

9.40 Intro to Neural Computation: pset1

Problem 1: Random walk

In P1 we consider a 500x1001 matrix X (randomwalk.mat) where each row represents the position of an object in microns as it takes a random walk for 1000 timesteps of 1ms.

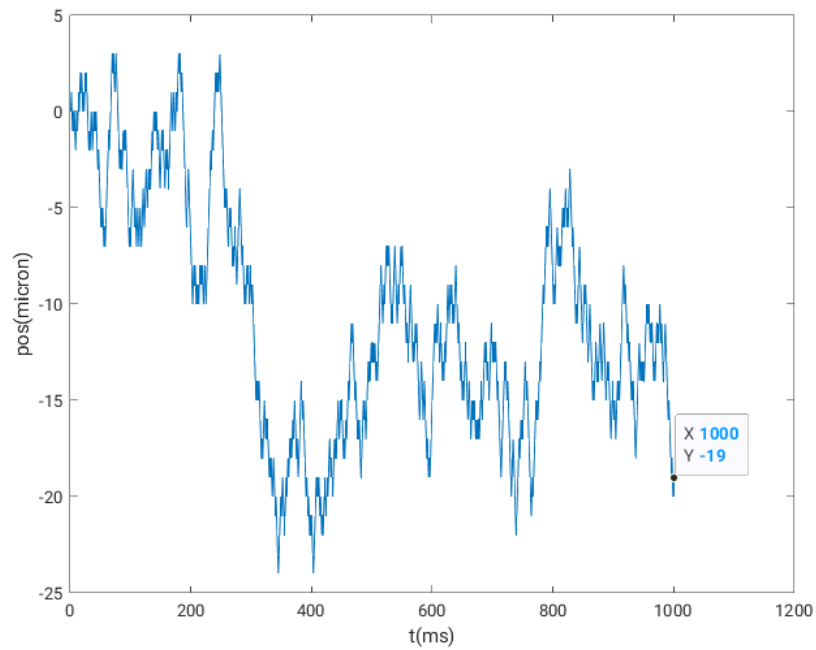
Q1: Look at col1 of X to determine the positions of all particles at $t=0$

Inspecting the first column of X we see that all particles starts at position 0.

Q2: What is the value of t for the last column of X ?

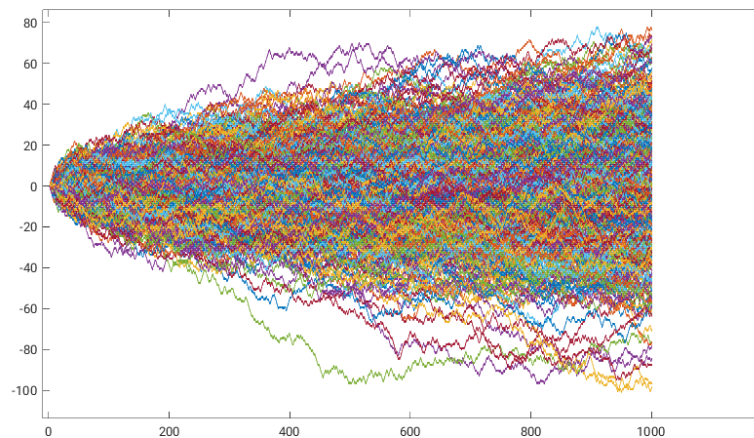
Since there are 1001 cols and we begin counting at $t = 0$ the 1000th col marks $t = 1000ms = 1s$

Q3:Plot the trajectory of particle 14 as a function of time



Like all the others it begins at $pos = 0$, 14 ends up at $pos = -19$

Q4:In a single figure plot the trajectories of all the particles as a function of time. What do you see? Why each particle follows a different trajectory?



We see that the trajectories when considered together resemble a normal distribution centered at 0.

At each time step each particle moves +1 or -1 with equal probability. So the probability of any particular trajectory is

$$p = \left(\frac{1}{2}\right)^{1000}$$

Let A be the event that at least two particles follow the same trajectory. Then the probability of A is given by:

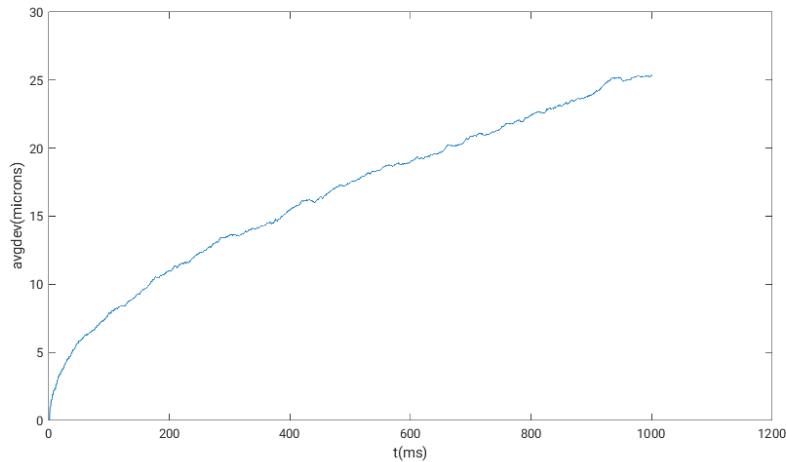
$$P(A) = \sum_{i=2}^{500} \binom{500}{i} * p^i$$

Which is an absurdly small number. To demonstrate this note that an upper bound for binomial coefficients is: $\binom{k}{i} \leq \left(\frac{ek}{i}\right)^i$. So we can write:

$$P(A) \leq \sum_{i=2}^{500} \left(\frac{e500}{i2^{1000}}\right)^i \leq 499 \frac{e500}{2^{1001}} \approx 2 \times 10^{-296}$$

So even a quite generous bound gives a probability which is as good as 0.

Q5: Plot the mean displacement as a function of time is there any trend in this plot? Explain.



The mean displacement seems to be growing logarithmically.