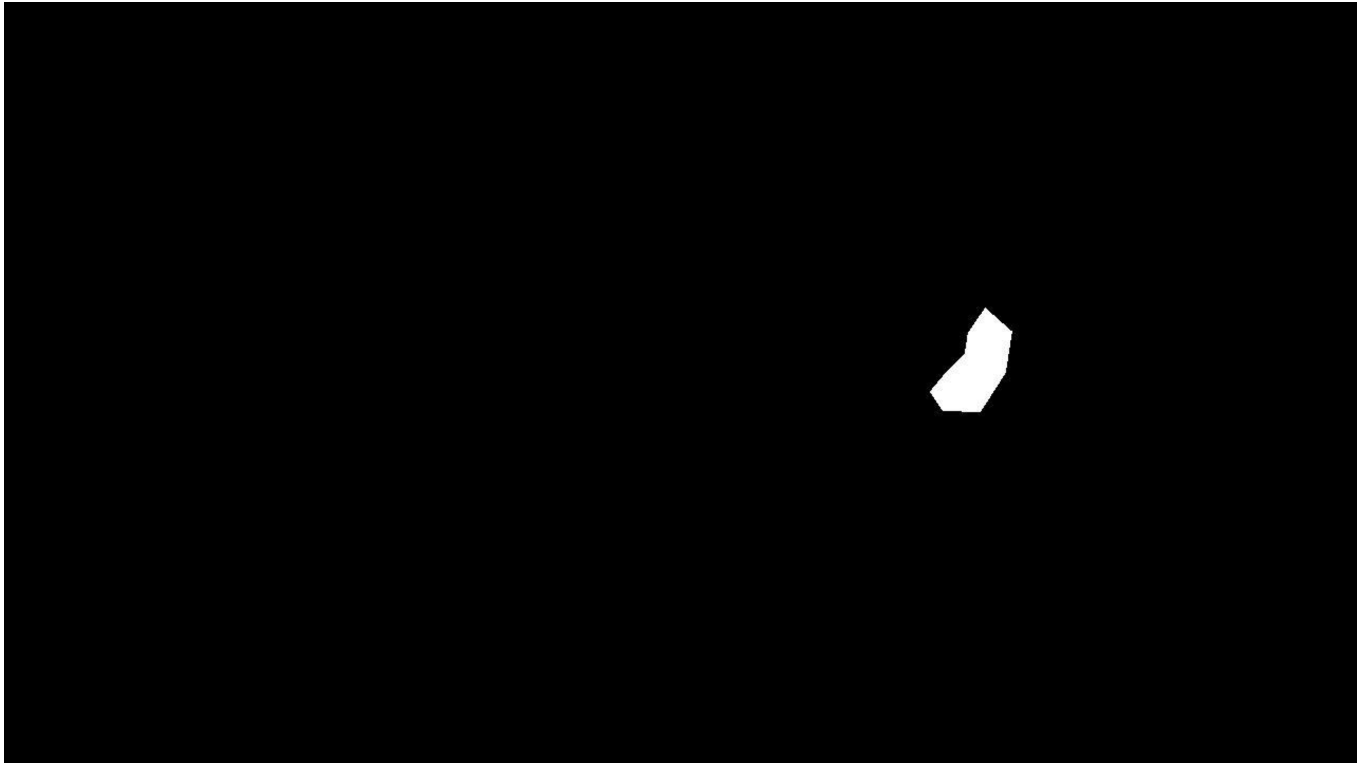

JPN 0 - 0 GRE
51:59
Continental
BAND-AID
2014 FIFA WORLD CUP BRAZIL





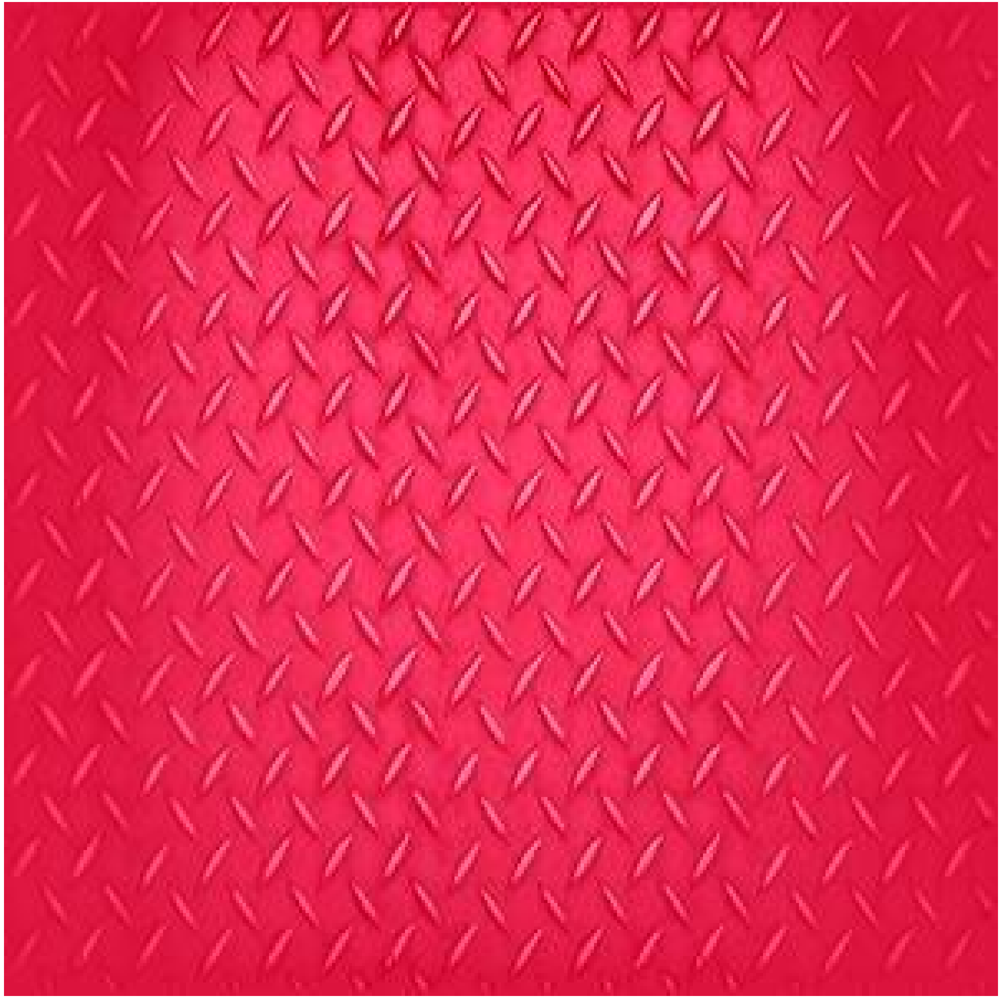

# TRANSPARENT OBJECTS

# OBJECT REMOVAL

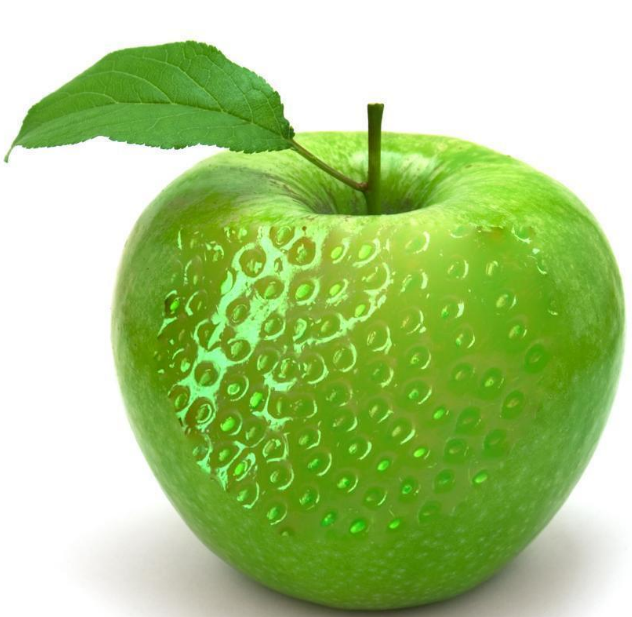



JPN 0 - 0 GRE
51:59
Continental
BAND-AID
2014 FIFA WORLD CUP BRAZIL

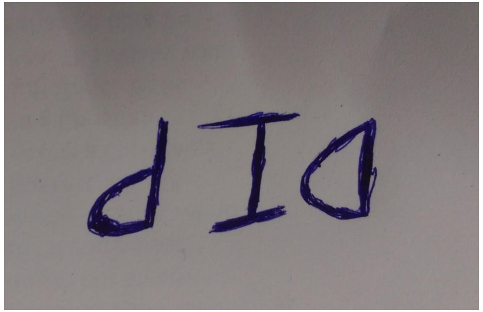
# TEXTURE REPLICATION

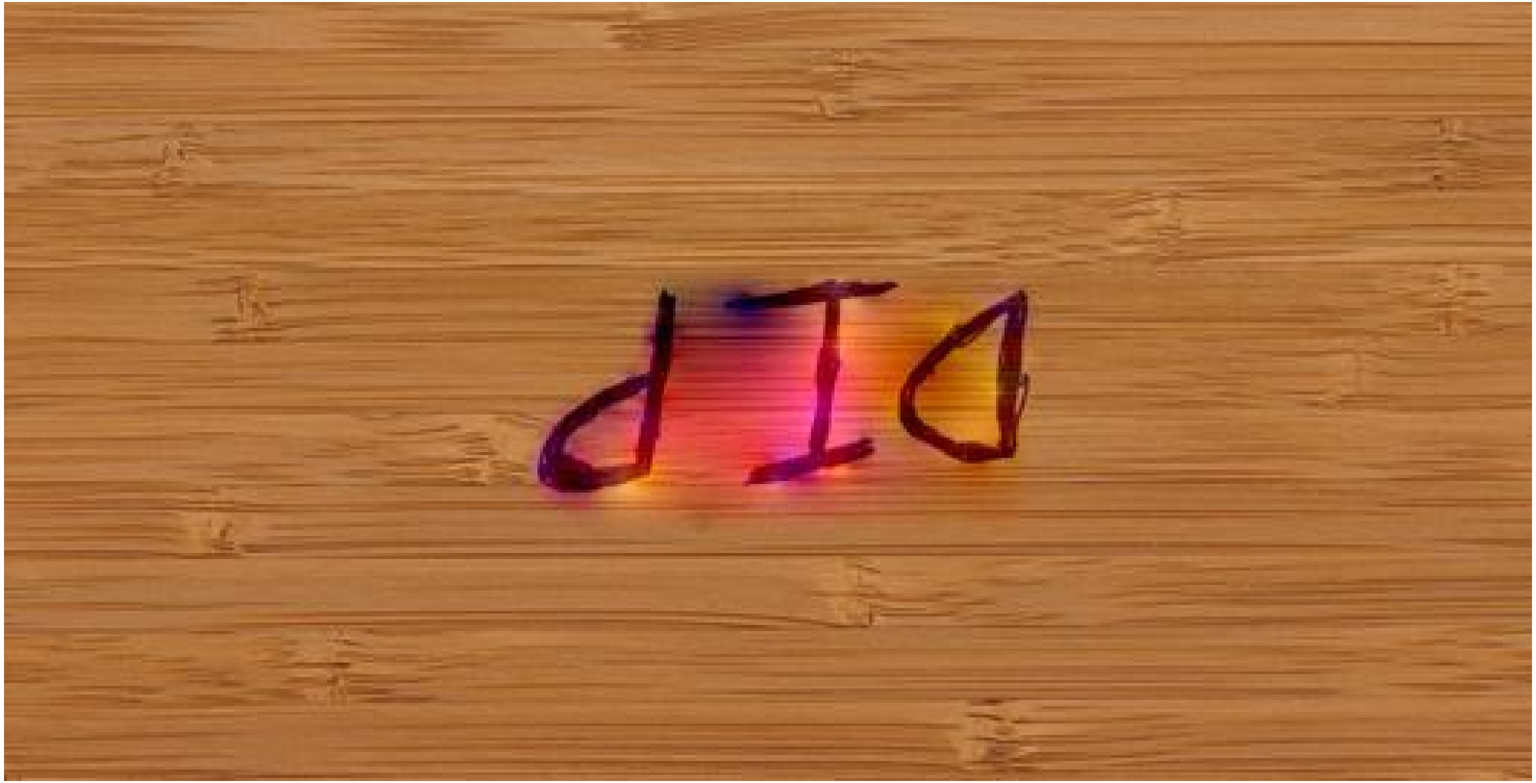


# TEXTURE SWAPPING

# OBJECTS WITH HOLES(a failure case for us)




dIa

# REFERENCES


- https://www.cs.jhu.edu/~misha/Fall07/Papers/Perez03.pdf
- http://eric-yuan.me/poisson-blending/
- https://www.youtube.com/watch?v=UcTJDamstdk