Ray-Primitive Intersections

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1 PARALLELOGRAM

Let us have a parallelogram P defined by a vertex and two vectors respectively: $\vec{c}, \vec{e_1}, \vec{e_2} \in \mathbb{R}^3; \vec{e_1} \nparallel \vec{e_2}, \vec{e_1} \neq \vec{0}, \vec{e_2} \neq \vec{0}$. Its parametric equation is:

$$\vec{p}(\alpha, \beta) = \vec{c} + \alpha\vec{e_1} + \beta\vec{e_2}; (\alpha, \beta) \in [0, 1]^2$$

Given a ray R with origin and direction $\vec{o}, \vec{d} \in \mathbb{R}^3; \vec{d} \neq \vec{0}$, with parametric equation:

$$\vec{r}(t) = \vec{o} + t\vec{d}; t \in [0, \infty)$$

We want to find the intersection $R \cap P$.

2 SOLUTION

The intersection $I = R \cap P$ is given as:

$$I = \{v \in \mathbb{R}^n | \vec{v} = \vec{r}(t) = \vec{p}(\alpha, \beta) \land (\alpha, \beta, t) \in [0, 1]^2 \times [0, \infty)\}$$

Thus we get the equation:

$$\vec{o} + t\vec{d} = \vec{c} + \alpha\vec{e_1} + \beta\vec{e_2}$$

$$\alpha\vec{e_1} + \beta\vec{e_2} - t\vec{d} = \vec{c} - \vec{o}$$

$$M(\alpha, \beta, t)^T = \vec{c} - \vec{o}$$

Where M is a matrix made up of $\vec{e_1}, \vec{e_2}, -\vec{d}$ as column vectors:

$$M = \begin{bmatrix} e_{11} & e_{21} & -d_1 \\ e_{12} & e_{22} & -d_2 \\ e_{13} & e_{23} & -d_3 \end{bmatrix}$$

As long as $\vec{e_1}, \vec{e_2}, \vec{d}$ are linearly independent M is invertible (we already know that $\vec{e_1}, \vec{e_2}$ are linearly independent).The vector $\vec{d}$ can be represented as a linear combination of $\vec{e_1}, \vec{e_2}$ only when the ray R is parallel to the parallelogram, so one gets either a line segment as an intersection, or no intersection. For simplicity we will ignore this edge case. Since M is invertible for $\vec{e_1}, \vec{e_2}, \vec{d}$ linearly independent, the unique solution of the matrix equation is given by:

$$(\alpha, \beta, t)^T = M^{-1}(\vec{c} - \vec{o})$$

Then the intersection of the ray R with the plane defined by $\vec{c}, \vec{e_1}, \vec{e_2}$ is given by $\vec{r}(t)$. For the intersection to be valid we need $t \geq 0$, and for the point $\vec{r}(t)$ to lie in the parallelogram we also need $(\alpha, \beta) \in [0, 1]^2$.