

ERROR AND CONVERGENCE RESULTS FOR MONOTONE CUBIC INTERPOLANTS

** EDELMAN CONSTRAINT **

=====

PCHIP

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*** 903 ***

2.9697e-05	4.4245e-06	4.0422e-07	1.0753e-07
2.7467	3.4523	1.9104	
4.2420e-05	3.9261e-06	4.7418e-07	5.8611e-08
3.4336	3.0496	3.0162	
4.1359e-05	4.3256e-06	4.4047e-07	1.0353e-07
3.2572	3.2958	2.0890	

*** 904 ***

3.2979e-04	8.6305e-05	2.1860e-05	5.4611e-06
1.9340	1.9811	2.0011	
3.8594e-04	1.0178e-04	2.5870e-05	6.4614e-06
1.9229	1.9762	2.0013	
3.3597e-04	8.9753e-05	2.2913e-05	5.7370e-06
1.9043	1.9698	1.9978	

*** 905 ***

2.3377e-04	6.0907e-05	2.2888e-05	2.3719e-06
1.9404	1.4120	3.2705	
2.3143e-04	6.2154e-05	1.9237e-05	1.3925e-06
1.8967	1.6919	3.7881	
1.7520e-04	5.6584e-05	1.5409e-05	2.0898e-06
1.6305	1.8766	2.8823	

*** 1003 ***

8.9254e-05	2.1749e-05	5.3838e-06	1.3693e-06
2.0370	2.0142	1.9752	
1.0539e-04	2.5358e-05	6.2639e-06	1.2950e-06
2.0552	2.0173	2.2741	
9.3691e-05	2.2860e-05	5.6609e-06	1.4398e-06
2.0351	2.0137	1.9752	

*** 1004 ***

3.2165e-05	9.3422e-06	6.2270e-06	2.0860e-07
1.7837	0.5852	4.8997	
1.1637e-05	1.9403e-06	9.9498e-08	1.3939e-08
2.5844	4.2854	2.8356	
3.4483e-05	9.1759e-06	3.9332e-06	1.8138e-07
1.9099	1.2222	4.4386	

*** 1012 ***

1.3970e-05	1.4804e-06	1.6951e-07	1.9891e-08
3.2383	3.1266	3.0912	
2.6132e-08	1.9064e-09	2.7088e-10	1.7708e-11
3.7769	2.8152	3.9351	
2.0868e-05	2.2292e-06	2.5620e-07	3.0117e-08
3.2267	3.1211	3.0886	

*** 1092 ***

2.4845e-04	6.1167e-05	1.5234e-05	3.7463e-06
2.0222	2.0055	2.0238	
2.9178e-04	7.1462e-05	1.6999e-05	4.2031e-06
2.0296	2.0718	2.0159	
2.5999e-04	6.4243e-05	1.6015e-05	3.9391e-06
2.0169	2.0041	2.0234	

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Nonuniform $O(h^2)$
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*** 903 ***

6.1111e-05	5.8453e-06	6.4694e-07	1.0840e-07
3.3861	3.1756	2.5773	
6.4674e-05	5.6463e-06	6.6575e-07	8.1316e-08
3.5178	3.0843	3.0334	
5.3106e-05	4.7259e-06	5.1918e-07	8.6550e-08
3.4902	3.1863	2.5846	

*** 904 ***

2.9574e-04	3.3685e-05	4.2810e-06	9.0770e-07
3.1341	2.9761	2.2377	
8.6835e-05	9.5615e-06	1.1190e-06	1.3511e-07
3.1830	3.0950	3.0501	
2.3110e-04	2.6378e-05	3.4157e-06	7.2807e-07
3.1311	2.9491	2.2300	

*** 905 ***

7.6966e-05	2.1300e-05	1.9834e-05	6.0465e-07
1.8534	0.1029	5.0357	
1.0030e-04	2.3979e-05	1.5687e-05	5.5280e-07
2.0645	0.6122	4.8267	
9.1945e-05	2.3906e-05	1.4222e-05	5.9219e-07
1.9434	0.7493	4.5859	

*** 1003 ***

9.3558e-06	1.1369e-06	2.7817e-07	1.8892e-08
3.0407	2.0311	3.8801	
2.5759e-05	3.5073e-06	1.0286e-06	8.8074e-08
2.8766	1.7697	3.5458	
2.5718e-05	3.5036e-06	1.0373e-06	9.1501e-08
2.8759	1.7560	3.5029	

*** 1004 ***

2.3891e-06	3.8130e-07	1.4951e-07	1.0456e-08
2.6475	1.3507	3.8378	
5.4875e-06	5.3595e-07	5.8024e-08	1.3533e-08
3.3560	3.2074	2.1002	
9.8989e-06	1.6973e-06	5.2281e-07	3.5863e-08
2.5441	1.6988	3.8657	

*** 1012 ***

3.3037e-05	3.5333e-06	4.0630e-07	4.7772e-08
3.2250	3.1204	3.0883	
2.0360e-08	1.5477e-09	1.1439e-09	2.6794e-11
3.7175	0.4362	5.4159	
2.6203e-05	2.8057e-06	3.2282e-07	3.7967e-08
3.2233	3.1196	3.0879	

*** 1092 ***

2.7933e-05	6.1487e-06	1.5342e-06	1.8410e-07
2.1836	2.0028	3.0589	
7.0348e-05	1.4537e-05	4.2325e-06	6.5281e-07
2.2748	1.7801	2.6968	
7.0237e-05	1.4541e-05	4.2672e-06	6.8110e-07
2.2721	1.7687	2.6473	

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=====
Nonuniform O(h^3) [Lagrange Form]
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```
*** 903 ***
1.9157e-05      7.8440e-07      4.4987e-08      3.9292e-09
      4.6101      4.1240      3.5172
2.1790e-05      8.5025e-07      4.8246e-08      2.9021e-09
      4.6797      4.1394      4.0552
1.9336e-05      7.2174e-07      4.0482e-08      3.5005e-09
      4.7437      4.1561      3.5317
```

```
*** 904 ***
1.2105e-04      7.6157e-06      5.1071e-07      1.0274e-07
      3.9904      3.8984      2.3135
4.1641e-05      2.8693e-06      1.8555e-07      1.1569e-08
      3.8592      3.9508      4.0035
1.0301e-04      6.5637e-06      4.5164e-07      8.1873e-08
      3.9722      3.8613      2.4637
```

```
*** 905 ***
9.4228e-05      2.3881e-05      1.9834e-05      5.5832e-07
      1.9803      0.2679      5.1507
1.0990e-04      2.6000e-05      1.5687e-05      5.0337e-07
      2.0796      0.7289      4.9618
9.5889e-05      2.4614e-05      1.4222e-05      5.7930e-07
      1.9619      0.7914      4.6177
```

```
*** 1003 ***
3.1239e-07      3.2965e-08      8.0259e-09      1.4880e-10
      3.2443      2.0382      5.7532
1.8984e-07      1.1231e-08      6.8554e-10      3.9165e-11
      4.0792      4.0341      4.1296
2.7573e-07      2.2343e-08      5.4552e-09      1.4057e-10
      3.6254      2.0341      5.2783
```

```
*** 1004 ***
1.5573e-07      8.5715e-09      7.9265e-10      1.4332e-10
      4.1833      3.4348      2.4675
1.1632e-07      7.8721e-09      4.2501e-10      3.8069e-11
      3.8852      4.2112      3.4808
1.2790e-07      7.6219e-09      5.0429e-10      2.1957e-10
      4.0687      3.9178      1.1996
```

```
*** 1012 ***
2.7171e-06      1.2128e-05      1.0134e-04      3.7996e-04
      -2.1582      -3.0629      -1.9066
4.7623e-08      2.5330e-09      2.6637e-10      1.4747e-11
      4.2328      3.2493      4.1750
2.4725e-06      1.3857e-05      8.5674e-05      2.8407e-04
      -2.4866      -2.6283      -1.7293
```

```
*** 1092 ***
2.7933e-05      6.1487e-06      1.5342e-06      1.0995e-07
      2.1836      2.0028      3.8026
2.4440e-05      3.3834e-06      6.0911e-07      2.1395e-08
      2.8527      2.4737      4.8313
2.0646e-05      4.6264e-06      1.1472e-06      1.1274e-07
      2.1579      2.0118      3.3470
```

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=====
Nonuniform O(h^4) [Lagrange Form]
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```

```
*** 903 ***
1.2806e-05      2.5059e-07      8.3392e-09      3.3603e-10
      5.6754      4.9093      4.6332
1.4482e-05      2.6585e-07      8.6028e-09      3.4008e-10
      5.7675      4.9497      4.6609
1.2535e-05      2.1138e-07      6.2843e-09      2.3083e-10
      5.8900      5.0719      4.7669
```

```
*** 904 ***
6.3001e-05      2.6399e-04      2.6500e-03      1.4586e-03
      -2.0670      -3.3274      0.8614
1.2785e-05      3.6927e-06      3.4063e-05      3.9184e-04
      1.7917      -3.2055      -3.5240
6.3142e-05      2.5120e-04      2.0595e-03      1.1436e-03
      -1.9922      -3.0354      0.8487
```

```
*** 905 ***
1.0016e-04      2.4234e-05      1.9834e-05      7.0667e-06
      2.0472      0.2891      1.4888
1.1320e-04      2.6266e-05      1.5687e-05      4.4095e-06
      2.1076      0.7436      1.8309
9.7461e-05      2.4711e-05      1.4222e-05      6.4090e-06
      1.9797      0.7970      1.1500
```

```
*** 1003 ***
1.5408e-07      3.2965e-08      8.0259e-09      1.2497e-08
      2.2246      2.0382      -0.6388
1.0300e-07      1.6591e-08      2.7118e-09      1.2842e-08
      2.6342      2.6130      -2.2435
1.0303e-07      2.2343e-08      5.4552e-09      1.1443e-08
      2.2051      2.0341      -1.0688
```

```
*** 1004 ***
1.3628e-07      5.1801e-09      1.1000e-09      8.8497e-09
      4.7174      2.2355      -3.0082
1.1632e-07      4.3987e-09      4.5597e-10      3.5478e-09
      4.7249      3.2701      -2.9599
1.1690e-07      5.2360e-09      8.8043e-10      6.5136e-09
      4.4806      2.5722      -2.8872
```

```
*** 1012 ***
3.7797e-02      2.1679e-02      1.0352e-02      5.2879e-03
      0.8020      1.0664      0.9692
4.9650e-08      3.0491e-09      5.6786e-10      1.0907e-11
      4.0253      2.4248      5.7022
2.9988e-02      1.7074e-02      8.1926e-03      4.1946e-03
      0.8126      1.0594      0.9658
```

```
*** 1092 ***
2.7933e-05      6.1487e-06      1.5342e-06      1.0954e-07
      2.1836      2.0028      3.8079
2.4440e-05      3.3834e-06      6.0893e-07      2.1758e-08
      2.8527      2.4741      4.8066
2.0646e-05      4.6264e-06      1.1472e-06      1.1263e-07
      2.1579      2.0118      3.3484
```

=====
Original Hyman
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*** 903 ***

6.2491e-05	8.6177e-06	9.5419e-08	3.0180e-07
2.8583	6.4969	-1.6612	
6.6381e-05	2.2962e-06	1.0127e-07	2.4764e-08
4.8534	4.5030	2.0319	
5.5195e-05	7.4630e-06	8.3455e-08	2.3162e-07
2.8867	6.4826	-1.4727	

*** 904 ***

1.8324e-04	1.1502e-05	3.7305e-06	1.0904e-06
3.9938	1.6244	1.7745	
2.0964e-05	1.2018e-06	1.0980e-07	1.1288e-07
4.1247	3.4522	-0.0399	
1.5923e-04	1.0042e-05	3.2209e-06	8.7498e-07
3.9870	1.6405	1.8801	

*** 905 ***

1.3429e-04	1.5225e-04	3.5915e-05	3.1487e-06
-0.1810	2.0838	3.5118	
1.1523e-04	1.5285e-04	1.9014e-05	1.2840e-06
-0.4075	3.0070	3.8884	
9.7132e-05	1.0624e-04	2.4474e-05	2.4715e-06
-0.1293	2.1180	3.3078	

*** 1003 ***

4.6288e-05	3.6564e-05	1.1376e-06	8.0948e-07
0.3402	5.0064	0.4909	
4.7218e-05	3.6362e-05	1.1577e-06	6.9985e-07
0.3769	4.9731	0.7261	
3.2872e-05	2.3447e-05	1.0052e-06	5.3420e-07
0.4875	4.5438	0.9120	

*** 1004 ***

7.3291e-05	2.0711e-05	1.5905e-05	3.5537e-07
1.8233	0.3809	5.4840	
2.1394e-05	6.3202e-06	2.4146e-08	5.2921e-10
1.7591	8.0320	5.5118	
5.1860e-05	1.5379e-05	1.0764e-05	2.6319e-07
1.7537	0.5147	5.3540	

*** 1012 ***

6.9546e-07	2.5672e-08	6.7969e-10	1.3246e-10
4.7597	5.2392	2.3593	
2.0360e-08	1.5477e-09	5.7279e-10	4.3406e-13
3.7175	1.4340	10.3659	
6.3630e-07	2.3963e-08	5.2120e-10	1.0763e-10
4.7308	5.5228	2.2758	

*** 1092 ***

8.7713e-05	6.1376e-05	2.7745e-05	3.7804e-06
0.5151	1.1454	2.8756	
8.8852e-05	5.9398e-05	2.6541e-05	3.5586e-06
0.5810	1.1622	2.8989	
6.4829e-05	4.2360e-05	1.7914e-05	2.4579e-06
0.6140	1.2416	2.8656	

=====
Mixed Hyman
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*** 903 ***

5.3677e-05	1.1035e-05	2.0061e-07	3.0180e-07
2.2822		5.7816	-0.5892
5.8659e-05	1.8598e-06	7.8775e-08	2.4764e-08
4.9791		4.5613	1.6695
5.0279e-05	9.9243e-06	1.7817e-07	2.3162e-07
2.3409		5.7996	-0.3785

*** 904 ***

1.6120e-04	6.0569e-06	3.0791e-06	1.0904e-06
4.7342		0.9761	1.4976
2.0964e-05	1.2018e-06	1.0980e-07	1.1288e-07
4.1247		3.4522	-0.0399
1.4511e-04	5.6271e-06	2.7734e-06	8.7498e-07
4.6887		1.0207	1.6643

*** 905 ***

1.3429e-04	1.5225e-04	3.5915e-05	3.1487e-06
-0.1810		2.0838	3.5118
1.1523e-04	1.5285e-04	1.9014e-05	1.2840e-06
-0.4075		3.0070	3.8884
9.7132e-05	1.0624e-04	2.4474e-05	2.4715e-06
-0.1293		2.1180	3.3078

*** 1003 ***

4.6288e-05	3.6564e-05	1.1376e-06	8.0948e-07
0.3402		5.0064	0.4909
4.7218e-05	3.6362e-05	1.1577e-06	6.9985e-07
0.3769		4.9731	0.7261
3.2872e-05	2.3447e-05	1.0052e-06	5.3420e-07
0.4875		4.5438	0.9120

*** 1004 ***

7.3291e-05	2.0711e-05	1.5905e-05	3.5537e-07
1.8233		0.3809	5.4840
2.1394e-05	6.3202e-06	2.4146e-08	5.2921e-10
1.7591		8.0320	5.5118
5.1860e-05	1.5379e-05	1.0764e-05	2.6319e-07
1.7537		0.5147	5.3540

*** 1012 ***

4.8826e-07	2.2468e-08	1.1219e-09	5.7676e-11
4.4417		4.3239	4.2818
2.0360e-08	1.5477e-09	5.7279e-10	4.3406e-13
3.7175		1.4340	10.3659
4.7935e-07	2.2226e-08	1.1214e-09	4.1123e-11
4.4307		4.3089	4.7692

*** 1092 ***

8.7713e-05	6.1376e-05	2.7745e-05	3.7804e-06
0.5151		1.1454	2.8756
8.8852e-05	5.9398e-05	2.6541e-05	3.5586e-06
0.5810		1.1622	2.8989
6.4829e-05	4.2360e-05	1.7914e-05	2.4579e-06
0.6140		1.2416	2.8656

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=====
Original Hyman [Precomputed Mapping]
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```
*** 903 ***
6.2491e-05      8.6552e-06      8.1483e-08      2.1483e-07
      2.8520      6.7309      -1.3986
6.6381e-05      2.2962e-06      1.0127e-07      2.4764e-08
      4.8534      4.5030      2.0319
5.5195e-05      7.3966e-06      9.1722e-08      3.0583e-07
      2.8996      6.3335      -1.7374
```

```
*** 904 ***
1.8324e-04      1.0766e-05      3.4844e-06      8.5586e-07
      4.0891      1.6275      2.0255
2.0964e-05      1.2018e-06      1.0980e-07      1.1288e-07
      4.1247      3.4522      -0.0399
1.5923e-04      1.0580e-05      3.3956e-06      1.0518e-06
      3.9117      1.6396      1.6907
```

```
*** 905 ***
1.3429e-04      1.2035e-04      1.9453e-05      2.3116e-06
      0.1581      2.6292      3.0730
1.1523e-04      1.5285e-04      1.9014e-05      1.2840e-06
      -0.4075      3.0070      3.8884
9.7132e-05      1.5325e-04      3.2771e-05      3.0549e-06
      -0.6579      2.2254      3.4233
```

```
*** 1003 ***
4.6288e-05      3.3710e-05      1.1407e-06      8.0655e-07
      0.4575      4.8851      0.5001
4.7218e-05      3.6362e-05      1.1577e-06      6.9985e-07
      0.3769      4.9731      0.7261
3.2872e-05      3.5992e-05      1.2616e-06      7.7192e-07
      -0.1308      4.8344      0.7087
```

```
*** 1004 ***
7.3291e-05      1.9366e-05      1.3788e-05      3.2849e-07
      1.9201      0.4901      5.3914
2.1394e-05      6.3202e-06      2.4146e-08      5.2921e-10
      1.7591      8.0320      5.5118
5.1860e-05      2.0088e-05      1.5971e-05      3.5543e-07
      1.3683      0.3309      5.4897
```

```
*** 1012 ***
6.9546e-07      2.4731e-08      5.5472e-10      1.2413e-10
      4.8136      5.4784      2.1599
2.0360e-08      1.5477e-09      5.7279e-10      4.3406e-13
      3.7175      1.4340      10.3659
6.3630e-07      2.4608e-08      6.5326e-10      1.1465e-10
      4.6925      5.2353      2.5104
```

```
*** 1092 ***
8.7713e-05      3.8440e-05      2.7666e-05      3.3981e-06
      1.1902      0.4745      3.0253
8.8852e-05      5.9398e-05      2.6541e-05      3.5586e-06
      0.5810      1.1622      2.8989
6.4829e-05      5.7774e-05      2.6483e-05      3.7773e-06
      0.1662      1.1253      2.8096
```

```
=====
Mixed Hyman [Precomputed Mapping]
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```

```
*** 903 ***
5.3677e-05      1.1092e-05      1.8968e-07      2.1483e-07
      2.2748      5.8697      -0.1796
5.8659e-05      1.8598e-06      7.8775e-08      2.4764e-08
      4.9791      4.5613      1.6695
5.0279e-05      9.8391e-06      1.8579e-07      3.0583e-07
      2.3533      5.7268      -0.7191
```

```
*** 904 ***
1.6120e-04      5.5968e-06      2.8443e-06      8.5586e-07
      4.8481      0.9765      1.7326
2.0964e-05      1.2018e-06      1.0980e-07      1.1288e-07
      4.1247      3.4522      -0.0399
1.4511e-04      5.9039e-06      2.9166e-06      1.0518e-06
      4.6194      1.0174      1.4714
```

```
*** 905 ***
1.3429e-04      1.2035e-04      1.9453e-05      2.3116e-06
      0.1581      2.6292      3.0730
1.1523e-04      1.5285e-04      1.9014e-05      1.2840e-06
      -0.4075      3.0070      3.8884
9.7132e-05      1.5325e-04      3.2771e-05      3.0549e-06
      -0.6579      2.2254      3.4233
```

```
*** 1003 ***
4.6288e-05      3.3710e-05      1.1407e-06      8.0655e-07
      0.4575      4.8851      0.5001
4.7218e-05      3.6362e-05      1.1577e-06      6.9985e-07
      0.3769      4.9731      0.7261
3.2872e-05      3.5992e-05      1.2616e-06      7.7192e-07
      -0.1308      4.8344      0.7087
```

```
*** 1004 ***
7.3291e-05      1.9366e-05      1.3788e-05      3.2849e-07
      1.9201      0.4901      5.3914
2.1394e-05      6.3202e-06      2.4146e-08      5.2921e-10
      1.7591      8.0320      5.5118
5.1860e-05      2.0088e-05      1.5971e-05      3.5543e-07
      1.3683      0.3309      5.4897
```

```
*** 1012 ***
4.8826e-07      2.1478e-08      1.0420e-09      3.3003e-11
      4.5067      4.3655      4.9806
2.0360e-08      1.5477e-09      5.7279e-10      4.3406e-13
      3.7175      1.4340      10.3659
4.7935e-07      2.2782e-08      1.1626e-09      5.4069e-11
      4.3951      4.2925      4.4264
```

```
*** 1092 ***
8.7713e-05      3.8440e-05      2.7666e-05      3.3981e-06
      1.1902      0.4745      3.0253
8.8852e-05      5.9398e-05      2.6541e-05      3.5586e-06
      0.5810      1.1622      2.8989
6.4829e-05      5.7774e-05      2.6483e-05      3.7773e-06
      0.1662      1.1253      2.8096
```



```
=====
Nonuniform O(h^3) [Newton Form]
=====
```

```
*** 903 ***
1.9157e-05      7.8440e-07      4.4987e-08      2.7151e-09
      4.6101      4.1240      4.0504
2.1790e-05      8.5025e-07      4.8246e-08      2.9021e-09
      4.6797      4.1394      4.0552
1.9336e-05      7.2174e-07      4.0482e-08      2.4229e-09
      4.7437      4.1561      4.0625
```

```
*** 904 ***
1.9154e-04      9.7194e-06      5.5194e-07      6.4621e-08
      4.3006      4.1383      3.0944
2.0005e-04      9.3819e-06      5.2707e-07      3.1607e-08
      4.4144      4.1538      4.0597
1.8602e-04      9.0182e-06      5.0103e-07      5.7860e-08
      4.3665      4.1699      3.1143
```

```
*** 905 ***
1.1088e-04      2.5074e-05      1.9834e-05      5.5308e-07
      2.1447      0.3383      5.1643
1.1916e-04      2.6898e-05      1.5687e-05      4.9791e-07
      2.1473      0.7779      4.9776
1.0030e-04      2.4942e-05      1.4222e-05      5.7784e-07
      2.0077      0.8104      4.6213
```

```
*** 1003 ***
3.1808e-07      3.2965e-08      8.0259e-09      8.3294e-11
      3.2704      2.0382      6.5903
4.5598e-07      2.2883e-08      1.8842e-09      7.5715e-11
      4.3166      3.6022      4.6373
4.4062e-07      2.2343e-08      5.4552e-09      1.2275e-10
      4.3016      2.0341      5.4738
```

```
*** 1004 ***
3.5067e-07      2.6882e-08      1.1335e-09      1.9962e-10
      3.7054      4.5677      2.5054
2.6658e-07      2.6917e-08      9.7246e-10      1.0843e-10
      3.3080      4.7907      3.1648
4.0605e-07      2.5921e-08      1.7827e-09      5.5901e-10
      3.9695      3.8620      1.6731
```

```
*** 1012 ***
1.6256e-06      8.5075e-08      5.8019e-09      3.0345e-10
      4.2561      3.8741      4.2570
3.2460e-08      2.3518e-09      1.1439e-09      2.6683e-11
      3.7868      1.0398      5.4219
1.8204e-06      9.2802e-08      5.5776e-09      3.0896e-10
      4.2940      4.0564      4.1741
```

```
*** 1092 ***
2.7933e-05      6.1487e-06      1.5342e-06      1.0930e-07
      2.1836      2.0028      3.8111
2.4440e-05      3.3834e-06      6.0837e-07      2.0880e-08
      2.8527      2.4755      4.8647
2.0646e-05      4.6264e-06      1.1472e-06      1.1257e-07
      2.1579      2.0118      3.3493
```

** M3 CONSTRAINT **

=====
Nonuniform $O(h^2)$
=====

*** 903 ***

6.1111e-05	5.8453e-06	6.4694e-07	1.0840e-07
3.3861	3.1756	2.5773	
6.4674e-05	5.6463e-06	6.6575e-07	8.1316e-08
3.5178	3.0843	3.0334	
5.3106e-05	4.7259e-06	5.1918e-07	8.6550e-08
3.4902	3.1863	2.5846	

*** 904 ***

2.9574e-04	3.3685e-05	4.2810e-06	9.0770e-07
3.1341	2.9761	2.2377	
8.6835e-05	9.5615e-06	1.1190e-06	1.3511e-07
3.1830	3.0950	3.0501	
2.3110e-04	2.6378e-05	3.4157e-06	7.2807e-07
3.1311	2.9491	2.2300	

*** 905 ***

3.4996e-05	6.3694e-06	6.5328e-07	7.5601e-08
2.4580	3.2854	3.1112	
8.9370e-05	1.0952e-05	1.7790e-06	1.9793e-07
3.0286	2.6221	3.1680	
9.1945e-05	1.3827e-05	2.1521e-06	2.2573e-07
2.7333	2.6837	3.2531	

*** 1003 ***

9.3558e-06	1.1369e-06	2.7817e-07	1.8892e-08
3.0407	2.0311	3.8801	
2.5759e-05	3.5073e-06	1.0286e-06	8.8074e-08
2.8766	1.7697	3.5458	
2.5718e-05	3.5036e-06	1.0373e-06	9.1501e-08
2.8759	1.7560	3.5029	

*** 1004 ***

2.3891e-06	3.8130e-07	1.4951e-07	1.0456e-08
2.6475	1.3507	3.8378	
5.4875e-06	5.3595e-07	5.8024e-08	1.3533e-08
3.3560	3.2074	2.1002	
9.8989e-06	1.6973e-06	5.2281e-07	3.5863e-08
2.5441	1.6988	3.8657	

*** 1012 ***

3.3037e-05	3.5333e-06	4.0630e-07	4.7772e-08
3.2250	3.1204	3.0883	
2.0360e-08	1.5477e-09	1.1439e-09	2.6794e-11
3.7175	0.4362	5.4159	
2.6203e-05	2.8057e-06	3.2282e-07	3.7967e-08
3.2233	3.1196	3.0879	

*** 1092 ***

2.0955e-05	3.9578e-06	1.1947e-06	1.8410e-07
2.4045	1.7281	2.6981	
7.0348e-05	1.4537e-05	4.2325e-06	6.5281e-07
2.2748	1.7801	2.6968	
7.0237e-05	1.4541e-05	4.2672e-06	6.8110e-07
2.2721	1.7687	2.6473	

```

**      M4 CONSTRAINT      **
=====
Nonuniform O(h^3) [Newton Form]
=====

```

```
*** 903 ***
```

```

1.9157e-05      7.8440e-07      4.4987e-08      2.7151e-09
      4.6101      4.1240      4.0504
2.1790e-05      8.5025e-07      4.8246e-08      2.9021e-09
      4.6797      4.1394      4.0552
1.9336e-05      7.2174e-07      4.0482e-08      2.4229e-09
      4.7437      4.1561      4.0625

```

```
*** 904 ***
```

```

1.9154e-04      9.7194e-06      5.5194e-07      6.4621e-08
      4.3006      4.1383      3.0944
2.0005e-04      9.3819e-06      5.2707e-07      3.1607e-08
      4.4144      4.1538      4.0597
1.8602e-04      9.0182e-06      5.0103e-07      5.7860e-08
      4.3665      4.1699      3.1143

```

```
*** 905 ***
```

```

2.6364e-05      1.9138e-06      9.6602e-08      5.2238e-09
      3.7841      4.3082      4.2089
3.2344e-05      2.1323e-06      1.0077e-07      7.1873e-09
      3.9230      4.4033      3.8094
3.3079e-05      1.8611e-06      9.8883e-08      1.0477e-08
      4.1517      4.2343      3.2385

```

```
*** 1003 ***
```

```

3.1808e-07      1.8911e-08      1.4989e-09      7.4993e-11
      4.0721      3.6573      4.3210
4.5598e-07      2.2883e-08      1.8842e-09      7.5715e-11
      4.3166      3.6022      4.6373
4.4062e-07      2.2052e-08      1.7343e-09      7.5132e-11
      4.3205      3.6685      4.5288

```

```
*** 1004 ***
```

```

3.5067e-07      2.6882e-08      1.1335e-09      1.9962e-10
      3.7054      4.5677      2.5054
2.6658e-07      2.6917e-08      9.7246e-10      1.0843e-10
      3.3080      4.7907      3.1648
4.0605e-07      2.5921e-08      1.7827e-09      5.5901e-10
      3.9695      3.8620      1.6731

```

```
*** 1012 ***
```

```

1.6256e-06      8.5075e-08      5.8019e-09      3.0345e-10
      4.2561      3.8741      4.2570
3.2460e-08      2.3518e-09      1.1439e-09      2.6683e-11
      3.7868      1.0398      5.4219
1.8204e-06      9.2802e-08      5.5776e-09      3.0896e-10
      4.2940      4.0564      4.1741

```

```
*** 1092 ***
```

```

9.1728e-06      3.6981e-07      1.2308e-08      1.2500e-09
      4.6325      4.9091      3.2996
1.1178e-05      4.8185e-07      1.8376e-08      2.8221e-09
      4.5360      4.7127      2.7030
9.8910e-06      4.6625e-07      1.7512e-08      3.9999e-09
      4.4069      4.7347      2.1303

```

```
=====
Nonuniform O(h^4) [Newton Form]
=====
```

```
*** 903 ***
```

```
1.2806e-05      2.5059e-07      8.3388e-09      3.3728e-10
      5.6754      4.9093      4.6278
1.4482e-05      2.6585e-07      8.6026e-09      3.4203e-10
      5.7675      4.9497      4.6526
1.2535e-05      2.1138e-07      6.2845e-09      2.5282e-10
      5.8900      5.0719      4.6356
```

```
*** 904 ***
```

```
2.4014e-04      5.7645e-06      3.1059e-07      4.3219e-07
      5.3805      4.2141      -0.4766
2.3407e-04      3.4968e-06      9.4906e-08      3.5199e-09
      6.0648      5.2034      4.7529
2.3949e-04      5.0929e-06      2.9793e-07      4.8464e-07
      5.5553      4.0954      -0.7019
```

```
*** 905 ***
```

```
2.3682e-05      1.1079e-06      5.5142e-08      1.3492e-09
      4.4179      4.3285      5.3529
2.8355e-05      1.4602e-06      6.1276e-08      1.3460e-09
      4.2794      4.5747      5.5086
3.0387e-05      1.8083e-06      1.0522e-07      1.8572e-09
      4.0708      4.1032      5.8241
```

```
*** 1003 ***
```

```
1.6608e-07      1.1554e-08      9.1014e-10      4.0202e-11
      3.8454      3.6661      4.5008
3.4639e-07      1.3584e-08      1.4202e-09      3.0424e-11
      4.6724      3.2577      5.5448
3.3401e-07      1.3456e-08      1.3402e-09      5.1230e-11
      4.6336      3.3277      4.7093
```

```
*** 1004 ***
```

```
3.7661e-07      2.0729e-08      1.0211e-09      6.8368e-11
      4.1833      4.3435      3.9006
2.8077e-07      2.2680e-08      1.1338e-09      1.7660e-11
      3.6299      4.3222      6.0045
3.8275e-07      1.9441e-08      1.6343e-09      5.2153e-11
      4.2992      3.5724      4.9698
```

```
*** 1012 ***
```

```
1.8733e-06      8.3304e-07      3.3608e-07      1.5648e-07
      1.1691      1.3096      1.1029
4.7623e-08      2.5330e-09      1.7147e-09      2.6258e-11
      4.2328      0.5629      6.0290
1.8657e-06      8.3402e-07      3.7118e-07      1.7022e-07
      1.1616      1.1680      1.1247
```

```
*** 1092 ***
```

```
6.3624e-05      1.4542e-06      1.9917e-08      6.5473e-10
      5.4513      6.1901      4.9269
8.2468e-05      1.9121e-06      2.9916e-08      9.6508e-10
      5.4306      5.9981      4.9541
7.3714e-05      1.7683e-06      3.2666e-08      9.0209e-10
      5.3815      5.7585      5.1784
```

NOTES

Grid sizes are [64 128 256 512] for 900-series functions and [128 256 512 1024] for 1000-series functions. Errors and convergence rates are reported at midpoints, t points and s points, respectively. The mappings were precomputed at 64 grid points for 900-series functions and 128 grid points for 1000-series functions.