Quantum Computing for the very curious, summary

Veikko Nyfors

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In this document I am wrapping up the Strangeworks' paced repetition document named 'Quantum computing for the very curious'

1 Matrix fundamentals relating to base states

Dirac notation

$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix} = e_0,\ |1\rangle = \begin{bmatrix}0\\1\end{bmatrix} = e_1,\ \langle 0| = \begin{bmatrix}1 & 0\end{bmatrix},\ \langle 1| = \begin{bmatrix}0 & 1\end{bmatrix}$$

$$|\psi\rangle^\dagger = \langle\psi|$$

Matrix addition

$$\frac{1+i}{2}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle = \begin{bmatrix}\frac{1+i}{2}\\ \frac{i}{\sqrt{2}}\end{bmatrix}$$

Unitarity

$$U^\dagger U = I, \quad U^\dagger = (U^T)^*, \quad \begin{bmatrix}a & b\\ c & d\end{bmatrix}^\dagger = \begin{bmatrix}a^* & c^*\\ d^* & d^*\end{bmatrix},\ I = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

2 Gates

Pauli gates: X(/NOT), Y and Z

$$NOT|0\rangle = |1\rangle \quad NOT|1\rangle = |0\rangle, \quad NOT(\alpha|0\rangle + \beta|1\rangle) = \alpha|0\rangle + \beta|1\rangle,$$

$$X = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},\ \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}1 & 0\end{bmatrix} = \begin{bmatrix}0 & 1\end{bmatrix},\ \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\end{bmatrix}$$

$$XX|\psi\rangle = \psi$$

$$Y = \begin{bmatrix}0 & -i\\ i & 0\end{bmatrix},\ Y|0\rangle = i|1\rangle,\ Y|1\rangle = -i|0\rangle$$

$$Z = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, Z|0\rangle = |0\rangle, Z|1\rangle = -|1\rangle$$

Hadamard Gate

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \quad H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$H(\alpha|0\rangle + \beta|1\rangle) = \alpha\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) + \beta\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) = \frac{\alpha+\beta}{\sqrt{2}}|0\rangle + \frac{\alpha-\beta}{\sqrt{2}}|1\rangle$$

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix},\ H = H^{\dagger},\ HH = HH^{\dagger} = I$$

$$\Rightarrow HH|0\rangle = |0\rangle,\ HH|1\rangle = |1\rangle,\ HH|\psi\rangle = |\psi\rangle$$

$$\frac{|0\rangle + |1\rangle}{\sqrt{2}} \quad \text{———}\boxed{\mathbf{H}}\text{———}\boxed{\mathbf{M}}) = |0\rangle$$

$$\frac{|0\rangle - |1\rangle}{\sqrt{2}} \quad \text{———}\boxed{\mathbf{H}}\text{———}\boxed{\mathbf{M}}) = |1\rangle$$

Rotation

$$R_y(\theta) = \begin{bmatrix}\cos(\theta/2) & -\sin(\theta/2)\\ \sin(\theta/2) & \cos(\theta/2)\end{bmatrix}$$