Quantum Computing for the very curious, summary

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27.01.2022

In this document I am wrapping up the Strangeworks' paced repetition document named 'Quantum computing for the very curious'

1 Matrix fundamentals relating to base states

Dirac notation

$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix} = e_0,\ |1\rangle = \begin{bmatrix}0\\1\end{bmatrix} = e_1,\ \langle 0| = \begin{bmatrix}1 & 0\end{bmatrix},\ \langle 1| = \begin{bmatrix}0 & 1\end{bmatrix}$$

$$|\psi\rangle^\dagger = \langle\psi|$$

Matrix addition

$$\frac{1+i}{2}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle = \begin{bmatrix}\frac{1+i}{2}\\ \frac{i}{\sqrt{2}}\end{bmatrix}$$

Unitarity

$$U^\dagger U = I, \quad U^\dagger = (U^T)^*, \quad \begin{bmatrix}a & b\\ c & d\end{bmatrix}^\dagger = \begin{bmatrix}a^* & c^*\\ d^* & d^*\end{bmatrix},\ I = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Tensor product

$$|0\rangle \otimes |1\rangle = \begin{bmatrix}1\\0\end{bmatrix} \otimes \begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}1\begin{bmatrix}0\\1\end{bmatrix}\\ 0\begin{bmatrix}0\\1\end{bmatrix}\end{bmatrix} = \begin{bmatrix}0\\1\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \otimes \begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} = \begin{bmatrix}1\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} & 1\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\\ 1\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} & -1\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\end{bmatrix} = \begin{bmatrix}1 & 1 & 1 & 1\\ 1 & -1 & 1 & -1\\ 1 & 1 & -1 & -1\\ 1 & -1 & -1 & 1\end{bmatrix}$$

2 Gates

2.1 Pauli gates: X(/NOT), Y and Z

$$NOT|0\rangle = |1\rangle \quad NOT|1\rangle = |0\rangle, \quad NOT(\alpha|0\rangle + \beta|1\rangle) = \alpha|0\rangle + \beta|1\rangle,$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$XX|\psi\rangle = \psi$$

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \; Y|0\rangle = i|1\rangle, \; Y|1\rangle = -i|0\rangle$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, Z|0\rangle = |0\rangle, Z|1\rangle = -|1\rangle$$

2.2 Hadamard Gate

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$H(\alpha|0\rangle + \beta|1\rangle) = \alpha\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) + \beta\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) = \frac{\alpha + \beta}{\sqrt{2}}|0\rangle + \frac{\alpha - \beta}{\sqrt{2}}|1\rangle$$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \; H = H^\dagger, \; HH = HH^\dagger = I$$

$$\Rightarrow HH|0\rangle = |0\rangle, \; HH|1\rangle = |1\rangle, \; HH|\psi\rangle = |\psi\rangle$$

Applying Hadamard to a qubit in Hadamard state, gives back the basis state Hadamard was originally applied to

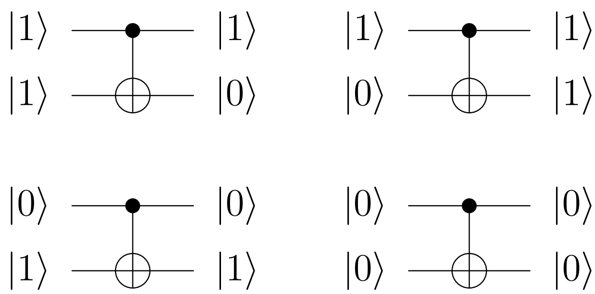
$$\tfrac{|0\rangle+|1\rangle}{\sqrt{2}} \quad -\boxed{H}-\boxed{\mathbf{M}} \quad = |0\rangle$$

$$\tfrac{|0\rangle-|1\rangle}{\sqrt{2}} \quad -\boxed{H}-\boxed{\mathbf{M}} \quad = |1\rangle$$

Two-qubit state is given by the tensor product, i.e. for below two qubit Hadamard system

$$|H|0\rangle|H|1\rangle\rangle = H|0\rangle \otimes H|1\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \otimes \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} * \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} * -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} * \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} * -\frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}$$

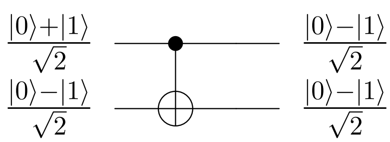
2.3 CNOT Gate



In general superposition, doesn't seem to modify control bit

$$\alpha\,|00\rangle + \beta\,|01\rangle + \gamma\,|10\rangle + \delta\,|11\rangle \to \alpha\,|00\rangle + \beta\,|01\rangle + \gamma\,|11\rangle + \delta\,|10\rangle$$

But in this specific Hadamard case, seemingly does it



$$\frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\end{bmatrix}$$

2.4 Rotation

$$R_y(\theta) = \begin{bmatrix}\cos(\theta/2) & -\sin(\theta/2)\\ \sin(\theta/2) & \cos(\theta/2)\end{bmatrix}$$