

UNIVERSITY OF TWENTE.



## CONTRACTS FOR CONCURRENT SOFTWARE WITH VERCORS

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## PLAN FOR TODAY

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- Permission annotations for shared memory
- Fork-join concurrency
- Parallel blocks
- Reasoning about locks
- Reasoning about atomics

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2 Voer de code van het evenement in de bovenste banner in

Evenementcode  
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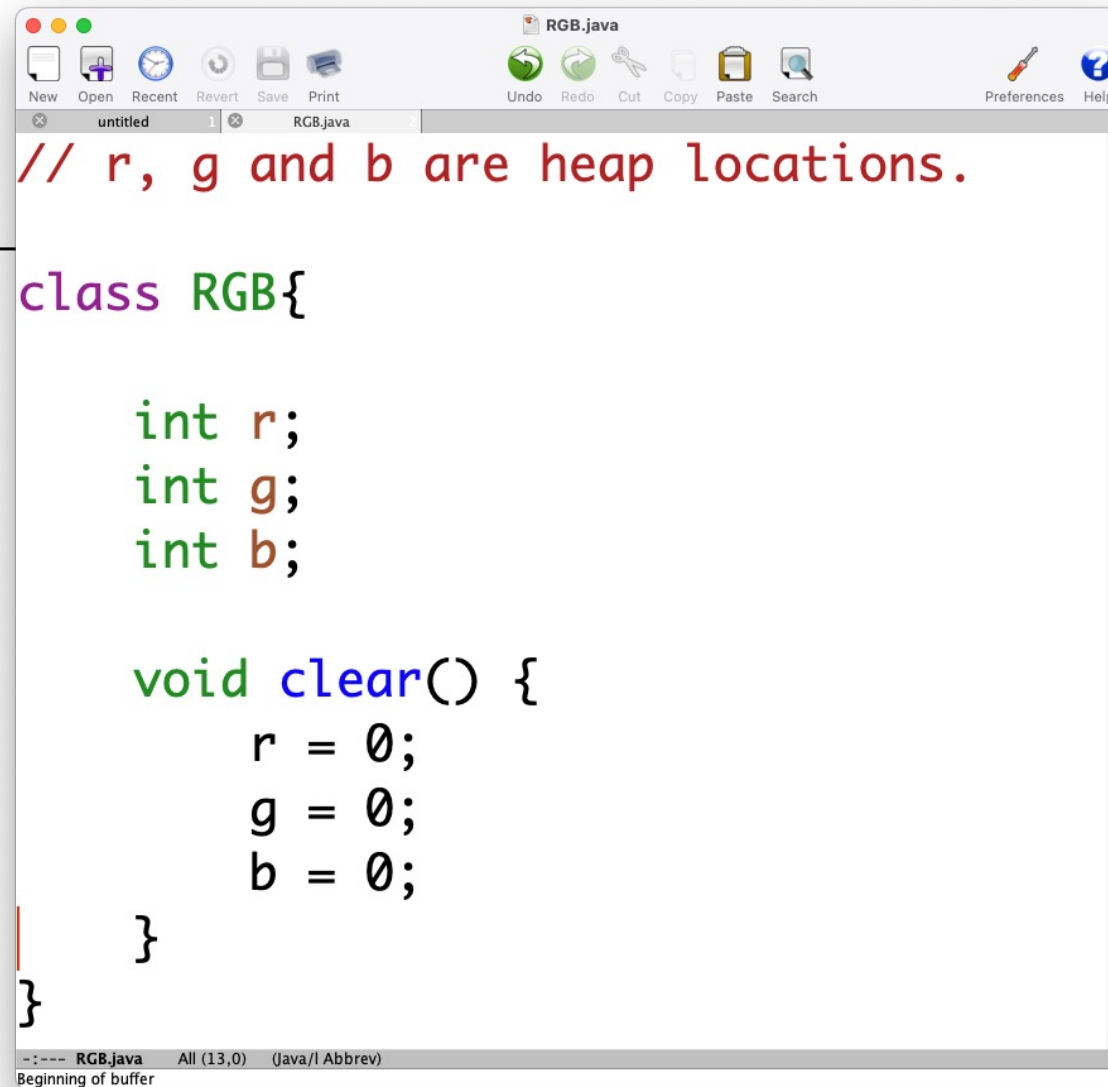
Stemmen - / 1 + Berichten 100%

0

## WHY PERMISSION ANNOTATIONS?

---

- Concurrent programs: data on the heap is shared by threads
- Data races should be avoided (multiple threads accessing and updating the heap at the same moment)
- Permission annotations capture this:
  - $\text{Perm}(x.f, 1)$  or  $\text{Perm}(x.f, \text{write})$  – exclusive access
  - $\text{Perm}(x.f, 1/2)$  or  $\text{Perm}(x.f, \text{read})$  -- shared read-only access
- Permissions can be split and combined
- Soundness of the logic: total number of permissions to a location never more than 1



```
// r, g and b are heap locations.

class RGB{

    int r;
    int g;
    int b;

    void clear() {
        r = 0;
        g = 0;
        b = 0;
    }
}
```

The screenshot shows a Java IDE window titled "RGB.java". The menu bar includes "New", "Open", "Recent", "Revert", "Save", "Print", "Undo", "Redo", "Cut", "Copy", "Paste", "Search", "Preferences", and "Help". The toolbar contains icons for these actions. The code editor shows the following code:

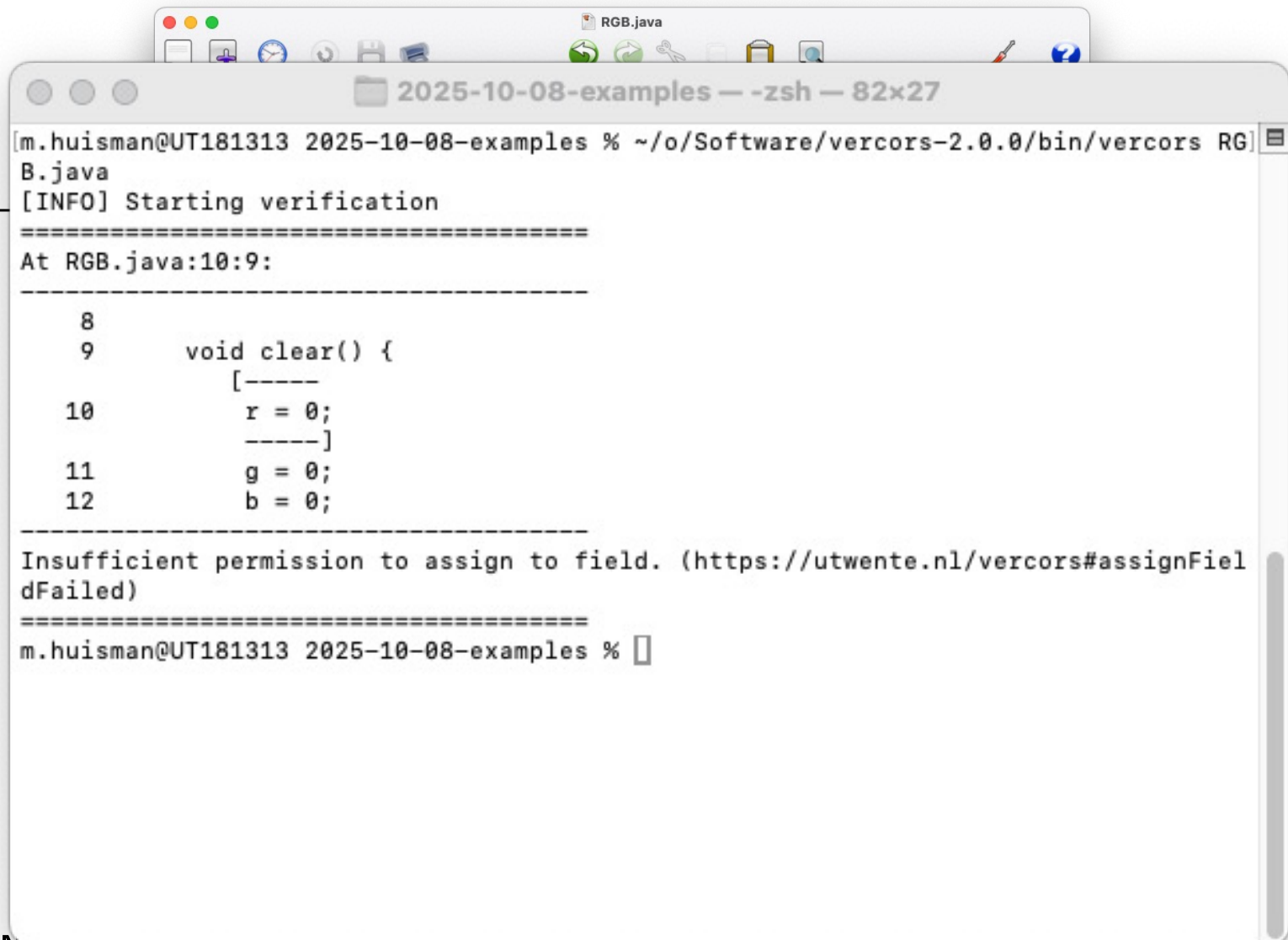
```
// r, g and b are heap locations.

class RGB{

    int r;
    int g;
    int b;

    void clear() {
        r = 0;
        g = 0;
        b = 0;
    }
}
```

The status bar at the bottom indicates the file is "RGB.java", the cursor is at "All (13,0)", and the language is "Java/I Abbrev". The text "Beginning of buffer" is also visible.



The image shows a macOS terminal window titled "2025-10-08-examples — -zsh — 82x27". The terminal displays the execution of the VerCors tool on a file named "B.java". The output shows the start of a verification process, followed by a code snippet from "RGB.java" at line 10:9. The code defines a "clear()" method that sets variables "r", "g", and "b" to 0. The verification fails with the error message: "Insufficient permission to assign to field. (https://utwente.nl/vercors#assignFieldFailed)".

```
[m.huisman@UT181313 2025-10-08-examples % ~/o/Software/vercors-2.0.0/bin/vercors RGB.java
[INFO] Starting verification
=====
At RGB.java:10:9:
-----
    8
    9     void clear() {
      10         [-----
            r = 0;
            -----]
      11         g = 0;
      12         b = 0;
      -----
Insufficient permission to assign to field. (https://utwente.nl/vercors#assignFieldFailed)
=====
m.huisman@UT181313 2025-10-08-examples %
```

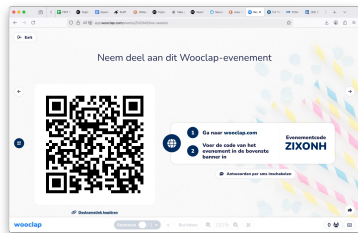
```
// Permissions for ownership/access accounting

class RGB{

    int r;
    int g;
    int b;

    /*@
     @ requires Perm(r, write);
     @ requires Perm(g, write);
     @ requires Perm(b, write);
     @*/
    void clear() {
        r = 0;
        g = 0;
        b = 0;
    }
}
```

Which permissions  
are needed here?



```
RGB-use.java
New Open Recent Revert Save Print Undo Redo Cut Copy Paste Search
RGB-use.java RGB-use-solution.java 5 RGB-use-alt-solution.java 6
// Giving up and returning permissions

class RGB{

    int r;
    int g;
    int b;

    void clear() {
        r = 0;
        g = 0;
        b = 0;
    }

    void main() {

        clear();
        r = 255;
    }
}
--- RGB-use.java Top (10,18) (Java/I Abbrev)
Wrote /Users/m.huisman/ownCloud/Documents/Research/VerCors/presentations/2025-10-08-examples/RGB-use.java
```



```
RGB-use-solution.java
New Open Recent Revert Save Print Undo Redo Cut Copy Paste Search
RGB-use.java 4 RGB-use-solution.java RGB-use-alt-solution.java 6
// Giving up and returning permissions

class RGB{

    int r;
    int g;
    int b;

    /*@ requires Perm(r, write);
    @ requires Perm(g, write);
    @ requires Perm(b, write);
    @ ensures Perm(r, write);
    @ ensures Perm(g, write);
    @ ensures Perm(b, write);
    @*/
    void clear() {
        r = 0;
        g = 0;
        b = 0;
    }

    /*@ requires Perm(r, write);
    @ requires Perm(g, write);
    @ requires Perm(b, write);
    @ ensures Perm(r, write);
    @ ensures Perm(g, write);
    @ ensures Perm(b, write);
    @*/
    void main() {
        clear();

        r = 255;
    }
}

-- RGB-use-solution.java All (30,17) (Java/I Abbrev)
```

# WHAT ARE PERMISSIONS?

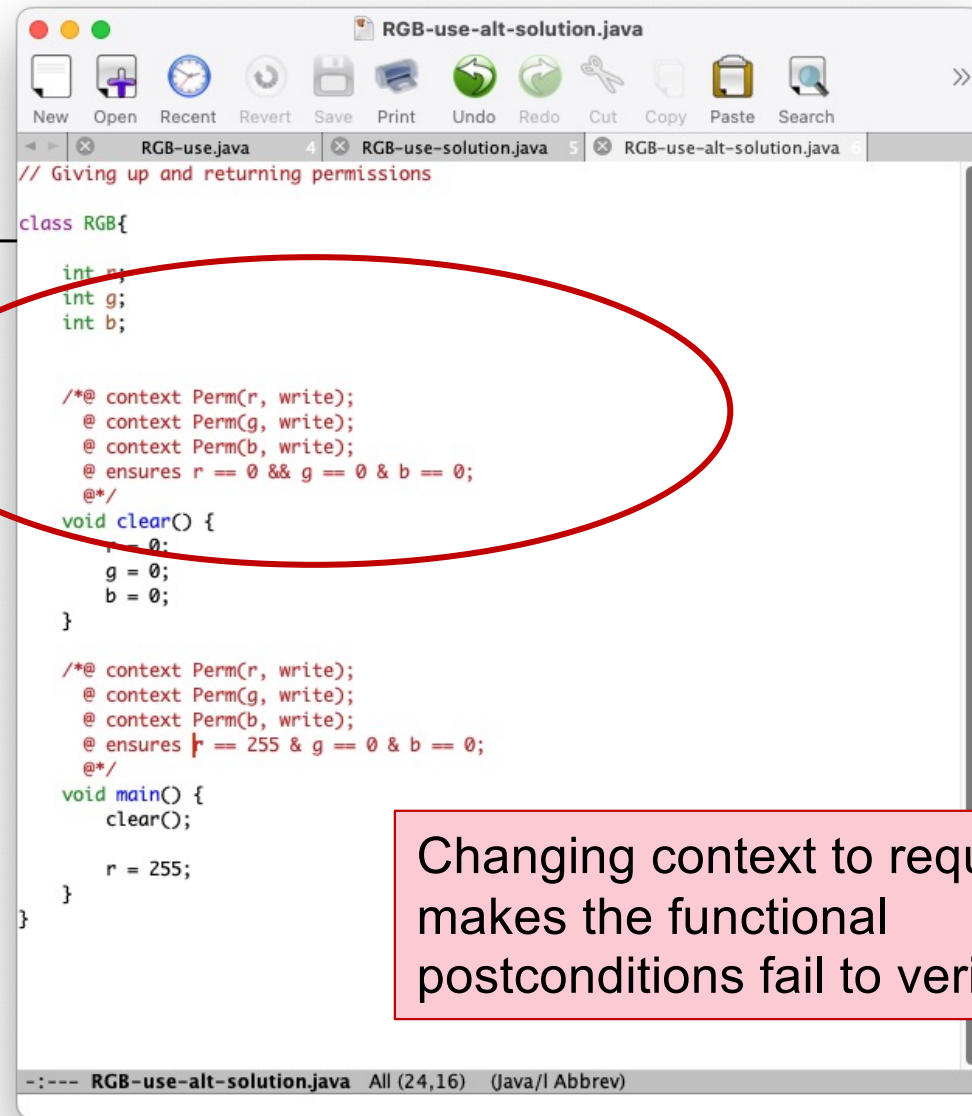
---

- Permissions are “resources”
- When you call a method, you give up the permissions specified in its requirements
- When you come back from a call, you get back the permissions specified by the postcondition
- Permissions for objects are created in its constructor

Often permission annotations are the same in pre- and postcondition:

Context keyword

Functional properties can only be given about state that you can access (i.e. have permission for)



```
// Giving up and returning permissions
class RGB{
    int r;
    int g;
    int b;

    /*@ context Perm(r, write);
    @ context Perm(g, write);
    @ context Perm(b, write);
    @ ensures r == 0 && g == 0 & b == 0;
    @*/
    void clear() {
        r = 0;
        g = 0;
        b = 0;
    }

    /*@ context Perm(r, write);
    @ context Perm(g, write);
    @ context Perm(b, write);
    @ ensures r == 255 & g == 0 & b == 0;
    @*/
    void main() {
        clear();

        r = 255;
    }
}
```

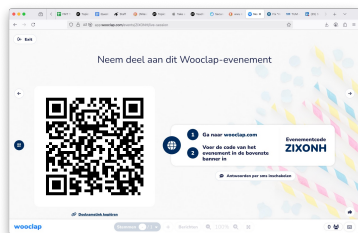
Changing context to requires makes the functional postconditions fail to verify

# HOW TO SPECIFY THIS FUNCTION?



```
increase-annotated.java
New Open Recent Revert Save Print Undo Redo Cut Copy Paste Search Preferences Help
example5.java 7 demo1.pvl 8 increase.java 9 increase-annotated.java 10
class Counter {
    int count;
    /*@
    @ requires Perm(count, 1);
    @ ensures Perm(count, 1\2);
    @ ensures count == \old(count) + n;
    @ */
    void incr(int n) {
        count = count + n;
    }
}
```

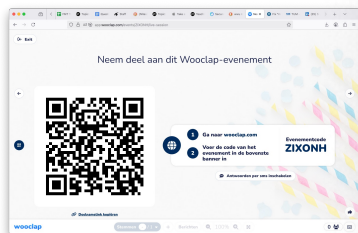
What will happen  
when we try to  
verify this?



```
increase-twice.java
New Open Recent Revert Save Print Undo Redo Cut Copy Paste Search Preferences Help
demo1.pvl 8 increase.java 9 increase-annotated.java 0 increase-twice.java
class Counter {
    int count;
    /*@
    @ requires Perm(count, 1);
    @ ensures Perm(count, 1\2);
    @ ensures count == \old(count) + n;
    @ */
    void incr(int n) {
        count = count + n;
    }

    /*@
    @ requires Perm(count, 1);
    @ ensures Perm(count, 1\2);
    @ ensures count == \old(count) + 2 * n;
    @ */
    void incr2(int n) {
        incr(n);
        incr(n);
    }
}
-:--- increase-twice.java All (19,25) (Java/I Abbrev)
```

What will happen  
when we try to  
verify this?



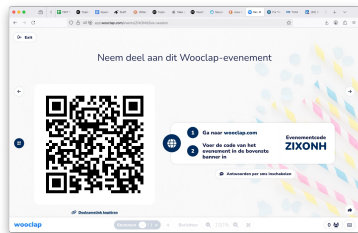
```
2025-10-08-examples — -zsh — 89x30

m.huisman@UT181313 2025-10-08-examples % ~/o/Software/vercors-2.0.0/bin/vercors increase-
twice.java
[INFO] Starting verification
=====
At increase-twice.java:19:3:
-----
17 void incr2(int n) {
18     incr(n);
19     [-----
20         incr(n);
21         -----]
-----
[1/2] Precondition may not hold, since ...
-----
At increase-twice.java:4:18:
-----
2 int count;
3 /*@
4     @ requires Perm(count, 1);
5     @ ensures Perm(count, 1\2);
6     @ ensures count == \old(count) + n;
-----
[2/2] ... there might not be enough permission to exhale this amount (https://utwente.nl/
vercors#preFailed:perm)
=====
m.huisman@UT181313 2025-10-08-examples %
```

```
class Clear {  
    /*@  
    @ requires A != null;  
    @ ensures A != null;  
    @ context (\forall int j; 0 <= j && j < A.length; Perm(A[j], write));  
    @ ensures (\forall int j; 0 <= j && j < A.length; A[j] == 0);  
    @ */  
    void clear(int[] A) {  
        /*@  
        @ loop_invariant A != null && 0 <= i && i <= A.length;  
        @ loop_invariant (\forall int j; 0 <= j && j < i; A[j] == 0);  
        @ */  
        for (int i = 0; i < A.length; i++) {  
            A[i] = 0;  
        }  
    }  
}
```



Which permissions  
need to be added to  
loop invariants?

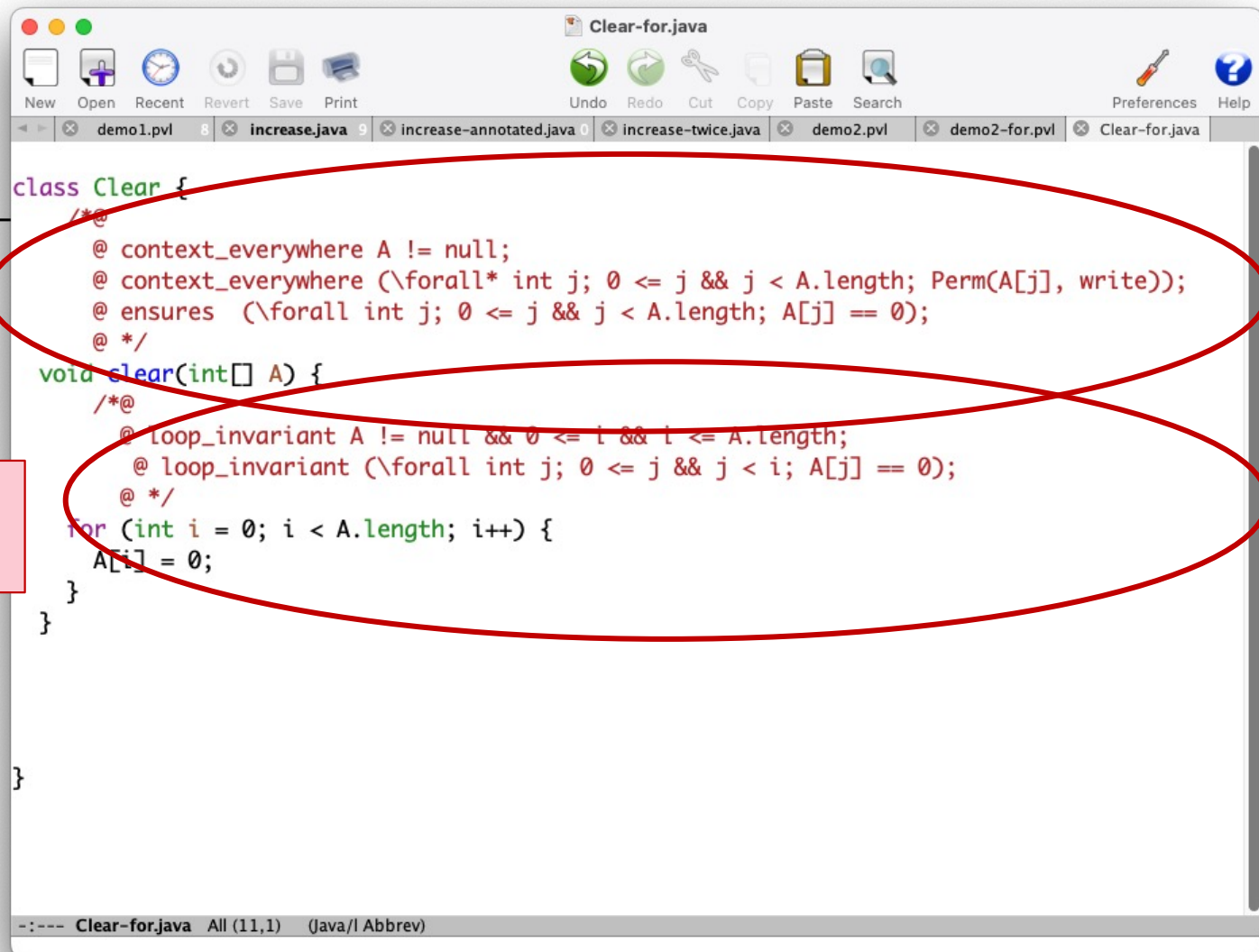


```
Clear-for.java
2025-10-08-examples — -zsh — 89x30
m.huisman@UT181313 2025-10-08-examples % ~/o/Software/vercors-2.0.0/bin/vercors Clear-for
.java
[INFO] Starting verification
=====
At Clear-for.java:12:59:
-----
10      /*@
11      @ loop_invariant A != null && 0 <= i && i <= A.length;
12      @ loop_invariant (\forall int j; 0 <= j && j < i; A[j] == 0);
13      @ */
14      for (int i = 0; i < A.length; i++) {

There may be insufficient permission to access the array. (https://utwente.nl/vercors#arrayPerm)
=====
m.huisman@UT181313 2025-10-08-examples %
```



Common pattern:  
Context\_everywhere



```
class Clear {
    /*@
    @ context_everywhere A != null;
    @ context_everywhere (\forall int j; 0 <= j && j < A.length; Perm(A[j], write));
    @ ensures (\forall int j; 0 <= j && j < A.length; A[j] == 0);
    @ */
    void clear(int[] A) {
        /*@
        @ loop_invariant A != null && 0 <= i && i <= A.length;
        @ loop_invariant (\forall int j; 0 <= j && j < i; A[j] == 0);
        @ */
        for (int i = 0; i < A.length; i++) {
            A[i] = 0;
        }
    }
}
```

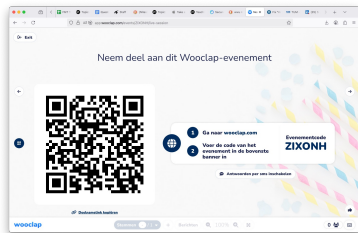
## FORK-JOIN CONCURRENCY

---

- Threads give away permissions when forking a thread
- When joining a thread, permissions are handed back to the joining thread
- Specified as contract of the run method
- If multiple threads can join, extra bookkeeping needed to ensure that permissions do not get duplicated (by means of join-token, which describes the share that the current thread will obtain)

# FIBONACCI

- Which permissions for the run method?
- Which permissions for the constructor?
- Functional behaviour



```
fibonacci-bare.pvl — 2025-10-08-examples (git: master)
fibonacci.pvl
fibonacci-bare.pvl

1  class Fib {
2    int input, output;
3
4    run {
5      if (input<2) {
6        output = 1;
7      } else {
8        Fib f1 = new Fib(input-1);
9        Fib f2 = new Fib(input-2);
10       fork f1; fork f2;
11       join f1; join f2;
12       output = f1.output + f2.output;
13     }
14   }
15
16   constructor(int n){
17     input = n;
18   }
19 }
20
21
22
```

Line: 22 | Vercors PVL | Tab Size: 4 | join

```
1  pure int fib(int n)=n<2?1:fib(n-1)+fib(n-2);
2
3  class Fib {
4    int input, output;
5
6    requires Perm(input,read) ** Perm(output,write);
7    ensures  Perm(input,read) ** Perm(output,write) ** output==fib(input);
8    run {
9      if (input<2) {
10       output = 1;
11      } else {
12       Fib f1 = new Fib(input-1);
13       Fib f2 = new Fib(input-2);
14       fork f1; fork f2;
15       join f1; join f2;
16       output = f1.output + f2.output;
17     }
18   }
19
20   ensures Perm(input,write) ** Perm(output,write) ** input==n;
21   constructor(int n){
22     input = n;
23   }
24 }
25
26
27
```

Line: 26 | Vercors PVL | Tab Size: 4 | join

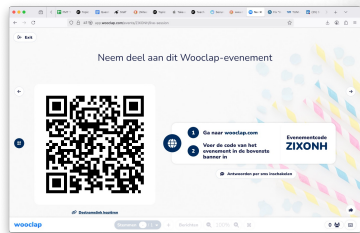
# PARALLEL BLOCKS

---

- Create  $n$  parallel threads, each executing the same code on their own data
- Massive parallelism (as in GPU's)
- Permissions should be distributed over the different threads
- Barriers for synchronization
- Barriers redistribute permissions

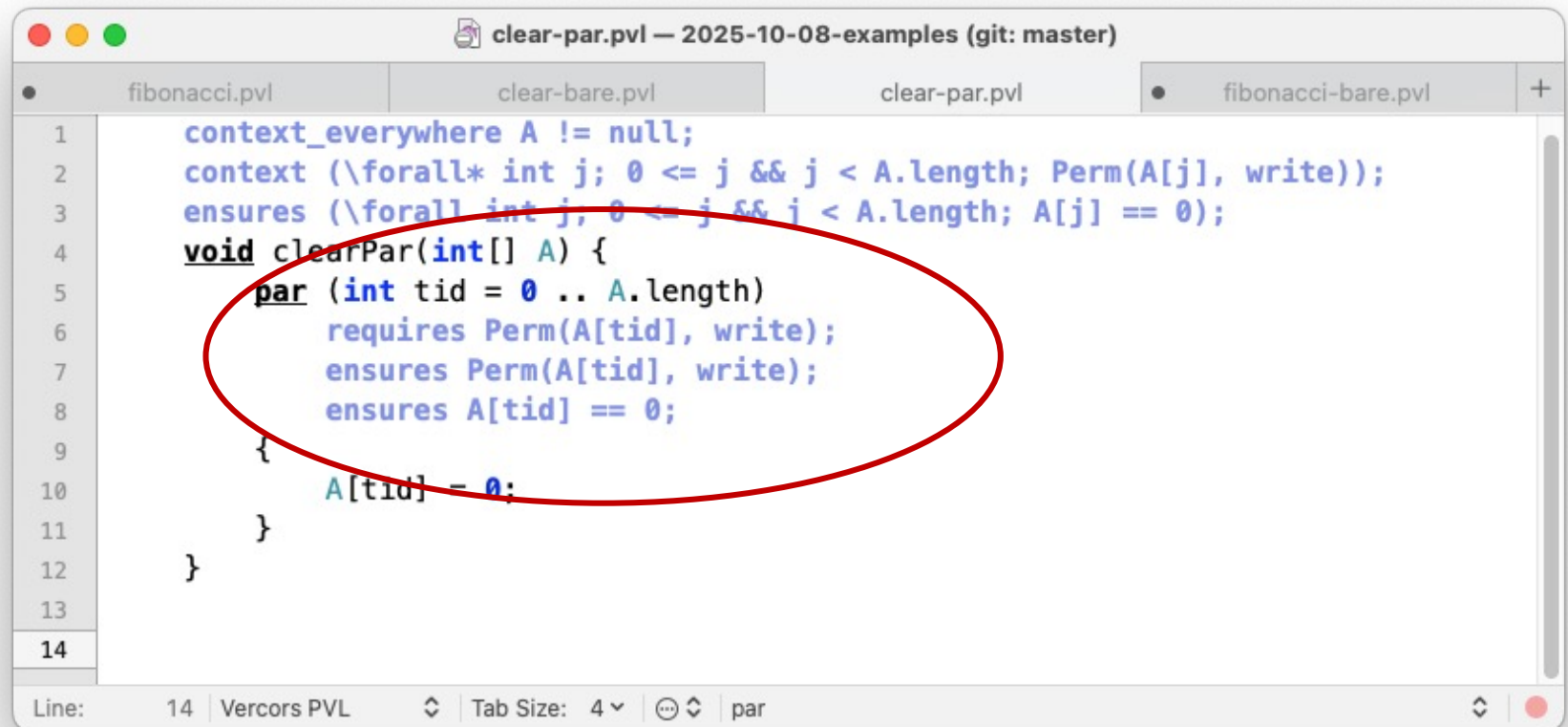
# PARALLEL CLEAR

What is the contract per thread?



```
clear-bare.pvl — 2025-10-08-examples (git: master)
fibonacci.pvl  clear-par.pvl  clear-bare.pvl  fibonacci-bare.pvl  +
1      context_everywhere A != null;
2      context (\forall int j; 0 <= j && j < A.length; Perm(A[j], write));
3      ensures (\forall int j; 0 <= j && j < A.length; A[j] == 0);
4      void clearPar(int[] A) {
5          par (int tid = 0 .. A.length)
6              {
7                  A[tid] = 0;
8              }
9      }
10
11
12
Line: 12  Vercors PVL  Tab Size: 4  par
```

# PARALLEL CLEAR



```
clear-par.pvl — 2025-10-08-examples (git: master)
fibonacci.pvl  clear-bare.pvl  clear-par.pvl  fibonacci-bare.pvl  +
1  context_everywhere A != null;
2  context (\forall* int j; 0 <= j && j < A.length; Perm(A[j], write));
3  ensures (\forall int j; 0 <= j && j < A.length; A[j] == 0);
4  void clearPar(int[] A) {
5      par (int tid = 0 .. A.length)
6          requires Perm(A[tid], write);
7          ensures Perm(A[tid], write);
8          ensures A[tid] == 0;
9      }
10     A[tid] = 0;
11 }
12 }
13
14
```

Line: 14 | Vercors PVL | Tab Size: 4 | par

## How to specify the effect of the barrier?





```
barrier-example.pvl — 2025-10-08-examples (git: master)
barrier-bare.pvl  barrier-example.pvl  +
1  context_everywhere a != null && b != null && c != null;
2  context_everywhere tcount >= 0;
3  context_everywhere a.length == tcount && b.length == tcount && c.length == tcount;
4  requires (\forall i; 0 <= i && i < tcount; Perm(a[i], write));
5  requires (\forall i; 0 <= i && i < tcount; Perm(b[i], write));
6  requires (\forall i; 0 <= i && i < tcount; Perm(c[i], 1\4)); // Perm(c[i], read) also possible (1)
7  void main(int tcount, int[] a, int[] b, int[] c) {
8      par fwd (int tid=0..tcount)
9          requires Perm(a[tid], write);
10         requires Perm(b[tid], write);
11         requires Perm(c[tid], 1\4); // Perm(c[tid], read) also possible (2)
12     {
13         b[tid]=c[tid];
14         barrier(fwd) {
15             context 0 <= tid && tid < tcount;
16             // losing all permissions:
17             context Perm(a[tid], write);
18             requires Perm(b[tid], write);
19             requires Perm(c[tid], 1\4); // Perm(c[tid], read) also possible (3)
20             ensures Perm(b[tid], 1\4); // Perm(b[tid], read) also possible (1)
21             ensures tid>0 ==> Perm(b[tid-1], 1\4); // Perm(b[tid-1], read) also possible (2)
22         }
23         if(tid>0) {
24             a[tid]=b[tid-1]+b[tid];
25         } else {
26             a[tid]=b[tid];
27         }
28     }
29 }
30
```

Line: 15:3 | Vercors PVL | Tab Size: 4 | barrier

# REASONING ABOUT LOCKS

---

- Shared state protected by a lock
- Lock invariant specifies which shared state is protected by the lock
- Lock invariant has to be explicitly “committed” (typically done in the constructor)
- Acquiring the lock transfers the lock invariant to the owner of the lock
- Releasing the lock transfer the lock invariant back to the “neutral” state
- Releasing the lock means that you do not know anything about the shared state anymore
- Reentrant locks: special care needed to reason about when lock invariant is (not) transferred

# PARALLEL SUM

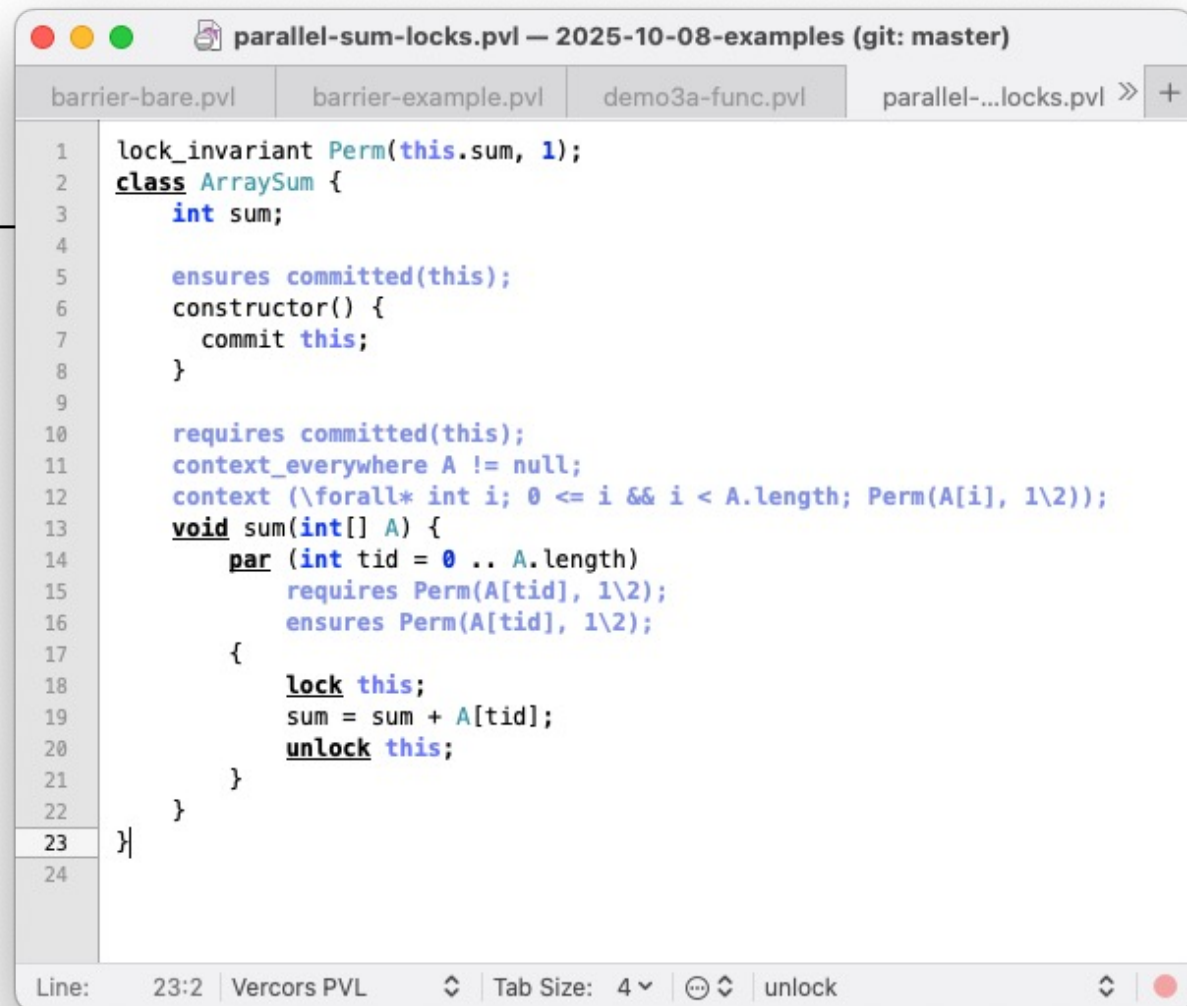
- Notice the committed annotations
- Which data is protected by the lock?
- What is the lock invariant?
- Do we need other permission annotations?



```
parallel-sum-locks-bare.pvl — 2025-10-08-examp...
barrier-bare.pvl  barrier-...mple.pvl  parallel...are.pvl  >> +

1  class ArraySum {
2    int sum;
3
4    ensures committed(this);
5    constructor() {
6      commit this;
7    }
8
9    requires committed(this);
10   context_everywhere A != null;
11   void sum(int[] A) {
12     par (int tid = 0 .. A.length)
13     {
14       lock this;
15       sum = sum + A[tid];
16       unlock this;
17     }
18   }
19 }
20 |
```

Line: 20 Verc... Tab Size: 4 unlock



```
1 lock_invariant Perm(this.sum, 1);
2 class ArraySum {
3   int sum;
4
5   ensures committed(this);
6   constructor() {
7     commit this;
8   }
9
10  requires committed(this);
11  context_everywhere A != null;
12  context (\forall i; 0 <= i && i < A.length; Perm(A[i], 1\2));
13  void sum(int[] A) {
14    par (int tid = 0 .. A.length)
15      requires Perm(A[tid], 1\2);
16      ensures Perm(A[tid], 1\2);
17    {
18      lock this;
19      sum = sum + A[tid];
20      unlock this;
21    }
22  }
23 }
24
```

Line: 23:2 Vercors PVL Tab Size: 4 unlock

# REASONING ABOUT ATOMICS

---

- Similar ideas to locks
- Atomic code blocks: invariant dynamically associated to it

The screenshot shows a code editor window titled "parallel-sum-atomic-bare.pvl — 2025-10-08-examples (git: master)". The editor has tabs for "barrier-bare.pvl", "barrier-example.pvl", "demo3a-func.pvl", and "parallel-s...c-bare.pvl". The code is as follows:

```
1  class ArraySum {
2      int sum;
3
4      context_everywhere A != null;
5      context (\forall i; 0 <= i && i < A.length; Perm(A[i], 1\2));
6      context Perm(this.sum, write);
7      void sum(int[] A) {
8          invariant inv(Perm(this.sum, write)) {
9              par (int tid = 0 .. A.length)
10                 requires Perm(A[tid], 1\2);
11                 ensures Perm(A[tid], 1\2);
12                 {
13                     atomic(inv) {
14                         sum = sum + A[tid];
15                     }
16                 }
17             }
18         }
19     }
20 }
```

A red circle highlights the invariant block starting at line 8: `invariant inv(Perm(this.sum, write)) {`. The status bar at the bottom shows "Line: 20 | Vercors PVL | Tab Size: 4 | void".

Can we also give a functional specification for this function?

The screenshot shows a VerCors PVL editor window titled "parallel-sum-func.pvl — 2025-10-08-examples (git: master)". The editor displays a PVL program for a parallel array sum. Red annotations highlight several key parts of the code:

- Class Definition and Initial Requirements:** Lines 1-7 are circled, showing the `class ArraySum` with a `sum` field, and initial requirements for `A` and the `sum_contrib` function.
- Context Setup:** Lines 10-17 are circled, showing the `given` block where `contrib` is introduced and various context requirements are set.
- Invariant:** Lines 20-24 are circled, showing the `invariant inv` block that maintains the invariant that the sum of `contrib` equals the total sum of `A`.
- Parallel Execution:** Lines 25-30 are circled, showing the `par` block that executes the summation for each thread `tid` in parallel.
- Atomic Update:** Lines 31-34 are circled, showing the `atomic` block where the `sum` is updated and `contrib[tid]` is set to `A[tid]`.

```

1 class ArraySum {
2   int sum;
3
4   requires A != null;
5   requires 0 <= i && i <= A.length;
6   requires (\forallall* int j; 0 <= j && j < A.length; Perm(A[j], 1\2));
7   pure int sum_contrib(int[] A, int i) =
8     (i == A.length) ? 0 : A[i] + sum_contrib(A, i + 1);
9
10  given int[A.length] contrib;
11  context_everywhere A != null;
12  context_everywhere contrib != null;
13  context_everywhere contrib.length == A.length;
14  context (\forallall* int i; 0 <= i && i < A.length; Perm(A[i], 1\2));
15  context (\forallall* int i; 0 <= i && i < A.length; Perm(contrib[i], 1));
16  context Perm(this.sum, write);
17  requires (\forallall* int i; 0 <= i && i < A.length; contrib[i] == 0);
18  void sum(int[] A) {
19
20    invariant inv(
21      Perm(this.sum, write) **
22      (\forallall* int i; 0 <= i && i < contrib.length; Perm(contrib[i], 1\2)) **
23      this.sum == sum_contrib(contrib, 0))
24    {
25      par (int tid = 0 .. A.length)
26        requires Perm(A[tid], 1\2);
27        requires Perm(contrib[tid], 1\2) ** contrib[tid] == 0;
28        ensures Perm(A[tid], 1\2);
29        ensures Perm(contrib[tid], 1\2) ** contrib[tid] == A[tid];
30      {
31        atomic(inv) {
32          sum = sum + A[tid];
33          contrib[tid] = A[tid];
34        }
35      }
36    }
37  }
38 }
39

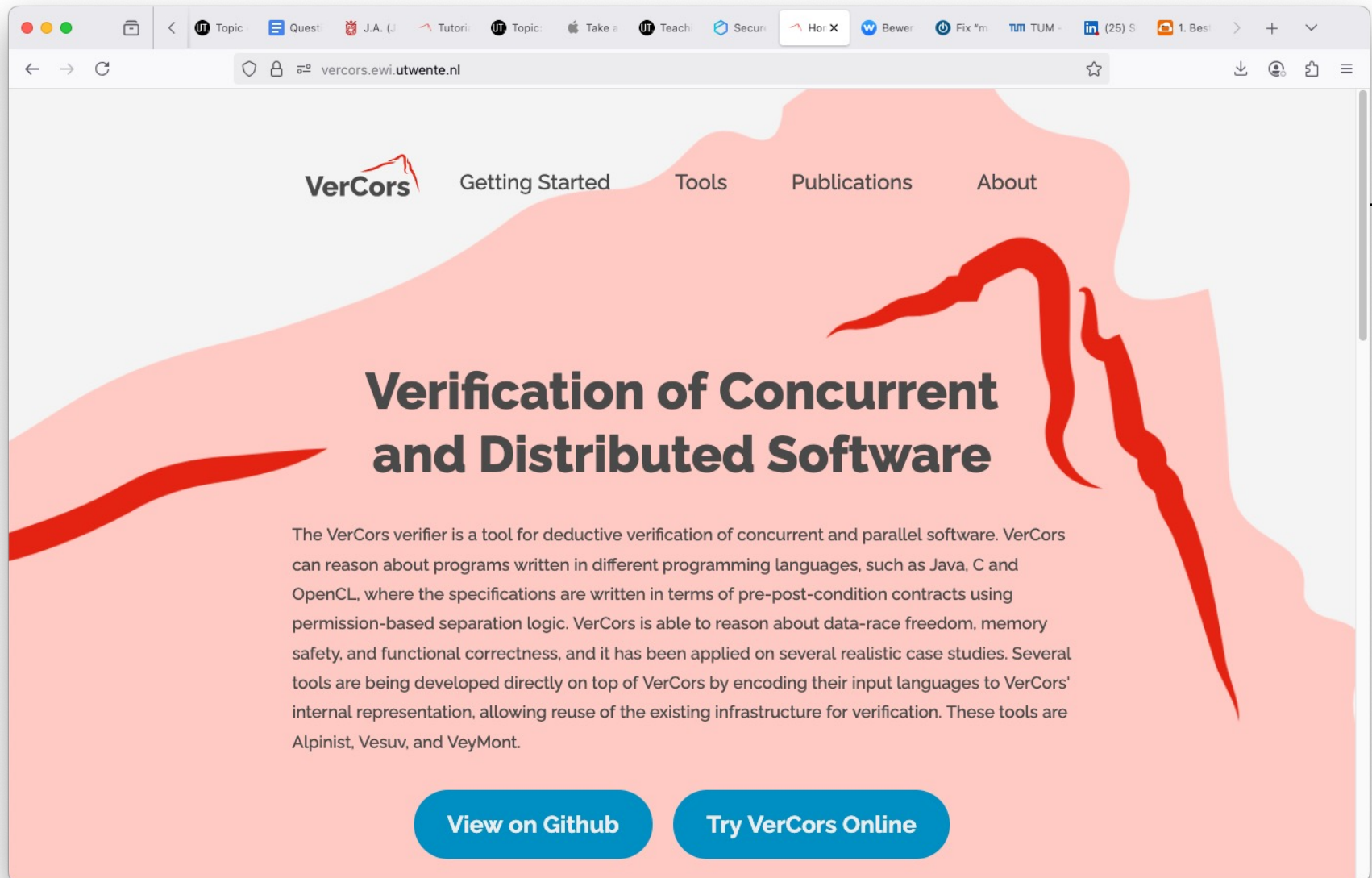
```



# FORK-JOIN SUM

```
1  class Worker {
2    Main main;
3    int val;
4
5    requires m != null;
6    ensures Perm(main, 1) ** main != null ** main == m;
7    ensures Perm(val, 1) ** val == v;
8    ensures idle(this);
9    constructor(Main m, int v) {
10     this.main = m;
11     this.val = v;
12   }
13
14   context Perm(main, 1\2) ** main != null;
15   requires committed(main);
16   context Perm(val, 1\2);
17   run {
18     lock main;
19     main.sum = main.sum + val;
20     unlock main;
21   }
22 }
23
24 lock_invariant Perm(this.sum, 1);
25 class Main {
26   int sum;
27
28   ensures committed(this);
29   constructor() {
30     commit this;
31   }
32
33   requires committed(this);
34   void sum(seq<int> xs) {
35     if (0 < |xs|) {
36       Worker w = new Worker(this, xs.head);
37       fork w;
38       sum(xs.tail);
39       join w;
40     }
41   }
42 }
43
```





The screenshot shows a web browser window displaying the VerCors website. The browser's address bar shows the URL `vercors.ewi.utwente.nl`. The website has a navigation bar with the VerCors logo and links for "Getting Started", "Tools", "Publications", and "About". The main heading is "Verification of Concurrent and Distributed Software". Below this, a paragraph describes the VerCors verifier as a tool for deductive verification of concurrent and parallel software, supporting languages like Java, C, and OpenCL. It mentions its application in reasoning about data-race freedom, memory safety, and functional correctness, and lists several tools built on top of VerCors: Alpinist, Vesuv, and VeyMont. At the bottom, there are two blue buttons: "View on Github" and "Try VerCors Online".

VerCors

Getting Started Tools Publications About

# Verification of Concurrent and Distributed Software

The VerCors verifier is a tool for deductive verification of concurrent and parallel software. VerCors can reason about programs written in different programming languages, such as Java, C and OpenCL, where the specifications are written in terms of pre-post-condition contracts using permission-based separation logic. VerCors is able to reason about data-race freedom, memory safety, and functional correctness, and it has been applied on several realistic case studies. Several tools are being developed directly on top of VerCors by encoding their input languages to VerCors' internal representation, allowing reuse of the existing infrastructure for verification. These tools are Alpinist, Vesuv, and VeyMont.

[View on Github](#) [Try VerCors Online](#)

# VERIFYTHIS 2019: SPARSE MATRIX MULTIPLICATION

---

Sparse matrices represented using the coordinate list format: non-zero values of the matrix are stored in a sequence of triplets containing the row, the column, and the corresponding value

Algorithm which computes the multiplication of a vector of values (encoded as a sequence) with a sparse matrix

```
y <- (0, ..., 0)
for every element (r, c, v) of m do
  y (c) <- y (c) + x (r) * v
done
```

Verify that the concurrent algorithm does not exhibit concurrency issues (data-races, deadlocks, . . . ).

```

1  /*
2  * Performs a multiplication of sparse matrix a (M x N) with vector x (M) and stores the result in vector y (N).
3  *
4  * Dimensions: |x| = m, |y| = n, |a| = (m, n)
5  * Notation:   (row, column, value)
6  */
7  context_everywhere \matrix(a, a.length, 3);           // Read-permission of 'a'
8  context_everywhere (\forall int idx; 0 <= idx && idx < a.length; Perm(a[idx][0], read) ** Perm(a[idx][1], read) ** Perm(a[idx][2], read));
9  context_everywhere x != null && y != null;
10 context_everywhere Perm(x[*], read);                  // Read-permission on 'x'
11 context Perm(y[*], write);                            // Write-permission on 'y'
12 context_everywhere a.length > 0 && x.length > 0 && y.length > 0;
13 context_everywhere (\forall int i = 0 .. a.length; 0 <= a[i][0] && a[i][0] < x.length); // Well-formedness of 'a'
14 context_everywhere (\forall int i = 0 .. a.length; 0 <= a[i][1] && a[i][1] < y.length);
15 // ensures mat_mult_spec(a, x, y);
16 void vec_mat_mult_par(int[] a, int[] x, int[] y) {
17
18     // Create a thread for every element in the output-vector.
19     par threads (int tid = 0 .. y.length)
20         context Perm(y[tid], write);
21         // ensures y[tid] == spec_elem(a, x, tid, a.length);
22     {
23         // Clear y[tid]
24         y[tid] = 0;
25
26         // assert y[tid] == spec_elem(a, x, tid, 0);
27
28         // Iterate over all elements of the matrix
29         loop_invariant 0 <= e && e <= a.length;
30         loop_invariant Perm(y[tid], write);
31         // loop_invariant y[tid] == spec_elem(a, x, tid, e);
32         for (int e = 0; e < a.length; e++) {
33             // Multiply and add all elements that contribute to y[tid] (i.e. all elements that belong to column 'tid' of the matrix).
34             if (a[e][1] == tid) {
35                 y[tid] = y[tid] + (a[e][2] * x[a[e][0]]);
36             }
37             // assert y[tid] == spec_elem(a, x, tid, e+1);
38         }
39     }
40
41     // assert (\forall int idx = 0 .. y.length; y[idx] == spec_elem(a, x, idx, a.length));
42
43 }
44

```

Line: 39:6 Vercors PVL Tab Size: 4 v if